Theory Seminar (Feb. 2019)

Finite Larmor Radius effects at the H-Mode pedestal and the related Force-Free State

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Abstract

We will first relate our previous calculations on the radial electric field at the H-mode pedestal with the actual magnetic fusion experimental measurements. We will then discuss the new pressure balance due to the E × B current, which is induced by the resulting radial electric field, and its impact on the gyrokinetic MHD equations as well as their relationship to the force-free steady state.

Basic references:

"Nonlinear, three-dimensional magnetohydrodynamics of noncircular tokamaks," H. R. Strauss, Phys. Fluids **19**, 134 (1976).

"Dynamics of high β tokamaks," H. R. Strauss, Phys. Fluids **20**, 1354 (1977).

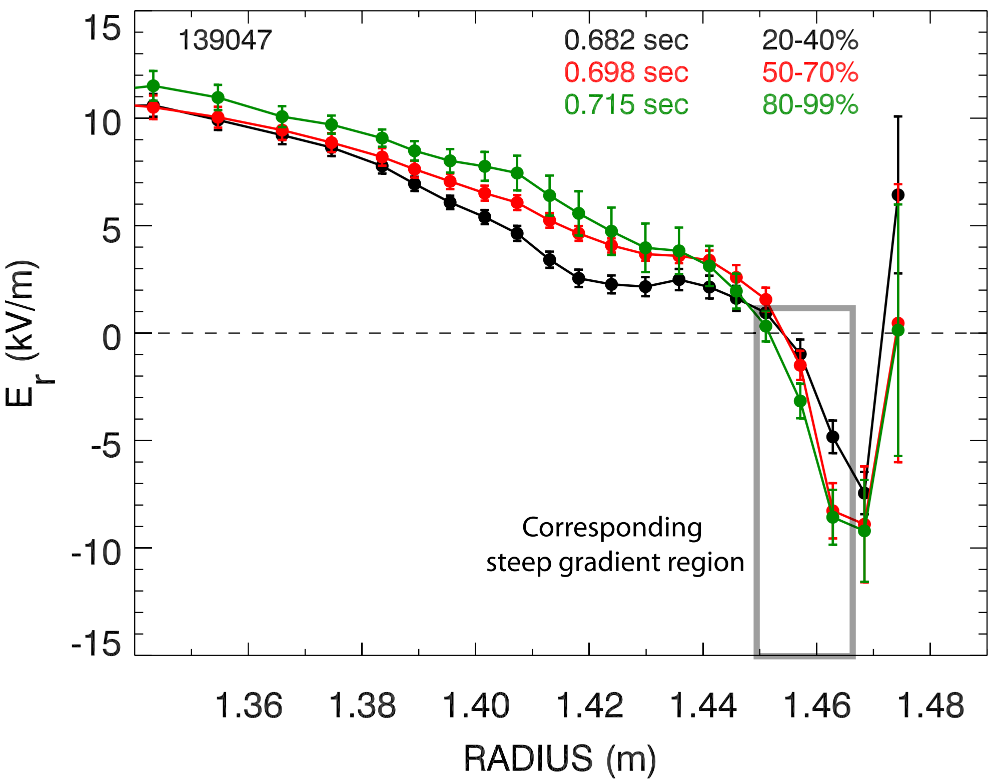
"Gyrokinetic Approach in Particle Simulation," W. W. Lee, Phys. Fluids **26,** 556 (1983).

"MHD for collisionless plasmas from the gyrokinetic perspective," W. W. Lee, Phys. Plasmas **23**, 070705 (2016).

