Electron modeling for MHD instabilities in toroidal plasmas

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outline

Motivation

1. Fluid electron model for MHD mode

- **Kink mode**
- **Resistive and collisionless tearing mode**

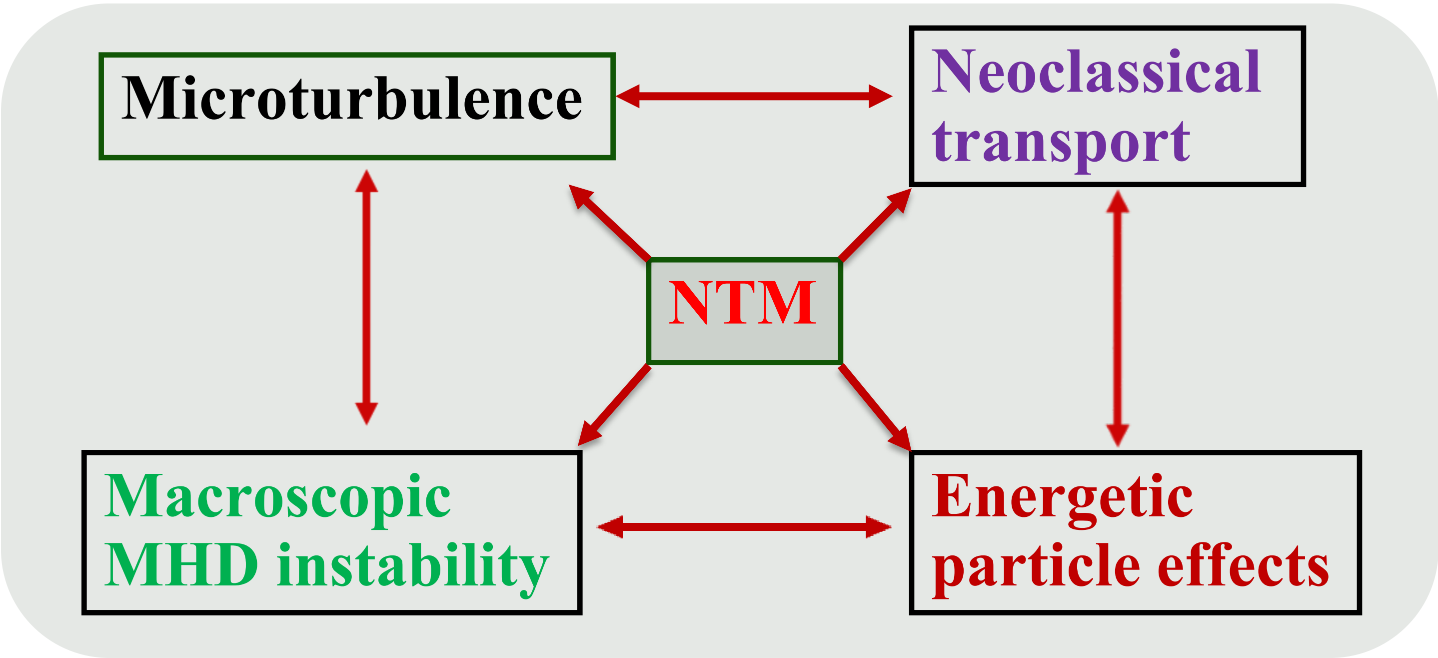
2. General kinetic electron model for MHD mode

- **Kinetic collisionless tearing mode**
- **Kinetic Ballooning mode**

Summary and discussion

Motivation

- Multiscale instabilities exist in fusion plasmas, macroscopic MHD instabilities: resistive kink, tearing mode, NTM may cause the disruption of plasmas, while microscopic instabilities, ITG、CTEM and MTM caused turbulence transport may limit the performance of confinement in large Tokamaks like ITER. Plenty of experimental evidence shows the strong cross scale coupling among these modes and particles. An example:



- Self-consistent simulation of the above low frequency and multiscale processes require accurate and efficient model of plasma spices, especially electron model which limit the shortest time step of the simulation.

1. Fluid Electron Model for MHD Mode

- For $\omega/k_{||} << v_{te}$, electron response mostly adiabatic (isothermal).
  First, estimate $\boldsymbol{E}_{||}$ using massless fluid electron

$$E_{||} = -\nabla_{||}(\phi + \phi_{ind}) = -\frac{T_e}{en_0}\nabla_{||}\delta n_e \qquad \frac{e\phi_{ind}}{T_e} = -\frac{e\phi}{T_e} + \frac{\delta n_e}{n_0}$$

  - ✓ Vector potential $A_{||}$ calculated from $E_{||}$ $\qquad \frac{\partial A_{||}}{\partial t} = \nabla_{||}\phi_{ind}$
  - ✓ Perturbed flow δu_e from Ampere's law $\qquad n_0 e\delta u_e = -\nabla_{\perp}^2 A_{||}$
  - ✓ Perturbed density δn_e from continuity equation $\qquad \frac{\partial \delta n_e}{\partial t} = -\nabla_{||} n_0 \delta u_e$
  - ✓ Electrostatic potential ϕ from Poisson equation using perturbed density δn_e

