Black Aurora, Bohm Diffusions, Quasi-Neutrality Mysteries Explained

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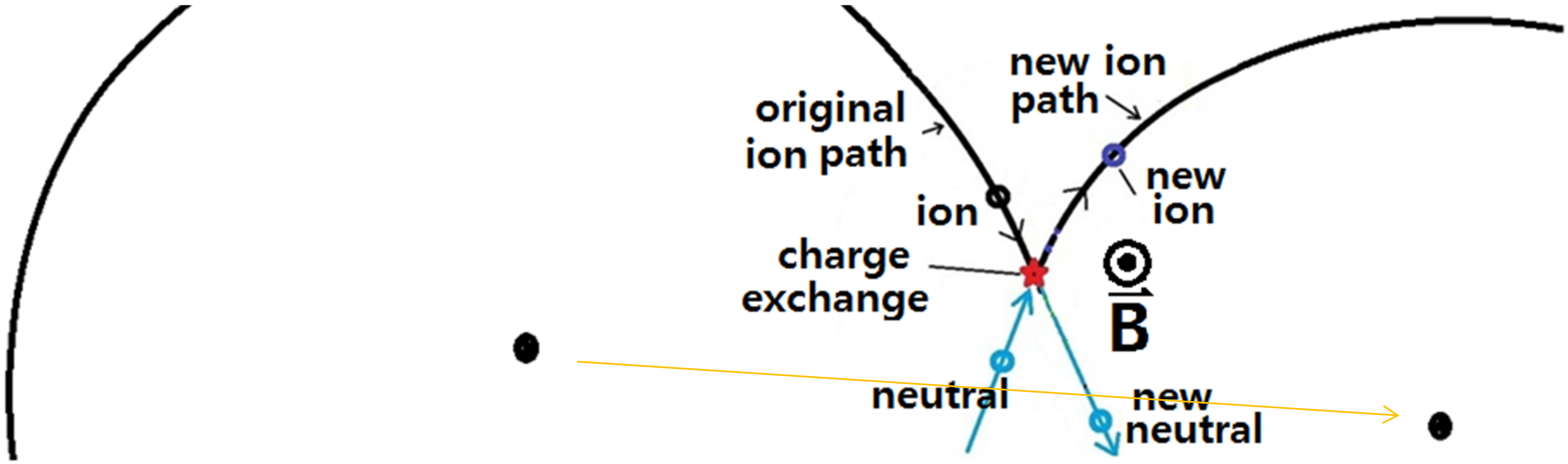
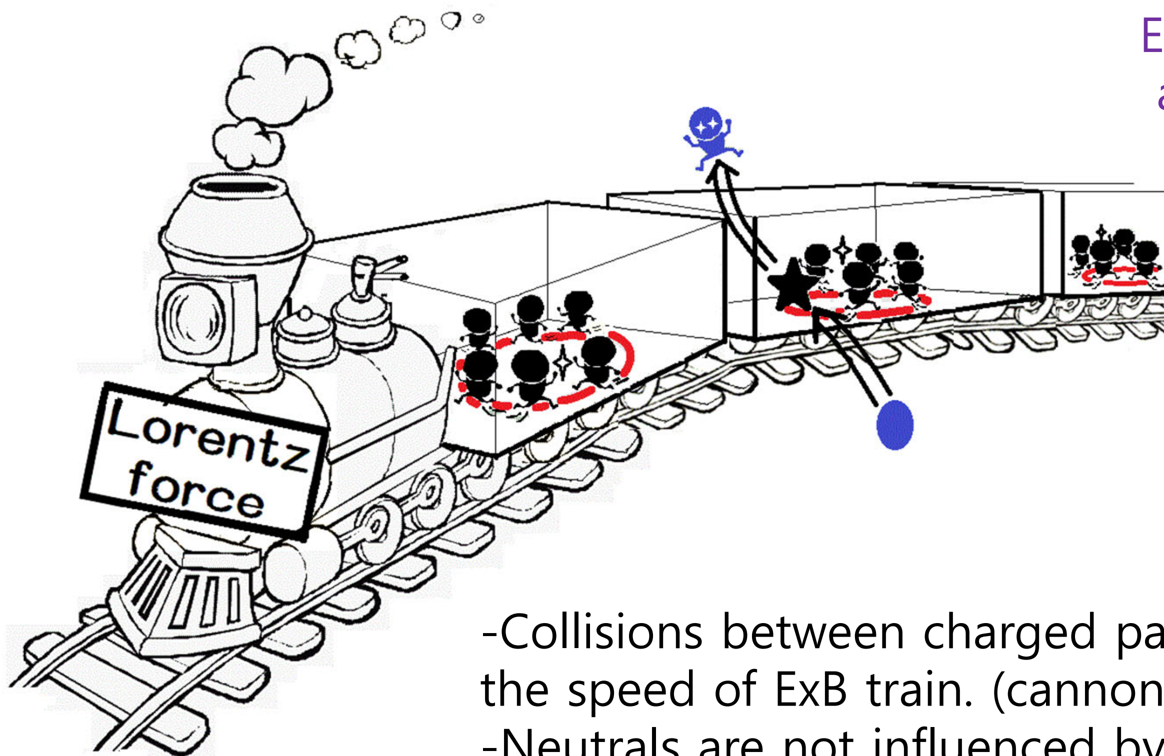
Introduction to Gyro-Center Shift (GCS) by ion-neutral collisions



1. GCS theory explained **E_r formation** at tokamak boundary [K.C. Lee, PoP 2006]
2. Explained **reversed motion** of cathode spot in **arc discharge** [K.C. Lee, PRL 2007]
3. Explained **L/H transition** & **turbulence** induced **diffusion** [K.C. Lee, PPCF 2009]
4. Measurement of turbulence induced diffusion on **NSTX** [K.C. Lee, *et al.*, JKPS* 2013]
5. Analysis of **quasi-neutrality violation** by ion-neutral collisions [K.C. Lee, JKPS* 2013]
6. Analysis of **Bohm diffusions** by GCS theory [K.C. Lee, IEEE* 2015]
7. Measurement and analysis of **poloidal rotation on KSTAR** [IAEA FEC 2014]
8. .**Black aurora and EEJ** analysis by GCS theory [Kwan Chul Lee, PoP 2017]

There are thousands times more collisions among ions and electrons (such as between passing particles and trapped particles).
Why is ion-neutral collision so important?



ExB of electrons and ions are same in tokamak

ExB train

-Collisions between charged particles cannot change the speed of ExB train. (cannonball in a railway carriage)
-Neutrals are not influenced by B-field or E-field,

-ExB train is in perpendicular direction,
-Collisions between charged particles can change in parallel direction, ex: bootstrap current

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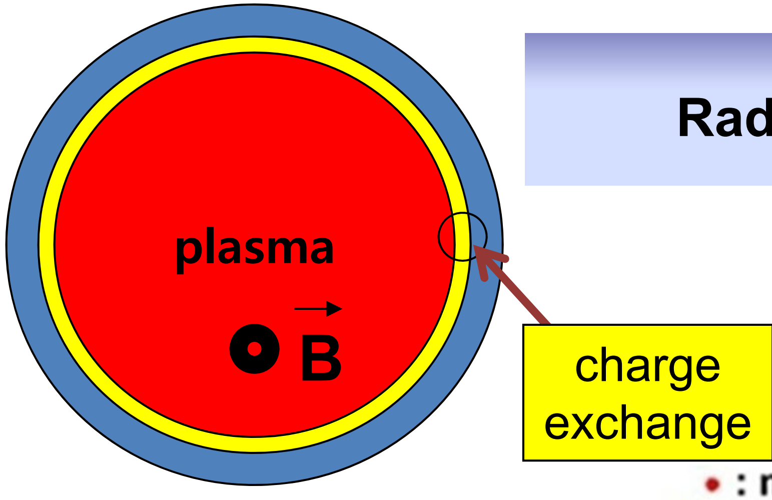
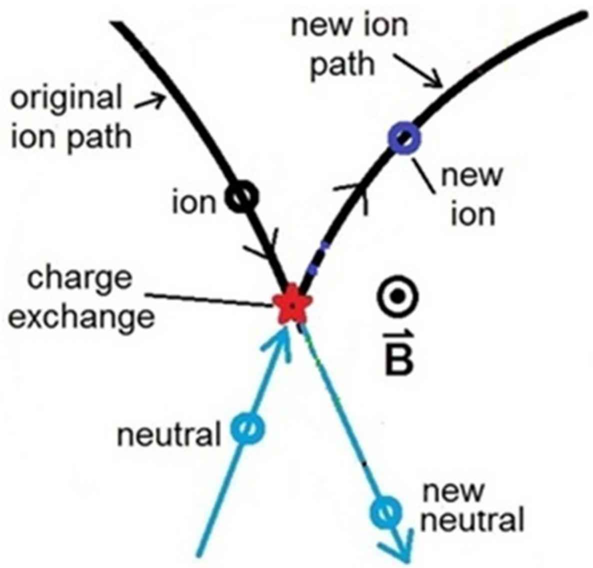
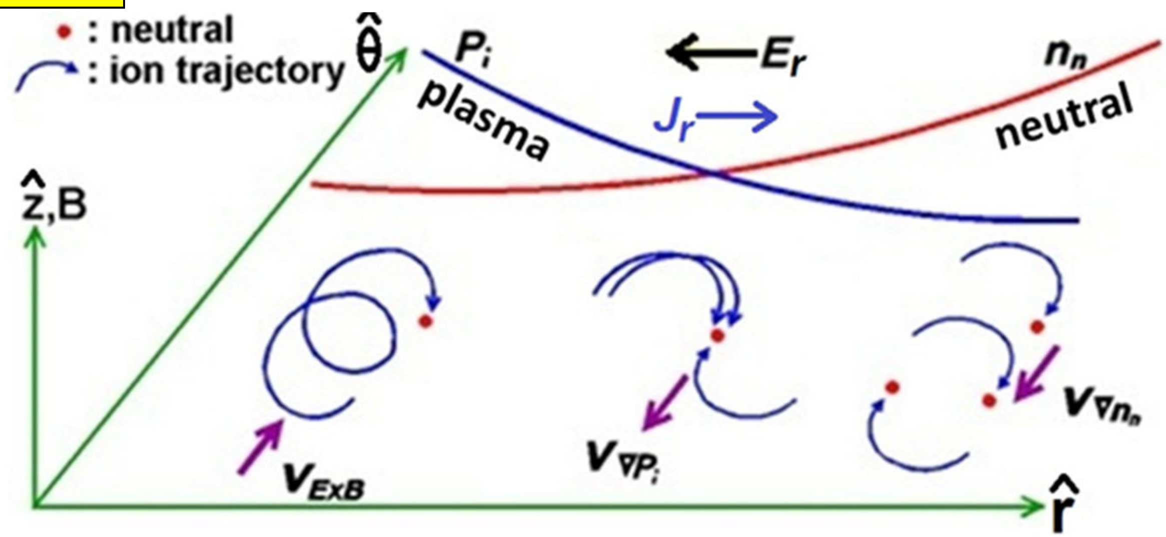