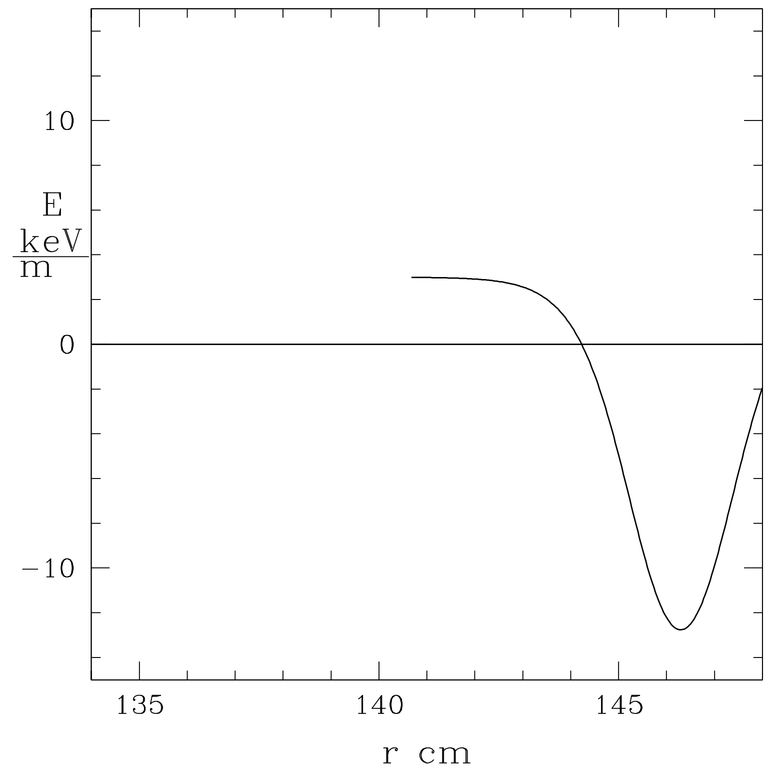
"Equilibrium potential well due to finite Larmor radius effects at the tokamak edge," W. W. Lee, R. B. White, Phys. Plasmas **24**, 081204 (2017).

NSTX Discharge and Theoretical Comparison

Radial electric field at the H-mode pedestal is believed to be caused by the charge separation resulting from the gyroradius differences between the electrons and the ions in the presence of sharp pressure gradients



A. Diallo *et al.*, Nucl. Fusion, **53**, 1 (2013)



Lee and White, Phys. Plasmas **24**, 081204 (2017)

Basis for Experimental Measurements
[K. Ida, PPCF 40, 1429 (1998)]

- Force Balance Equation: $$n_i e \mathbf{E} = \nabla p_i - n_i e \frac{\mathbf{V}_i \times \mathbf{B}}{c}$$
- Radial Direction: $$n_i e E_r \approx \frac{\partial p_i}{\partial r} - \frac{n_i e}{c}(V_{\theta i} B_T)$$
- Poloidal Drifts:

$$V_{\theta i} = V_{di} + V_{\perp i}$$

- Pressure balance

$$\frac{\partial p_i}{\partial r} = \frac{n_i e}{c}(V_{di} B_T)$$

- Radial Electric Field

$$E_r \approx -\frac{1}{c}(V_{\perp i} B_T)$$

Question: what are the different drifts active at the pedestal?

$$\boxed{V_{\perp i} = V_{\perp i}^{E\times B} + V_{\perp i}^{\nabla B} + V_{\perp i}^{\nabla\times B}}$$

[i.e., Lee and Qin, PoP '03]

$$V_{\perp i}^{\nabla B} = \frac{T_i c}{eB_T}\mathbf{b}\times\nabla ln B_T \qquad V_{\perp i}^{\nabla\times B} = \frac{T_i c}{eB_T}(\nabla\times\mathbf{b})_\perp$$

$$V_{\perp i}^{E\times B} = -cE_r/B_T$$

$$\boxed{E_r = \frac{1}{2n_i e}\frac{\partial p_i}{\partial r}}$$

[Lee and White, PoP '17 & '18]

$$V_{\perp i}^{E\times B} = -\frac{1}{2}\frac{c}{eB_T}\underline{\frac{1}{n_i}\frac{\partial p_i}{\partial r}} \quad \propto \quad \underline{\frac{\partial T_i}{\partial r} + \frac{T_i}{n_i}\frac{\partial n_i}{\partial r}}$$

- Answer: since T_i is small and magnetic gradients are small, it is most likely the dominant mechanism at the pedestal is

$$\boxed{V_{\perp i}^{E\times B}}$$

• Since the L and H modes have the same magnetic configuration with the same magnetic drifts at the edge region, the difference is in the existence of the E x B drift motion for the H-mode.