- Then, $\boldsymbol{E}_{||}$ is corrected by kinetic electron non-adiabatic response using split-weight scheme to reduce noise $\quad \delta f_e = f_M e^{e(\phi+\varphi_{ind})/T_e} + \delta g$

- Technical difficulties:

  1. No tearing mode due to the lowest order massless fluid electron,

  2. Electron response, especially passing electron, near the mode rational surface is not accurate.

[Lin and Chen, Phys. Plasmas **8**, 1447 (2001)
I. Holod, W. L. Zhang, Y. Xiao, and Z. Lin, Phys. Plasmas 16, 122307 (2009)]

Finite mass electron fluid model

Field equation:

$$\frac{n_0 e}{T_e}\rho_s^2\nabla_\perp^2\delta\phi = \delta n_e - \delta n_i$$

$$\frac{c}{4\pi n_0 e}\left(\nabla_\perp^2 - \frac{1}{D_e^2}\right)\delta A_{||} = \delta u_{e||c}$$

$$\delta u_{e||} = \frac{c}{4\pi n_0 e}\nabla_\perp^2\delta A_{||} + \delta u_{i||} - \frac{1}{n_0 e}(J_{ECCD} + J_{bs})$$

Dynamics equation:

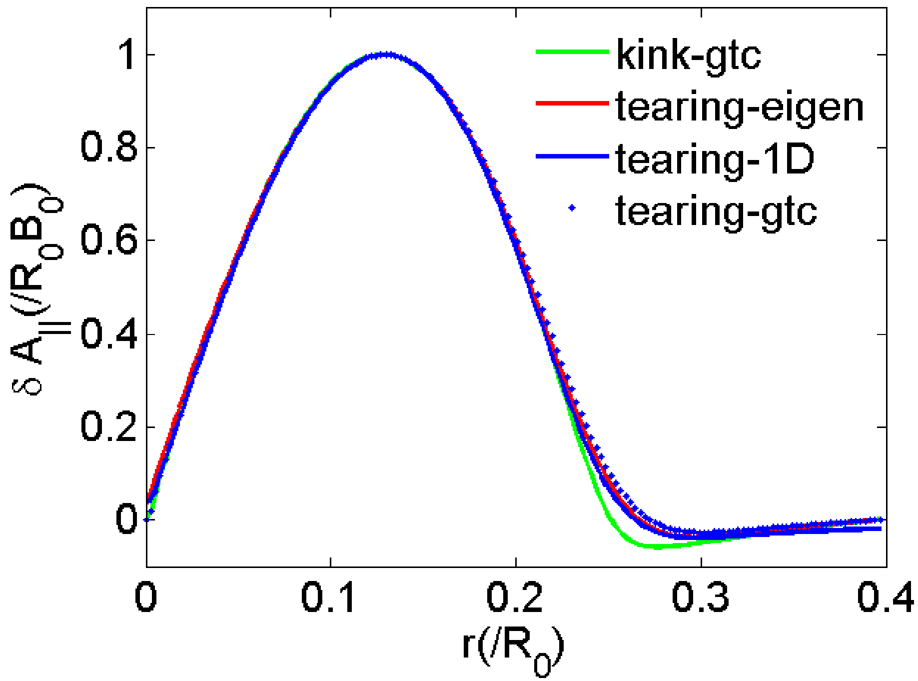
$$\frac{\partial}{\partial t}\delta n_e = -\mathbf{B}_0\cdot\nabla\left(\frac{n_0\delta u_{e||}}{B_0}\right) - \delta\mathbf{u}_E\cdot\nabla n_0 - \frac{\delta\mathbf{B}_\perp}{B_0}\cdot\nabla(n_0 u_{e0})$$

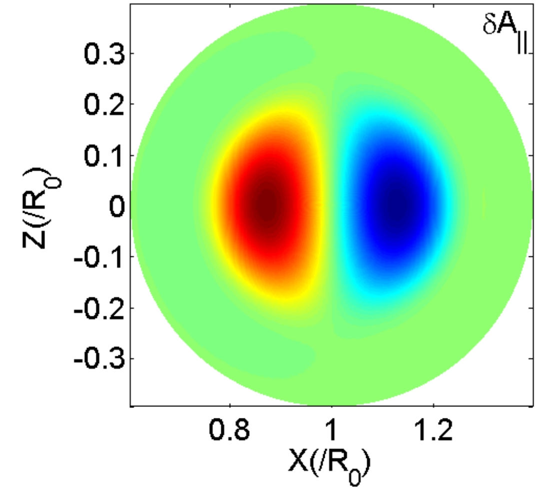
$$n_0 m_e\frac{\partial}{\partial t}\delta u_{e||c} = -\nabla_{||}\delta p_e + n_0 e\nabla_{||}\delta\phi - n_0 m_e \nu_{ei}\delta u_{e||}$$

