Appendix 1
ANOVA sum of squares and mean squares –
definitions and relations

We refer to Table 2, main text. Each row \((i = 1, \ldots, n)\) of the matrix normally corresponds to a "subject", while each column \((j = 1, \ldots, k)\) corresponds to a "measurement", "rater" or "condition", depending on the context. We have chosen the word "subject" for rows, and "measurement" for columns.

First, we define the mean value \(S_i\) for each subject (row) \(i\), the mean value \(M_j\) for each measurement (column) \(j\), and the total mean value \(\bar{x}\) of all the measured values \(x_{ij}\).

\[
S_i = \frac{1}{k} \sum_{j=1}^{k} x_{ij} \quad \text{(A1-1)}
\]

\[
M_j = \frac{1}{n} \sum_{i=1}^{n} x_{ij} \quad \text{(A1-2)}
\]

\[
\bar{x} = \frac{1}{n \cdot k} \sum_{i=1}^{n} \sum_{j=1}^{k} x_{ij} \quad \text{(A1-3)}
\]

The various sums of squares may now be defined in a symmetrical way by double sums:

\[
SST = \sum_{i=1}^{n} \sum_{j=1}^{k} (x_{ij} - \bar{x})^2 \quad \text{(A1-4)}
\]

(Sum of Squares, Total)

\[
SSBS = \sum_{i=1}^{n} \sum_{j=1}^{k} (S_i - \bar{x})^2 \quad \text{(A1-5)}
\]

(Sum of Squares Between Subjects)

\[
SSBM = \sum_{i=1}^{n} \sum_{j=1}^{k} (M_j - \bar{x})^2 \quad \text{(A1-6)}
\]

(Sum of Squares Between Measurements)

\[
SSWS = \sum_{i=1}^{n} \sum_{j=1}^{k} (x_{ij} - S_i)^2 \quad \text{(A1-7)}
\]

(Sum of Squares Within Subjects)

\[
SSWM = \sum_{i=1}^{n} \sum_{j=1}^{k} (x_{ij} - M_j)^2 \quad \text{(A1-8)}
\]

(Sum of Squares Within Measurements)
\[ SSE = SST - SSBS - SSBM \]  
(Sum of Squares, Error)  
(A1-9)

We have here for convenience expressed \( SSE \) (often called "residual" instead of "error") as the difference between \( SST \) and \( (SSBS + SSBM) \), although it may, like the others, be defined by means of a double summation. From the definitions (A1-4) – (A1-9) the following exact relation may be derived:

\[ SST = SSBS + SSWS = SSBM + SSWM \]  
(A1-10)

This may be used together with (A1-9) to derive other useful relations, for example

\[ SSWM = SSBS + SSE \]  
(A1-11)

\[ SSWS = SSBM + SSE \]  
(A1-12)

From the sums of squares the mean squares (MS) are calculated as follows:

\[ MST = \frac{SST}{n \cdot k - 1} \]  
(Mean Square, Total)  
(A1-13)

\[ MSBS = \frac{SSBS}{n - 1} \]  
(Mean Square Between Subjects)  
(A1-14)

\[ MSBM = \frac{SSBM}{k - 1} \]  
(Mean Square Between Measurements)  
(A1-15)

\[ MSWS = \frac{SSWS}{n \cdot (k - 1)} \]  
(Mean Square Within Subjects)  
(A1-16)

\[ MSWM = \frac{SSWM}{k \cdot (n - 1)} \]  
(Mean Square Within Measurements)  
(A1-17)

\[ MSE = \frac{SSE}{(n - 1) \cdot (k - 1)} \]  
(Mean Square, Error)  
(A1-18)

The denominators in (A1-13) – (A1-18) are the respective degrees of freedom (df).