Appendix S2: Decomposition of Simpson index

We use Pélissier and Couteron's [1] organization of data to derive the decomposition. We define an indicator $\mathbf{1}_{sk}$ such that it takes a value of 1 if individual k belongs to species s, and of zero otherwise. $\mathbf{1}_{sk}$ follows a Bernoulli distribution: either an individual belongs to species s or not. $\sum_{k=1}^{n} \mathbf{1}_{sk}$ follows a binomial distribution of variance $np_s(1-p_s)$. A standard partitioning of variance can be applied. Within each community, variance is $n_i p_{si}(1-p_{si})$. The expectation of $\sum_{k=1}^{n} \mathbf{1}_{sk}$ is np_s for pooled data, and that of within-community $\sum_{k=1}^{n_i} \mathbf{1}_{sk}$ is $n_i p_{si}$. We have then:

$$p_s(1-p_s) = \sum_i \frac{n_i}{n}\left[p_{si}(1-p_{si}) + (p_{si}-p_s)^2\right] \quad \text{(1)}$$

Summation of $p_s(1-p_s)$ over species provides Simpson's entropy:

$${}^2H_\gamma = \sum_i \frac{n_i}{n}\, {}^2_iH_\alpha + \sum_i \frac{n_i}{n} \sum_s (p_{si}-p_s)^2 \quad \text{(2)}$$

Assuming ${}^2H_\gamma = {}^2H_\alpha + {}^2H_\beta$ and α entropy is the weighted sum of within-community α entropy values, the second term of (2) can be identified to β entropy in the classical additive partitioning. ${}^2H_\beta$ is the weighted sum of contributions of communities, denoted ${}^2_iH_\beta = \sum_s (p_{si}-p_s)^2$. This additive partitioning of entropy is that of Nei [2] among others. As shown by Jost [3,4], ${}^2H_\beta$is constrained by ${}^2H_\alpha$ since γ entropy is limited to 1. Using ${}^2H_\beta$ or $G_{ST} = {}^2H_\beta / {}^2H_\gamma$ as a measure of differentiation has been shown to be erroneous [5-7].

References:

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