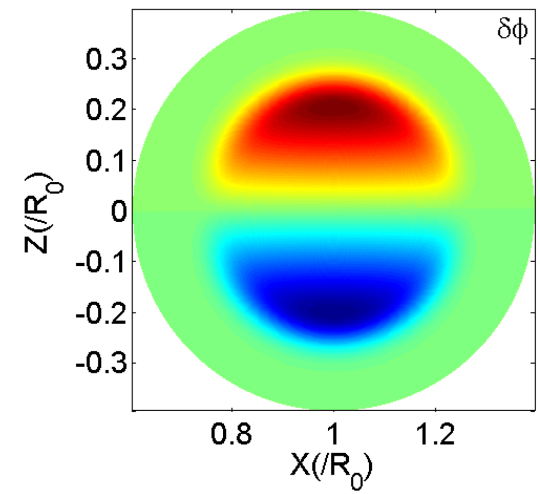
Eigen value and1D initial value approaches have been used to solve the model in cylinder geometry.

➢ (1,1) resistive kink tearing mode in cylinder

$q=0.8+3.2*r^2$, $a/R_0 = 1/2$, $B=1.0$**T**, $n_0=10^{20}$ /**m³**, $T_e=100$**ev**

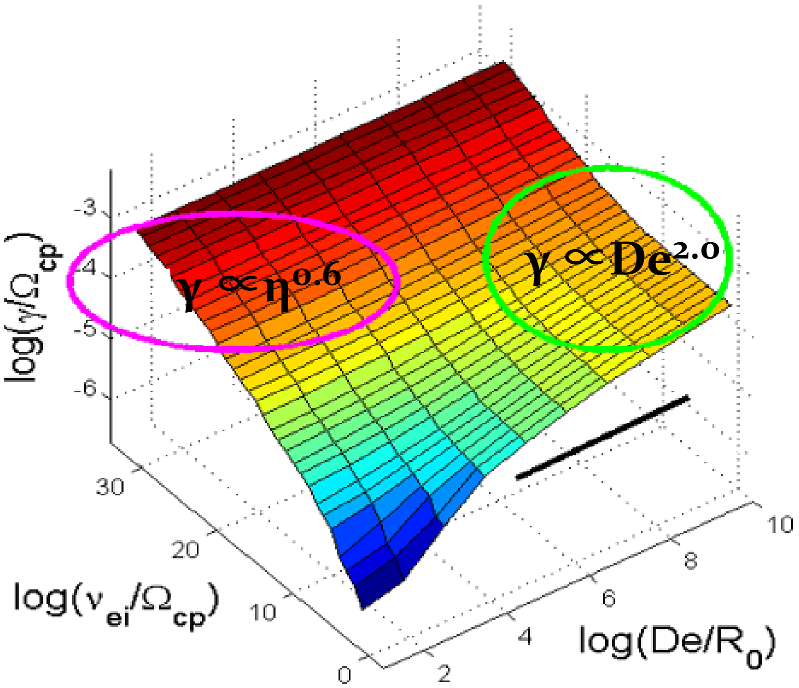


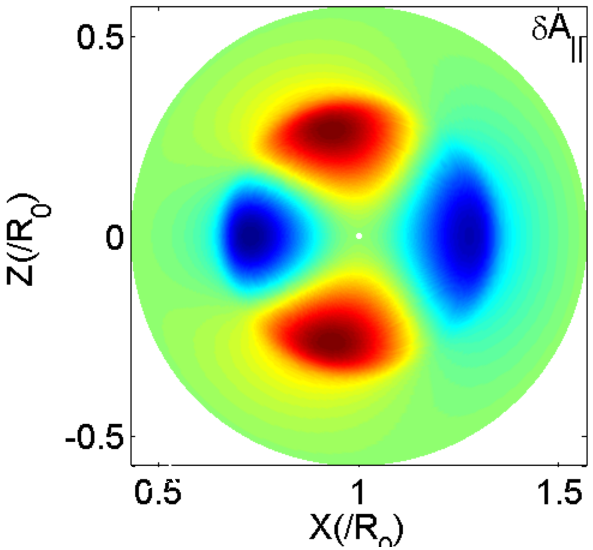


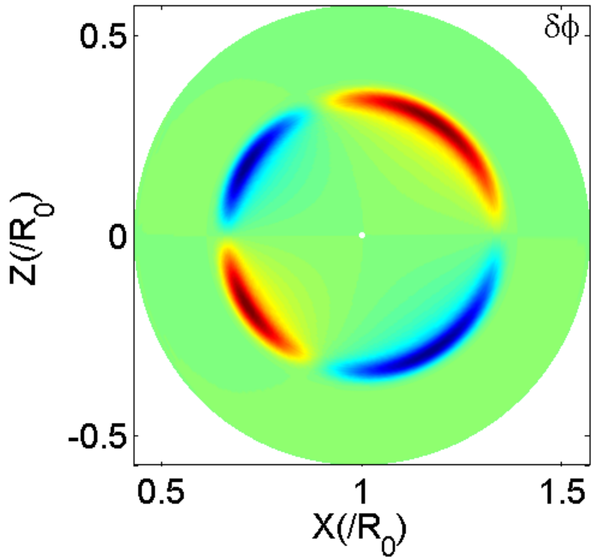


- $\gamma_{GTC}=0.066\Omega_A$, $\gamma_{eig}=0.074\ \Omega_A$, $\gamma_{init}=0.074\ \Omega_A$
- **Finite resistivity near the mode rational surface breaks the ideal MHD limit and causes finite $E_{||}$**

