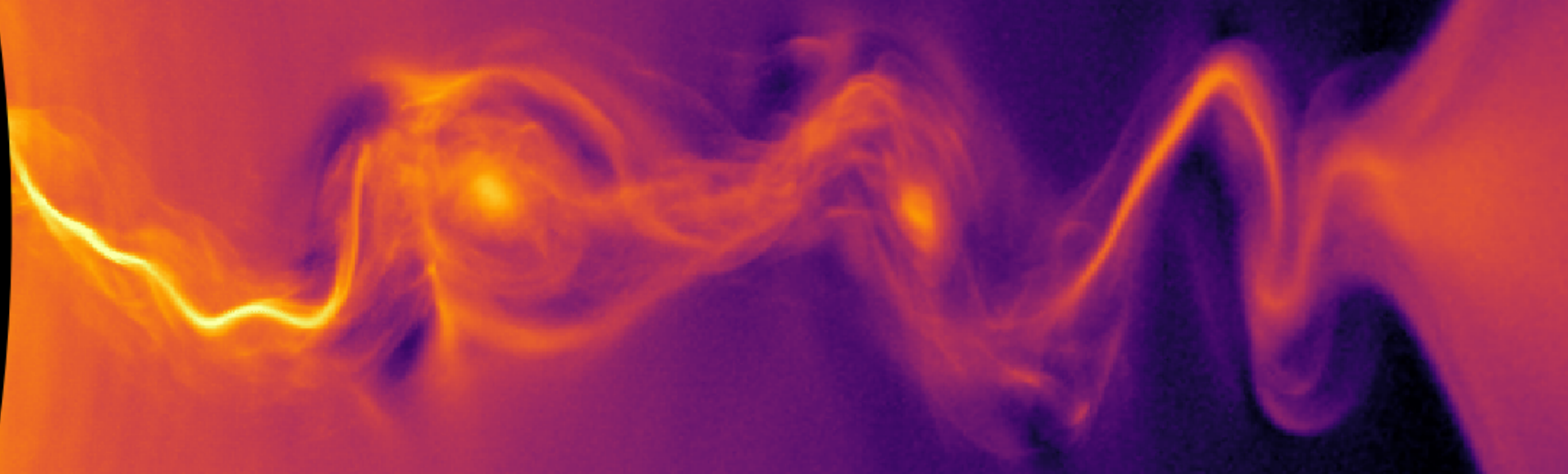


Simulating Relativistic Astroplasmas

from Microphysics to Global Dynamics

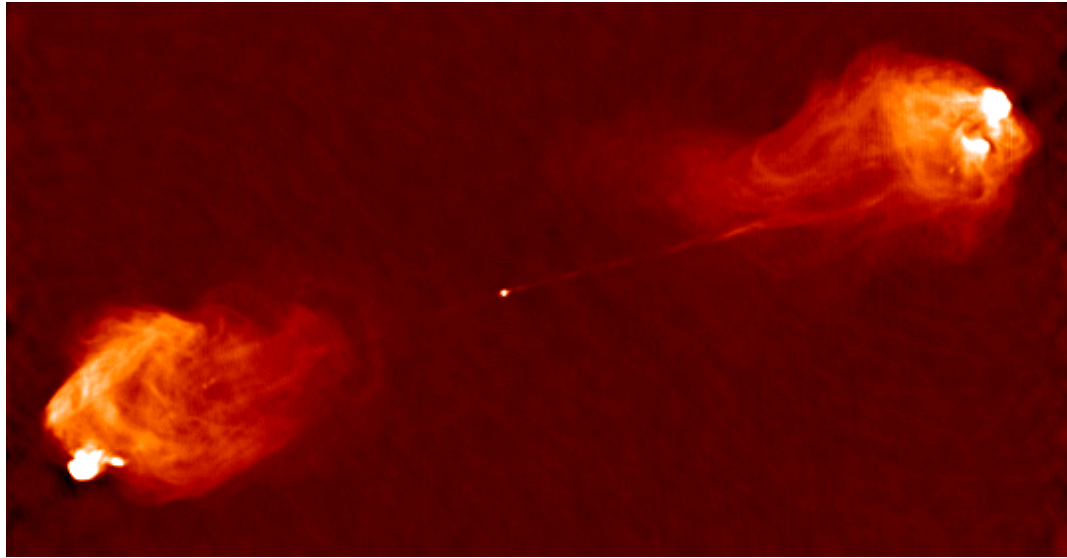


Kyle Parfrey

NASA Goddard Space Flight Center

Plasmas around black holes & neutron stars

VLA radio data



relativistic jets

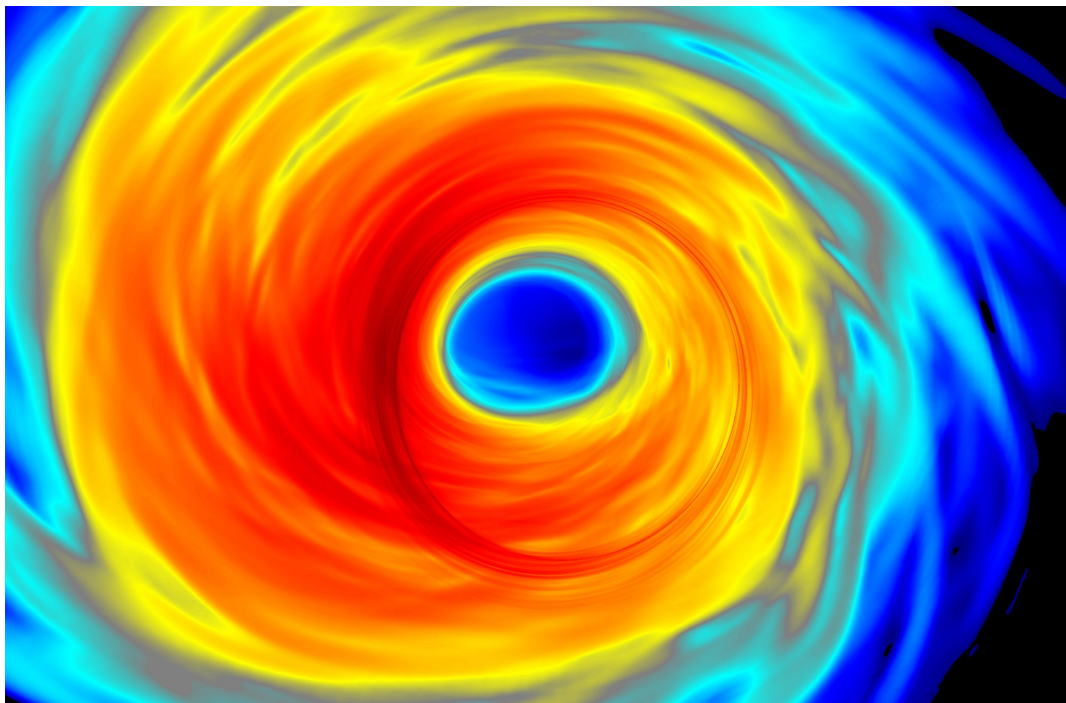
artist's conception (NASA)



magnetars

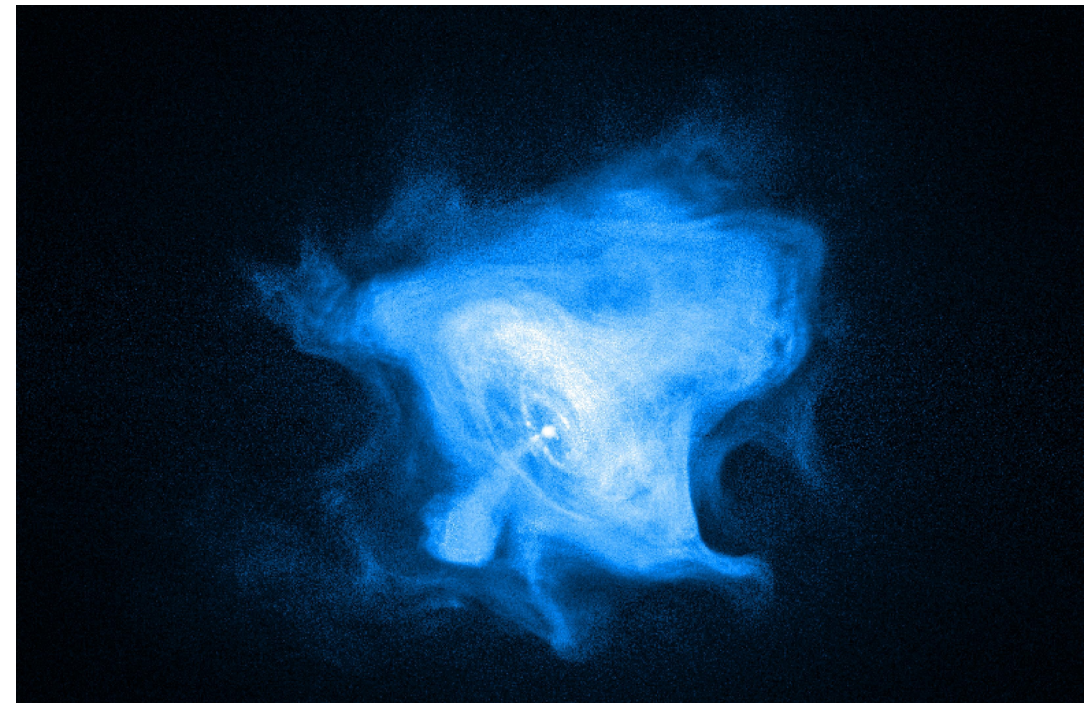
nature's most
extreme & exotic
environments

accretion discs



GRMHD simulation + GR ray tracing

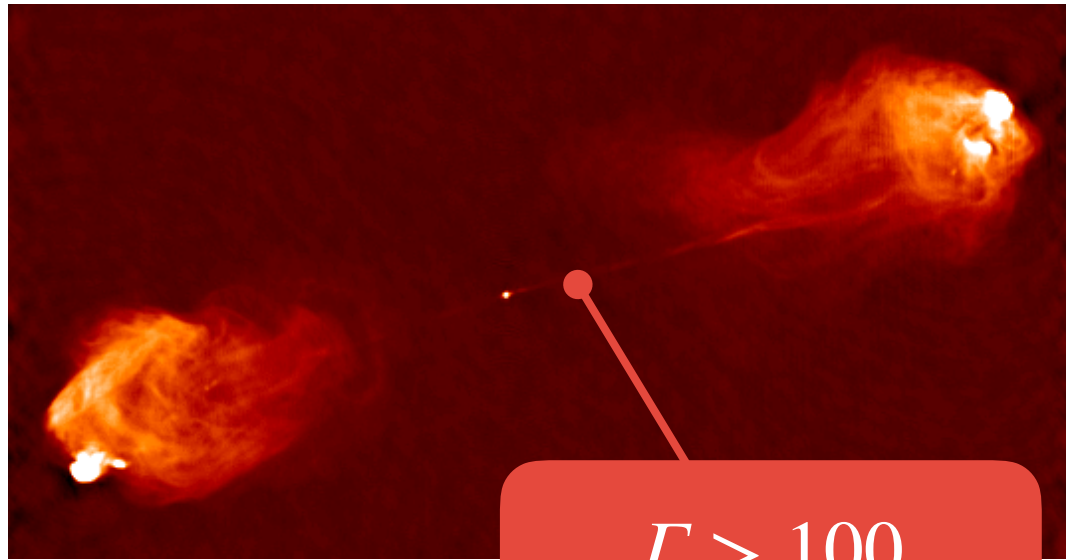
pulsars



Chandra X-ray data

Plasmas around black holes & neutron stars

VLA radio data



$$\Gamma > 100$$

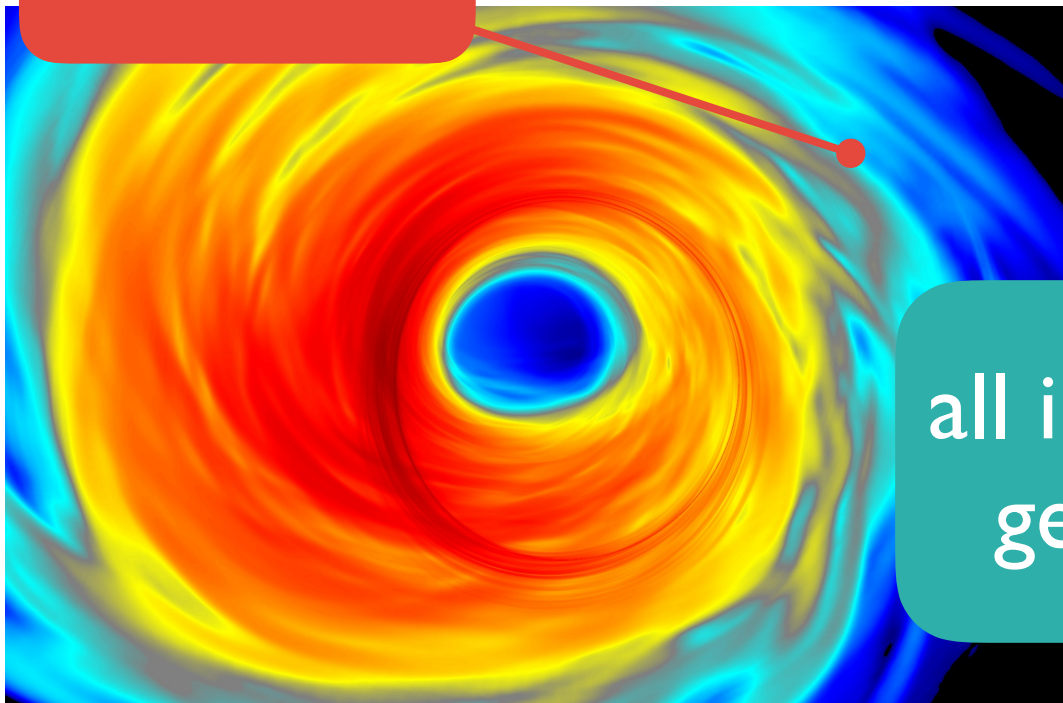
artist's conception (NASA)



$$B > 10^{14} \text{ G}$$

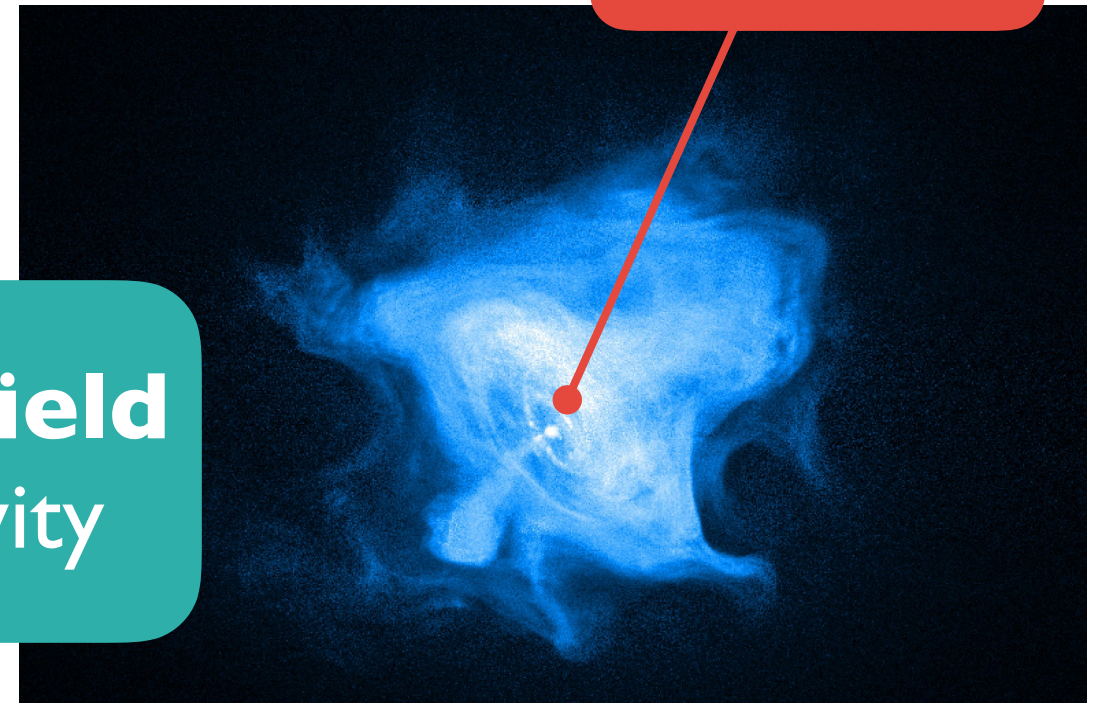
nature's most
extreme & exotic
environments

$$T > 10^9 \text{ K}$$



all in **strong-field**
general relativity

$$P \sim 1 \text{ ms}$$

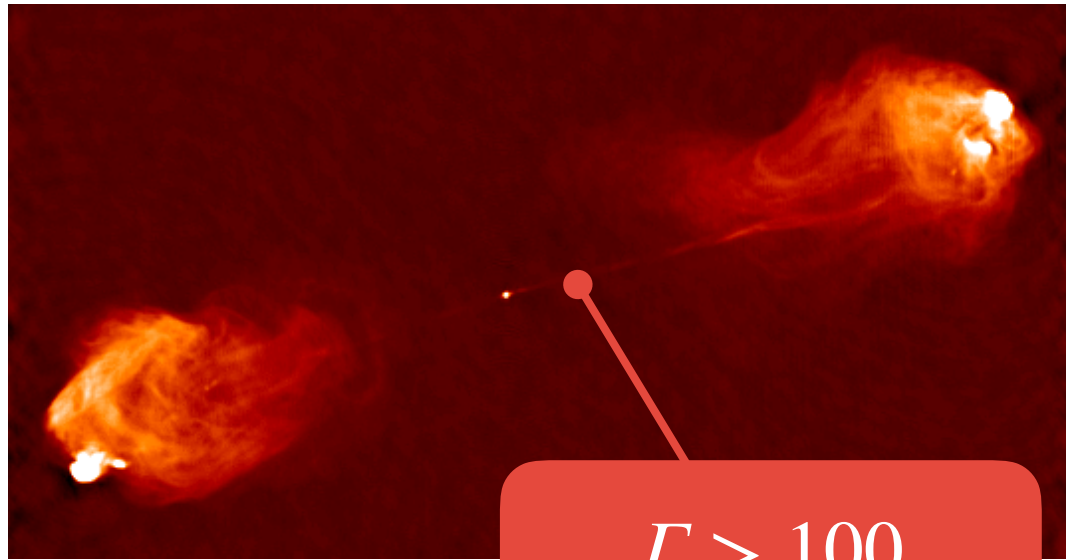


GRMHD simulation + GR ray tracing

Chandra X-ray data

Plasmas around black holes & neutron stars

VLA radio data



$$\Gamma > 100$$

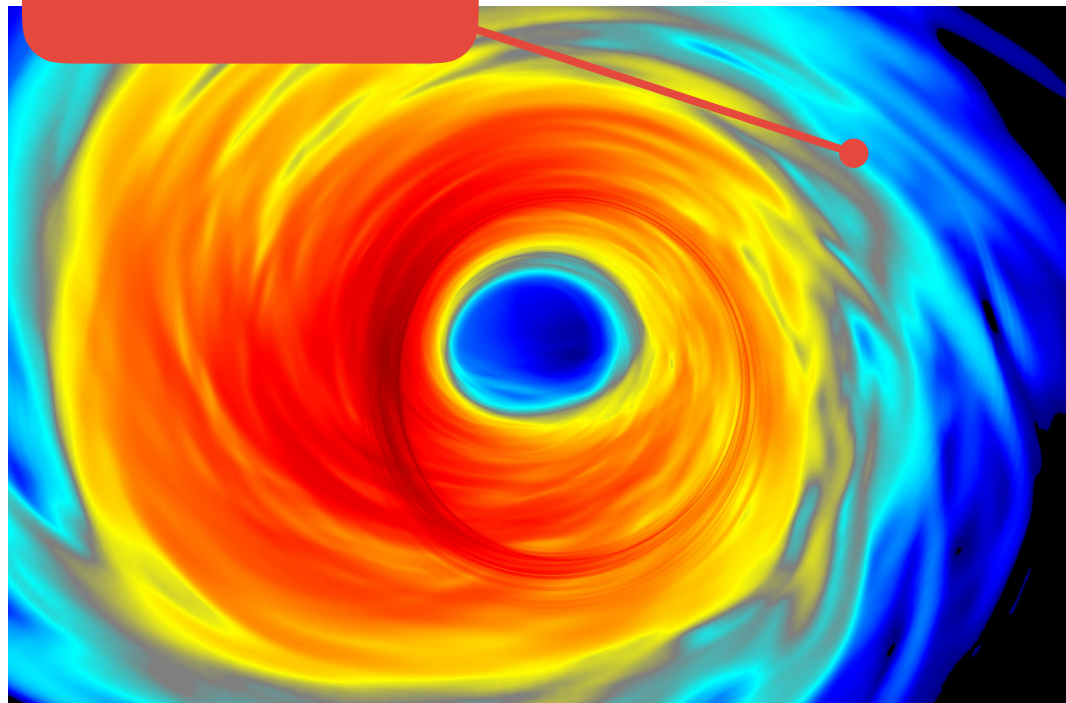
artist's conception (NASA)