• The E x B drift motion for the H-mode comes from the equilibrium zonal flow due to FLR effects in the region of sharp pressure gradient, but is not related to the turbulence.

• The E x B drift motion apparently facilitates the plasmas evolving into an optimum confinement configuration.

This is why!

Gyrokinetic Quasineutrality

[Lee and Kolesnikov, PoP **16**, 044506 (2009); Lee, PoP **23**, 070705 (2016); Lee and White, PoP **24**, 081204 (2017); Lee and White, PoP **25**, 054701 (2018).]

$$\bar{n}(\mathbf{x}) = \int \left(1 + \frac{1}{4}\frac{v_\perp^2}{\Omega^2}\nabla_\perp^2\right) F_{gc}(\mathbf{R}) dv_\| d\mu \qquad \frac{n_i|_{particle}}{n_i} = \underline{1 + \frac{1}{2}\rho_i^2 \frac{1}{p_i}\nabla_\perp^2 p_i}$$

related to gyroviscosity in 2 fluid mom eqts [Scott, 2007]

$$\boxed{\mathbf{v}_{E\times B} \approx -\frac{1}{2}\hat{\mathbf{b}} \times \frac{\nabla_\perp p_i}{p_i}\frac{cT_i}{eB}}$$

-- Equilibrium Zonal Flow

$$\mathbf{J}_\perp^{E\times B}(\mathbf{x}) = \sum_\alpha q_\alpha \langle \int \mathbf{v}_{E\times B}(\mathbf{R}) F_\alpha(\mathbf{R}) \delta(\mathbf{R} - \mathbf{x} + \rho) d\mathbf{R} d\mu dv_\| \rangle_\varphi$$

$$\mathbf{J}_\perp = \frac{c}{B}\hat{\mathbf{b}} \times \nabla p + e n_i \frac{\rho_i^2}{2}\left[\nabla_\perp^2 \mathbf{v}_{E\times B} + \frac{\mathbf{v}_{E\times B}}{p_i}\nabla_\perp^2 p_i\right]$$

-- Difference in gyroradius effects between ions and electrons

$$\boxed{\mathbf{J}_\perp \approx \frac{c}{B}\hat{\mathbf{b}} \times (\nabla p)\left[1 - \frac{1}{2}\rho_i^2 \frac{\nabla_\perp^2 p}{p}\right]}$$

-- FLR modification of pressure balance

Gyrokinetic MHD Equations

[Lee, PoP **23**, 070705 (2016); Lee, Hudson and Ma, PoP **24**, 124508 (2017)]

Vorticity Equation: $$\frac{d}{dt}\nabla_\perp^2\phi - 4\pi\frac{v_A^2}{c^2}\nabla\cdot(\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi\times\mathbf{b}\cdot\nabla$$

Ohm's Law: $$E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b}\cdot\nabla\phi \approx -\frac{1}{en_e}\frac{\partial p_{\parallel e}}{\partial x_\parallel} + \eta J_\parallel,$$

Equations of State: $$\frac{dp_{\parallel e}}{dt} = 2E_\parallel J_\parallel \qquad \frac{dp_\perp}{dt} = 0 \qquad p = nT$$

Energy Conservation: $$\frac{\partial}{\partial t}\int\frac{1}{8\pi}\left(|\nabla_\perp\phi|^2 + \frac{v_A^2}{c^2}|\nabla A_\parallel|^2\right)d\mathbf{x} = -\frac{v_A^2}{c^2}\int\mathbf{E}_\perp\cdot\mathbf{J}_\perp d\mathbf{x},$$

Ampere's Law: $$\nabla^2 A_\parallel = -\frac{4\pi}{c}J_\parallel$$

MHD equilibrium: $$\nabla\cdot(\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0$$

What happens when $\mathbf{J}_\perp \approx 0$?