➢ (2, 1) tearing mode simulation







This scaling agrees well with the FKR prediction. Fluid model can recover both the resistive and the collisionless tearing mode.

[Liu et al, POP, 2014, 2016]

2. general kinetic electron model of MHD Modes

- Parallel electric field $\boldsymbol{E}_{||} = -\nabla_{||}\phi - \frac{\partial A_{||}}{\partial t}$
- Need to calculate accurately scalar ϕ and vector potential $A_{||}$

$$\phi - \tilde{\phi} = -\sum_s 4\pi e Z_s \bar{n}_s \qquad \nabla_\perp^2 A_{||} = -\sum_s \frac{4\pi e}{c} Z_s \bar{n}_s u_s$$

- Ideal MHD $E_{||}$~0. $|E_{||}|/|\nabla_{||}\phi|$~$|E_{||}|/|\frac{\partial A_{||}}{\partial t}|$ $\sim(k_\perp \rho_s)^2 \ll 1$
- Physics: cancellation between electrostatic and inductive $E_{||}$
- Canonical momentum used as independent velocity variable to avoid explicit time derivative operation:

$$(\nabla_\perp^2 - \frac{\omega_{pe}^2}{c^2} - \frac{\omega_{pe}^2}{c^2})\frac{\partial \delta A_{\|}}{\partial t} = \frac{4\pi}{c}\frac{\partial}{\partial t}(n_{e0} e \delta u_{e||c} - n_{i0} q_i \delta u_{i||c})$$

- LHS: $|1^{st}\ term|/|2^{nd}\ term|\sim(k_\perp d_e)^2 \ll 1$. Small numerical error of RHS leads to large error of $A_{||}$
- Technical difficulty: Infamous "cancellation problem": calculate 2nd-order $\mathcal{O}(k_\perp^2 \rho_s d_e)^2$ term from 0th–order $\mathcal{O}(1)$ equation!

A Conservative Scheme Solving Exact DKE

- Calculate exact $\boldsymbol{E}_{||}$ by separating $A_{||}$ into adiabatic and non-adiabatic parts: $A_{||} = A_{||}^{A} + A_{||}^{NA}$

$$\frac{\partial A_{||}^{A}}{\partial t} = \nabla_{||}\phi_{ind}$$

$$\frac{e\phi_{ind}}{T_e} = -\frac{e\phi}{T_e} + \frac{\delta n_e}{n_0} - \frac{\delta\psi^A}{n_0}\frac{\partial n_0}{\partial\psi}$$

Alfven wave, IAW, drift wave

- Define electron adiabatic responses using adiabatic $\boldsymbol{E}_{||}$

$$\delta f_A = \frac{e(\phi + \phi_{ind})}{T_e} f_0 + \delta\psi^A \frac{\partial f_0}{\partial\psi}$$

$$L\delta h = -\delta L f_{e0} - L_0 \delta f_a$$
$$= -\left(\mathbf{v_E} + v_{\|}\frac{\boldsymbol{\delta}\mathbf{B}_{\perp}^{NA}}{B_0}\right)\cdot\nabla f_{e0} + \frac{ef_{e0}}{T_{e0}}\mathbf{v_c}\cdot\nabla\delta\phi - \frac{\partial\delta f_a}{\partial t} - \mathbf{v_d}\cdot\nabla\delta f_a + \frac{ev_{\|}f_{e0}}{cT_{e0}}\frac{\partial\delta A_{\|}^{NA}}{\partial t}$$

[Mitigation of the cancellation problem in the gyrokinetic particle-in-cell simulation of the global electromagnetic modes, Alexey Mishchenko, Alberto Bottino, Roman Hatzky, Eric Sonnendrücker, Ralf Kleiber and Axel Könies, Phys. Plasmas, 2017]

[A conservative scheme of drift kinetic electrons for gyrokinetic simulation of kinetic-MHD processes in toroidal plasmas, J. Bao, D. Liu, Z. Lin, Phys. Plasmas, 2017]