Radial current by ion-neutral collision

$$\vec{J} \times \vec{B} = n_i \nu_{i-n} m_i \upsilon^*$$



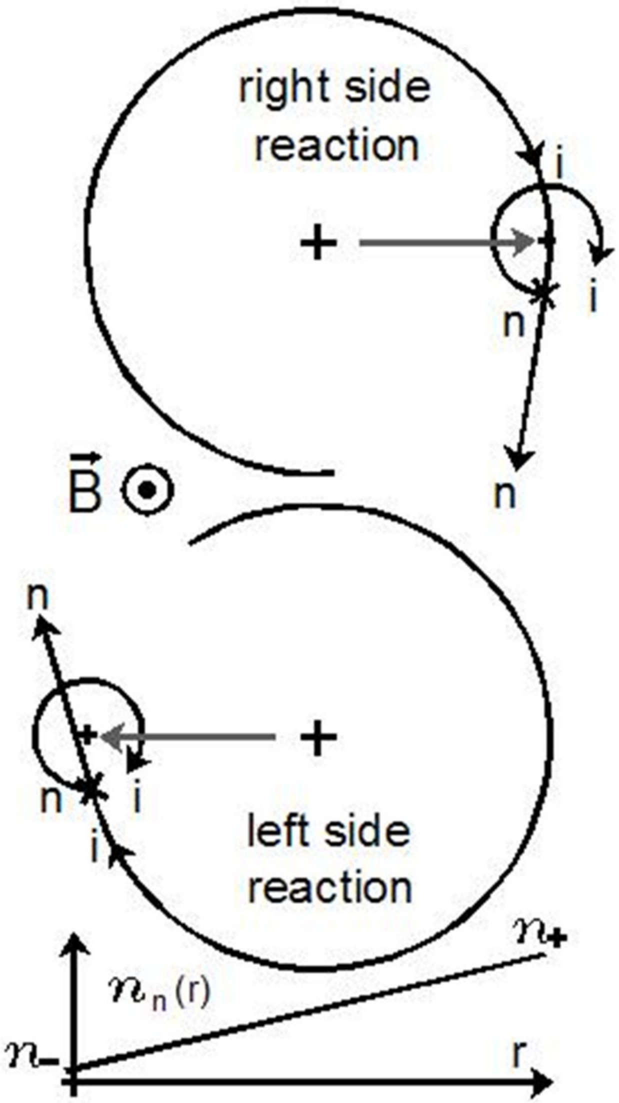


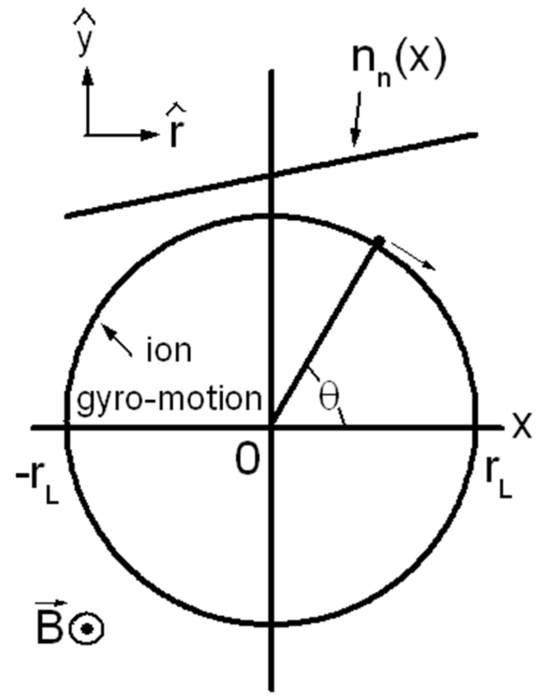
$$\nu_{i-n} = \sigma_{i-n} \upsilon_i n_n$$

$$J_r^{GCS} = \frac{n_i \nu_{i-n} m_i}{B} \Bigg(\underbrace{\frac{E}{B} - \frac{\nabla P_i}{qBn_i} + \frac{kT_i \nabla n_n}{qBn_n}}_{\upsilon^*} \Bigg)$$

v* by neutral density gradient

reaction rate = $\sigma_{i-n}\upsilon_i n_n$





$$\vec{\upsilon}_{i\perp} = \upsilon_{i\perp}\cos\theta\hat{y} + \upsilon_{i\perp}\sin\theta\hat{r}$$

$$\upsilon_{av} = \frac{\sigma_{cx}\upsilon_{i\perp}\oint \vec{\upsilon}_{i\perp}(\theta) n_n(\theta)d\theta}{\sigma_{cx}\upsilon_{i\perp}\oint n_n(\theta)d\theta} = \frac{1}{2} r_{Li}\upsilon_{i\perp}\frac{1}{n_n}\frac{\partial n_n}{\partial r}$$

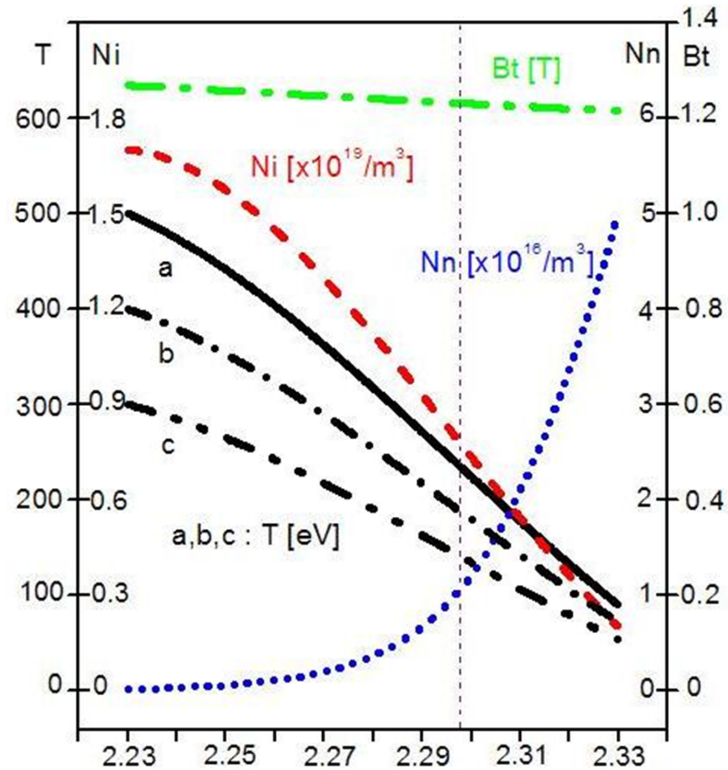
expectation value

$$= \frac{kT_i\nabla n_n}{eBn_n}$$

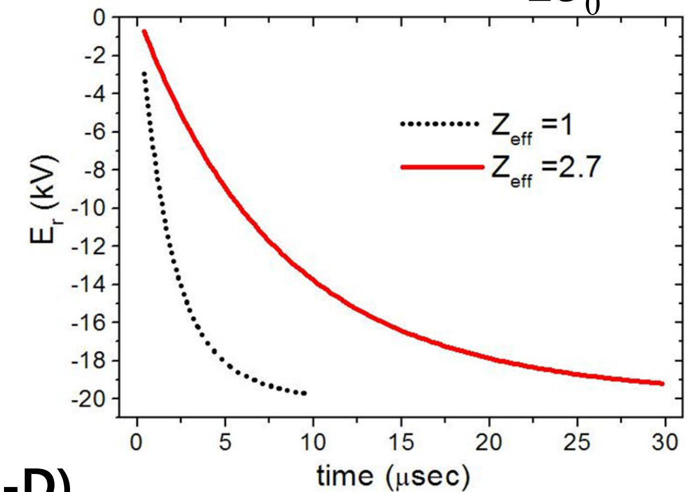
Example 1: E_r well of tokamak

$$J_r^{GCS} = en_i \frac{r_{Li}}{\lambda_{i-n}} \left(\frac{E}{B} - \frac{1}{eB}\frac{\nabla P_i}{n_i} + \frac{kT_i}{eB}\frac{\nabla n_n}{n_n} \right),$$

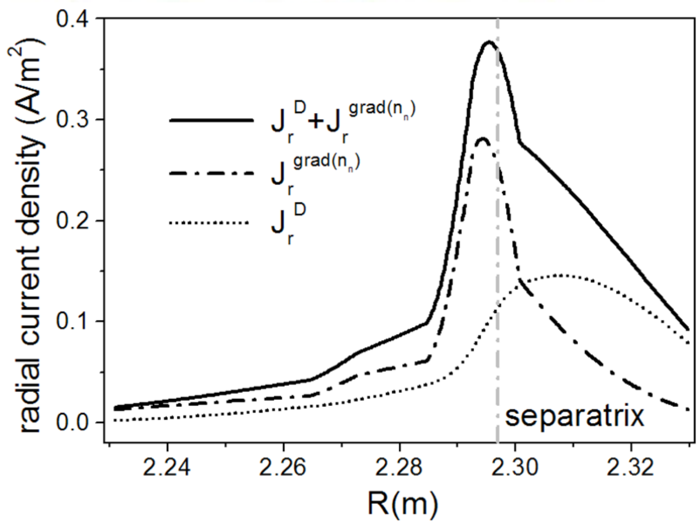
$$\nabla \cdot J = -\frac{d\rho}{dt}, \qquad \nabla E = \frac{\rho}{2\varepsilon_0}$$

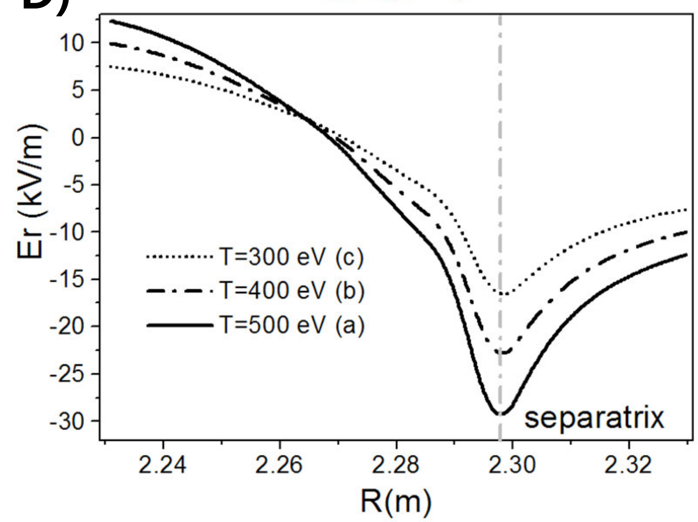


(K.C.Lee, PoP 06')

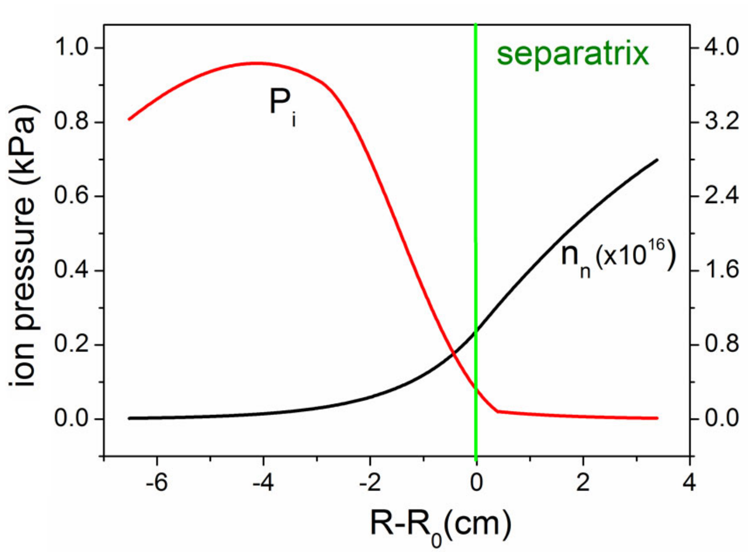


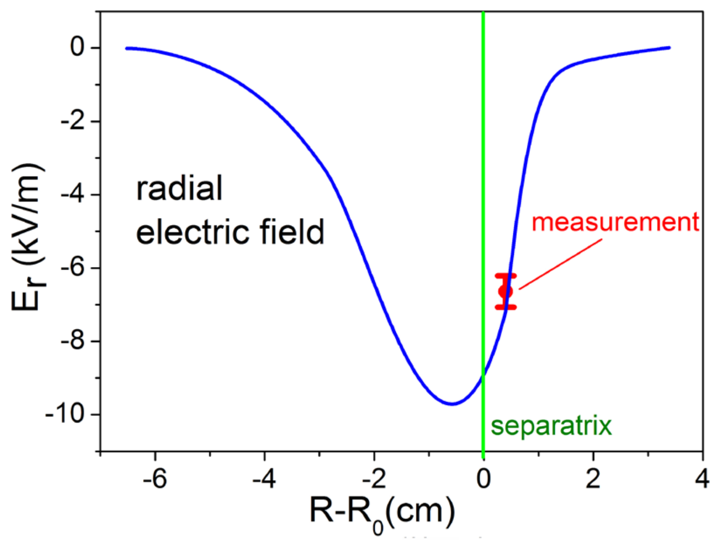
(DIII-D)





