

## S1 Appendix

**PDF of the length of a random vector with normal distributed components.**

**Theorem 1.** Let  $\mathbf{v} = (v_1, \dots, v_D)$  be a vector of dimension  $D \in \mathbb{N}^+$  with random components distributed independently according to  $v_i \sim \mathcal{N}(0; \sigma)$ . Then the probability density function of  $|\mathbf{v}|$  is

$$p_{|\mathbf{v}|}(x; \sigma, D) = k_D x^{D-1} \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad (1)$$

with normalizing constant

$$k_D = \frac{1}{2^{(D/2)-1} \sigma^D \Gamma\left(\frac{D}{2}\right)}. \quad (2)$$

*Proof.* The length of  $\mathbf{v}$  is distributed according to

$$p_{|\mathbf{v}|}(x; \sigma, D) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^D \int_{-\infty}^{\infty} dv_1 \cdots \int_{-\infty}^{\infty} dv_D \exp\left(-\frac{v_1^2}{2\sigma^2}\right) \cdots \exp\left(-\frac{v_D^2}{2\sigma^2}\right) \delta(|\mathbf{v}| - x). \quad (3)$$

By transforming to polar coordinates we obtain

$$\begin{aligned} p_{|\mathbf{v}|}(x; \sigma, D) &= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^D \int \int_0^{\infty} \delta(|\mathbf{v}| - x) |\mathbf{v}|^{D-1} \exp\left(-\frac{|\mathbf{v}|^2}{2\sigma^2}\right) d|\mathbf{v}| d\Omega_D \\ &= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^D \int x^{D-1} \exp\left(-\frac{x^2}{2\sigma^2}\right) d\Omega_D \\ &= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^D \cdot \frac{2\pi^{D/2}}{\Gamma\left(\frac{D}{2}\right)} \cdot x^{D-1} \exp\left(-\frac{x^2}{2\sigma^2}\right) \\ &= \frac{1}{2^{(D/2)-1} \sigma^D \Gamma\left(\frac{D}{2}\right)} \cdot x^{D-1} \exp\left(-\frac{x^2}{2\sigma^2}\right) \end{aligned} \quad (4)$$

with the  $D$ -dimensional solid angle differential  $d\Omega_D$ . □