A Conservative Scheme Solving Exact DKE

- Non-adiabatic part of $\boldsymbol{E}_{||}$ calculated via electron parallel momentum equation (generalized Ohm's law) using non-adiabatic response
- Recover collisionless tearing mode with current sheet at d_e scale
- Non-tearing modes: current screened by collisionless skin depth d_e
- No "cancellation problem" due to the adiabatic treatment of most of electron

$$\left(\nabla_\perp^2 - \frac{1}{d_e^2}\right)\frac{\partial A_{||}^{NA}}{\partial t} = \frac{1}{d_e^2}\chi_{||} - c\nabla_\perp^2\left(\nabla_{||}\delta\phi_{ind}\right)$$

$$\chi_{||} = \underbrace{-\frac{c}{en_0}\mathbf{b_0}\cdot\nabla\delta P_{||}^{NA}}_{\{III\}} \underbrace{-\frac{c}{en_0B_0}\boldsymbol{\delta}\mathbf{B}^{NA}\cdot\nabla P_{||0}}_{\{IV\}} \underbrace{-\frac{c}{en_0B_0}\boldsymbol{\delta}\mathbf{B}\cdot\nabla\delta P_{||}^{NA}}_{\{V\}} \underbrace{-\frac{c}{B_0}\boldsymbol{\delta}\mathbf{B}\cdot\nabla\delta\phi_{ind}}_{\{VI\}}$$

$$\underbrace{+\frac{c}{B_0}\boldsymbol{\delta}\mathbf{B}\cdot\nabla\langle\phi\rangle}_{\{VII\}} \underbrace{-\frac{cm_e}{en_0}\nabla\cdot\left[n_0\delta u_{||e}\left(3\mathbf{V_c}+\mathbf{V_g}\right)+n_0u_{||0}\mathbf{V_E}\right]}_{\{VIII\}} \underbrace{-\frac{cm_e}{en_0}\nabla\cdot\left(n_0\delta u_{||e}\mathbf{V_E}\right)}_{\{IX\}}$$

$$\underbrace{+\frac{c}{en_0}\frac{P_{||0}-P_{\perp0}}{B_0^2}\boldsymbol{\delta}\mathbf{B}\cdot\nabla B_0 + \frac{c}{en_0}\frac{\delta P_{||}^{NA}-\delta P_{\perp}^{NA}}{B_0^2}\mathbf{B_0}\cdot\nabla B_0}_{\{X\}}$$

The conservative scheme recovers collisionless tearing mode dispersion relation in drift kinetic electron model

Linearize the physics model of conservative scheme and solve the eigen value problem:

$$\nabla_\perp^4 \omega^2 \delta A_{||} = \nabla_\perp^2 [\frac{1}{D_e^2} \omega^2 \xi_e^2 Z'(\xi_e) \delta A_{||}] - \frac{m_i}{m_e} k_{||} \xi_e^2 Z'(\xi_e)(k_{||} \nabla_\perp^2 - k_{||}'') \delta A_{||}$$

Inner region equation:

$$\nabla_\perp^2 \delta A_{||} = \frac{1}{D_e^2} \xi_e^2 Z'(\xi_e) \delta A_{||}$$

Outer region equation:

$$\left(k_\parallel \nabla_\perp^2 - k_\parallel''\right) \delta A_\parallel = 0$$

Asymptotic matching to CTM dispersion relation:
Recover Drake and Lee 1977 result.

$$-i\omega = \gamma = \frac{D_e^2}{\sqrt{\pi}} |k_{||}' v_{te}| \Delta_o'$$

Collisionless tearing mode dispersion relation in finite mass fluid electron model

$$\nabla_\perp^2\left(k_{||}v_{te}^2-\frac{\omega^2}{k_{||}}\right)\nabla_\perp^2\delta A_{||}=\nabla_\perp^2\left(k_{||}''v_{te}^2-\frac{\omega^2\omega_{pe}^2}{k_{||}c^2}\right)\delta A_{||}+\frac{v_{te}^2}{\rho_s^2}(k_{||}\nabla_\perp^2-k_{||}'')\delta A_{||}$$

$$(k_{||}^2v_{te}^2-\omega^2)\frac{\partial^2}{\partial x^2}\delta A_{||}=\left(-\frac{\omega^2\omega_{pe}^2}{c^2}\right)\delta A_{||}$$