- Vorticity Equation: $\frac{d}{dt}\nabla_\perp^2\phi + 4\pi\frac{v_A^2}{c}\frac{\partial}{\partial x_\parallel}(\nabla^2 A_\parallel) = 0$
- Collisonless Ohm's Law: $E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \frac{\partial \phi}{\partial x_\parallel} = 0$

MHD stable for q > 1 [H. Strauss PoP ('76)], unstable for q > 1 when $\mathbf{J}_\perp \neq 0$ [H. Strauss PoP ('77)]

- Normal Modes: $\omega^2 = k_\parallel v_A^2$ — Alfven waves, weakly damped kinetically
- Modified Pressure balance: $(\nabla_\perp p_i)\left(1 - \frac{1}{2}\rho_i^2\frac{\nabla_\perp^2 p_i}{p_i}\right) \approx 0,$
- H-mode like pressure profile: $\nabla_\perp p_i \approx 0$ at the core; $p_i \propto exp(-\sqrt{2}r/\rho_i)$ at the edge
- Energy conservation: $\boxed{\frac{\partial}{\partial t}\int \frac{1}{8\pi}\left(|\nabla_\perp\phi|^2 + \frac{v_A^2}{c^2}|\nabla A_\parallel|^2\right) d\mathbf{x} \approx 0}$
- Force-free Steady State: $\boxed{\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}_\parallel}$ as a part of vorticity equation
- Woltjer/Taylor Equilibrium State: $\nabla \times \mathbf{B} = \mu\mathbf{B}$
- Beltrami Equilibrium State: $\nabla \times \mathbf{B} = 0$

Proceedings of the

NATIONAL ACADEMY OF SCIENCES

Volume 44 · Number 6 · June 15, 1958

A THEOREM ON FORCE-FREE MAGNETIC FIELDS

BY L. WOLTJER

UNIVERSITY OF CHICAGO

Communicated by S. Chandrasekhar, May 5, 1958

Chandrasekhar and Woltjer[1] have shown that the force-free magnetic fields with "constant α" can be related to a variational principle. Here α is the scalar function in the equation

$$\text{curl } \boldsymbol{H} = \alpha \boldsymbol{H}, \tag{1}$$

which characterizes the force-free fields. This equation follows from the definition of force-free magnetic fields as fields in which the Lorentz force vanishes.

Note that the fact that α is a constant in this equilibrium case is to satisfy the condition that $\nabla \cdot \mathbf{H} = 0$, which is not needed for our dynamic cases.

Energy exchange dynamics across L–H transitions in NSTX

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Nucl. Fusion **57** (2017) 066050 A. Diallo *et al*