E_r well of NSTX





▶Neutral density profile is analyzed by tangential camera data and DEGAS2 code [D. P. Stotler et al PoP 2015]

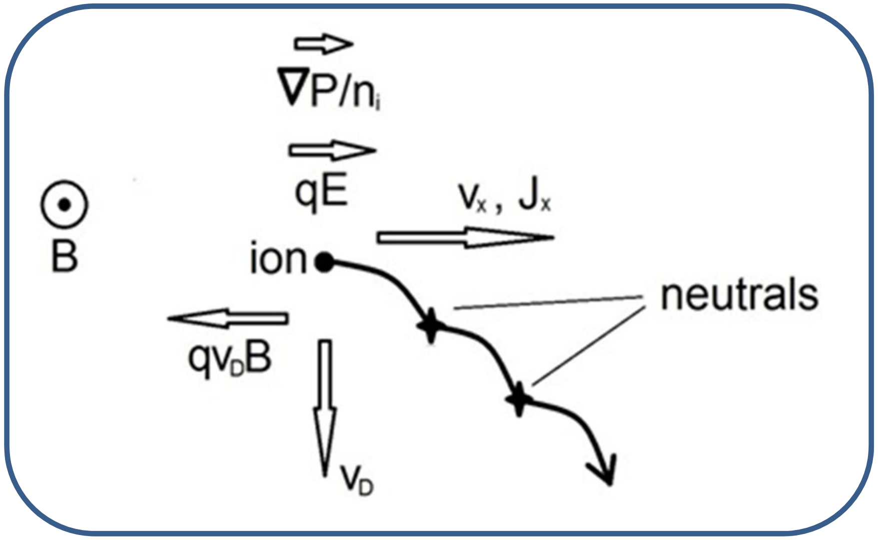
▶Er measurement provided by ERD: R. Bell

▶Er calculation by GCS current agreed with measurement

NSTX #142214: 0.4 sec

J_r^{GCS} for weakly ionized cases

general form of J_r^{GCS} for weakly ionized/magnetized plasmas ($r_{Li} \sim \lambda_{i-n}$)



$$\upsilon_D = \frac{1}{1 + r_{Li}^2 / \lambda_{i-n}^2}\left(\frac{E}{B} - \frac{\nabla p}{qBn_i}\right)$$

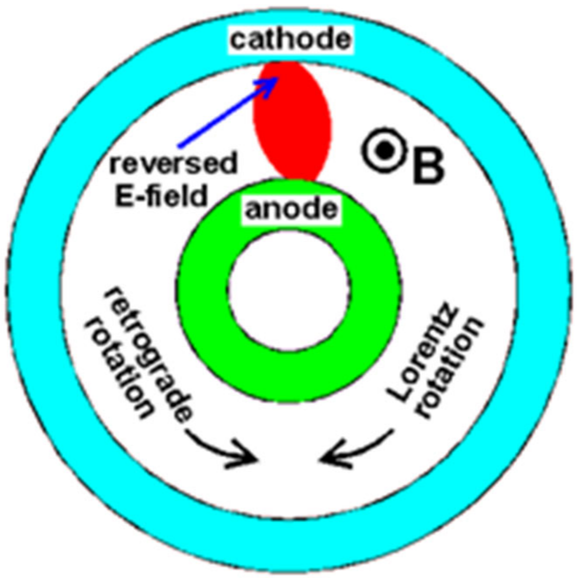
$$\theta = \frac{r_{Li}}{\lambda_{i-n}} = \frac{\nu_{i-n}}{\omega_{ci}} = \frac{1}{\omega_{ci}\tau_{i-n}}$$

$$\lambda_{i-n} = \frac{1}{\sigma_{i-n} n_n}$$

$$J_r^{GCS} = qn_i\theta\left[\frac{1}{1+\theta^2}\left(\frac{E}{B} - \frac{\nabla P_i}{qBn_i} - \upsilon_n\right) + \exp(-\theta)\frac{kT_i \nabla n_n}{qBn_n}\right]$$

probability of ions to have longer mean free path than ion gyro-radius

reversed motion of cathode spots in arc discharges



- first found by Stark 1903
- no explanation for both pressure and B-field dependence until 2007

Yuri P. Raizer

(1991)

Gas Discharge Physics

h) Spot in a magnetic field. One curious phenomenon was noticed by *Stark* in 1903, who observed the behavior of cathode spots on the surface of liquid mercury cathodes: in a magnetic field tangential to the surface, the spot moves in the direction opposite to the magnetic force $[i \times H]$ acting on the current. No satisfactory explanation of this effect has been found, even at the qualitative level (despite numerous attempts).

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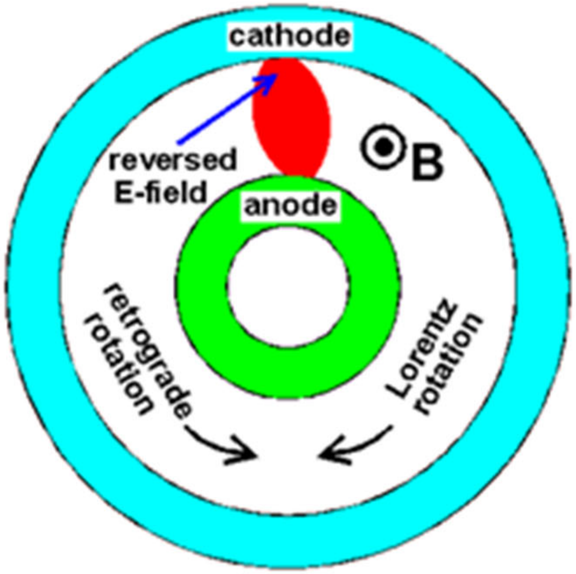
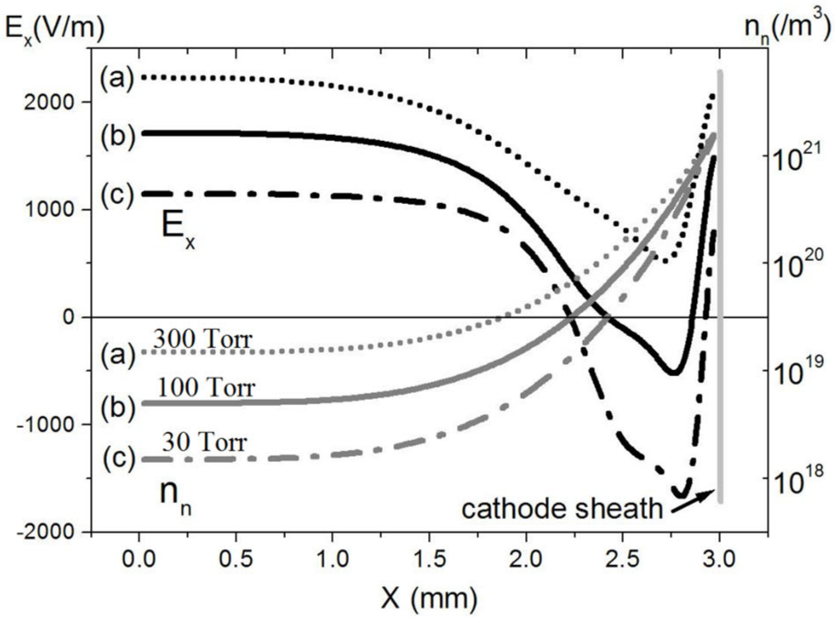
► reversed motion of cathode spots in arc discharges

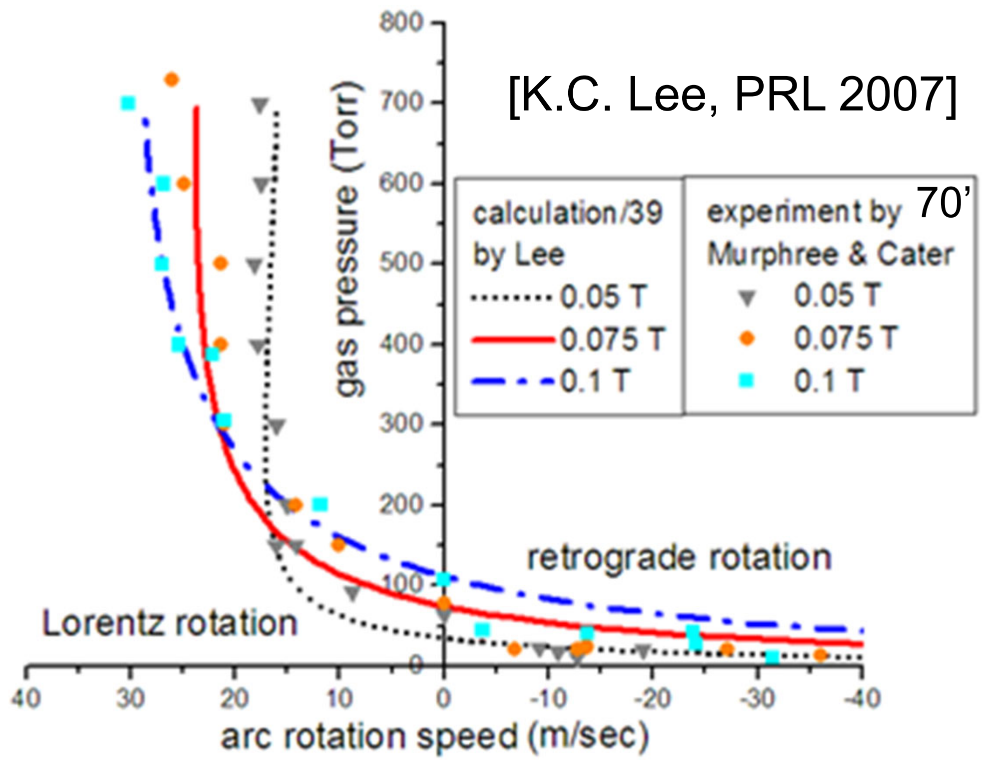


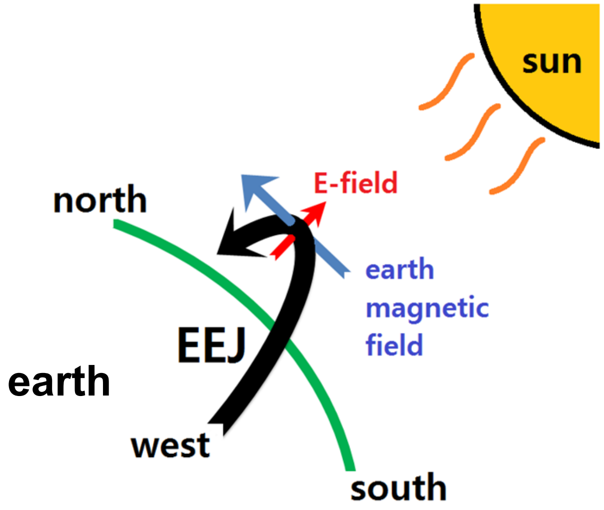
$$J_r^{GCS} = qn_i\theta[\frac{1}{1+\theta^2}(\frac{E}{B} - \frac{\nabla P_i}{qBn_i}) + \exp(-\theta)\frac{kT_i\nabla n_n}{qBn_n}]$$

$$\theta = \frac{r_{Li}}{\lambda_{i-n}}$$



[K.C. Lee, PRL 2007]