$$\left(k_\parallel\nabla_\perp^2-k_\parallel''\right)\delta A_\parallel=0$$

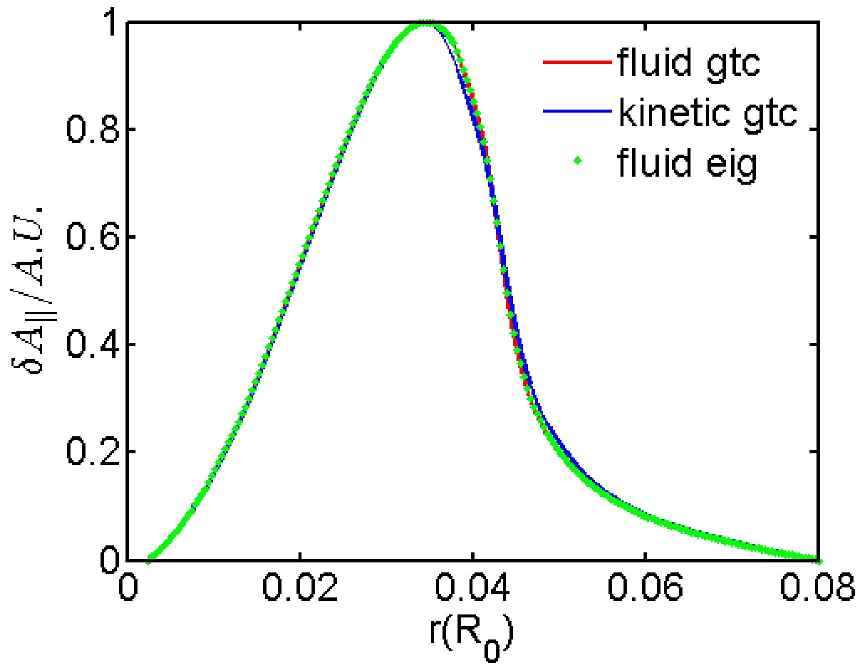
$$\Delta_o'=\lim_{|x|\to 0}\frac{\delta A_{||}'}{\delta A_{||}}=\frac{2}{a}\left(\frac{1}{k_y a}-k_y a\right)=\Delta_I'$$

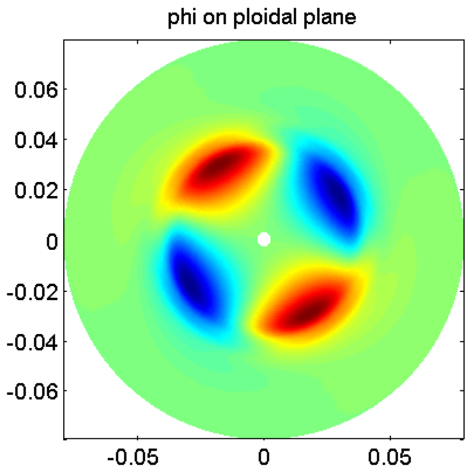
and

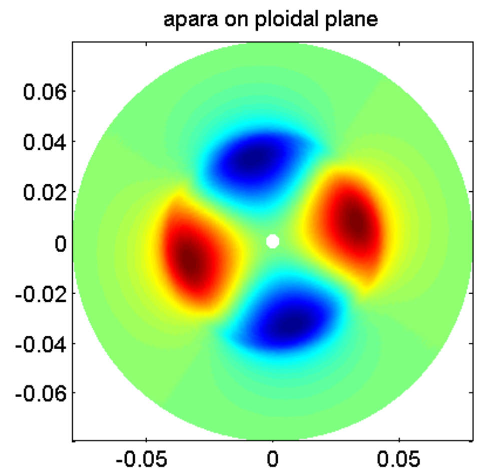
$$-i\omega=\gamma=\frac{D_e^2}{\pi}|k_{||}'v_{te}|\Delta_o'$$

- **Kinetic growth rate is $\pi^{1/2}$ times larger than fluid one.**
- **Reason: fluid calculation assumes two counter propagating beam electron background, kinetic model uses shifted Maxwellian electron.**