$$B > 10^{14} \text{ G}$$

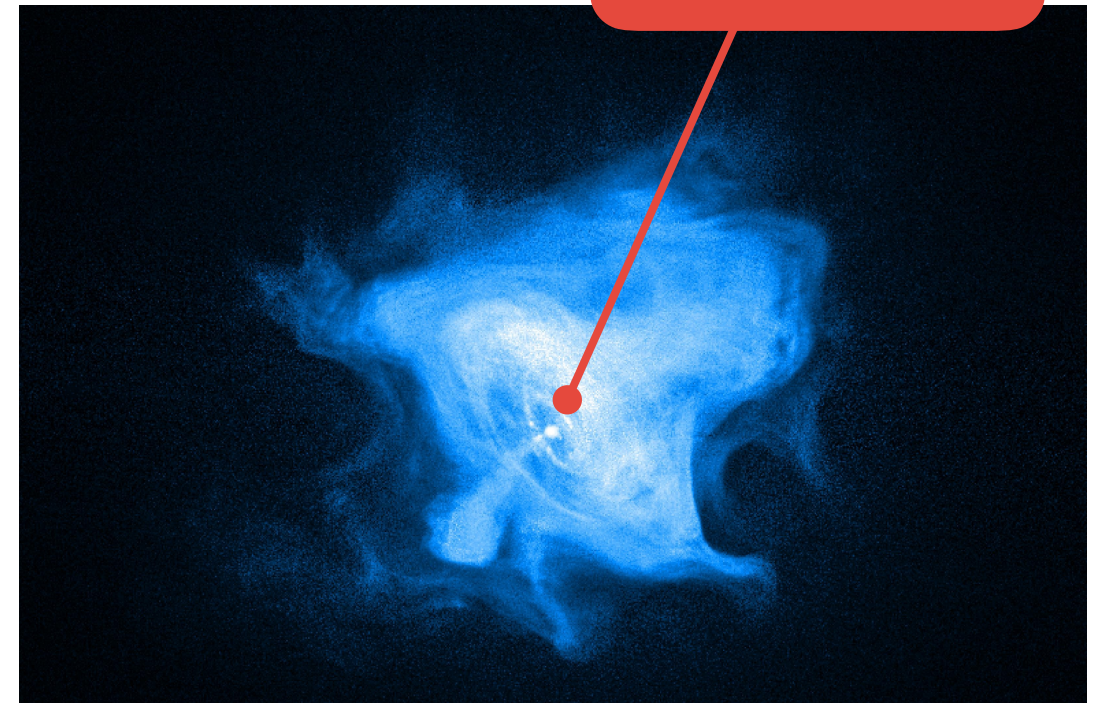
conditions place
new demands on
computational methods

$$T > 10^9 \text{ K}$$



GRMHD simulation + GR ray tracing

$$P \sim 1 \text{ ms}$$



Chandra X-ray data

I — Black-hole plasmas from first principles



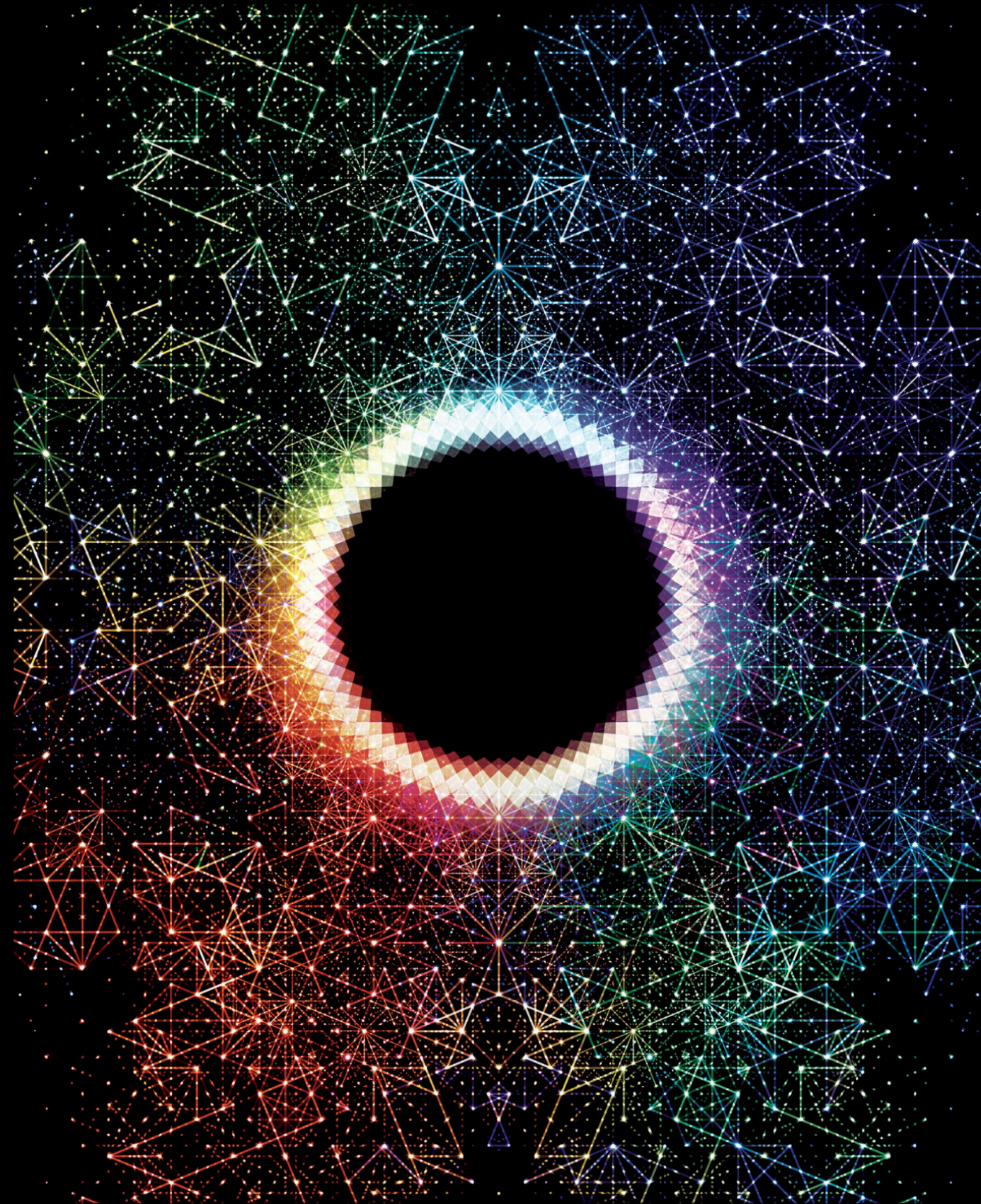
The New York Times Magazine

Account ▾

How Do You Take a Picture of a Black Hole? With a Telescope as Big as the Earth

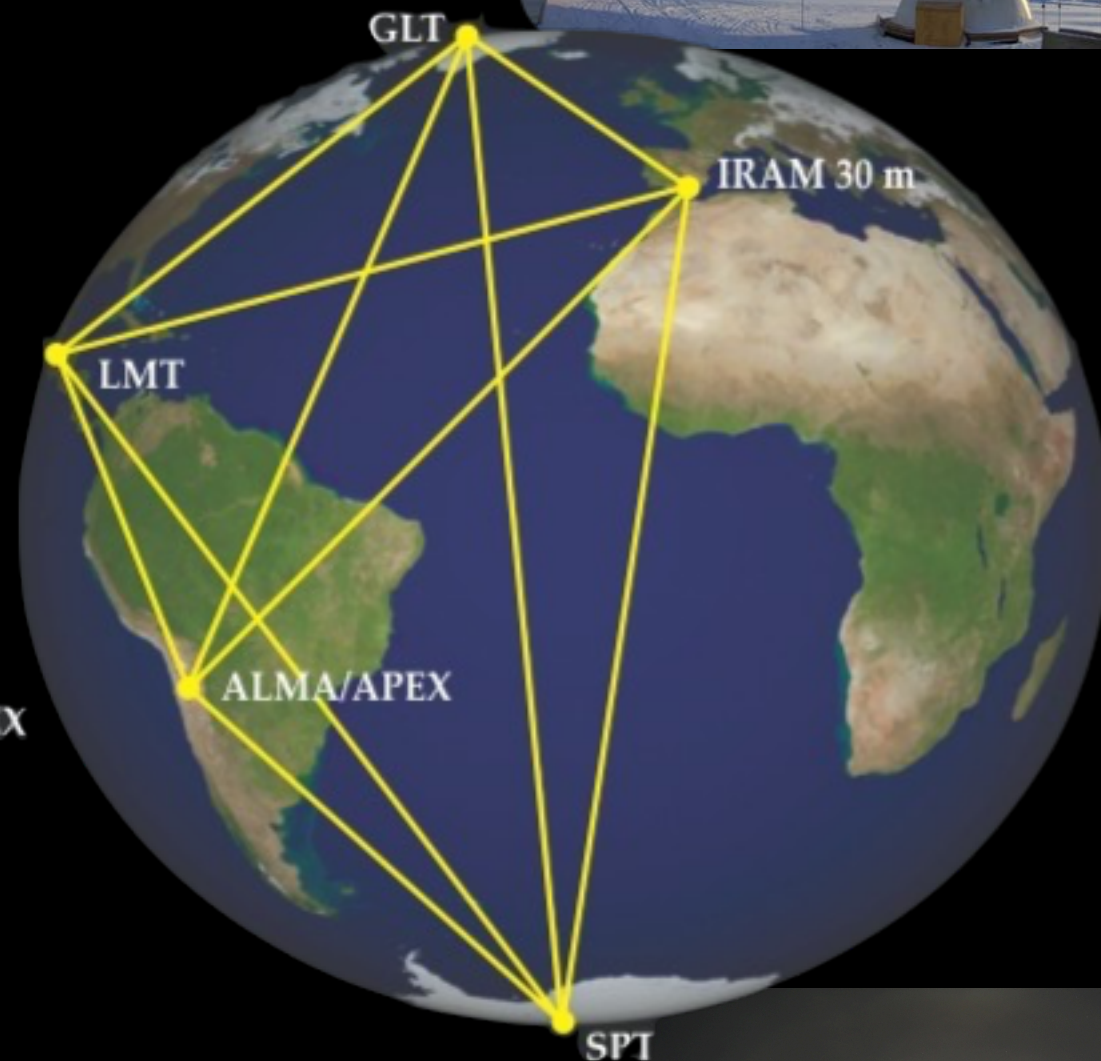
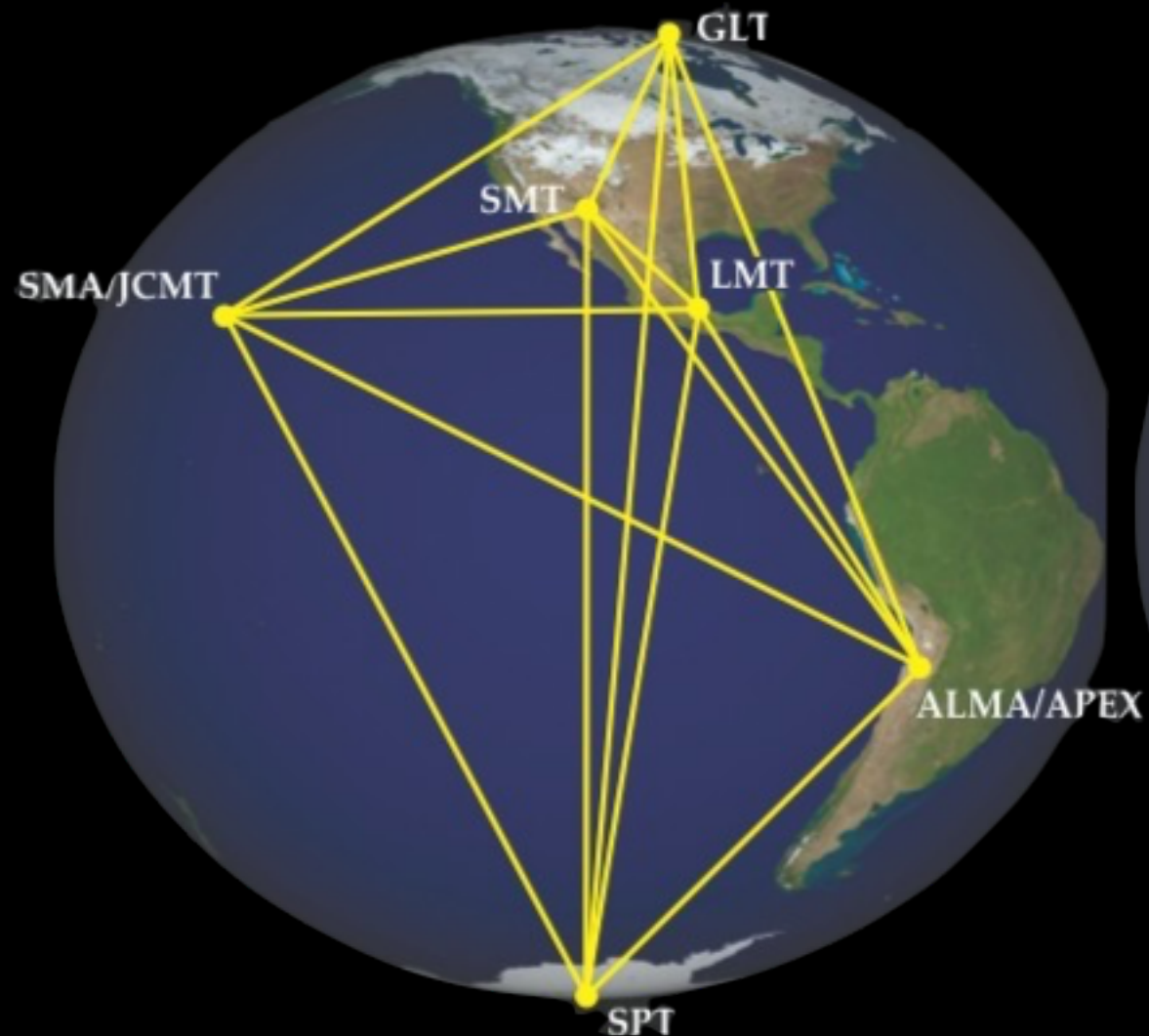
A planet-spanning virtual observatory, years in the making, could change how we think about space, time and the nature of reality. Will it work?