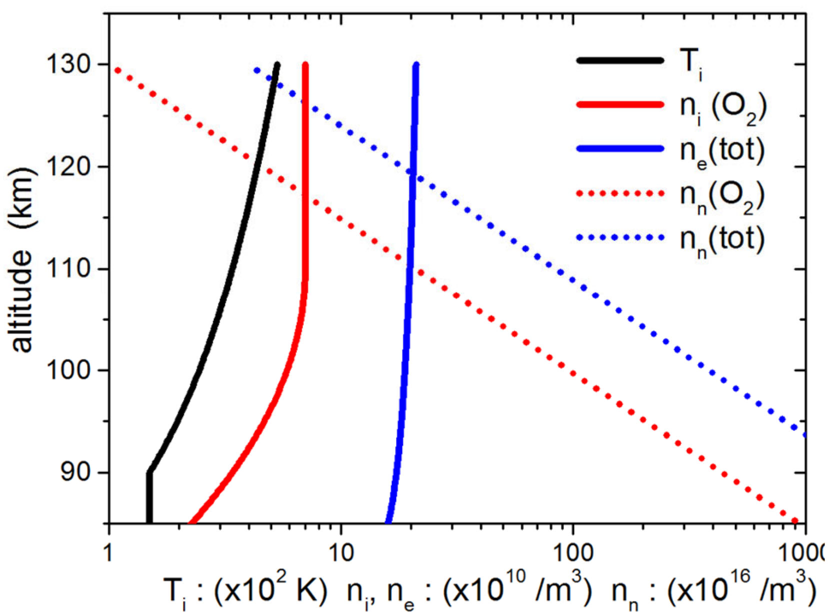
► equatorial electrojet (EEJ)

- east-west current structure on magnetic dip equator
- fluctuations with daily reverse
- extensively measured and studied [Forbes, 1981]
- difference of ion drift and electron drift of ExB generates Hall current (missing: origin of E_r)

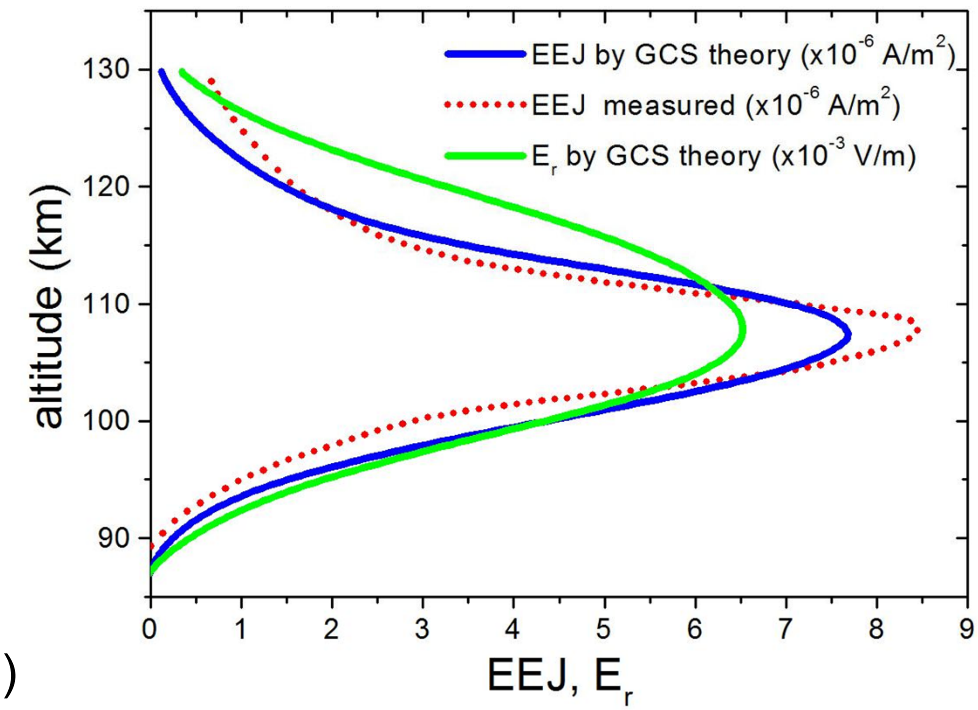
$$J_r^{GCS} = q n_i \frac{\theta}{1+\theta^2}\left(\frac{E}{B} - \upsilon_n\right)$$

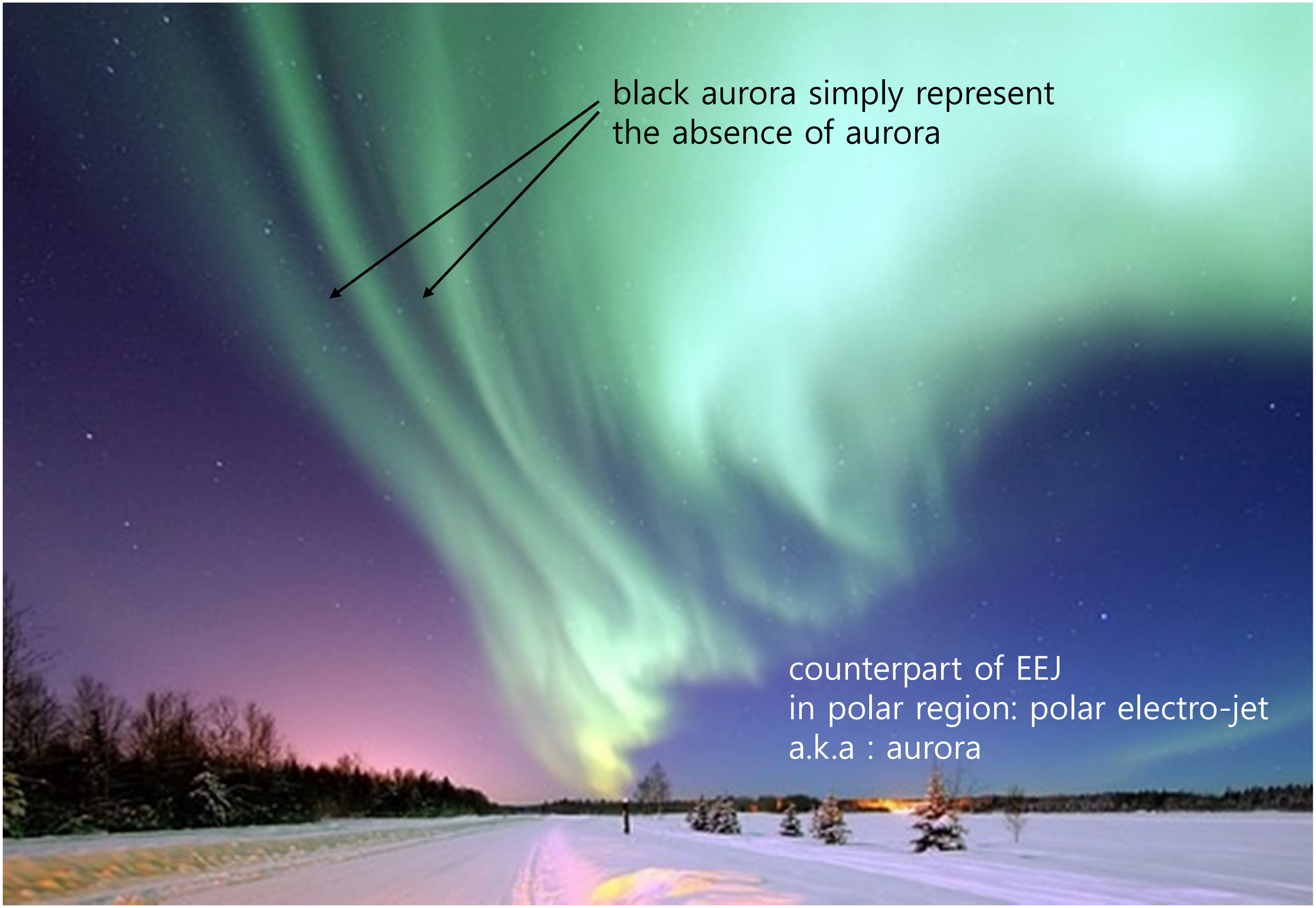
Dynamo [Rishbeth,1997]

~200m/sec westward (day time) neutral wind measured (Hysell JGR 2002)

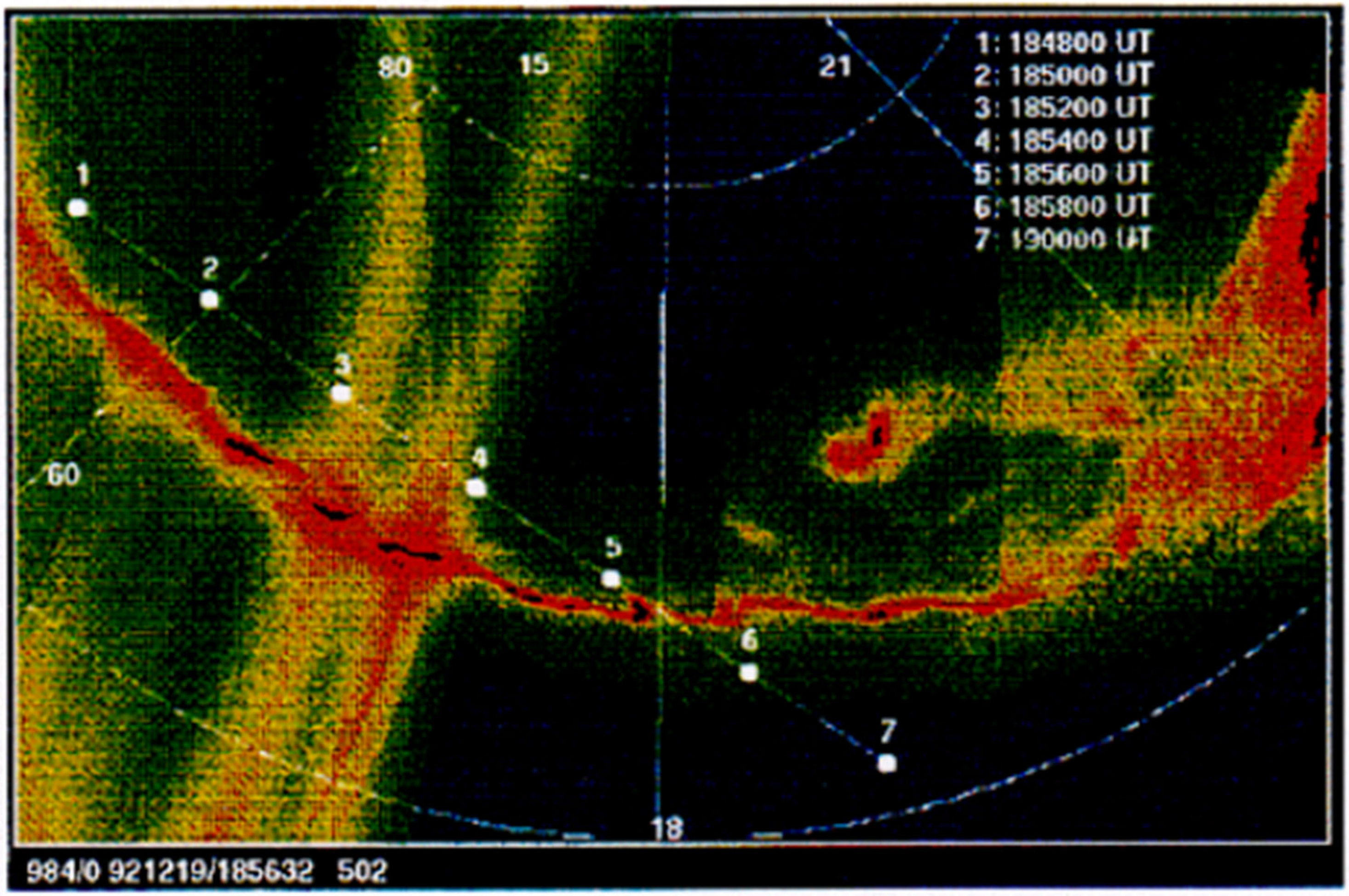


(satellite measurement Kelly 1989)