➢ kinetic collisionless tearing mode



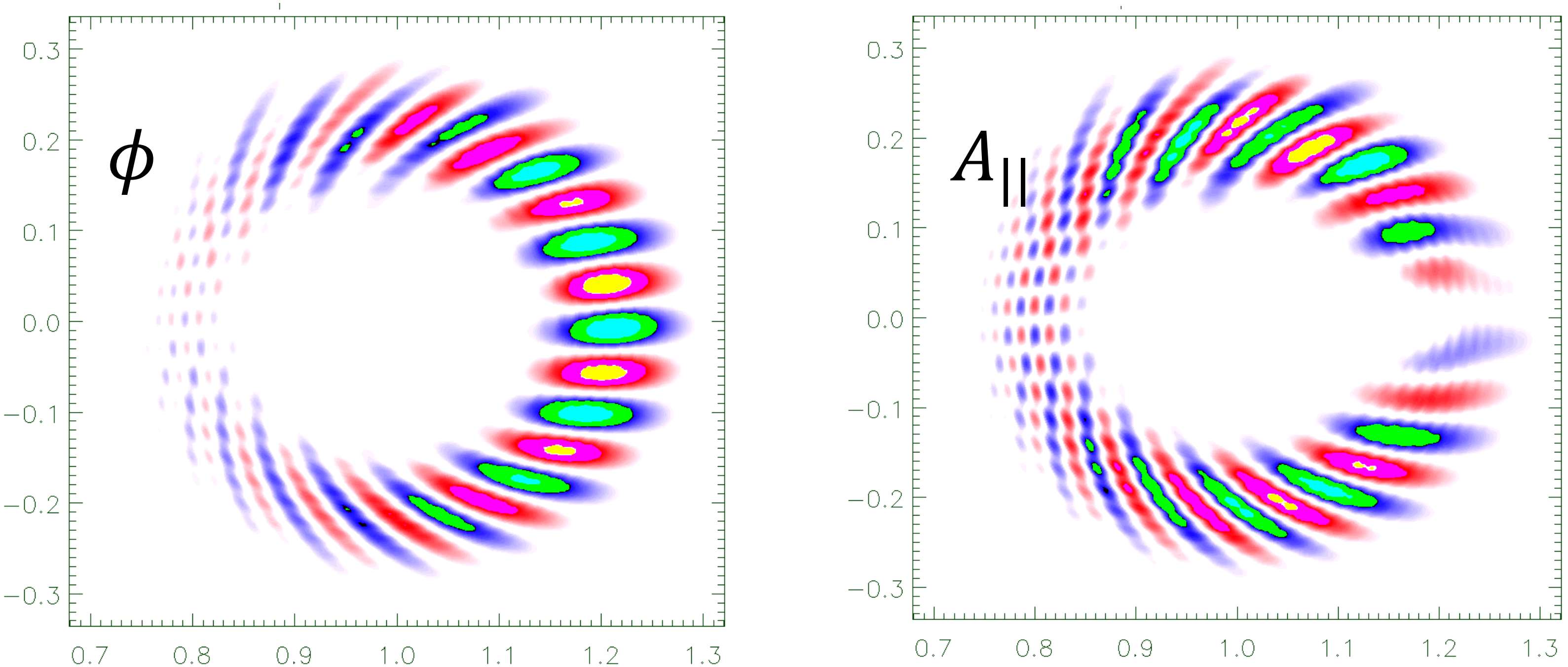




γ_{gtc}=0.0031(Cs/R_0) almost the $\pi^{1/2}$ of fluid growth rate. Kinetic picture of the collisionless tearing mode can be accurately captured through the DKE model.

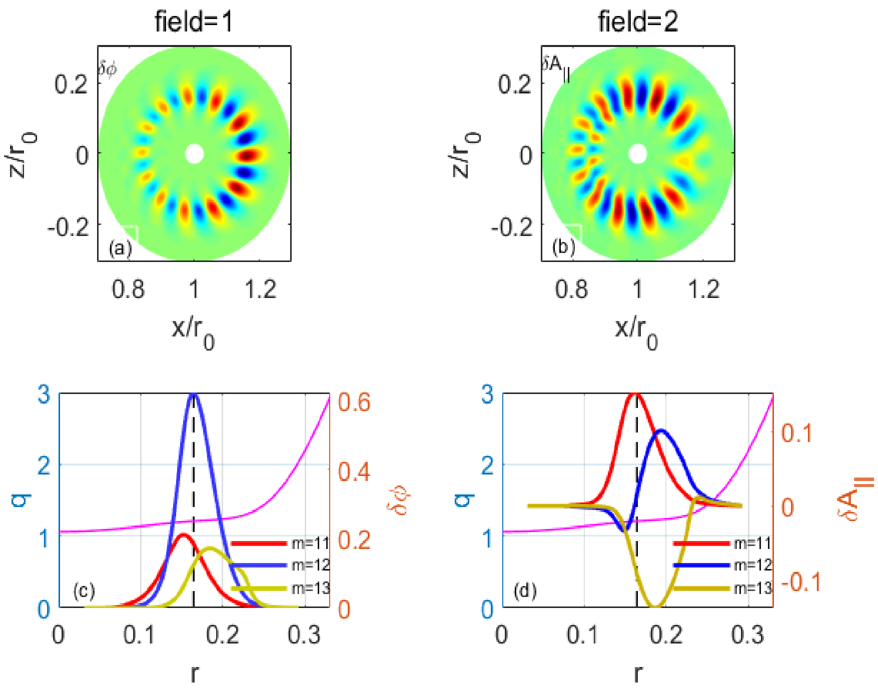
DKE simulation of Kinetic Ballooning Mode

$\beta_e = 2.0\%$

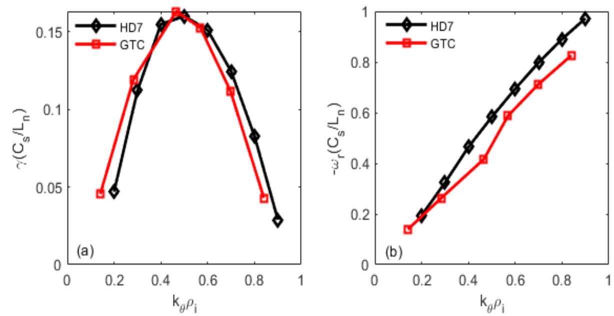


The drift kinetic electron faithfully captures kinetic ballooning mode in high $\boldsymbol{\beta_e}$ plasmas. Near the mode rational surface, accurate electron response is essential for low frequency MHD modes, especially for tearing Mode.