October 4, 2018





Event Horizon Telescope



Worldwide radio interferometry system

Greenland and South Pole telescopes
shown during April 2018 EHT run

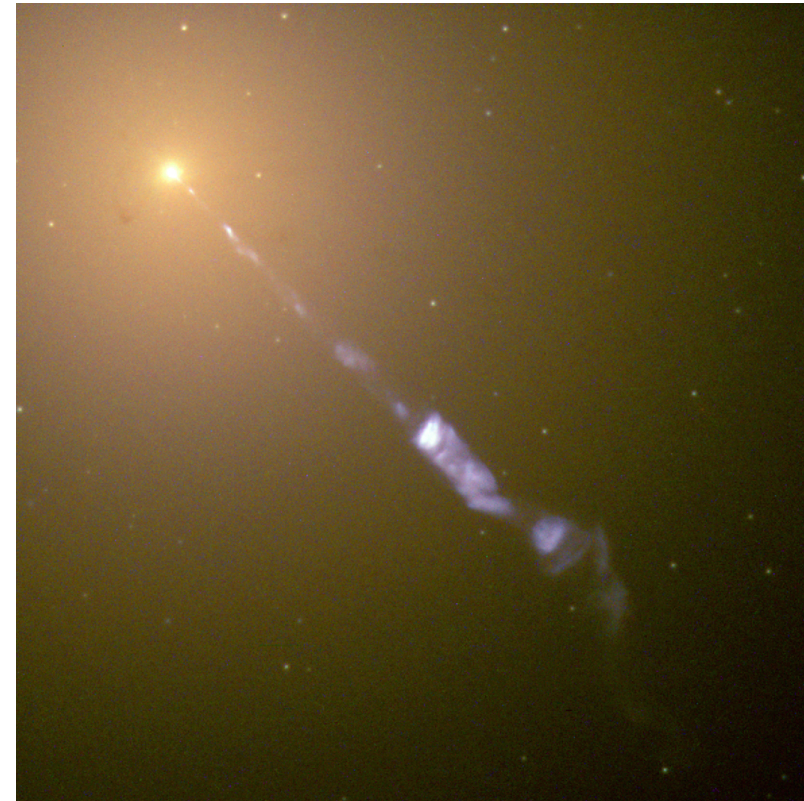
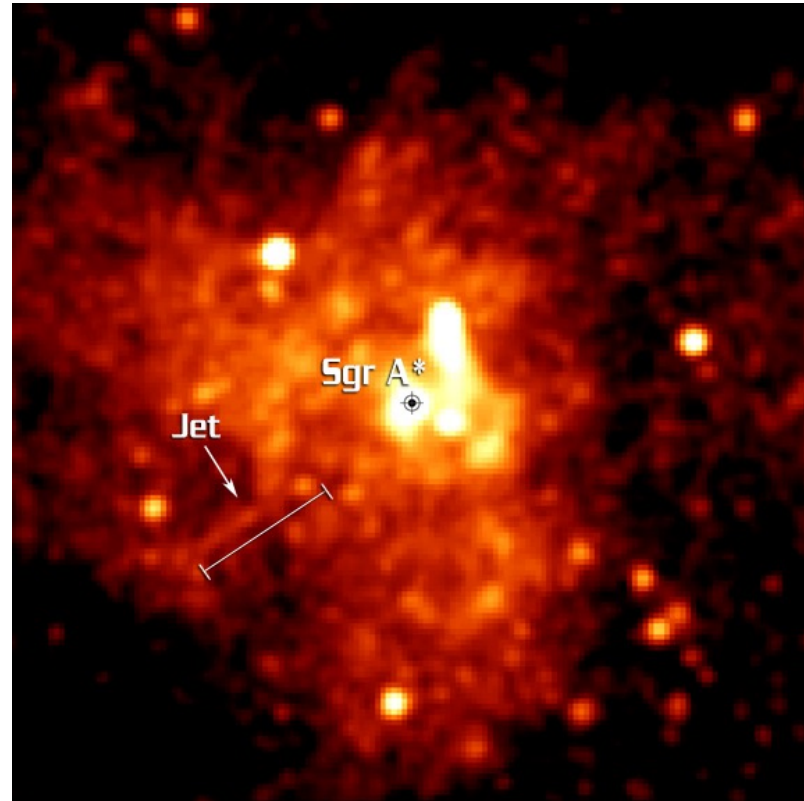
EHT Targets

mm VLBI imaging of accretion flows on horizon scales

Galactic Center (Sgr A*)

M87

Chandra
(X-ray)



Hubble
(visible)

$$r_g = GM_{\text{BH}}/c^2$$

0.05 AU

30 AU

~ horizon scale

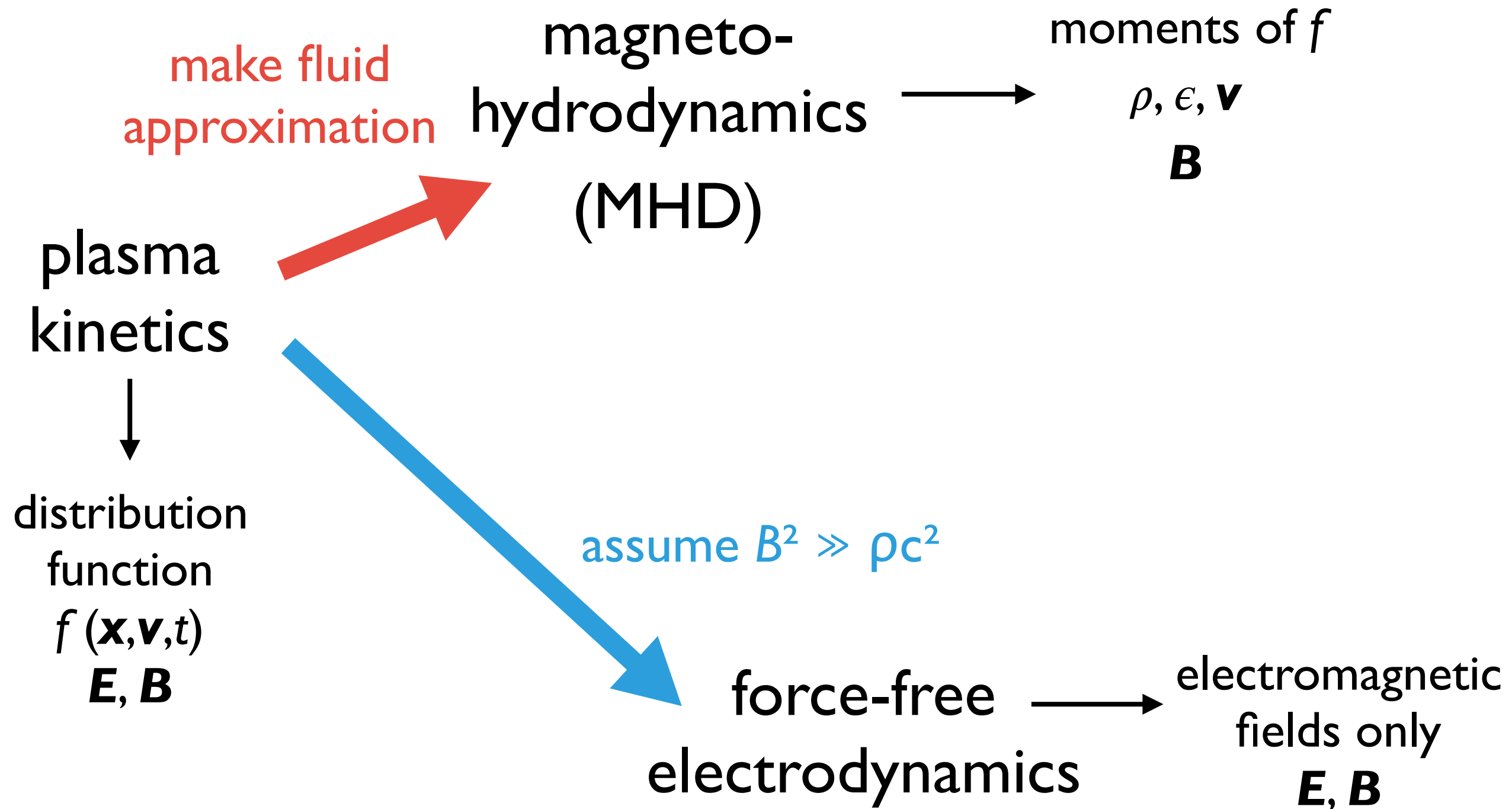
apparent size

$1.4 \times 10^{-9}^\circ$

$5.5 \times 10^{-10}^\circ$

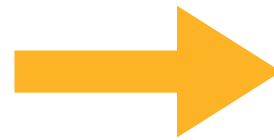
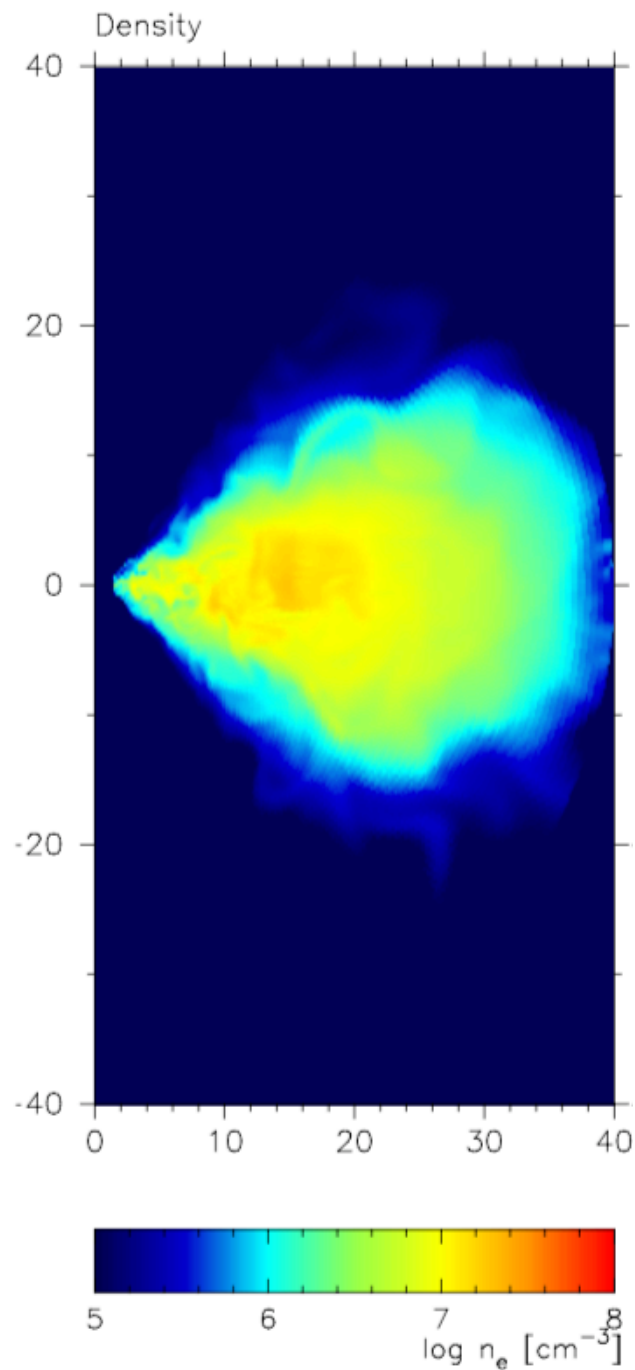


Plasma Descriptions

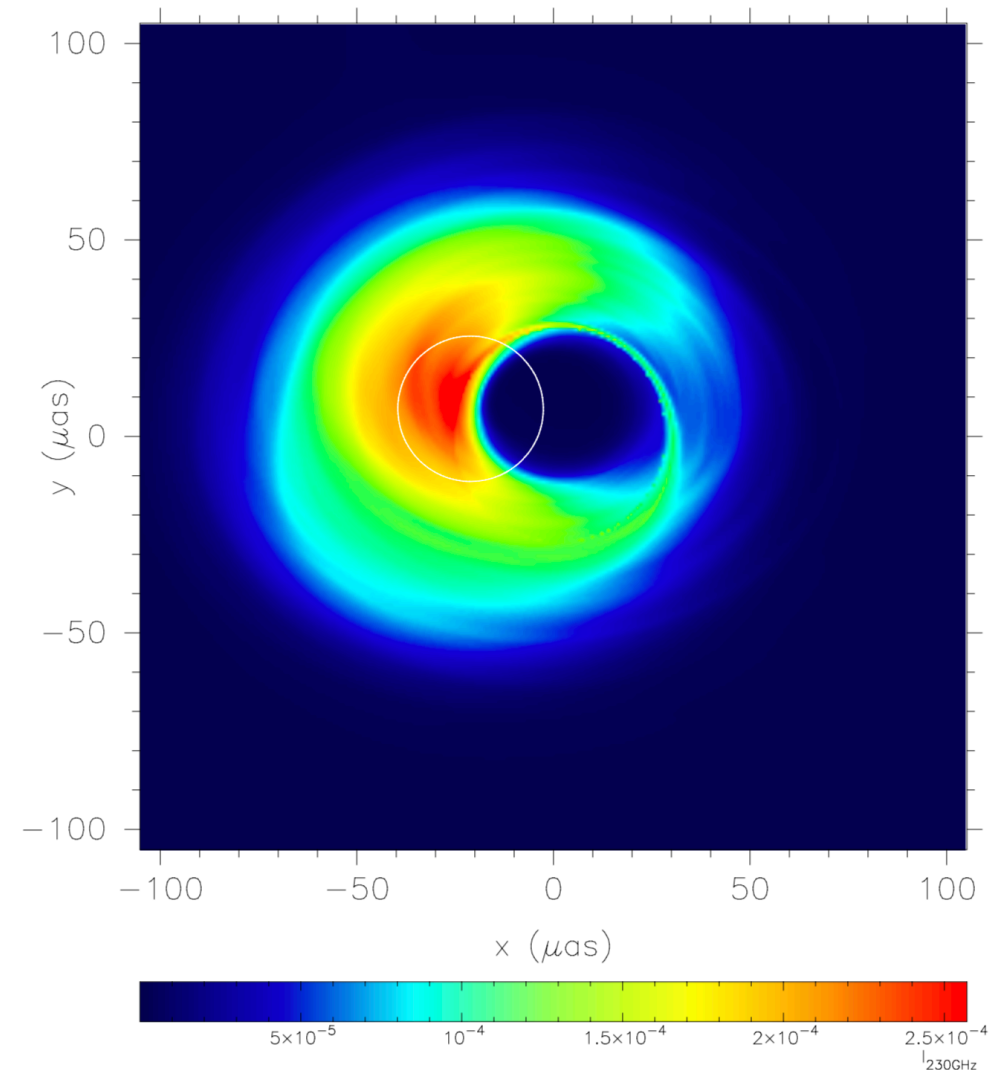


State of the art: GRMHD simulations

GRMHD
simulation



Moscibrodzka+ 2009



observational prediction
via ray-tracing

Includes all general-relativistic effects...

...but treats the accretion flow as a *fluid*

These flows expected to be collisionless

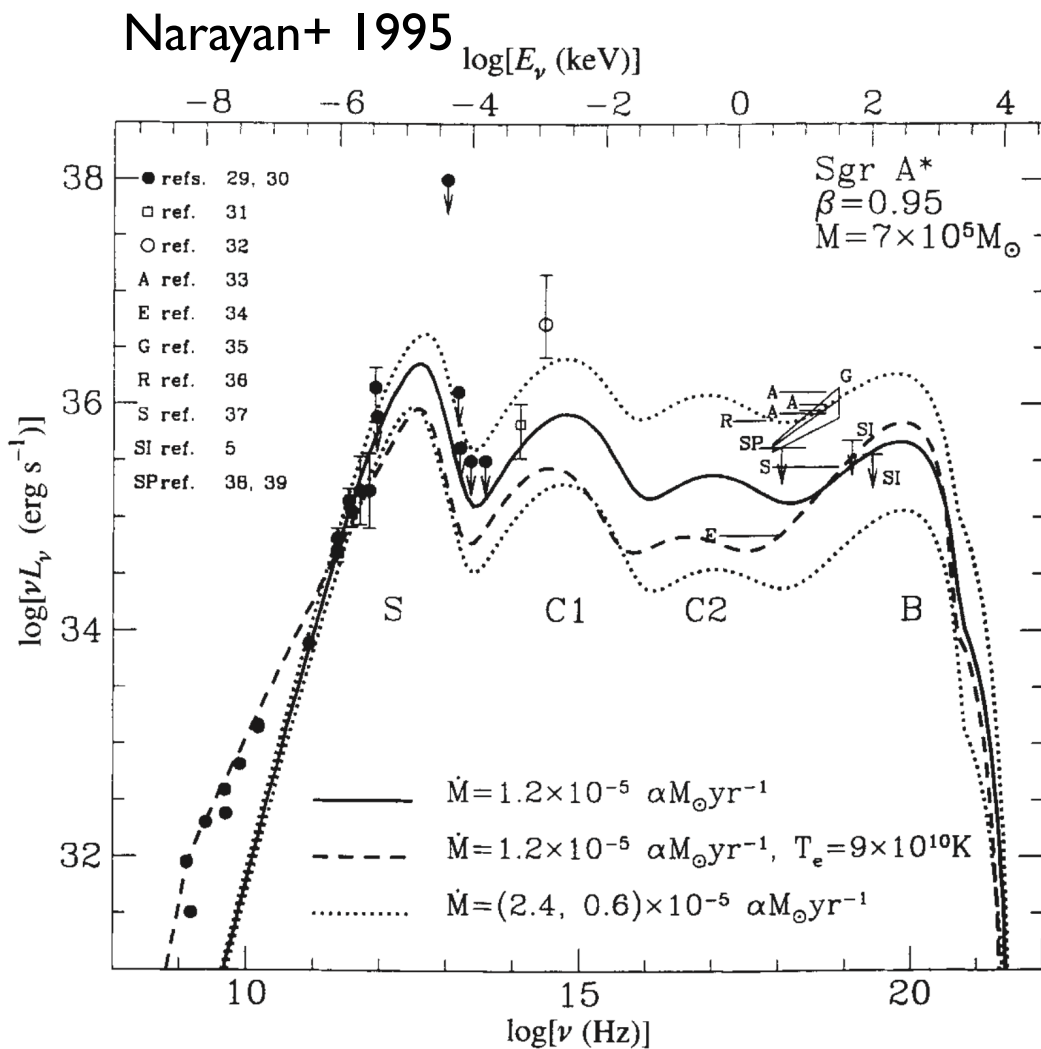
At low accretion rate, flow can have **low-density** structure where:

1. Collision mean free path $\gg$ system scales $\sim r_g$
2. Collision timescale $\gg$ accretion timescale

↓
electrons & protons have different T ,
and generally non-thermal distribution

+

no single bulk velocity (anisotropic)



fit of low-density ADAF
model to Sgr A*

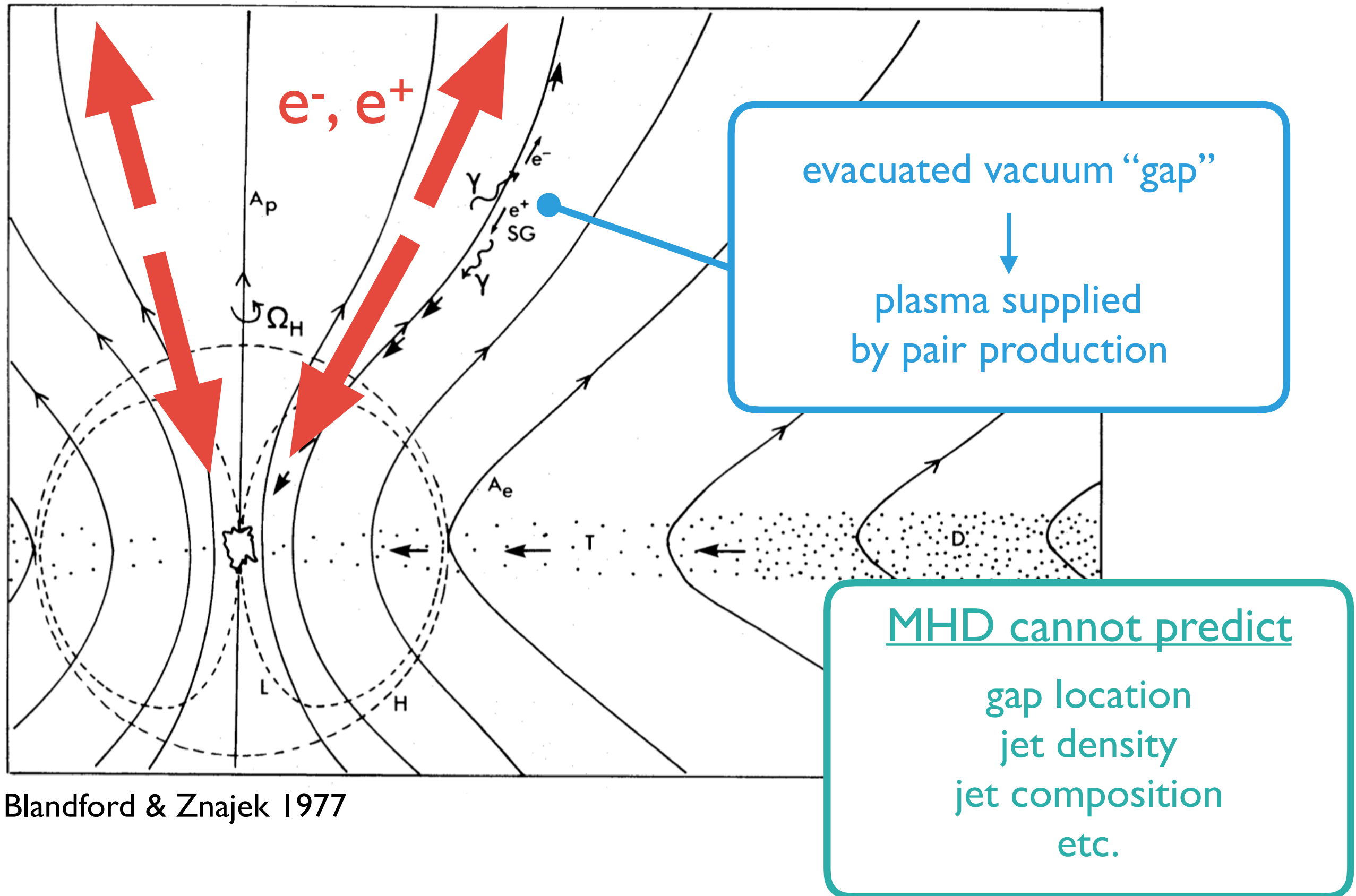
Standard MHD approximation not valid

Holds for

Sgr A* **M87**

many AGN & XRBs
disc coronae jets

Jet launching by rotating black holes



Collisionless physics: plasma kinetics

$$\partial_t \mathbf{E} = \nabla \times \mathbf{B} - \mathbf{J}$$

Maxwell's
equations

$$\nabla \cdot \mathbf{E} = \rho_e$$

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E}$$

$$\nabla \cdot \mathbf{B} = 0$$

+

(a) Continuum dynamics

$$\frac{\partial f_s}{\partial t} + \mathbf{v}_s \cdot \nabla f_s + q_s (\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{p}} = 0 \quad \begin{array}{l} s : \text{electrons,} \\ \text{ions} \end{array}$$