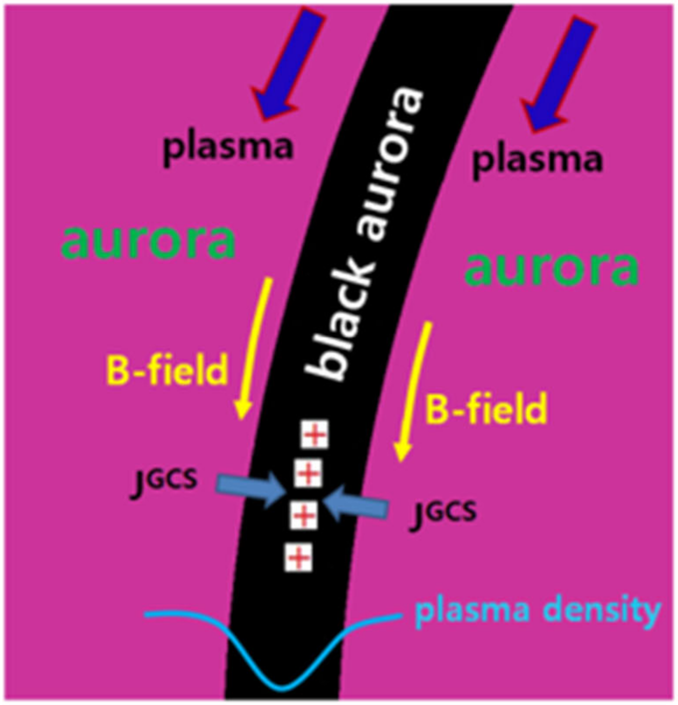


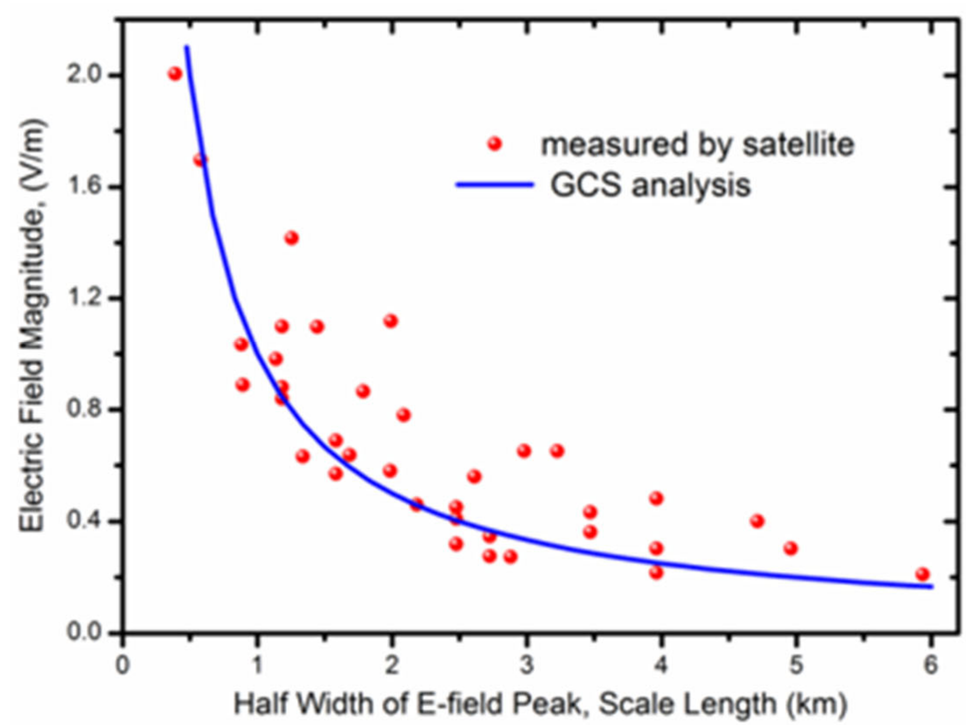

black aurora simply represent
the absence of aurora
counterpart of EEJ
in polar region: polar electro-jet
a.k.a : aurora

Strong E-field is found across the black aurora



Swedish satellite Freja observed strong cross E-field on black aurora [Marklund, PPCF 1997]





$$\frac{qn_i}{\omega_c \tau}\left(\frac{E}{B} - \frac{\nabla P_i}{qBn_i} + \frac{kT_i}{qB}\frac{\nabla n_n}{n_n}\right) \approx 0$$

$$E \approx kT_i/qL_n$$

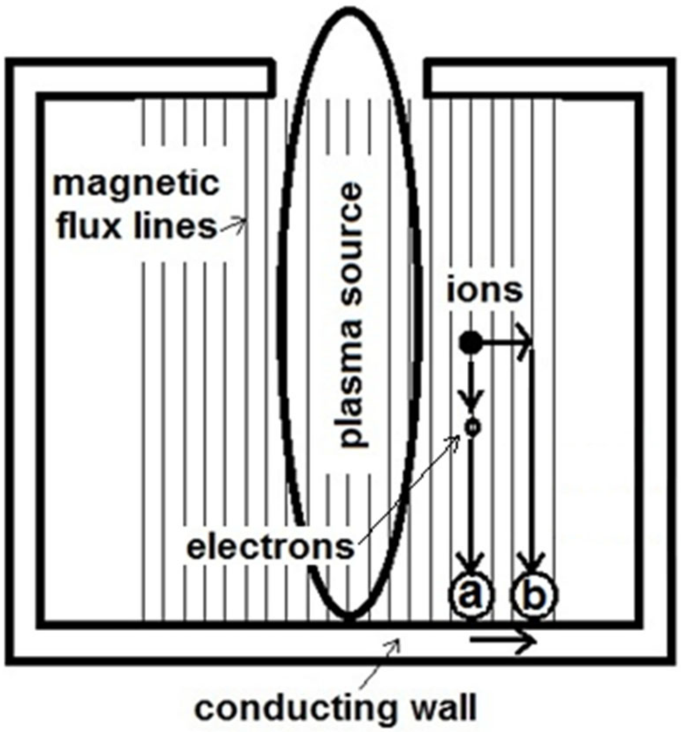
Agreements: 1. polarity
2. magnitude
3. inversely proportional to scale length

[Kwan Chul Lee, PoP 2017]

origin of Bohm diffusions

$$J_r^{GCS} = en_i \frac{r_{Li}}{\lambda_{i-n}}\left(\frac{E}{B} - \frac{1}{eB}\frac{\nabla P_i}{n_i} + \frac{kT_i}{eB}\frac{\nabla n_n}{n_n}\right),$$

short circuit effect : E => 0, $\Gamma = J^{GCS}/q$



$$\Gamma = n_i \frac{r_L}{\lambda_{cx}}\left[-\frac{kT_i \nabla n_i}{eBn_i} + \frac{kT_i \nabla n_n}{eBn_n}\right]$$

$$\Gamma = -D\nabla n_i$$

approximation of $\nabla n_i / n_i \approx -\nabla n_n / n_n$

[Goldston, 1995]

$$D = \frac{2r_L}{\lambda_{cx}}\frac{kT_i}{eB}$$

short circuit effect [Simon, 1959]

Bohm experiment : $\frac{1}{13} < \frac{r_{Li}}{\lambda_{i-n}} < \frac{1}{40}$ ⇨ $D = \left(\frac{1}{7} \sim \frac{1}{20}\right)\frac{kT_i}{eB}$

$$D_B = \frac{1}{16}\frac{kT_e}{eB}$$

Simon experiment : $\frac{r_{Li}}{\lambda_{i-n}} \approx \frac{1}{13}$

measured
D=0.92 m²/s

GCS theory
D=0.74 m²/s

GPI fluctuation analysis on NSTX [Diallo *et al*. NF (2017)]

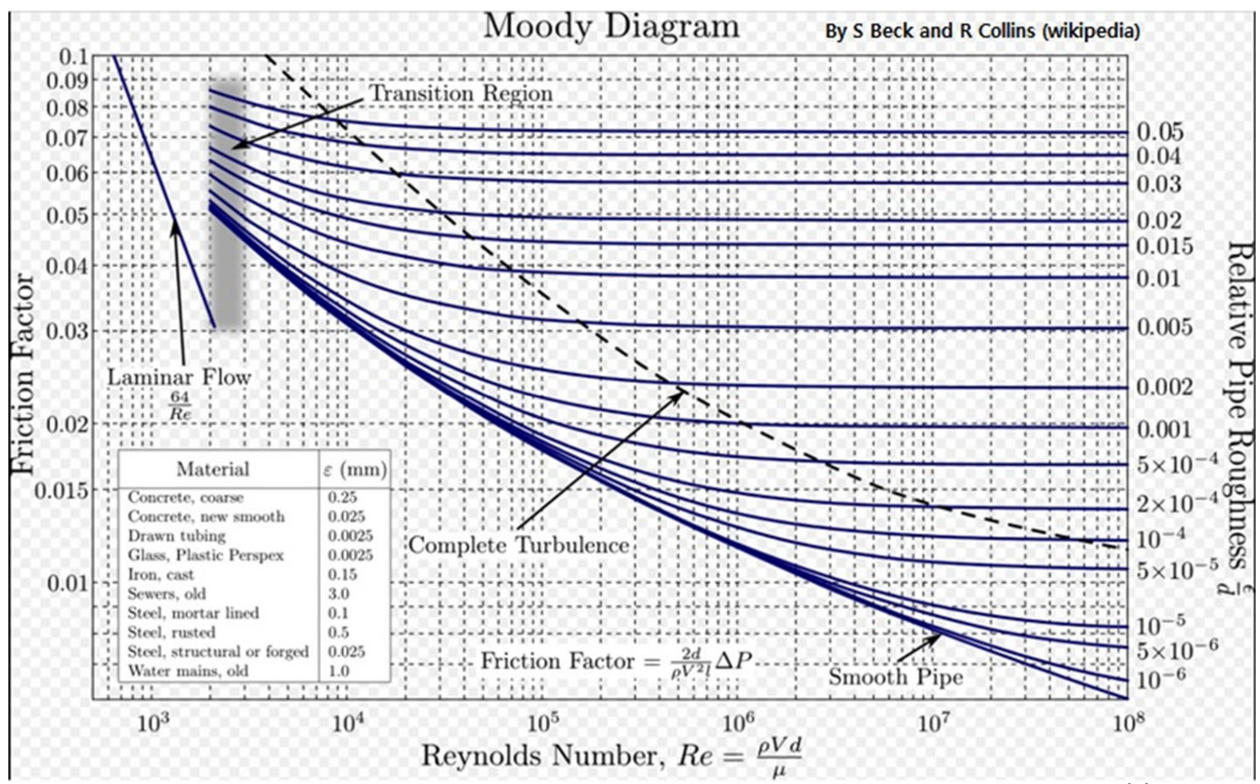
- poloidal velocities do not change prior to the L-H transition
- Reynolds work may not be large enough to explain the L-H transition on NSTX, contrary to the predator-prey model.

What is the real mechanism for L-H transition?