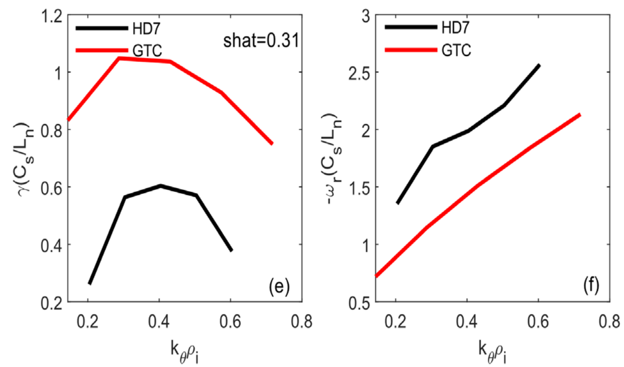
KBM in weak magnetic shear plasma



KBM mode structure in weak shear plasma



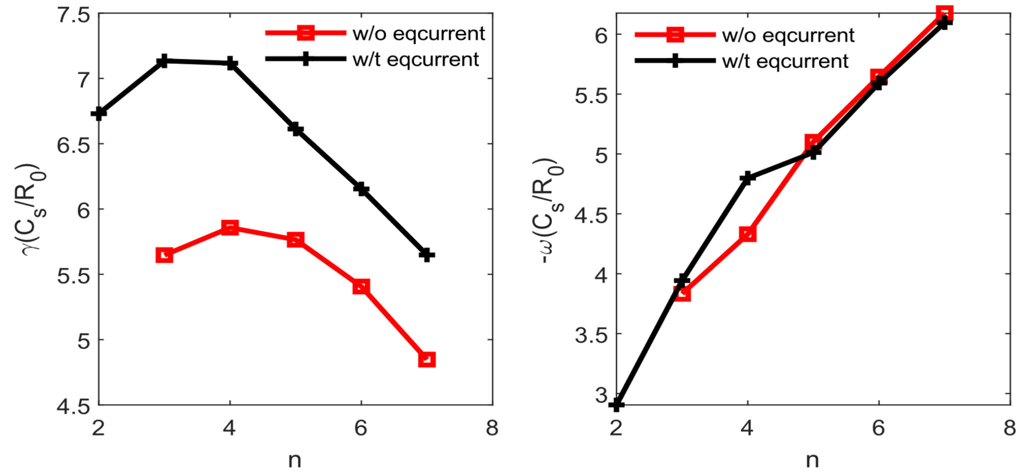
ITG spectrum



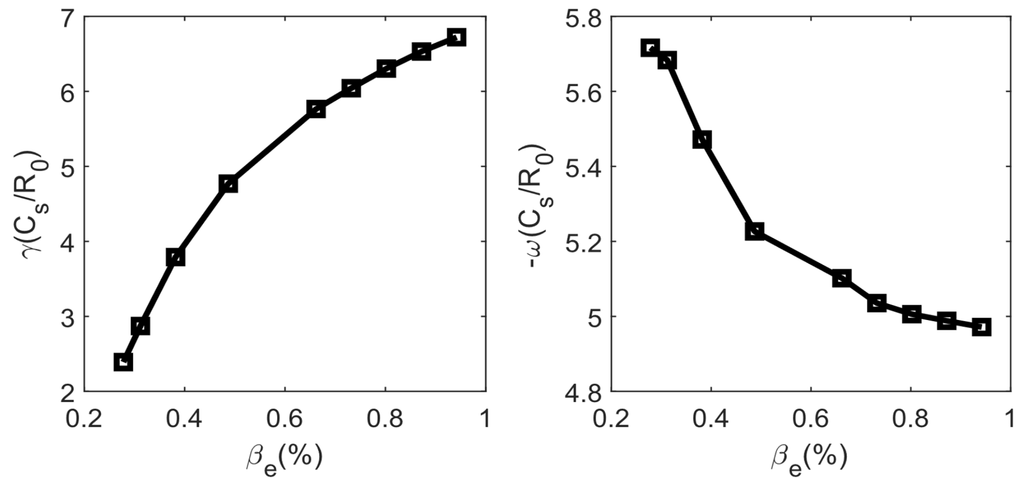
KBM spectrum

- **KBM mode structure is unconventional ballooning mode structure for low magnetic shear plasma, dominant modes exist,**
- **Traditional ballooning representation may break down for theoretical and eigen value approaches.**

KBM in weak magnetic shear plasma



KBM spectrum for low magnetic shear



KBM frequency and growth rate vs β_e

- KBM is sensitive to the equilibrium, especially the magnetic shear.
- The mode spectrum may shift towards to the low mode number side,
- The critical β for the excitation of KBM is low for weak magnetic shear plasma.

Progresses:

1. A kinetic electron and gyrokinetic ion model is developed for MHD modes in low beta plasmas;

2. Linear behavior of both collisional and collisionless tearing modes are investigated in toroidal geometry.

Some discussion:

1. Kinetic effects of thermal background and energetic particles on MHD modes.

2. Can a first principle model, especially kinetic electron model be used to self-consistently simulate the Multi-Scale MHD process, for an example, NTM.

Thanks very much!