Solve for distribution function $f(\mathbf{x}, \mathbf{p}, t)$

—Vlasov-Maxwell system

or

(b) Particle dynamics

$$\frac{d\mathbf{p}_i}{dt} = q_i (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) \quad \frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i \quad i = 1, \dots, N : \text{particles}$$

Solve for 6 fields: $\mathbf{E}, \mathbf{B}$ + (3D momentum space **or** N particles)

Collisionless physics: plasma kinetics

$$\partial_t \mathbf{E} = \nabla \times \mathbf{B} - \mathbf{J}$$

Maxwell's
equations

$$\nabla \cdot \mathbf{E} = \rho_e$$

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$$\nabla \cdot \mathbf{B} = 0$$

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Choose particle-in-cell (PIC) method

GRPIC I. – electromagnetic field equations

4D spacetime form

$$\begin{array}{llll} \nabla_\nu F^{\mu\nu} = I^\mu & \longrightarrow & \partial_t \mathbf{D} = \nabla \times \mathbf{H} - \mathbf{J} & \nabla \cdot \mathbf{D} = \rho_e \\ \nabla_\nu {}^*F^{\mu\nu} = 0 & & \partial_t \mathbf{B} = -\nabla \times \mathbf{E} & \nabla \cdot \mathbf{B} = 0 \end{array}$$

3+1
form

constitutive relations:

$$\begin{aligned} \mathbf{E} &= \alpha \mathbf{D} + \boldsymbol{\beta} \times \mathbf{B} \\ \mathbf{H} &= \alpha \mathbf{B} - \boldsymbol{\beta} \times \mathbf{D} \end{aligned}$$

Komissarov 2004

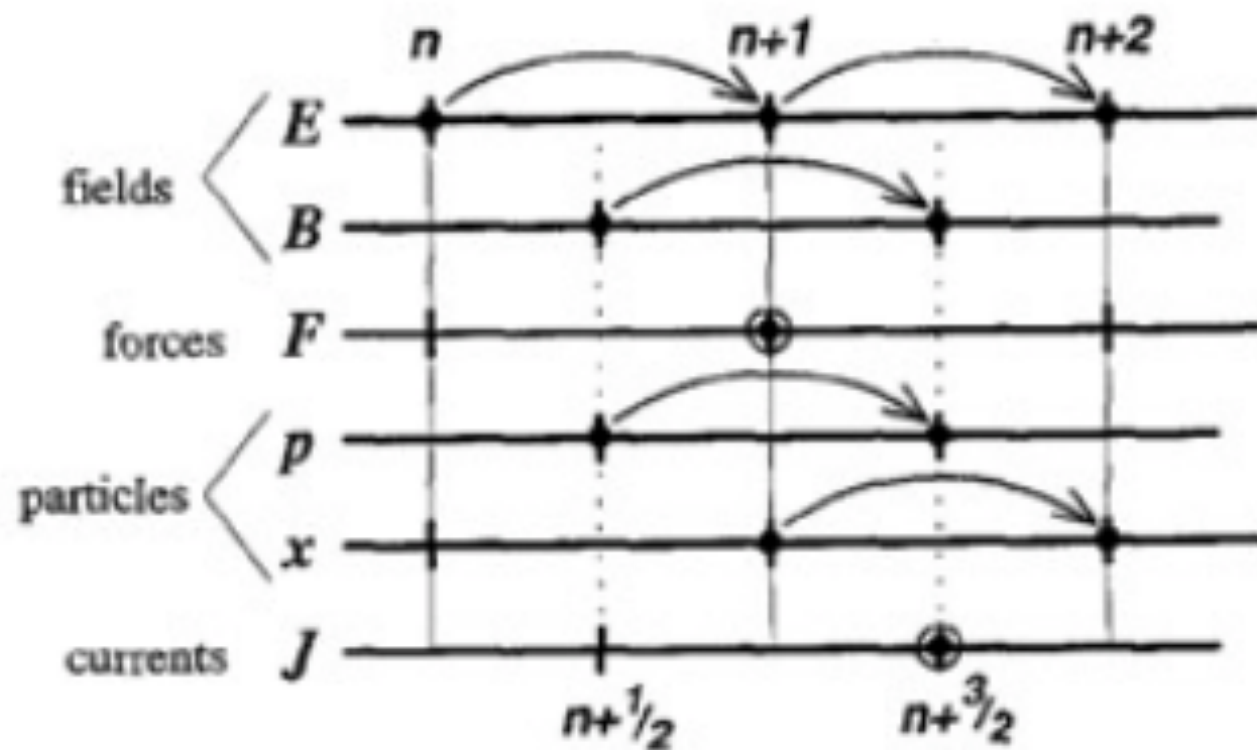
and particles determine current density $\mathbf{J}$

α and β^i are metric functions: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

$$= (\beta^2 - \alpha^2) dt^2 + 2\beta_i dx^i dt + \gamma_{jk} dx^j dx^k$$

Standard PIC algorithm for plasma physics

Leapfrog time integration scheme



Natural to *time-offset* and
leapfrog all quantities



2nd order algorithm with good
energy properties

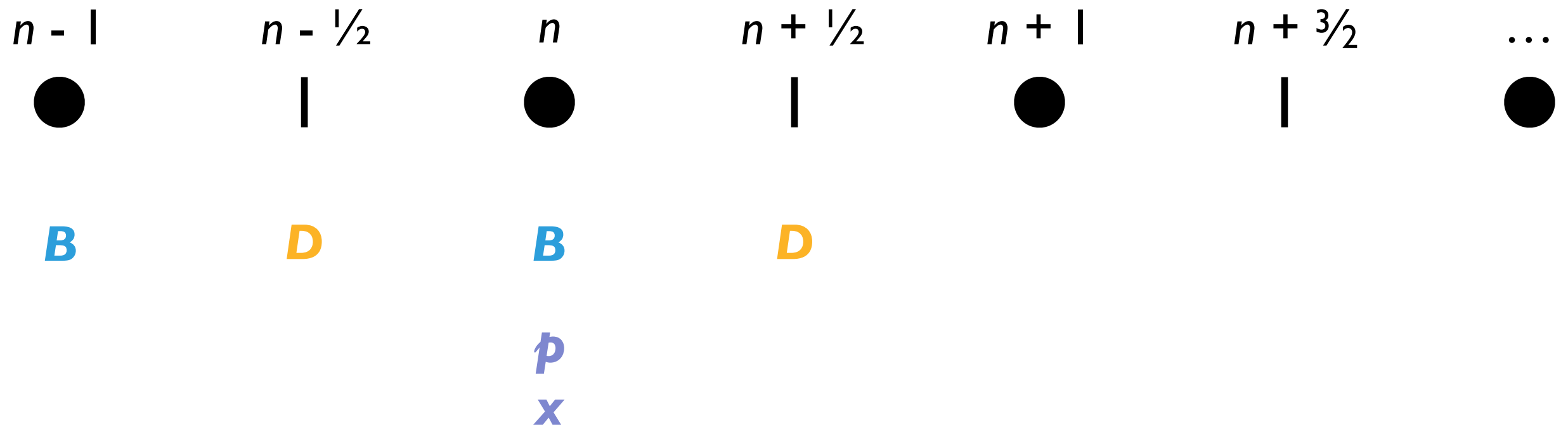
$$\partial_t \mathbf{E} = \nabla \times \mathbf{B} - \mathbf{J}$$

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E}$$

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}(\mathbf{E}, \mathbf{B}, \mathbf{p})$$

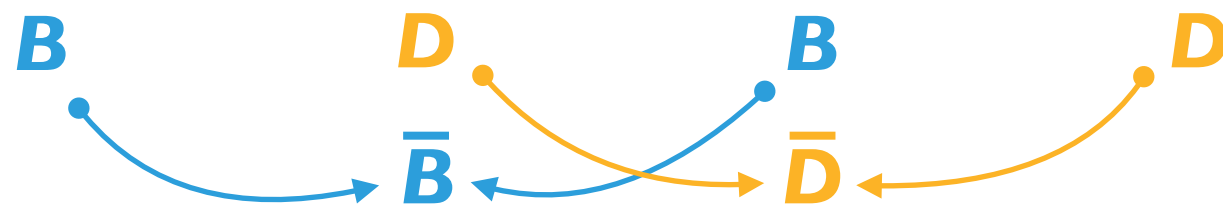
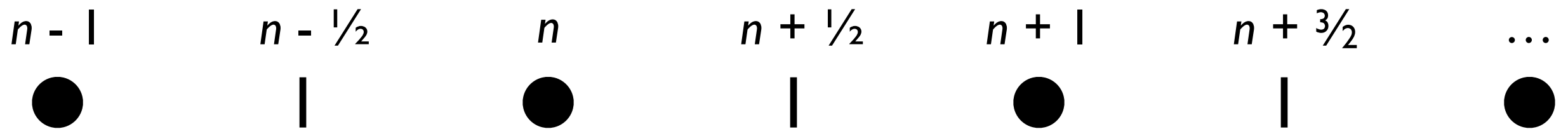
$$\frac{d\mathbf{x}}{dt} = \frac{\mathbf{p}}{m}$$

(i) evolve B from n to $n+1$



Start of timestep, know: $\left[\begin{array}{l} B^n, D^{n+1/2} \\ p^n, x^n \end{array} \right]$ and $B^{n-1}, D^{n-1/2}$

(i) evolve B from n to $n+1$

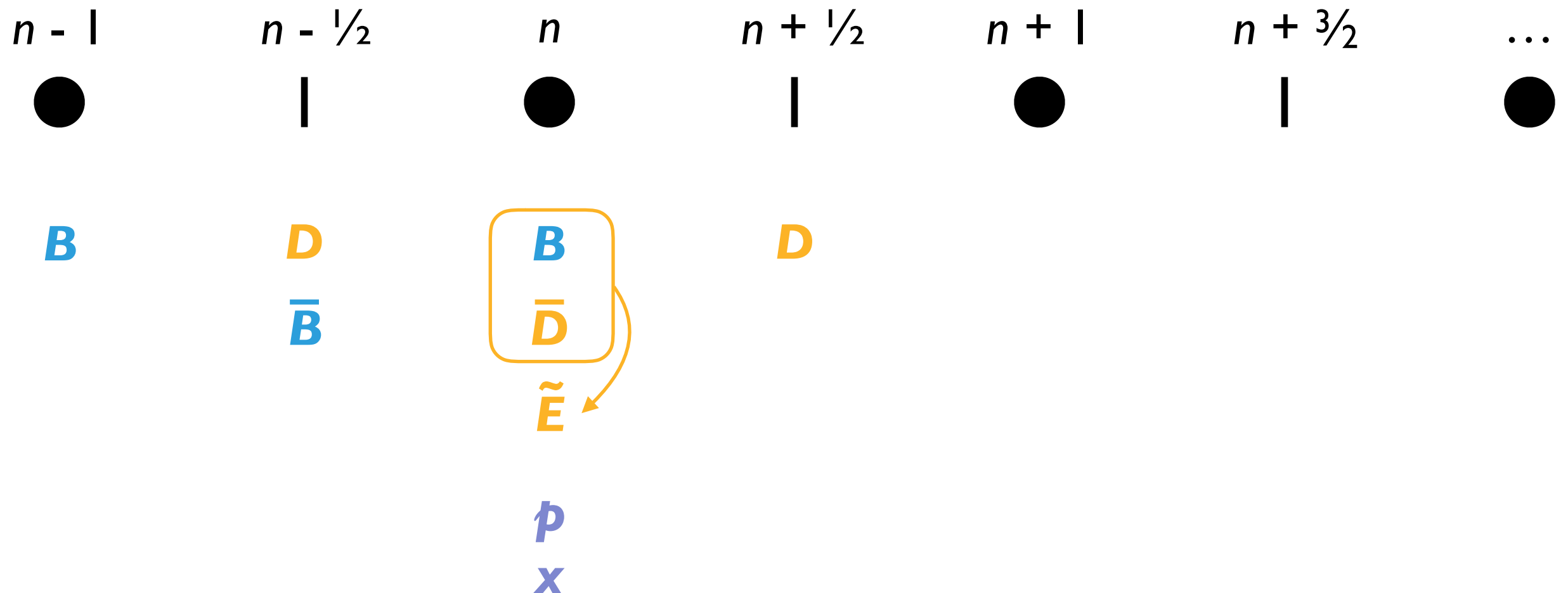


p
 x

$$\bar{B}^{n-1/2} = \frac{1}{2}(B^n + B^{n-1})$$

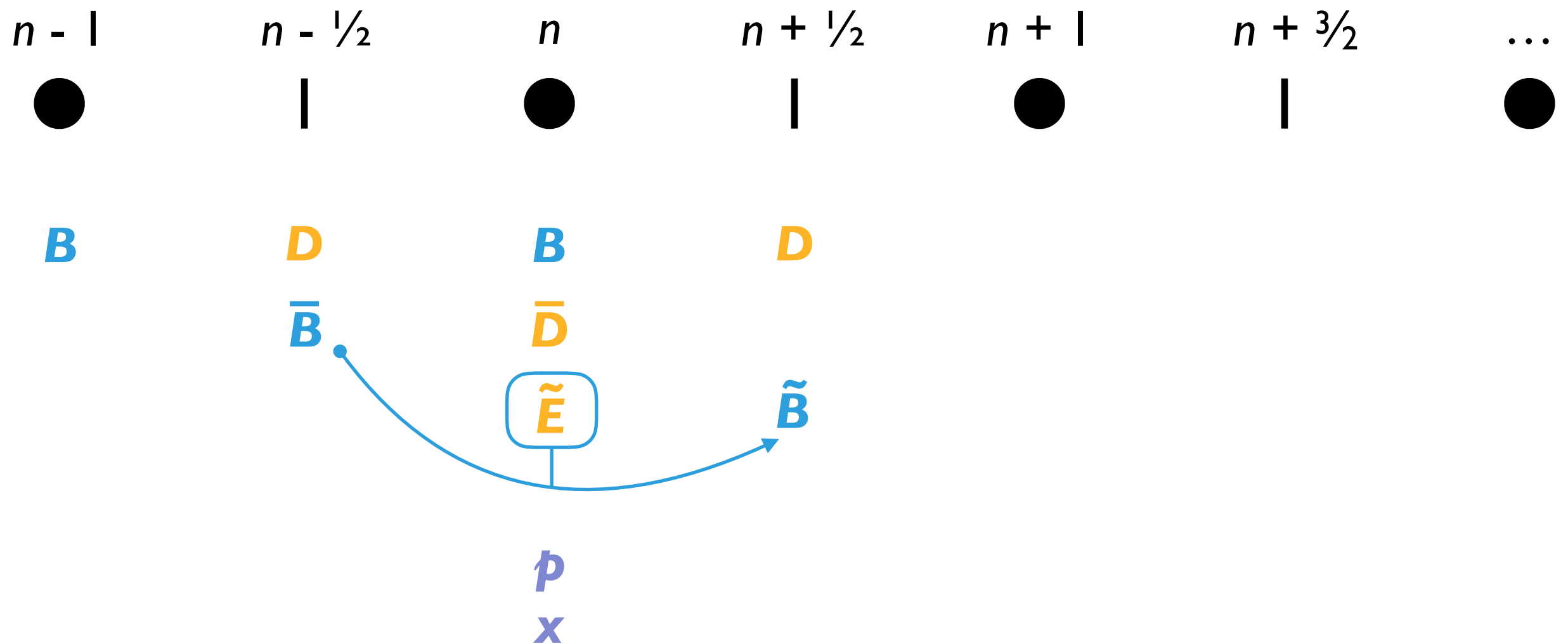
$$\bar{D}^n = \frac{1}{2}(D^{n+1/2} + D^{n-1/2})$$

(i) evolve B from n to $n+1$



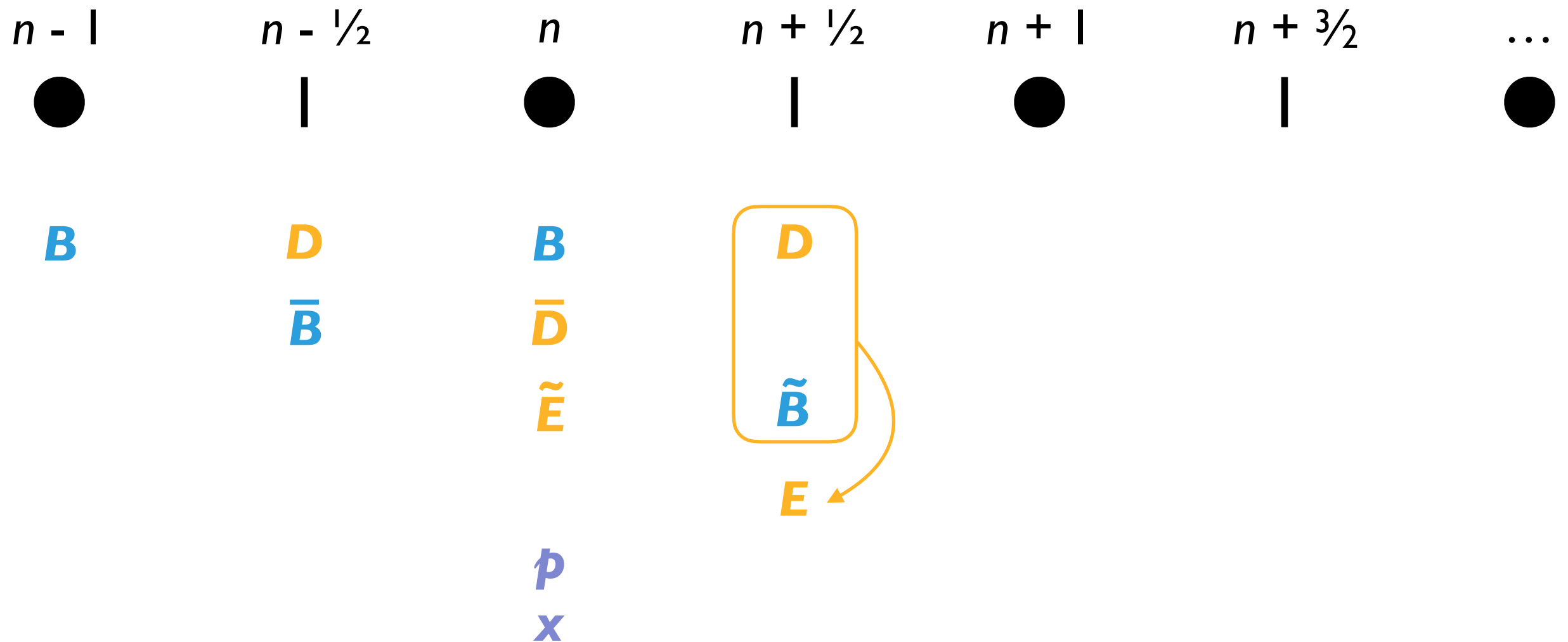
$$\tilde{E}^n = \alpha \bar{D}^n + \beta \times B^n$$