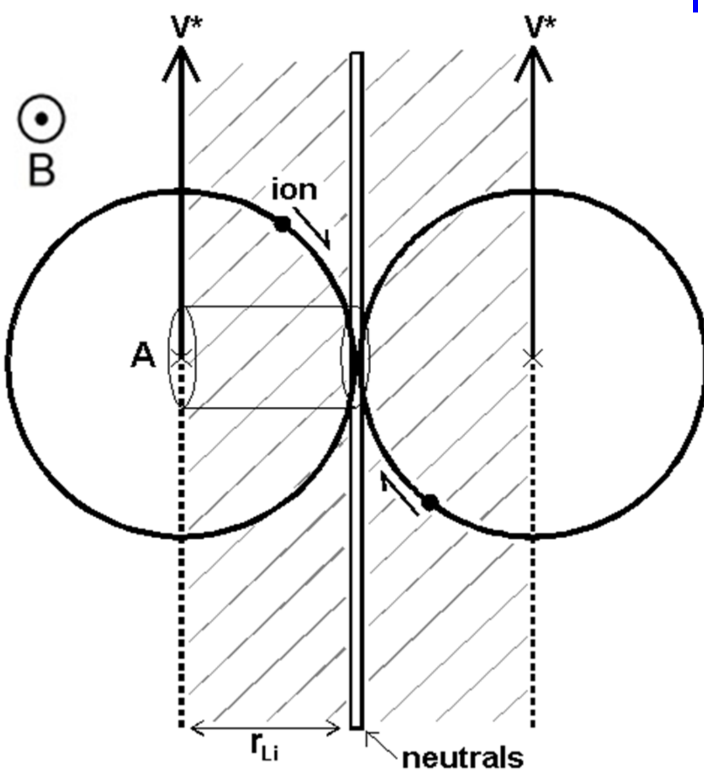
-learning by analogy

-in neutral fluids turbulence acts as a source of friction

-at tokamak boundary Er increases in H-mode



Reynolds number of ion-neutral friction



Inertial force

$$\mathrm{Re} \equiv \frac{n_i m_i \upsilon^{*2} / r_{Li}}{n_i m_i \nu_{i-n} \upsilon^*} = \frac{eB}{kT_i} \lambda_{i-n} \upsilon^* = \frac{\lambda_{i-n}}{r_{Li}} \frac{\upsilon^*}{\upsilon_i}$$

viscous force

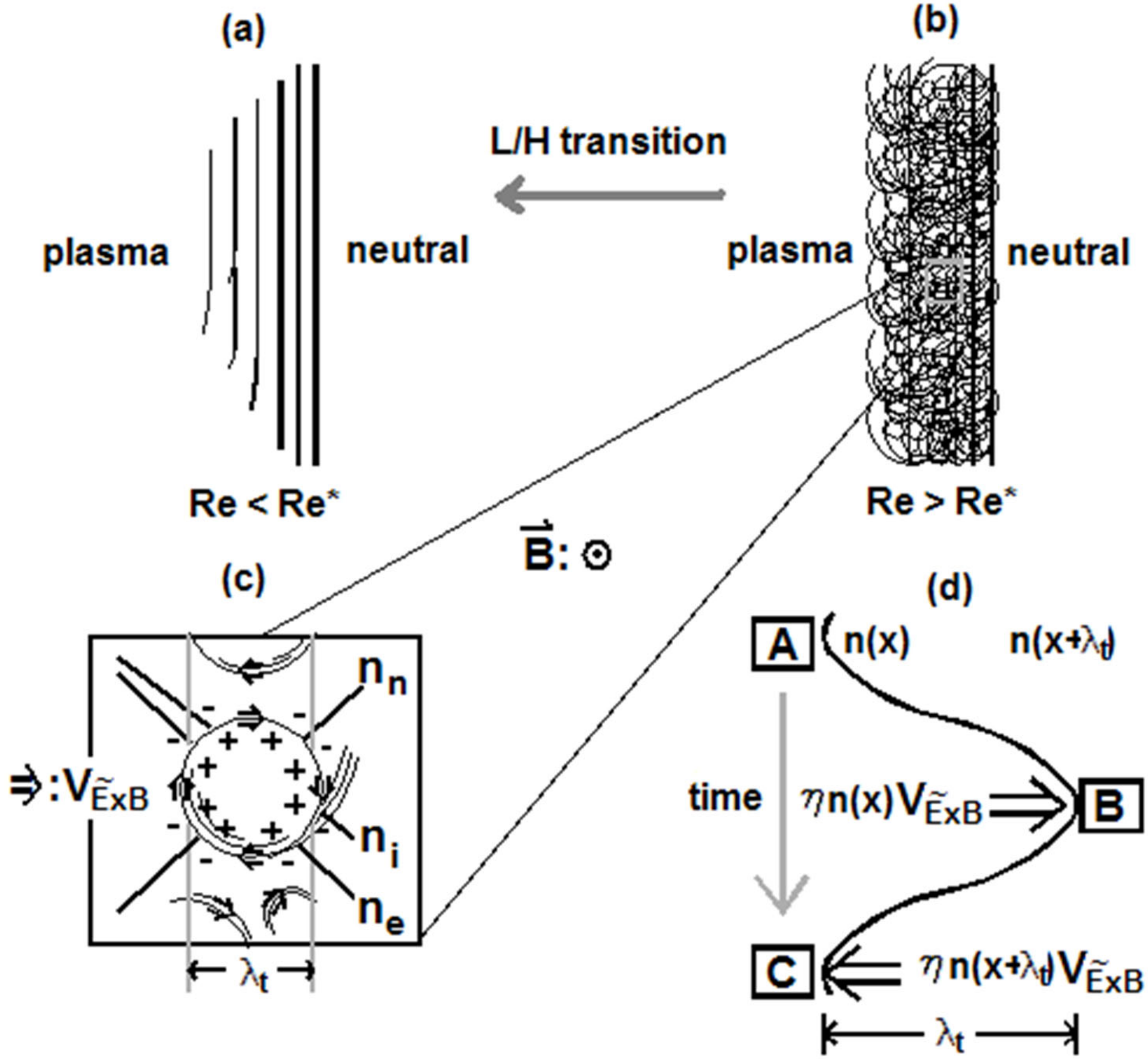
Re > Re* : turbulent flow
Re < Re* : laminar flow

Re*: critical Reynolds number

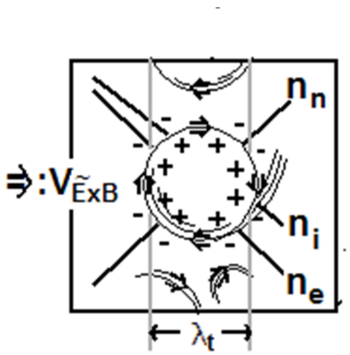
Re* ~ 2000

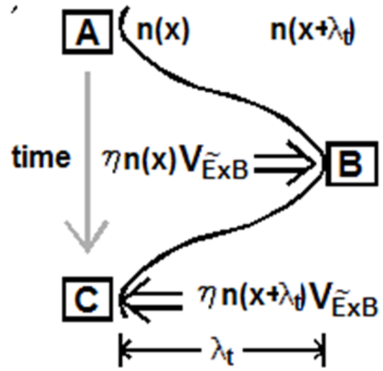
$$\upsilon^* = \frac{1}{1+\theta^2}\left(\frac{E}{B} - \frac{\nabla P_i}{qBn_i} - \upsilon_n\right) + \exp(-\theta)\frac{kT_i \nabla n_n}{qBn_n}$$

Turbulence diffusion coefficient



$$\eta \equiv \frac{\tilde{n}}{n},$$





Turbulence induced diffusion

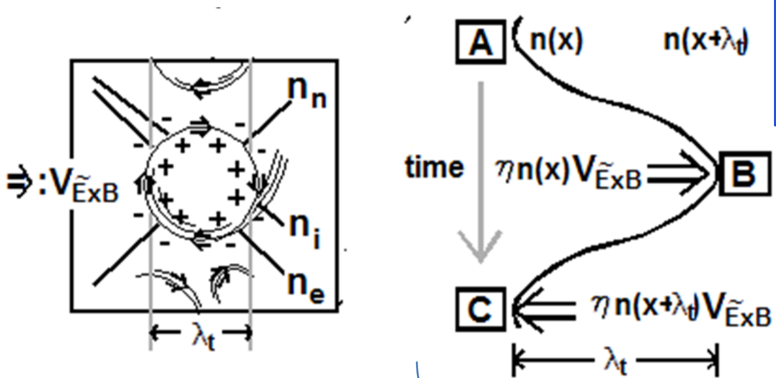
$$\eta \equiv \frac{\tilde{n}}{n}, \quad n' \equiv \frac{\partial n}{\partial x} < 0$$

	x	$x+\lambda_t$
[A]	$n_{i,e}(x) \equiv n_{i,e}$ $\times\eta$	$n_{i,e}(x+\lambda_t) = n_{i,e} + \lambda_t n'_{i,e}$
[B]	$n_{i,e} - \eta n_{i,e}$	$\times\eta$ $n_{i,e} + \lambda_t n'_{i,e} + \eta n_{i,e}$
[C]	$n_{i,e} - \cancel{\eta n_{i,e}} + \cancel{\eta n_{i,e}} + \eta\lambda_t n'_{i,e} + \cancel{\eta^2 n_{i,e}}$ $\approx n_{i,e}(x) + \eta\lambda_t n'_{i,e}$	$n_{i,e} + \lambda_t n'_{i,e} + \cancel{\eta n_{i,e}} - \cancel{\eta n_{i,e}} - \eta\lambda_t n'_{i,e} - \cancel{\eta^2 n_{i,e}}$ $\approx n_{i,e}(x+\lambda_t) - \eta\lambda_t n'_{i,e}$

- ► net movement of one cycle is $\eta\lambda_t \nabla n$: same result from L-R-L and R-L-R cycles
- ► turbulence induced charge diffusion : $-\eta\lambda_t \nabla\rho$

  - ► ion and electron move toward boundary => ***diffusion***
  - ► charge (ρ) moves toward core => ***dilution current =>*** ***Saturation by J^{GCS}***

Turbulence induced diffusion coefficient by GCS theory



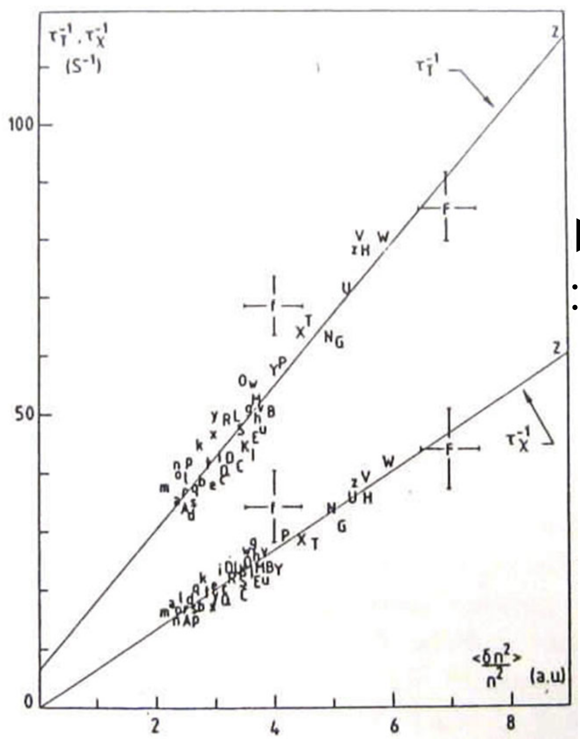
$$D = \frac{2}{\pi}\eta^2\frac{kT_e}{eB} \qquad \eta \equiv \frac{\tilde{n}}{n},$$

$$\Gamma = \partial n \cdot \tilde{\upsilon} \;\rightarrow\; D = \frac{\eta}{\pi}\frac{\tilde{E}\lambda_t}{B} \;\rightarrow\; D = \frac{2}{\pi}\eta^2\frac{kT_e}{eB}$$

∂n: $\eta\lambda_t\nabla n$; $\tilde{\upsilon}$: $\frac{1}{\pi}\frac{\tilde{E}}{B}$

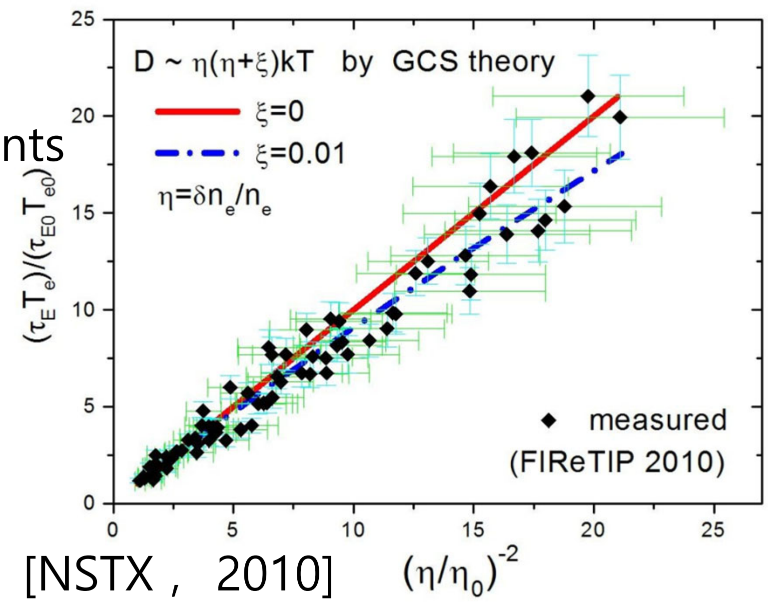
$$\tilde{E}\lambda_t \approx 2\eta\frac{kT_e}{e} \leftarrow \left(\frac{e\tilde{\phi}_t}{kT_e} \approx \frac{\tilde{n}_e}{n_e} : \text{Boltzmann relation}, \ \tilde{\phi}_t \approx \tilde{E}\frac{\lambda_t}{2}\right)$$

