S2 Appendix

The latent process \{X_n : n \geq 0\} is assumed to be INAR(1) given by \(X_{n+1} = \alpha \circ X_n + W_n\), where \(W_n \sim \text{Poisson}(\lambda_n)\), independent of \(X_n\). The expectation of \(X_{n+1}\) is computed as \(E(X_{n+1}) = \alpha E(X_n) + \lambda_{n+1}\) Therefore, the following representation can be derived:

\[
    \begin{align*}
    E(X_{n+2}) &= \alpha E(X_{n+1}) + \lambda_{n+2} = \alpha (\alpha E(X_n) + \lambda_{n+1}) + \lambda_{n+2} = \alpha^2 E(X_n) + \alpha \lambda_{n+1} + \lambda_{n+2} \\
    E(X_{n+3}) &= \alpha E(X_{n+2}) + \lambda_{n+3} = \alpha (\alpha^2 E(X_n) + \alpha \lambda_{n+1} + \lambda_{n+2}) + \lambda_{n+3} \\
    &= \alpha^3 E(X_n) + \alpha^2 \lambda_{n+1} + \alpha \lambda_{n+2} + \lambda_{n+3} \\
    &
    \vdots \\
    E(X_{n+k}) &= \alpha^k E(X_n) + \alpha^{k-1} \lambda_{n+1} + \alpha^{k-2} \lambda_{n+2} + \cdots + \lambda_{n+k} = \alpha^k E(X_n) + \sum_{i=1}^{k} \alpha^{k-i} \lambda_{n+i} \\
\end{align*}
\]

(S2.1)

On the other hand, \(E(X_n) \approx \frac{Y_n}{1 - \omega(1 - q_n)}\) since we can assume that \(E(Y_n) \approx Y_n\) and we know that \(E(Y_{n+k}) = E(X_{n+k})(1 - \omega(1 - q_{n+k}))\). Hence:

\[
    \begin{align*}
    E(Y_{n+1}) &= \frac{1 - \omega(1 - q_{n+1})}{1 - \omega(1 - q_n)} \alpha Y_n + (1 - \omega(1 - q_{n+1})) \lambda_{n+1} \\
    E(Y_{n+2}) &= \frac{1 - \omega(1 - q_{n+2})}{1 - \omega(1 - q_n)} \alpha^2 Y_n + (1 - \omega(1 - q_{n+2})) (\alpha \lambda_{n+1} + \lambda_{n+2}) \\
    &
    \vdots \\
    E(Y_{n+k}) &= \frac{1 - \omega(1 - q_{n+k})}{1 - \omega(1 - q_n)} \alpha^k Y_n + (1 - \omega(1 - q_{n+k})) \sum_{i=1}^{k} \alpha^{k-i} \lambda_{n+i}. \\
\end{align*}
\]

(S2.2)