(i) evolve B from n to $n+1$



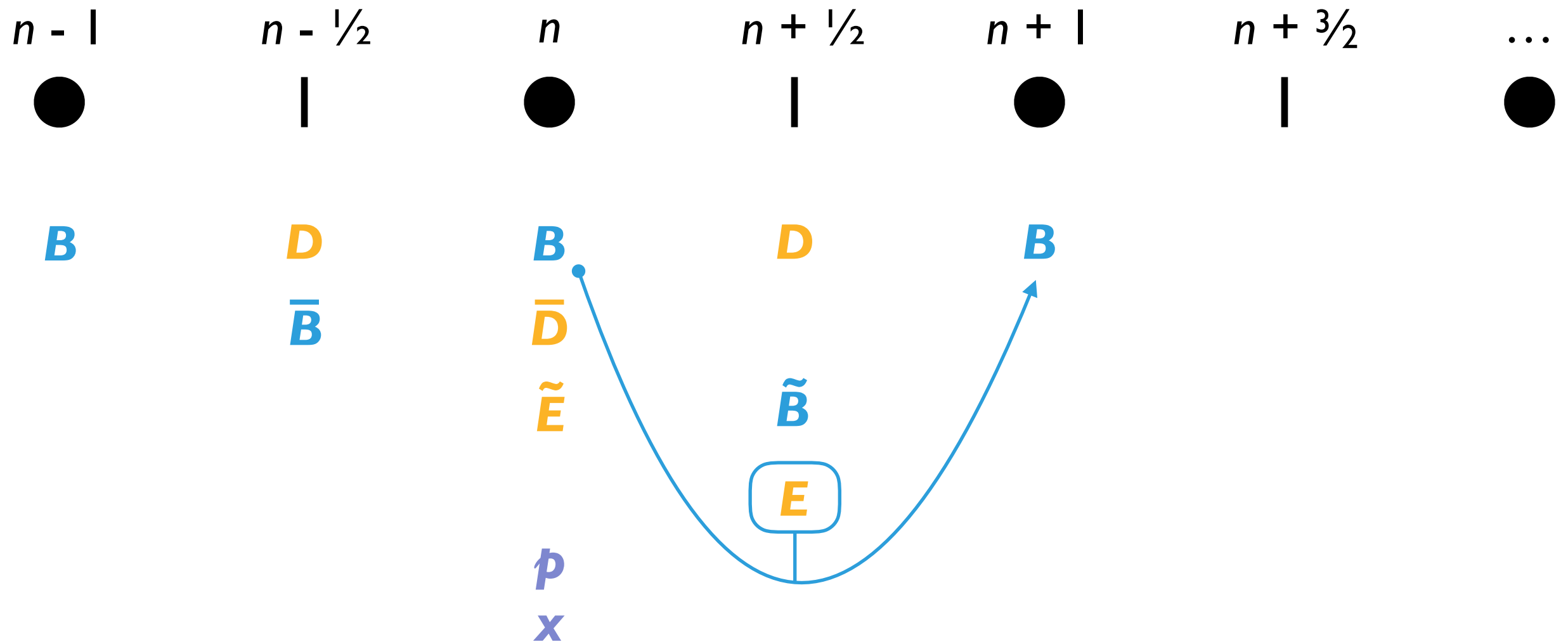
$$\tilde{B}^{n+1/2} = \bar{B}^{n-1/2} - \Delta t \nabla \times \tilde{E}^n$$

(i) evolve B from n to $n+1$



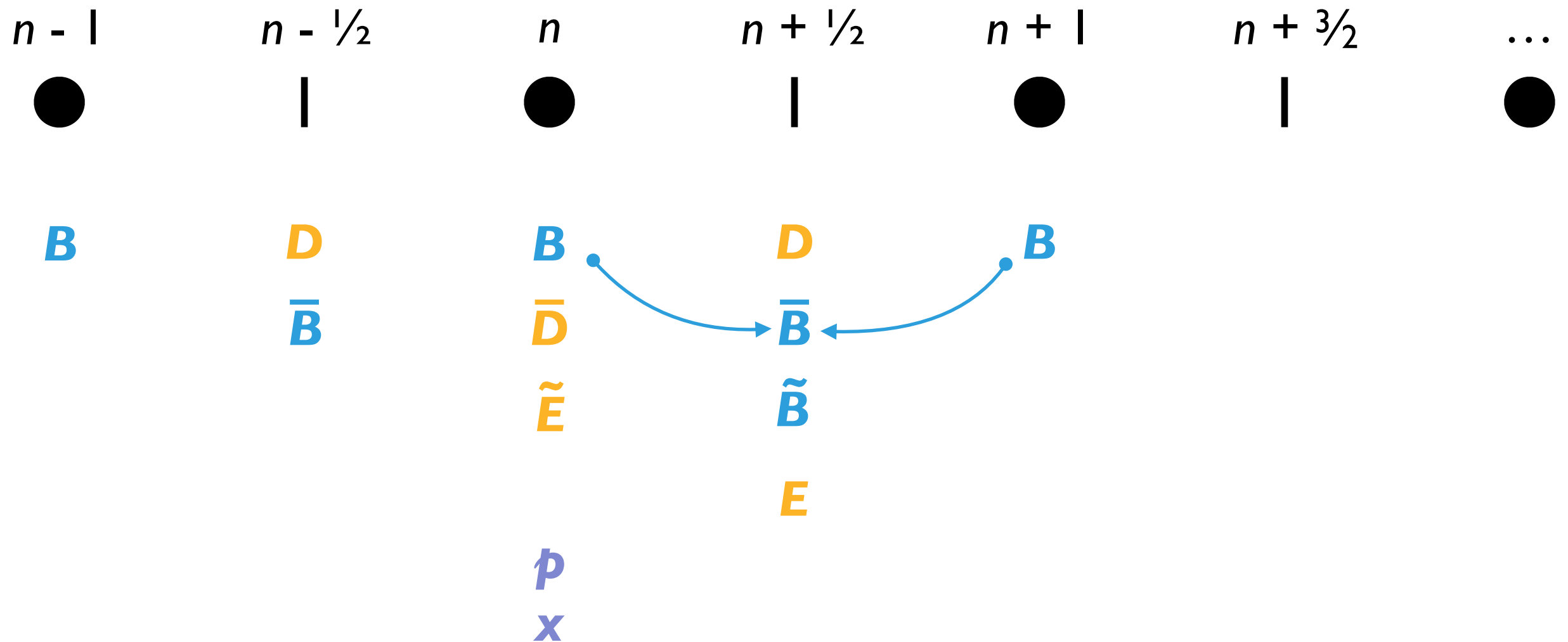
$$E^{n+1/2} = \alpha D^{n+1/2} + \beta \times \tilde{B}^{n+1/2}$$

(i) evolve B from n to $n+1$

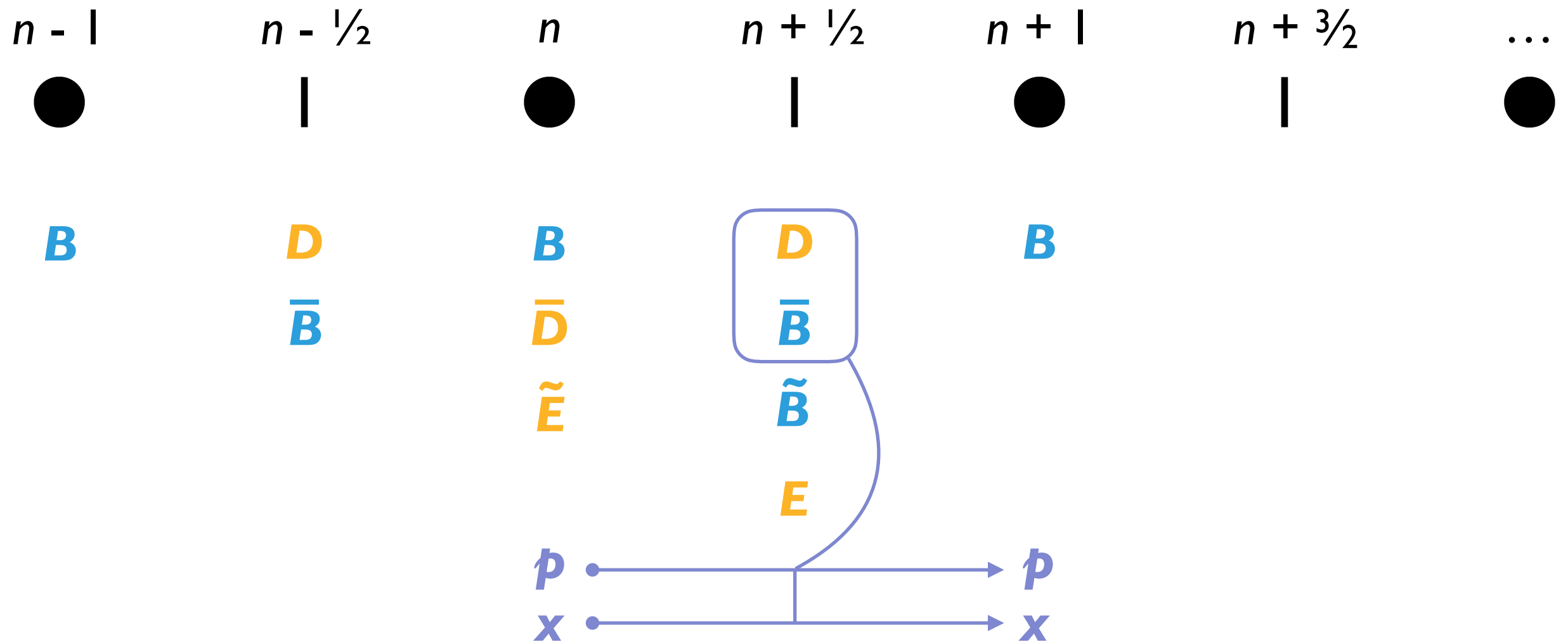


$$B^{n+1} = B^n - \Delta t \nabla \times E^{n+1/2}$$

(ii) evolve $\boldsymbol{p}, \boldsymbol{x}$ from n to $n+1$

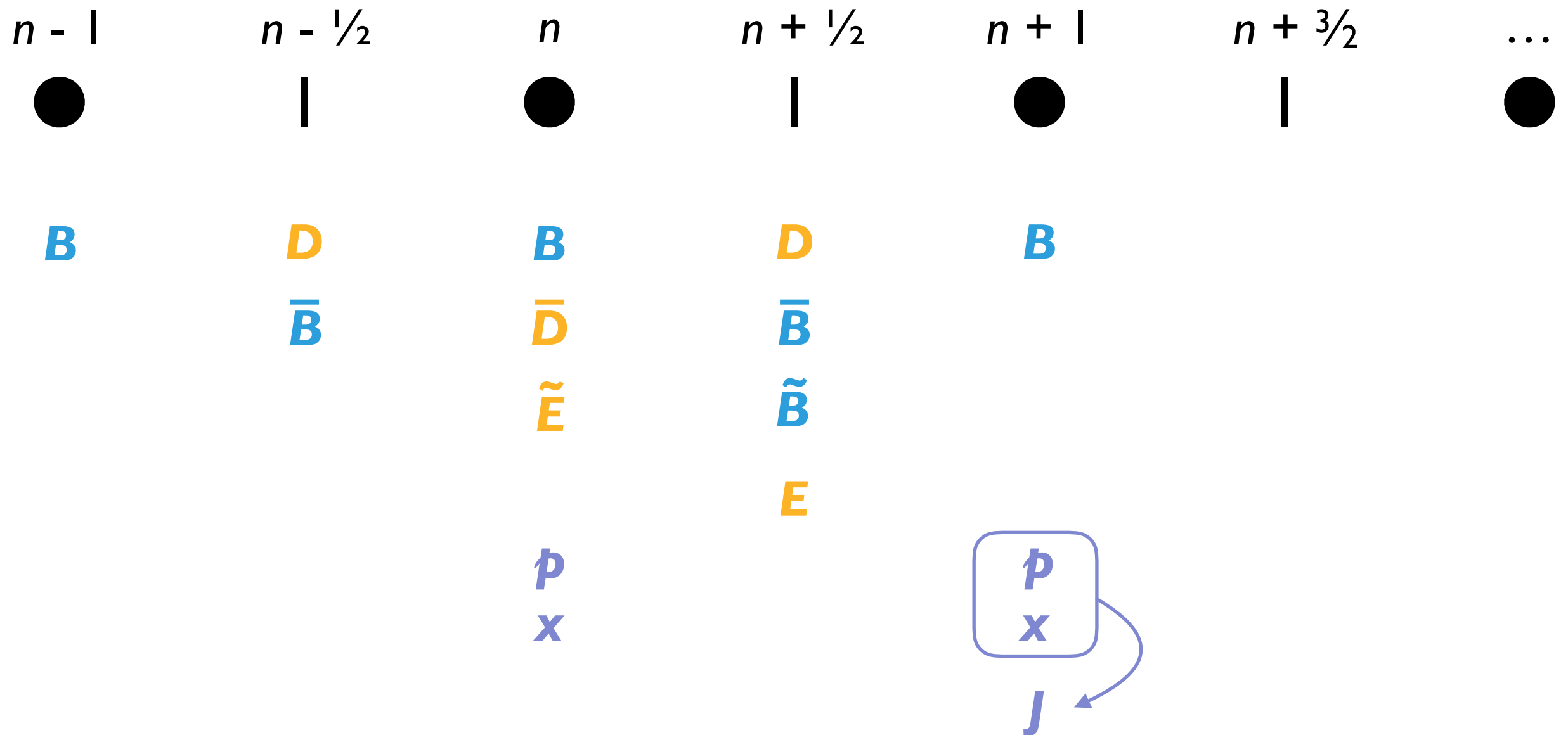


(ii) evolve $\boldsymbol{p}, \boldsymbol{x}$ from n to $n+1$



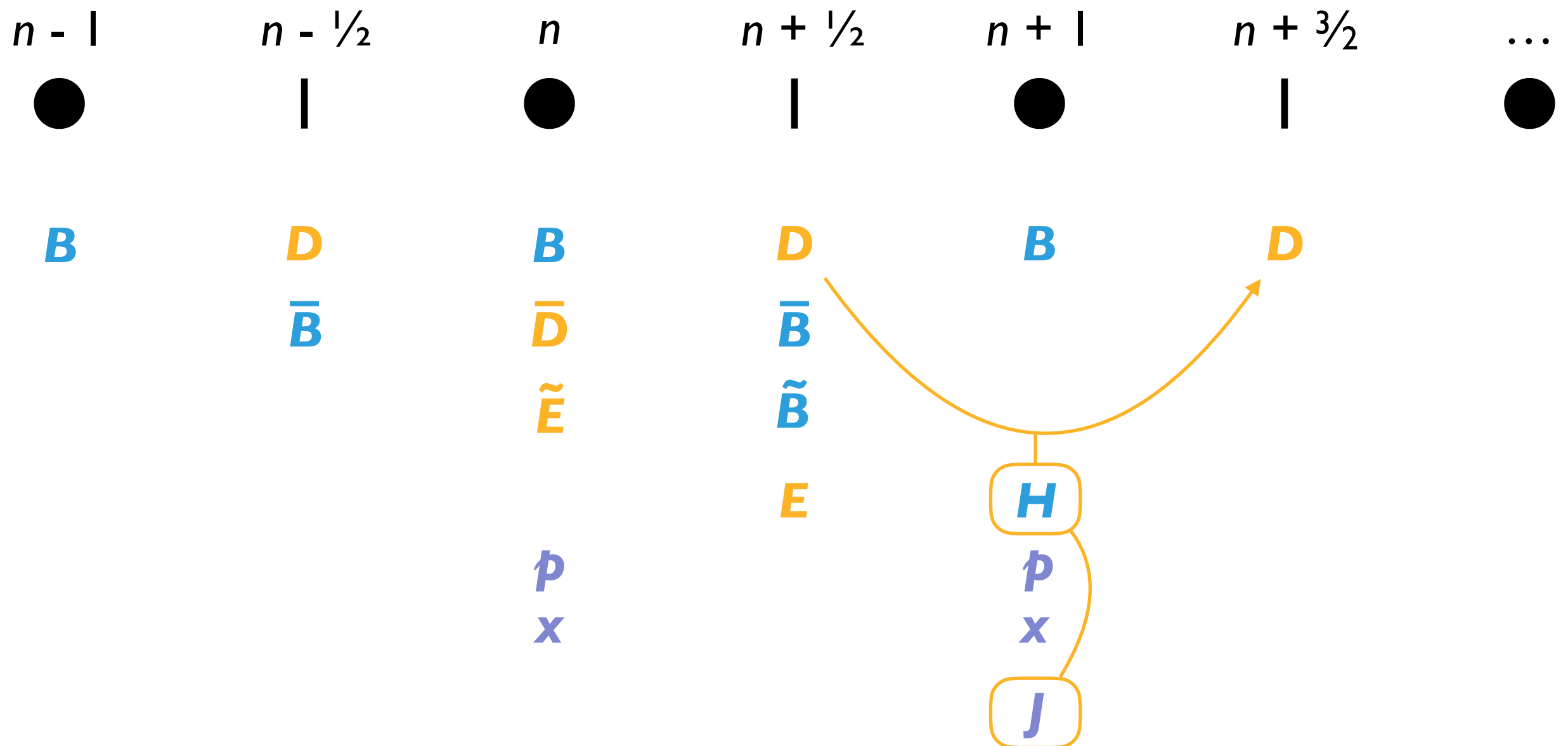
$$p^n, x^n \longrightarrow p^{n+1}, x^{n+1} \text{ using } \bar{B}^{n+1/2}, D^{n+1/2}$$