[TFR group, 1986]



► proportional to η^2

: agreed by experiments



[NSTX , 2010]

Equilibrium of radial current and Reynolds number

$$J_r^{GCS} = en_i \frac{r_{Li}}{\lambda_{i-n}} \left(\frac{E}{B} - \frac{1}{eB} \frac{\nabla P_i}{n_i} + \frac{kT_i}{eB} \frac{\nabla n_n}{n_n} \right),$$

ambipolar E-field and diffusion:
➡ Ideal case

$$\nabla \cdot J = -\frac{d\rho}{dt}, \quad \rightarrow \quad \nabla E = \frac{\rho}{2\varepsilon_0}$$

turbulence induced counter-current

$$(-\eta \lambda_t \nabla \rho)$$

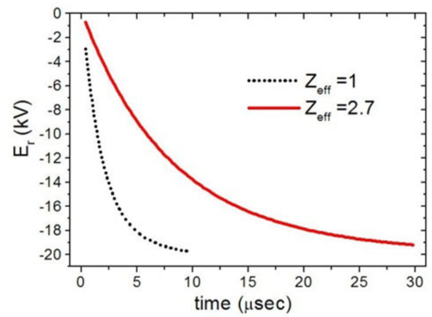
(saturation condition : $J_r^{GCS} = -D\nabla \rho$)

⬇

Reynolds number of ion-neutral collision

$$\mathrm{Re} = \frac{4}{\pi} \eta^2 \frac{\varepsilon_0 B}{m_i n_i (\sigma_{i-n} n_n)^2 \upsilon_i} \nabla^2 E$$

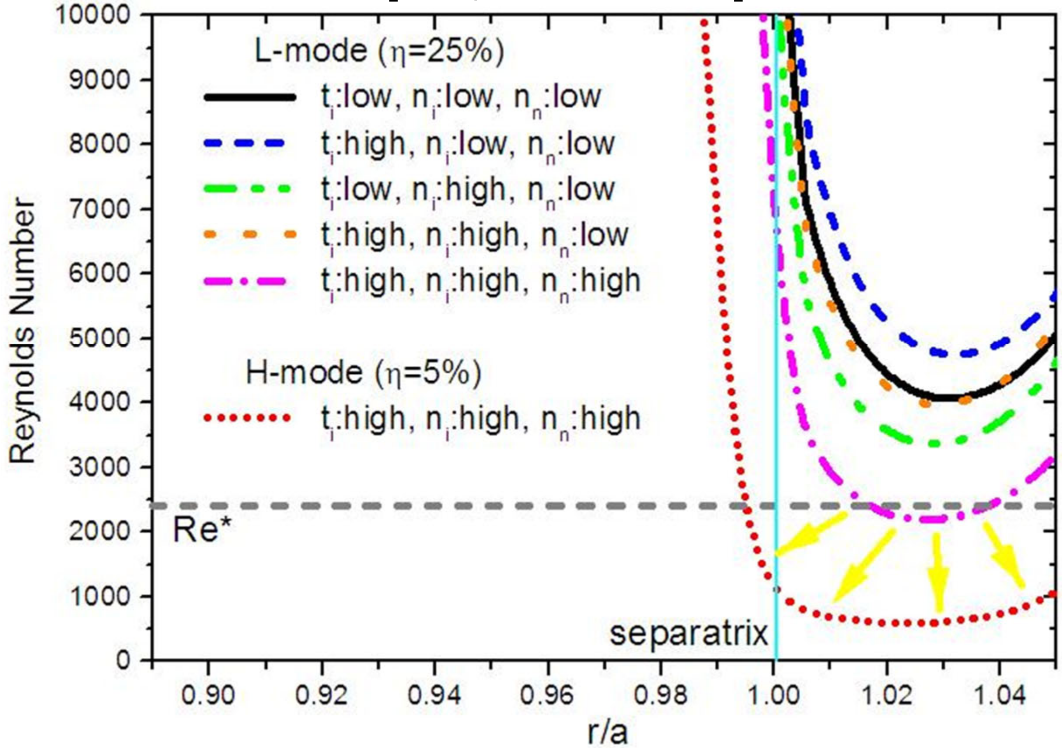
$$\mathrm{Re} = \frac{\lambda_{i-n}}{r_{Li}} \frac{\upsilon^*}{\upsilon_i}, \quad D = \frac{2}{\pi} \eta^2 \frac{kT_e}{eB}$$



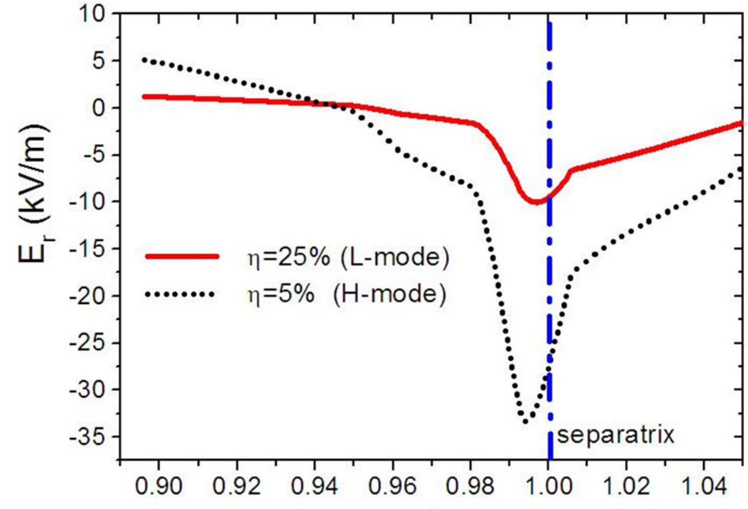
L/H transition of tokamaks

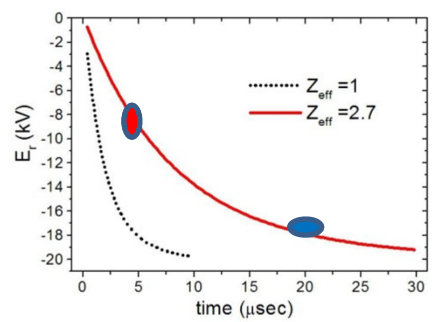
$$\mathrm{Re} = \frac{4}{\pi}\eta^2 \frac{\varepsilon_0 B}{m_i n_i (\sigma_{i-n} n_n)^2 \upsilon_i} \nabla^2 E$$

[Lee, PPCF 2009]



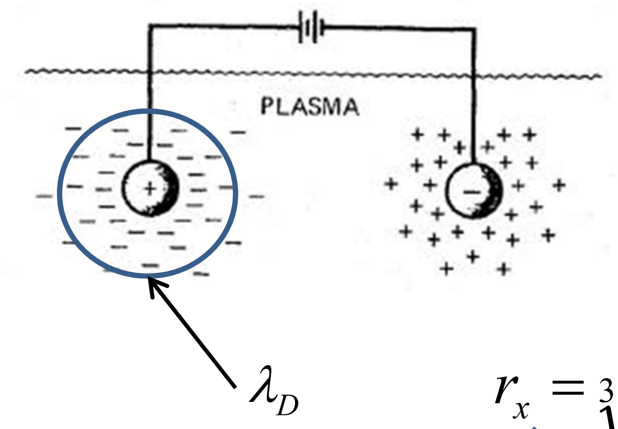
$$J_r^{GCS} = -D\nabla\rho$$





$$J_r^{GCS} = en_i \frac{r_{Li}}{\lambda_{i-n}}\left(\frac{E}{B} - \frac{1}{eB}\frac{\nabla P_i}{n_i} + \frac{kT_i}{eB}\frac{\nabla n_n}{n_n}\right), \qquad D = \frac{2}{\pi}\eta^2 \frac{kT_e}{eB}$$

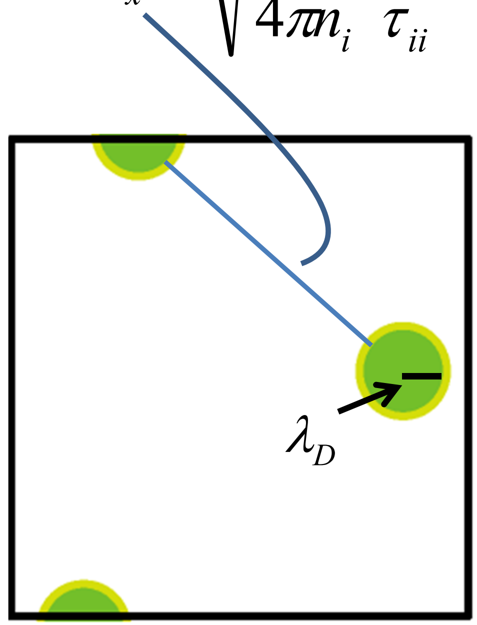
quasi-neutrality is <u>not</u> valid for most charge exchanges



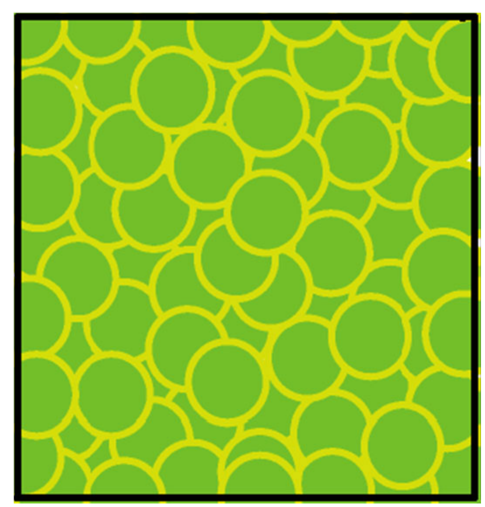
- **electric potential vanishes away out of Debye shielding : (screening effect)**
- **electric potential is effective inside Debye shielding**
- **when charge exchange rate is high enough => all space become field effective**

$$r_x = \sqrt[3]{\frac{3}{4\pi n_i}\frac{\tau_{cx}}{\tau_{ii}}}$$

: radius in which one charge exchange occurs in average

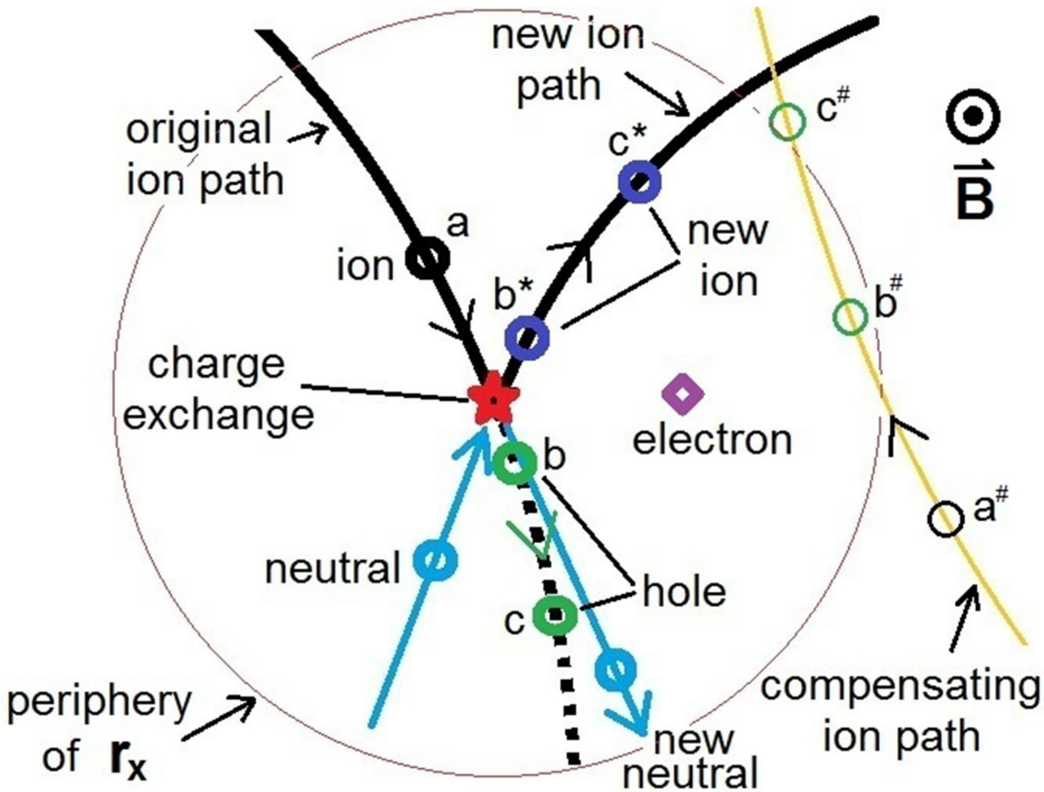


$\lambda_D / r_x < 1$



$\lambda_D / r_x > 1$

many test charges in a Debye shielding



ion-ion collisions make redistribution of ions and Debye shielding disappeared

$$\tau_{cx} > \tau_{ii}$$

$\frac{\tau_{cx}}{\tau_{ii}} =$ number of ions required to have one point charg on average among them