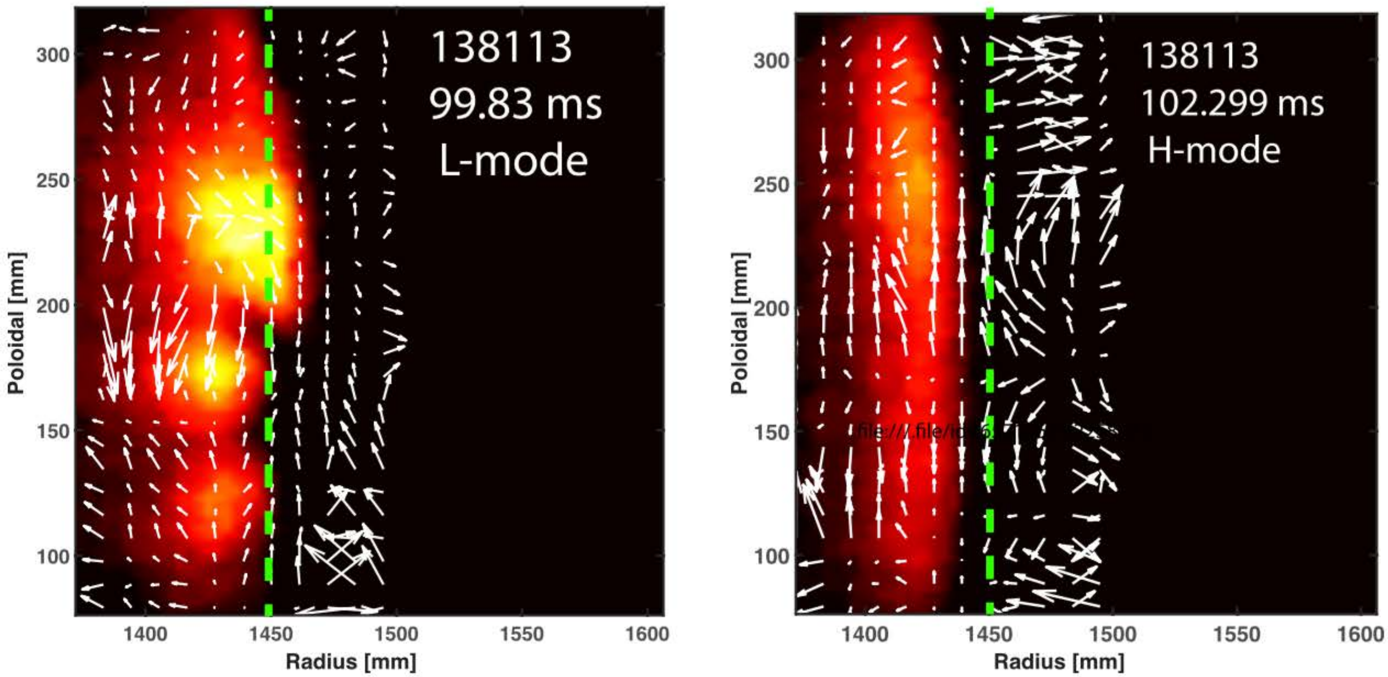
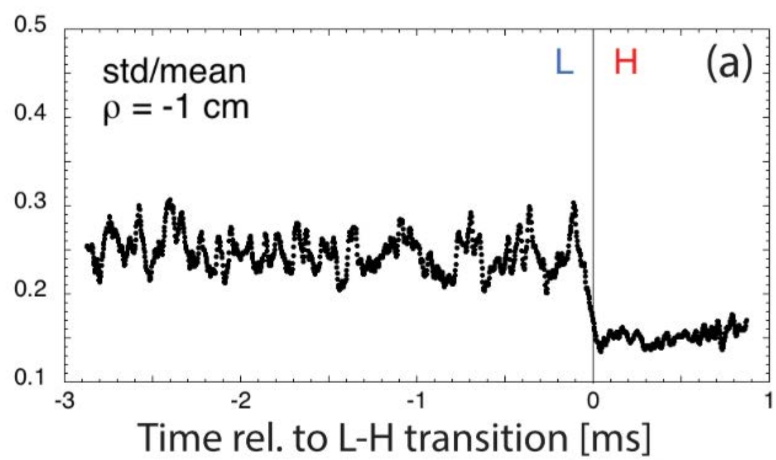


Figure 5. Examples of the GPI intensity images where the arrows represent the velocity vectors. The vertical green dotted line indicates the separatrix location (with ±1 cm uncertainty).



PHYSICS OF PLASMAS **23**, 042507 (2016)