(iii) evolve D from $n + \frac{1}{2}$ to $n + \frac{3}{2}$



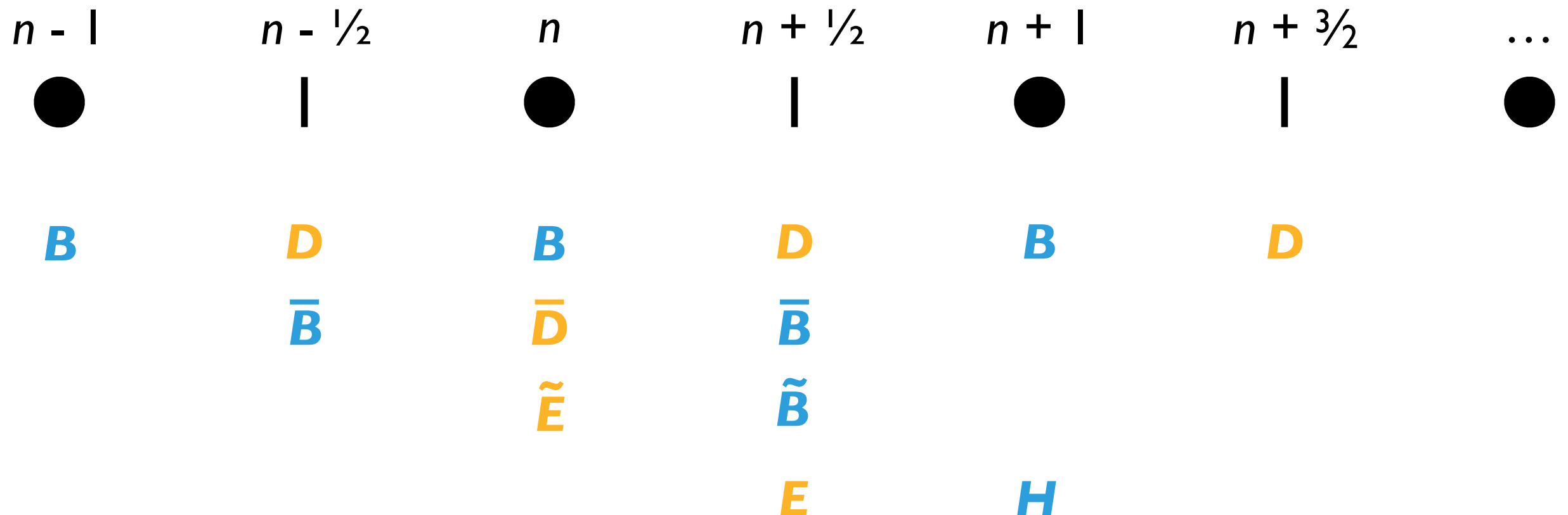
$$J^{n+1} = J(x^{n+1}, p^{n+1})$$

(iii) evolve D from $n + \frac{1}{2}$ to $n + \frac{3}{2}$



Find H^{n+1} using trapezoidal leapfrogs again
Find $D^{n+3/2}$ using H^{n+1} and J^{n+1}

(iii) evolve D from $n + 1/2$ to $n + 3/2$



- Goes to (energy conserving) leapfrog as $\beta \rightarrow 0$
- Gives better energy conservation for nonzero β
- One particle push per timestep

GRPIC 2. – particles

Start from **4D action**: $S = \int (-m ds + q A_\mu dx^\mu)$ where $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

find Lagrangian $L = -m\alpha/\Gamma + q A_j v^j + q A_t$

and Hamiltonian $H = \pi_i v^i - L$

canonical momentum: $\pi_i = p_i + q A_i$

kinetic momentum: p_i and $v^i = \frac{dx^i}{dt}$

$$\frac{dx^i}{dt} = \frac{\alpha}{m\Gamma} p^i - \beta^i$$

← Hamilton's equations give

$$\frac{dp_i}{dt} = -m\Gamma \partial_i \alpha + p_j \partial_i \beta^j - \frac{\alpha}{2\Gamma m} \partial_i (\gamma^{lm}) p_l p_m + q \left\{ \alpha D_i + \epsilon_{ijk} (v^j + \beta^j) B^k \right\}$$

↑
gravitational
acceleration

↑
~ extrinsic
curvature

↑
Lorentz force

Designing a particle integrator

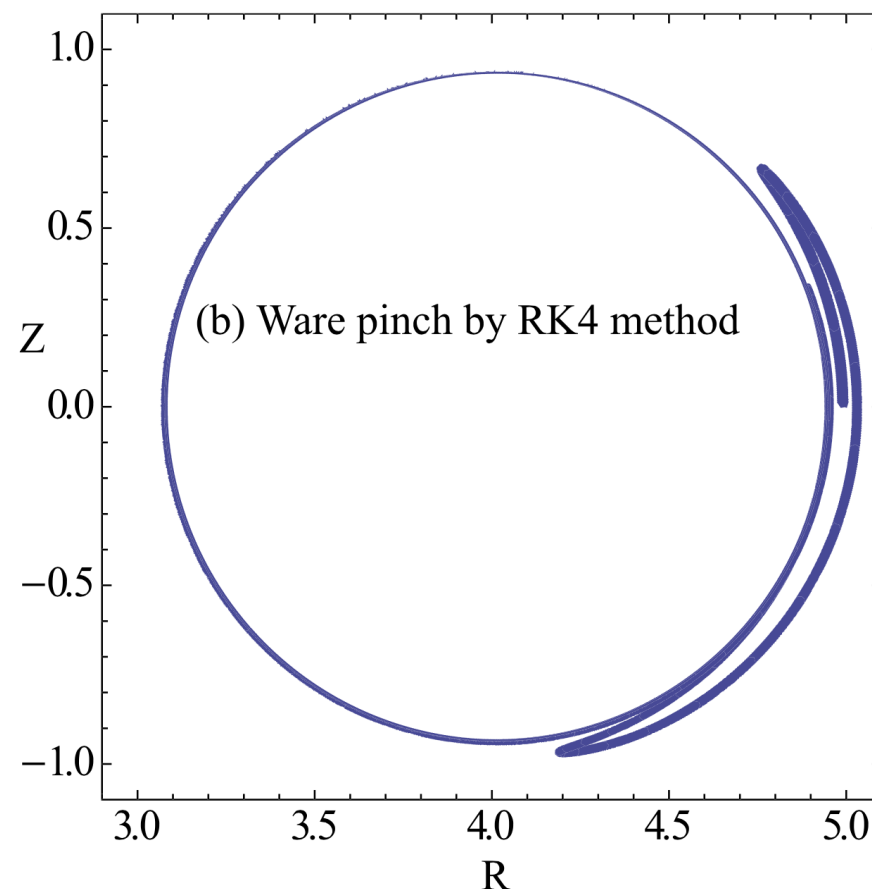
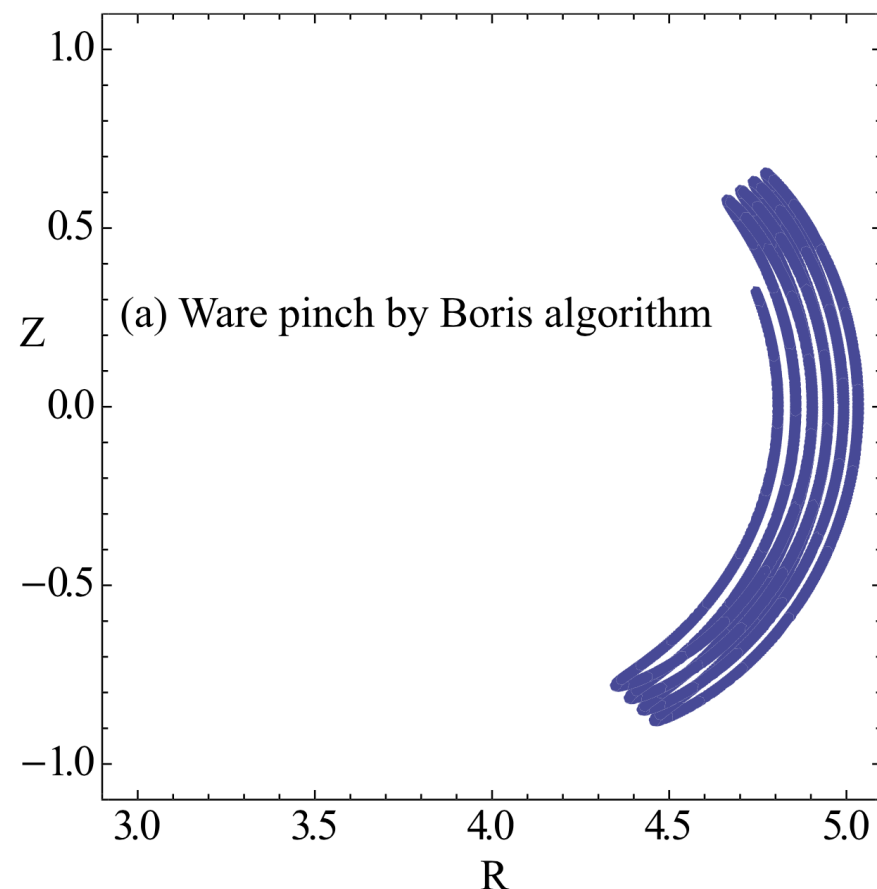
Ideally want **symplectic** integrator

preserves symplectic two-form: $s_{\mu\nu} = x_{\mu} \wedge p_{\nu}$

very good energy stability

Standard to use **Boris push** for Lorentz force

not symplectic, but volume preserving: $|x_{\mu} \wedge p_{\nu}|$ is maintained



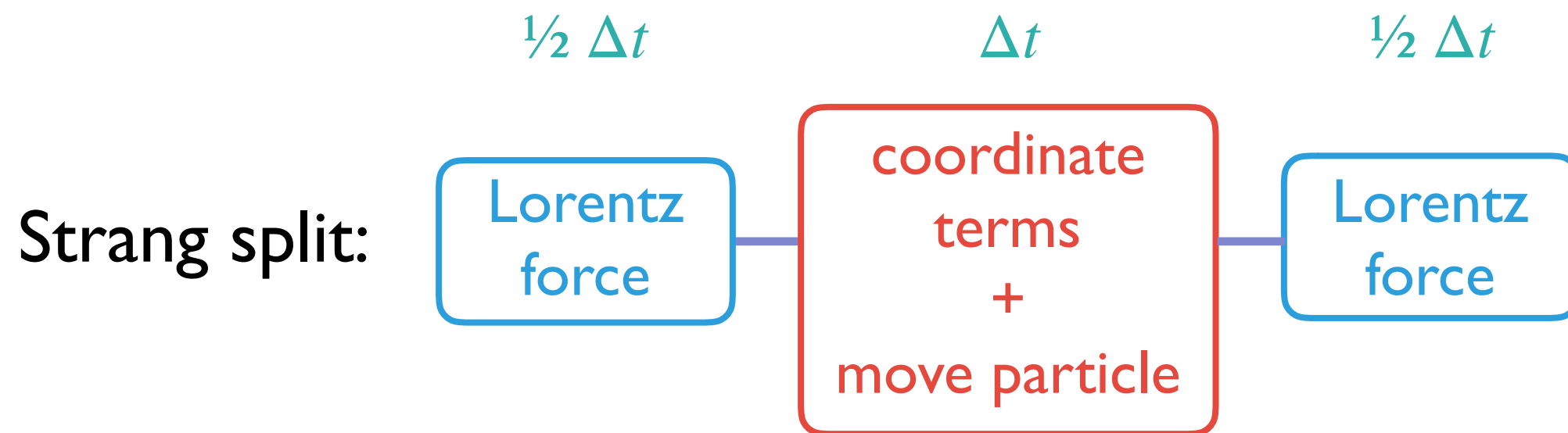
Qin+ 2013

*“Why is Boris Algorithm
So Good?”*

Particle integrator scheme

Requirements:

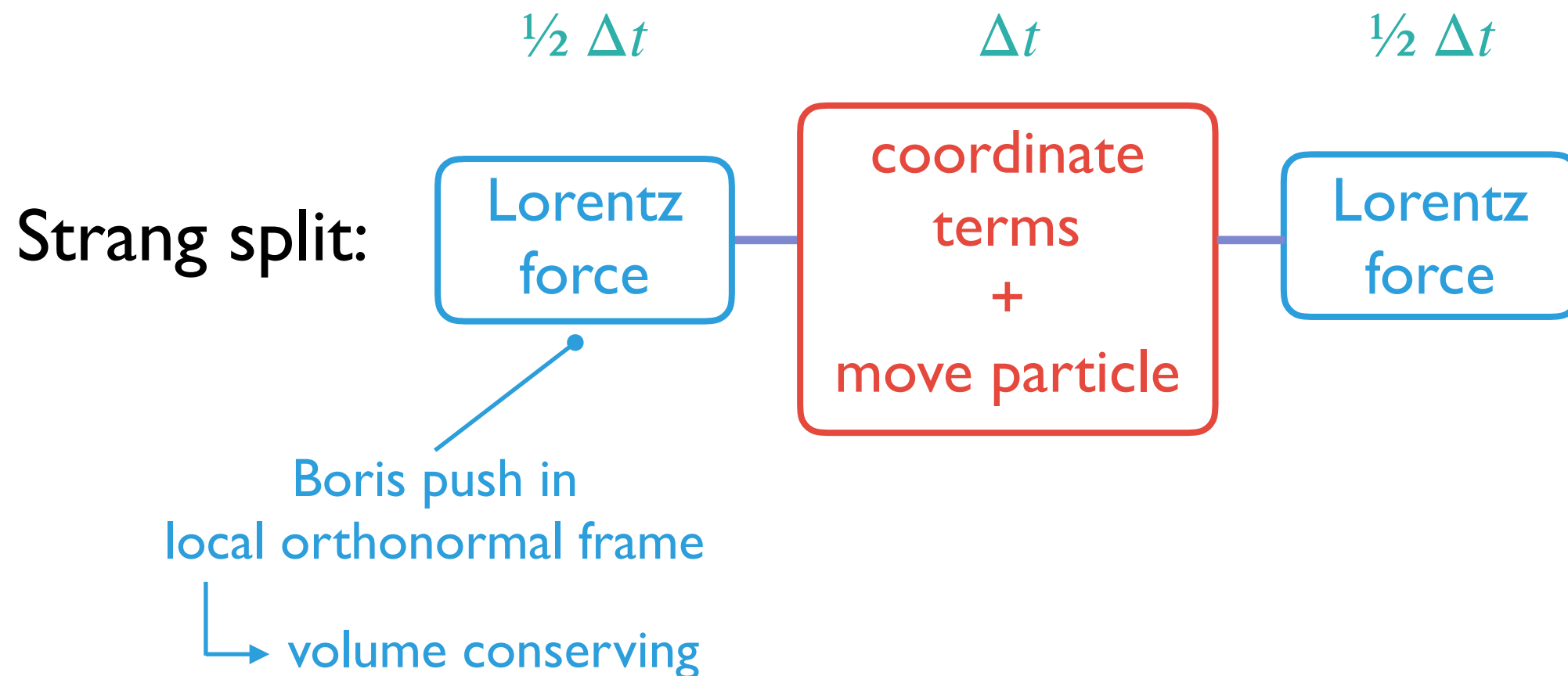
1. conserve phase-space volume, $|x \wedge p|$
2. time-symmetric



Particle integrator scheme

Requirements:

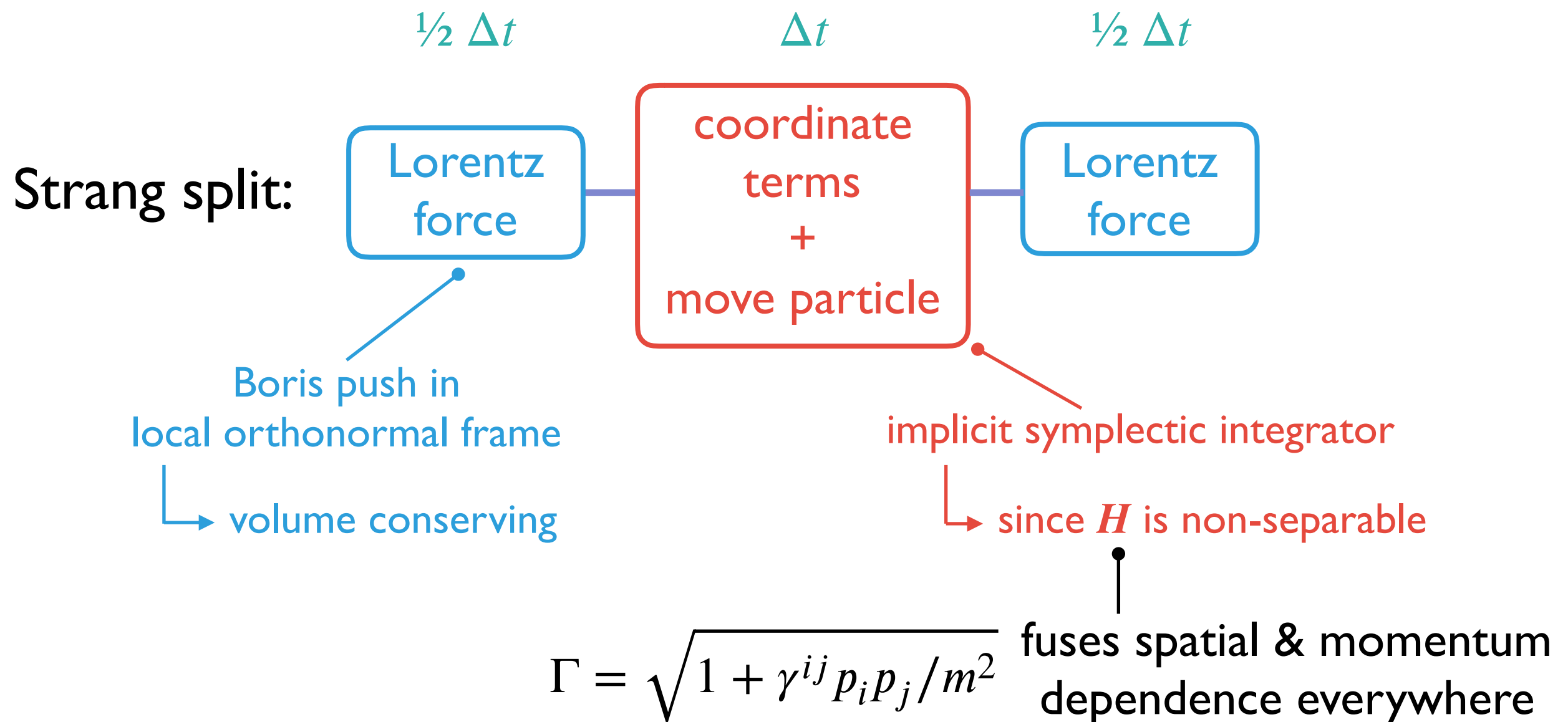
1. conserve phase-space volume, $|x \wedge p|$
2. time-symmetric



