Appendix 2: Formulae, empirical example, and proof regarding the Bice-Boxerman and modified Bice-Boxerman continuity of care indices

Let \( n_i \) be the number of visits to \( i \)th provider and \( n_j \) be the number of visits within the \( j \)th specialty. The overall number of visits, number of providers, number of specialties are given by \( n \), \( p \), and \( s \) respectively.

The Bice-Boxerman continuity of care index is given by:

\[
\frac{(\sum_{i=1}^{p} n_i^2) - n}{n^2 - n}
\]

The modified Bice-Boxerman continuity of care index used in this study is defined as:

\[
\frac{(\sum_{i=1}^{p} n_i^2) - n}{(\sum_{j=1}^{s} n_j^2) - n}
\]

The modified Bice-Boxerman continuity of care index assumes that providers belong to one and only one specialty.
Empirical Example

Figure 1: Behavior of the original and modified Bice-Boxerman indices with increasing visits to multiple specialties

The above figure displays the value of the Bice-Boxerman and modified Bice-Boxerman continuity indices under several scenarios involving visits to family physicians (FP) and cardiologists. The first set of bars representing a patient visiting the same family physician 10 times while the second set of bars represents a patient visiting one family physician 9 times and a different family physician once. The third set of bars represents a patient visiting one family physician 9 times, a different family physician once, and the same cardiologist twice, and the last set of bars represents the patient visiting one family physician 9 times, a different family physician once, and two different cardiologists twice each. The original Bice-Boxerman index drops in value across every scenario, including when then patient sees
only a single cardiologist, while the modified Bice-Boxerman index only drops in value when visits within a specialty are dispersed among multiple providers.

**Proof**

Let $n_i$ be the number of visits to $i$th provider, $n_j$ be the number of visits within the $j$th specialty, and $n_{jk}$ be the number of visits to $k$th provider within the $j$th specialty. The overall number of visits, number of providers, number of specialties, and number of providers within each specialty $j$ are given by $n, p, s$, and $r_j$ respectively.

Proof that the modified Bice-Boxerman (MBB) index is a weighted averaged of specialty-specific unmodified Bice-Boxerman indices (BB) where each specialty has the weight $(n^2_j - n_j)/(\Sigma_{j=1}^{s}(n^2_j) - n_j)$:

Assuming that each provider exists within only one specialty and that each $n_j \geq 2$ then:

\[
BB = \frac{(\Sigma_{i=1}^{p} n^2_i) - n}{n^2 - n}
\]

\[
BB_j = \frac{(\Sigma_{k=1}^{r} n^2_{jk}) - n_j}{n^2_j - n_j}
\]

\[
MBB = \frac{(\Sigma_{i=1}^{p} n^2_i) - n}{(\Sigma_{j=1}^{s} n^2_j) - n}
\]

\[
= \frac{(\Sigma_{j=1}^{s}(\Sigma_{k=1}^{r} n^2_{jk})) - \Sigma_{j=1}^{s} n_j}{(\Sigma_{j=1}^{s} n^2_j) - n}
\]
\[
\sum_{j=1}^{s} \left( \frac{(\sum_{k=1}^{r} n_{jk}^2) - n_j \left( \frac{n_j^2 - n_j}{n_j^2 - n_j} \right)}{(\sum_{j=1}^{s} n_j^2) - n} \right)
\]

\[
= \frac{\sum_{j=1}^{s} \left( BB_j \left( n_j^2 - n_j \right) \right)}{(\sum_{j=1}^{s} n_j^2) - n}
\]

\[
= \frac{BB_1 (n_1^2 - n_1)}{(\sum_{j=1}^{s} n_j^2) - n} + \frac{BB_2 (n_2^2 - n_2)}{(\sum_{j=1}^{s} n_j^2) - n} + \cdots + \frac{BB_s (n_s^2 - n_s)}{(\sum_{j=1}^{s} n_j^2) - n}
\]