Integrated fusion simulation with self-consistent core-pedestal coupling

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L. L. Lao,[1] J. M. Park,[2] D. L. Green,[2] W. Elwasif,[2] B. A. Grierson,[3] and C. Holland[4]
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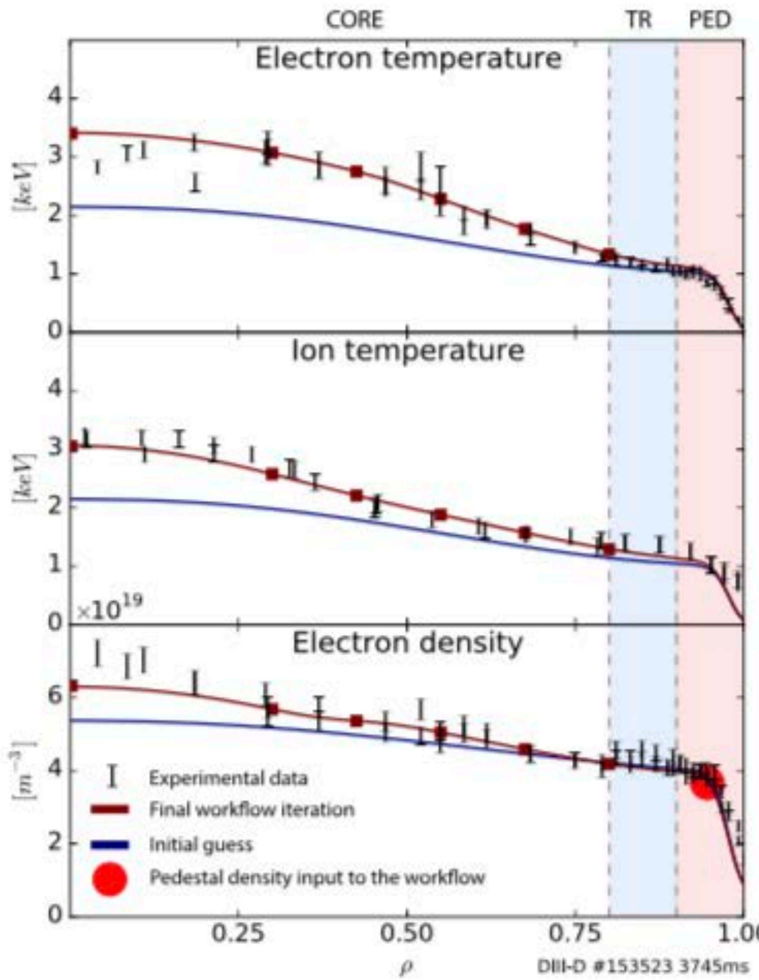

FIG. 4. Evolution of the simulated kinetic profiles for a DIII-D discharge and comparison with the experimental measurements. The profiles from the initial run of EPED are shown by the dark blue curves. The squares in these curves indicate the radii at which TGYRO performed the flux-matching calculations. The final step of the simulated profiles (dark red) is compared with experimental measurements, showing agreement across the entire plasma radius.

Plasma Phys. Control. Fusion 57 (2015) 014038 (12pp)

doi:10.1088/0741-3335/57/1/014038

The H-mode density limit in the full tungsten ASDEX Upgrade tokamak

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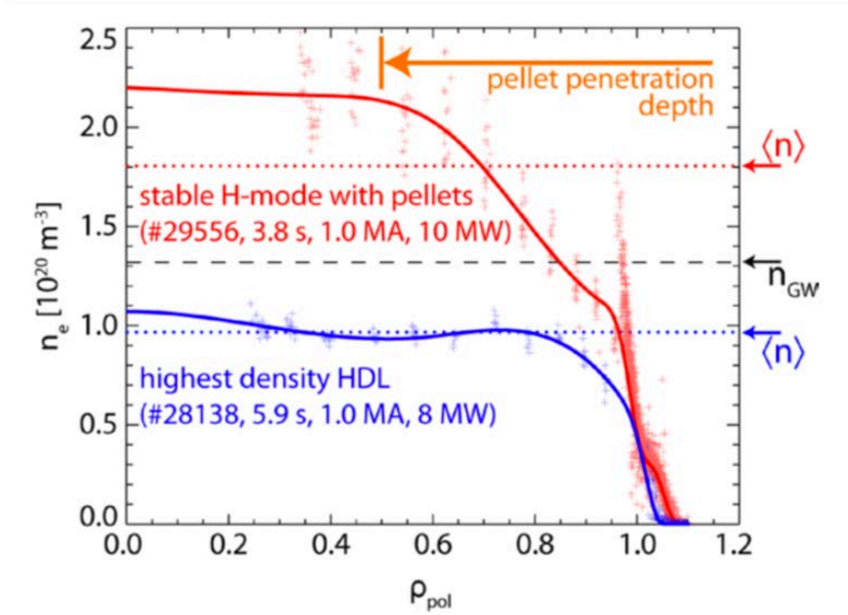

Figure 18. Density profiles of the HDL discharge with the highest density and a pellet fueled H-mode discharge. The central line averaged densities (H-1, dotted) and Greenwald density (dashed) are indicated.