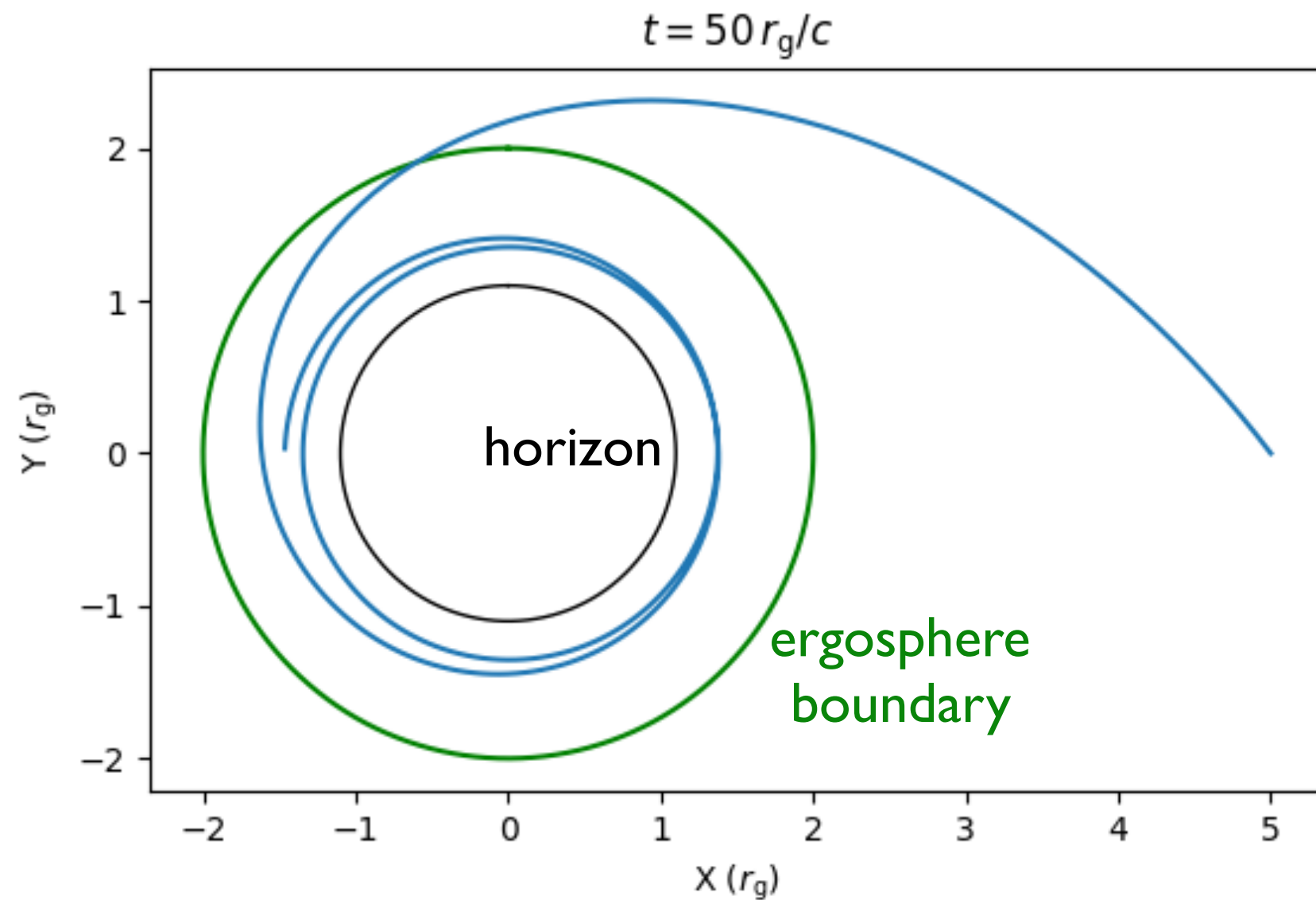
Particle integrator scheme

Requirements:

1. conserve phase-space volume, $|x \wedge p|$
2. time-symmetric



Geodesic tests — $B = 0$

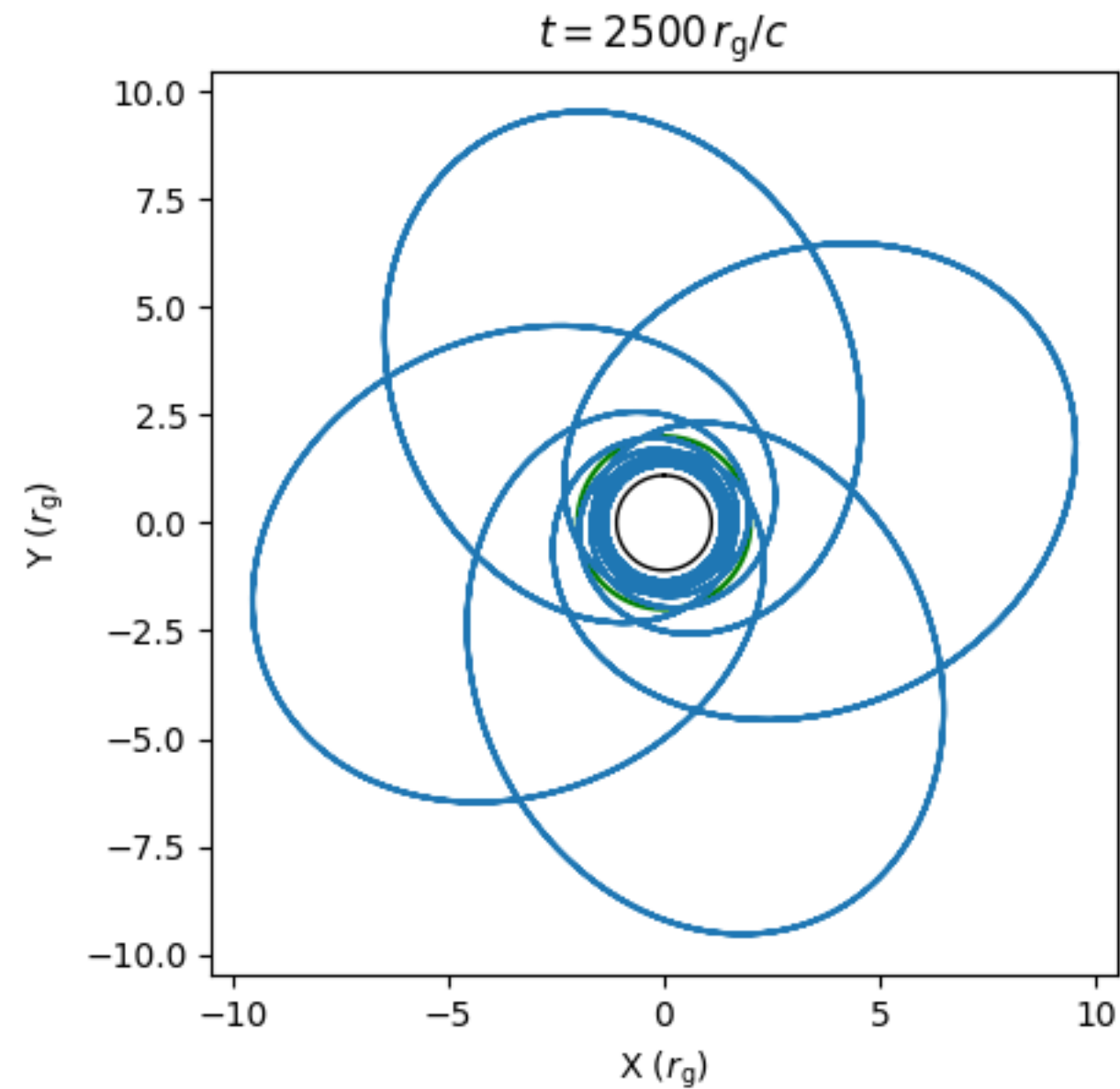
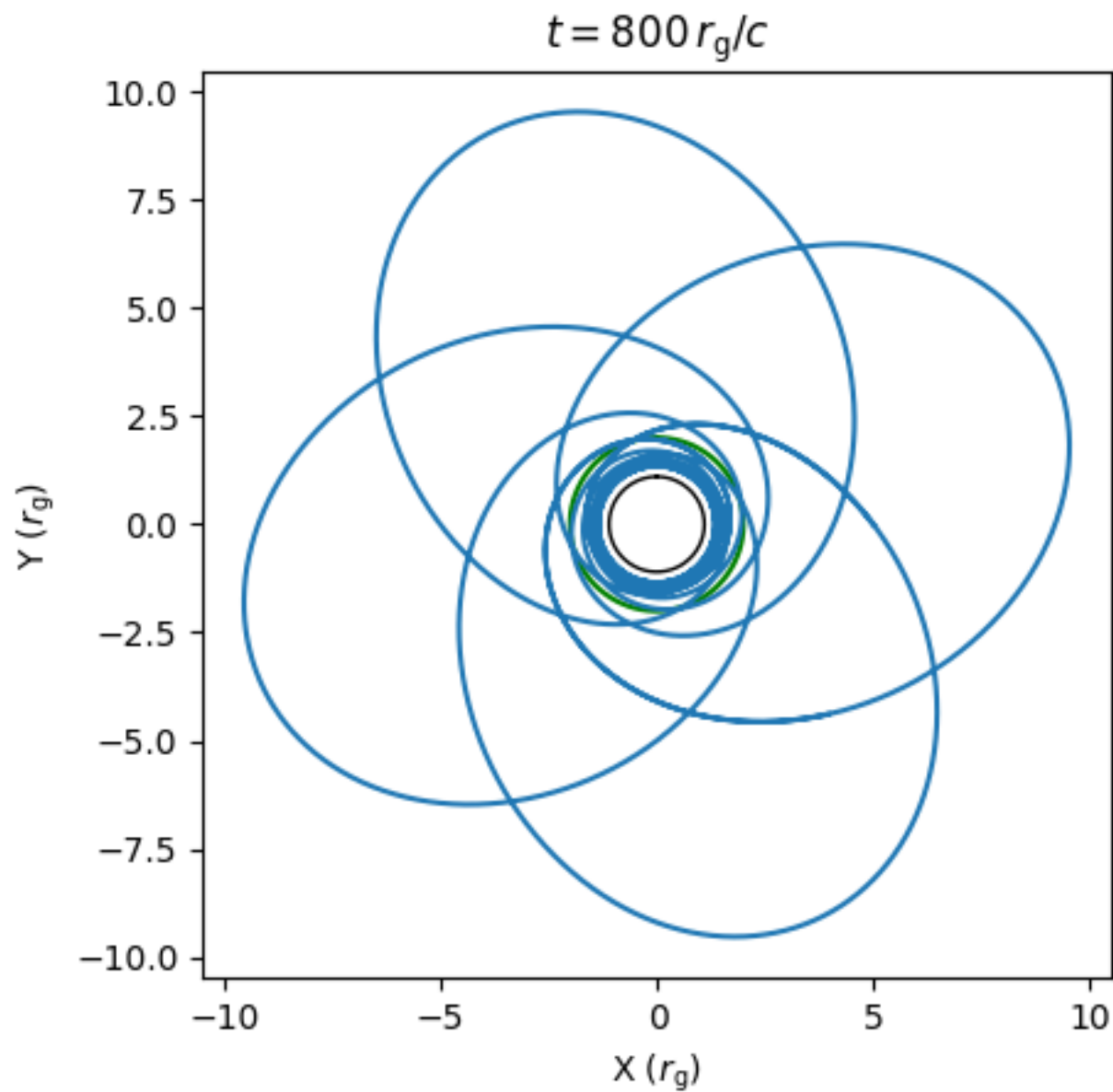


Rotating black hole
 $a = 0.995$

Prograde orbit with
 $L = u_\phi = 2$
 $E = -u_t = 0.915082$

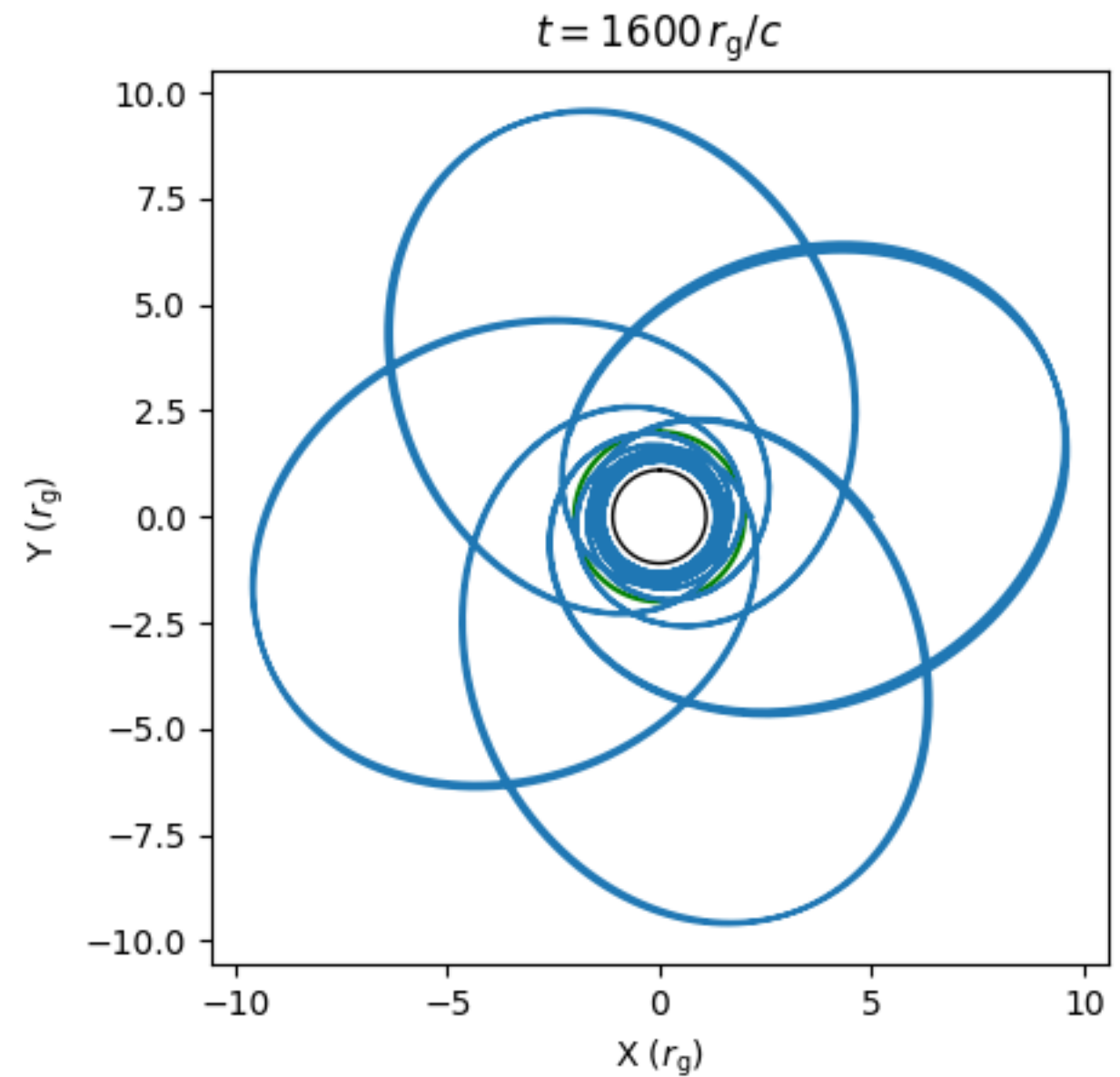
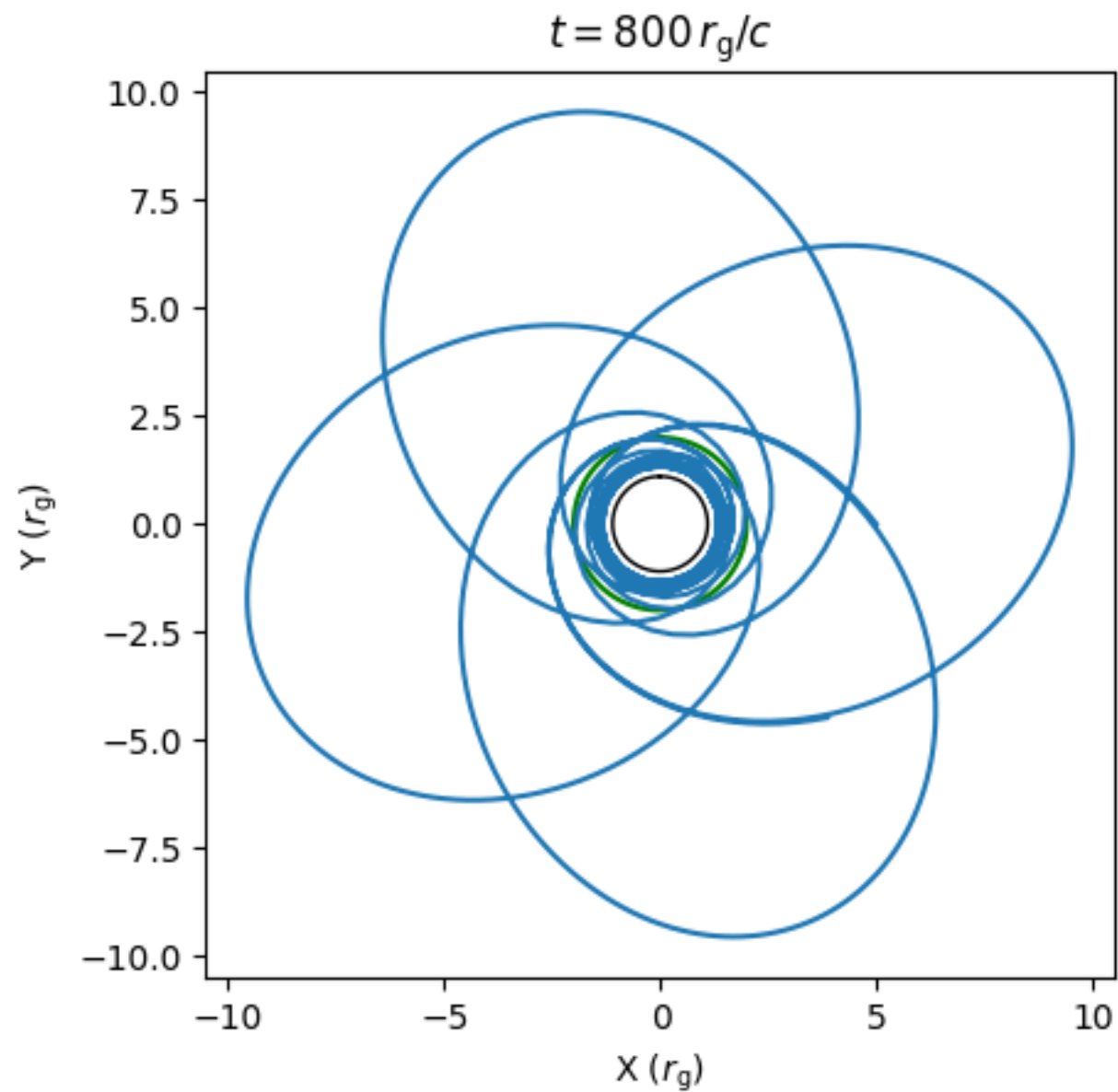
Symplectic integrator (implicit midpoint)
with $dt = 0.1 r_g/c$