$$\frac{4\pi}{3} r_x^3 n_i = \tau_{cx}/\tau_{ii}$$

$$r_x = \sqrt[3]{\frac{3}{4\pi n_i}\frac{\tau_{cx}}{\tau_{ii}}}$$

$$\frac{\lambda_D}{r_x} = \frac{2^{4/3}\pi^{5/6}\varepsilon_0^{7/6}T_e^{1/2}T_i^{2/3}\sigma_{cx}^{1/3}n_n^{1/3}}{e^{7/3}(\ln\Lambda)^{1/3}n_e^{1/3}} = (5 \sim 50)$$

polarization drift

$$E(t) = E_0 e^{i\omega t}$$

when $\omega << \omega_c$

$$\upsilon_p^{i,e} = \frac{m_{i,e}}{qB^2}\frac{\partial E}{\partial t}$$

$$\frac{\upsilon_{np}}{\upsilon_p^i} = \frac{\varepsilon_0 B^2}{m_i n_i} = \frac{\upsilon_A^2}{c^2} < 1/1000$$

=> quasi-neutrality

$$J^{GCS} = en_i\upsilon_i^{GCS},\ \nabla\cdot J = -\frac{d\rho}{dt},\quad \nabla E = \frac{\rho}{2\varepsilon_0}$$

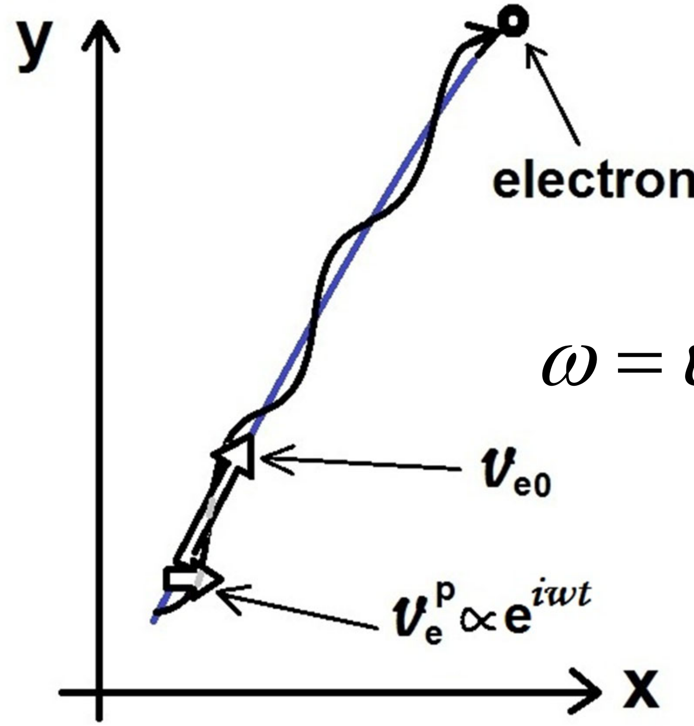
$$\upsilon_i^{GCS} = -(\varepsilon_0/en_i)\partial E/\partial t$$

But

$\omega >> \omega_c$

$$\upsilon_e^p = \frac{e}{m_e\omega^2}\frac{\partial E}{\partial t}$$

$$\frac{\varepsilon_0}{en_i}\frac{\partial E}{\partial t}\Bigg/\frac{e}{m_e}\frac{\partial E}{\partial t}\frac{r_x^2}{\upsilon_e^2} = \frac{\lambda_D^2}{r_x^2}$$



$$\omega = \upsilon_e / r_x$$

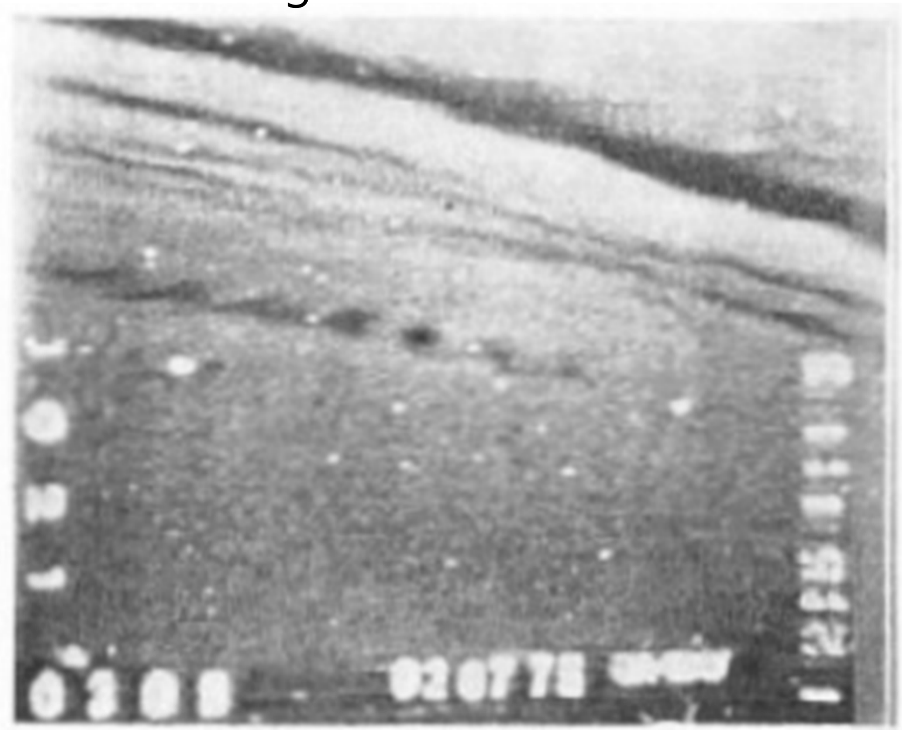
parameters of typical plasmas in which violation of quasi-neutrality takes place

	tokamak boundary	arc discharge	earth ionosphere
ion/neutral species	D^+/D	Ar^+/Ar	O_2^+/O_2
n_e/n_i (/m^3)	$10^{19}/10^{19}$	$10^{19}/10^{19}$	$1.7\times10^{11}/7\times10^{10}$
n_n (/m^3)	10^{16}	5×10^{20}	1.1×10^{17}
T_i/T_e (eV)	100/100	0.5/0.5	0.025/0.17
σ_{cx} (m^2)	2.5×10^{-19}	6.2×10^{-19}	1.0×10^{-18}
B (T)	1.2	7.5×10^{-2}	3×10^{-5}
ω_{ce} (/sec)	2.1×10^{11}	1.3×10^{10}	5.3×10^{6}
$\omega=\upsilon_e/r_x$ (/sec)	1.0×10^{13}	1.4×10^{12}	2.0×10^{9}
τ_{cx}/τ_{ii}	16	2.1	1
$\lambda_D=\sqrt{\varepsilon_0 T_e/e^2 n_e}$ (m)	2.3×10^{-5}	1.7×10^{-6}	7.4×10^{-3}
λ_D/r_x	**32**	**4.6**	**49**
υ_i^{GCS} (m/sec)	1.7×10^{-2}	1.4×10^{-3}	43
$\upsilon_e\tau_{ii}/3\upsilon_i^{GCS}\tau_{cx}$	9.2×10^{6}	5.9×10^{7}	2.3×10^{3}

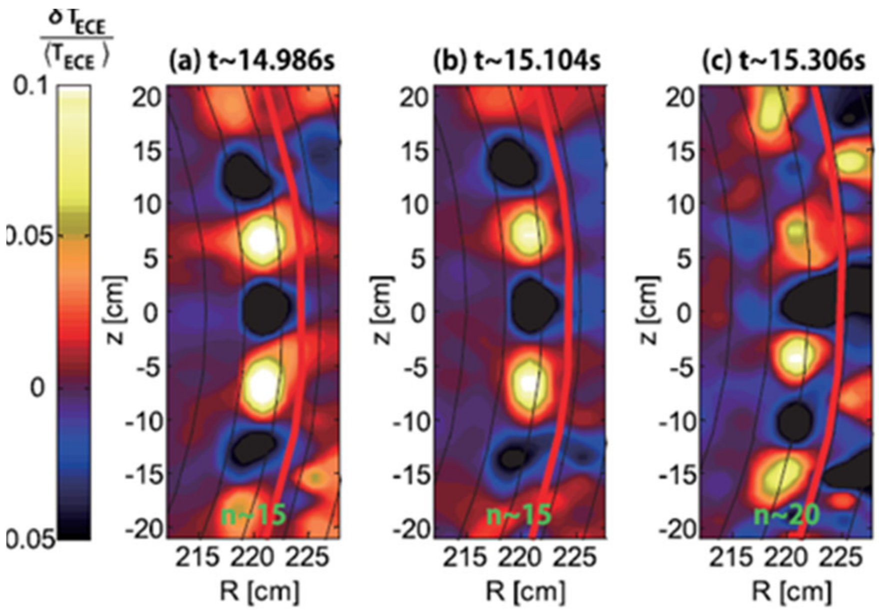
[K.C. Lee, JKPS (2013)]

two common features of black aurora and tokamak edge : 1. strong E-field, 2. breaking into circular structure

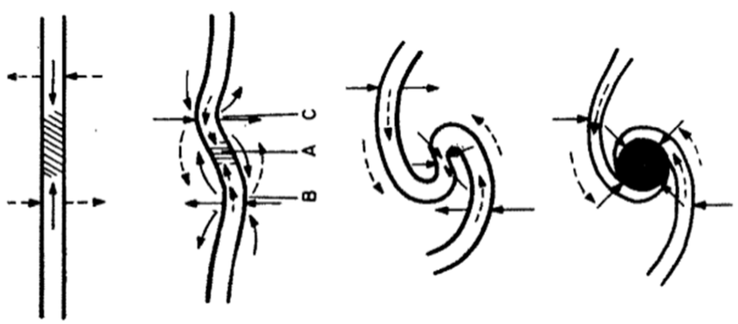
TV image of black aurora



ECEI measurement on KSTAR ELMs



[J.H. Lee, PRL 2016]



arc distortions of black aurora by ExB

[Hallinan and Davis, Planet. Space Sci. 1970]

A future work:
What is the role of strong E_r for the ELM triggering?

Conclusions

1. Gyro-Center Shift current is induced by ion-neutral collisions

2. E-field formations in tokamaks, arc discharge , EEJ and black aurora are explained by GCS current

3. Original Bohm experiment is explained by J_r^{GCS} and short circuit effect

4. turbulence-laminar transition by ion-neutral friction
   ⇨ L-H transition in tokamak

5. turbulence induced diffusion is analyzed

8. Criterion of quasi-neutrality validity for charge exchange is discussed