IOP PUBLISHING PLASMA PHYSICS AND CONTROLLED FUSION

Plasma Phys. Control. Fusion **54** (2012) 085018 (13pp) doi:10.1088/0741-3335/54/8/085018

Understanding the core density profile in TCV H-mode plasmas

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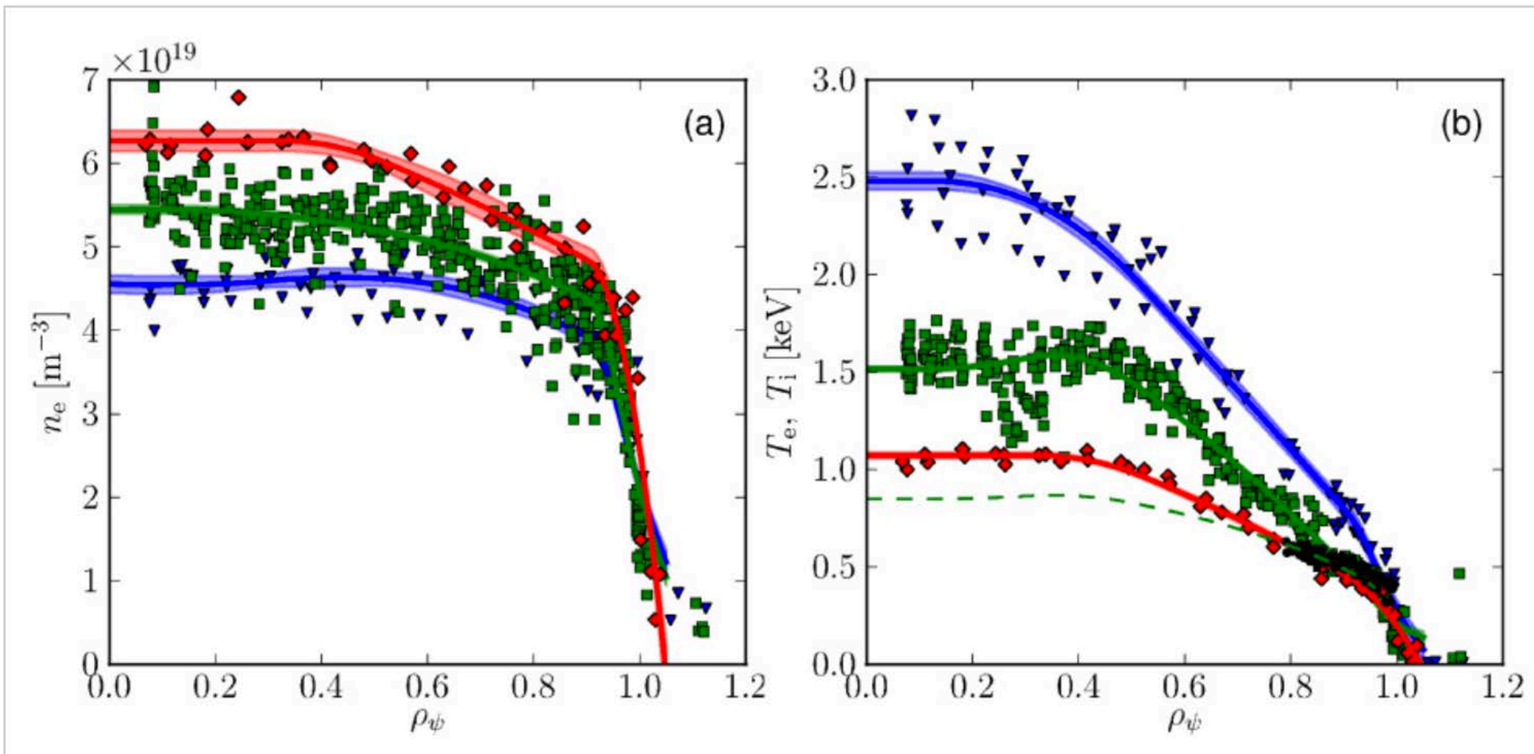

Conclusion

The H-modes are related to the spontaneous relaxation of magnetically confined plasmas to a nearly force-free steady state as described by time-dependent GK MHD equations, where

$$\boxed{\begin{gathered}\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}_{\parallel} \\ \mathbf{J}_{\perp} \approx 0\end{gathered}}$$

is a part of the vorticity equation.