Geodesic tests — $B = 0$

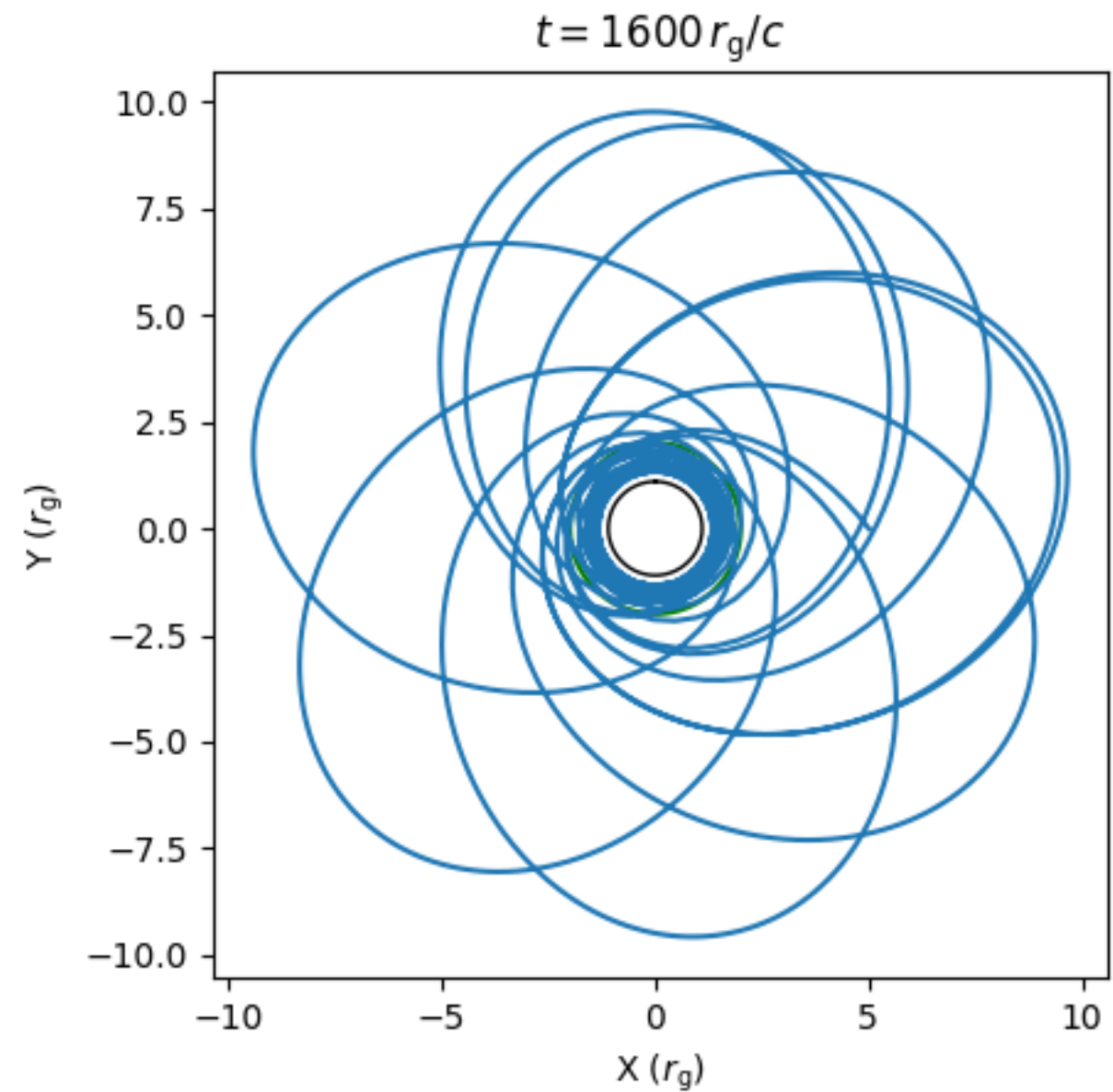
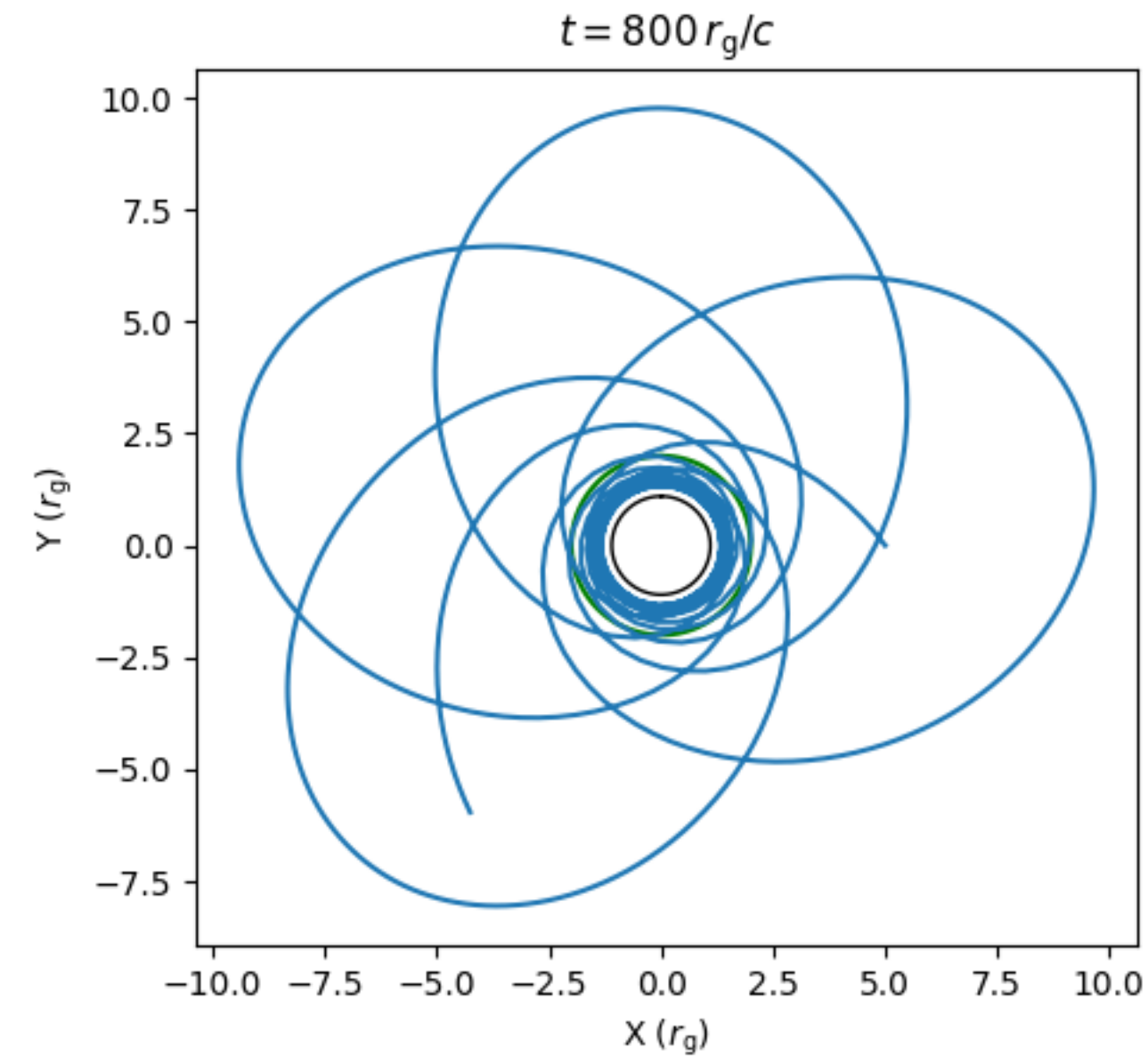


Periodic orbits: Levin & Perez-Giz 2008

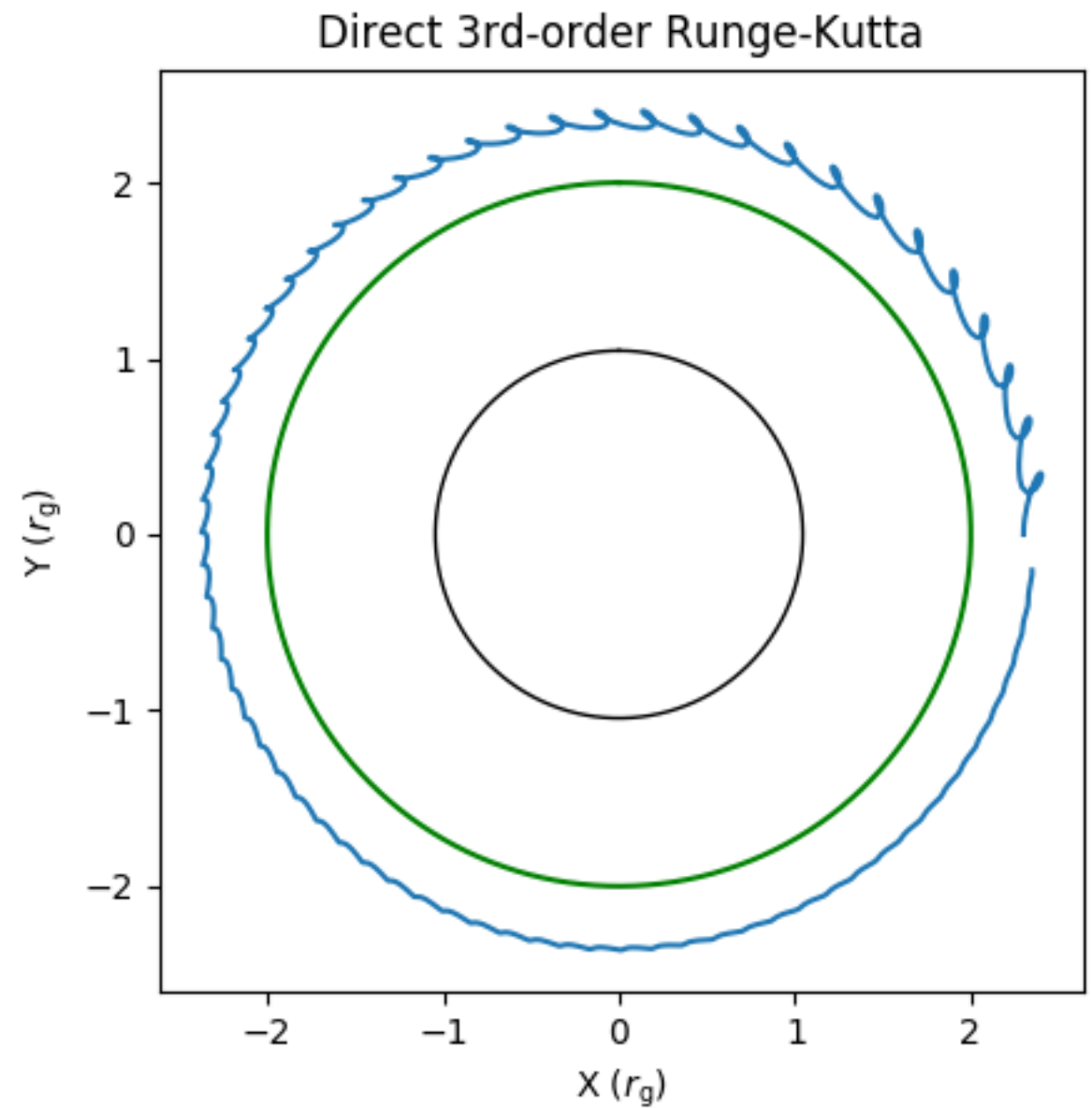
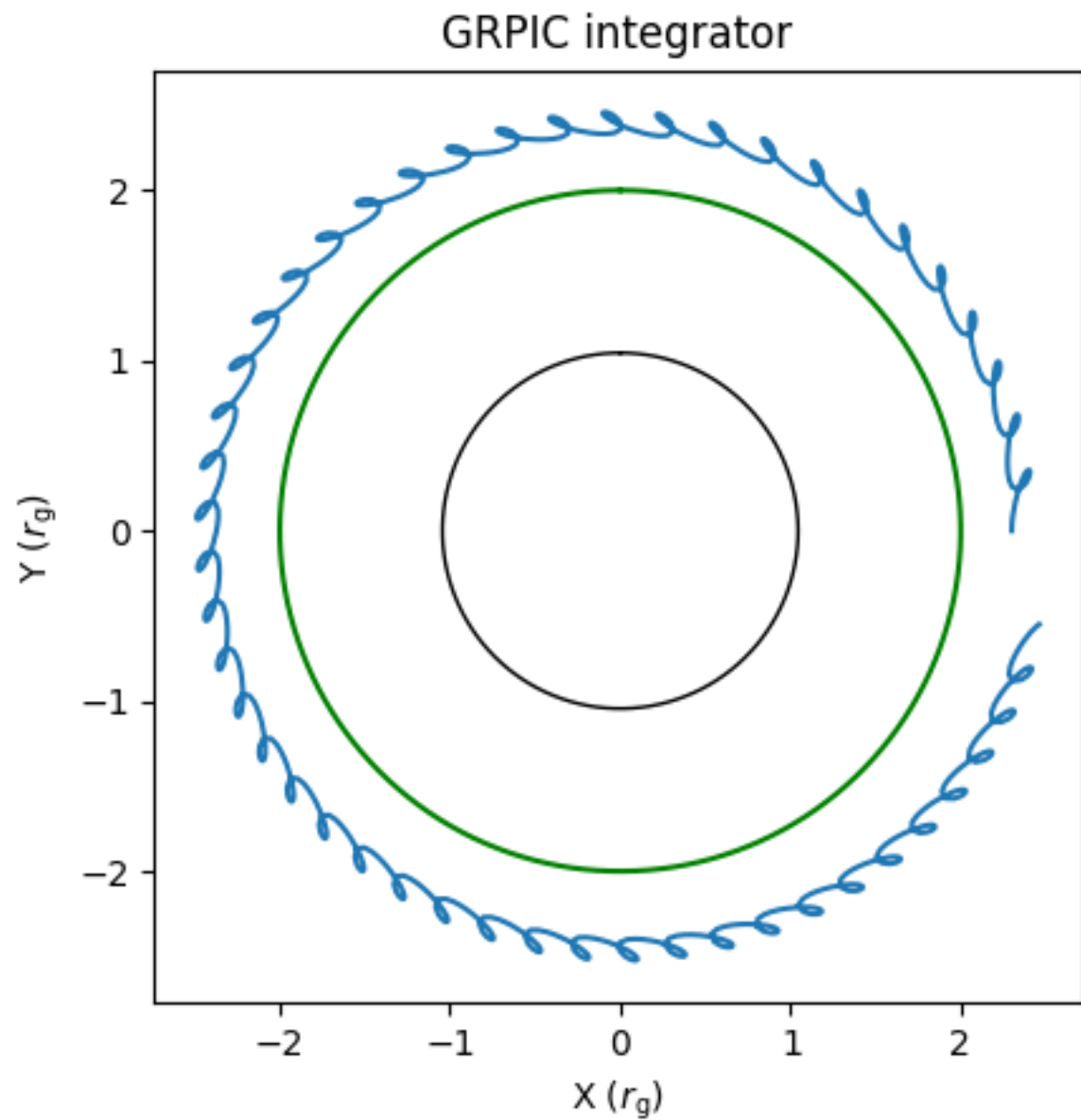
Symplectic integrator @ $dt = r_g/c$



3rd-order Runge-Kutta @ $dt = r_g/c$



Add ***B*** and ***D*** – particle in “Wald” vacuum field



Kerr metric: $a = 0.999$

equatorial plane

Kinetic problem set-up

Characteristic quantities

Larmor (gyro) radius: $r_{L,0} = 10^{-3} r_g \longrightarrow B_0$ uniform field strength at infinity

$$n_0 = \frac{\Omega_H B_0}{4\pi c e} \longrightarrow \sigma_0 = \frac{B_0^2}{4\pi n_0 m c^2} \approx 2000 \quad \text{strongly magnetized}$$

Scale hierarchy: $r_{L,0} \ll \delta_0 \ll r_g$

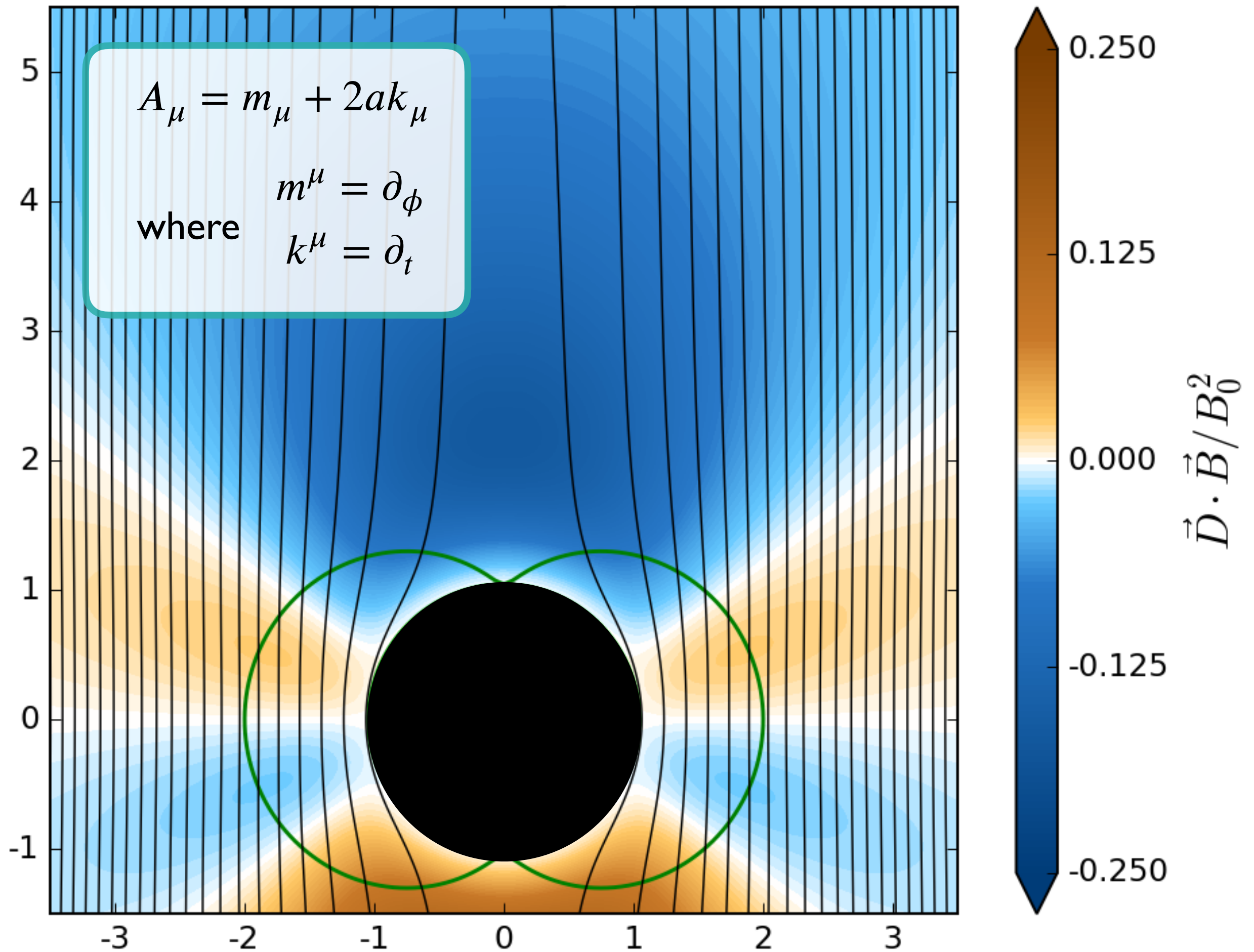
 plasma skin depth

Injection

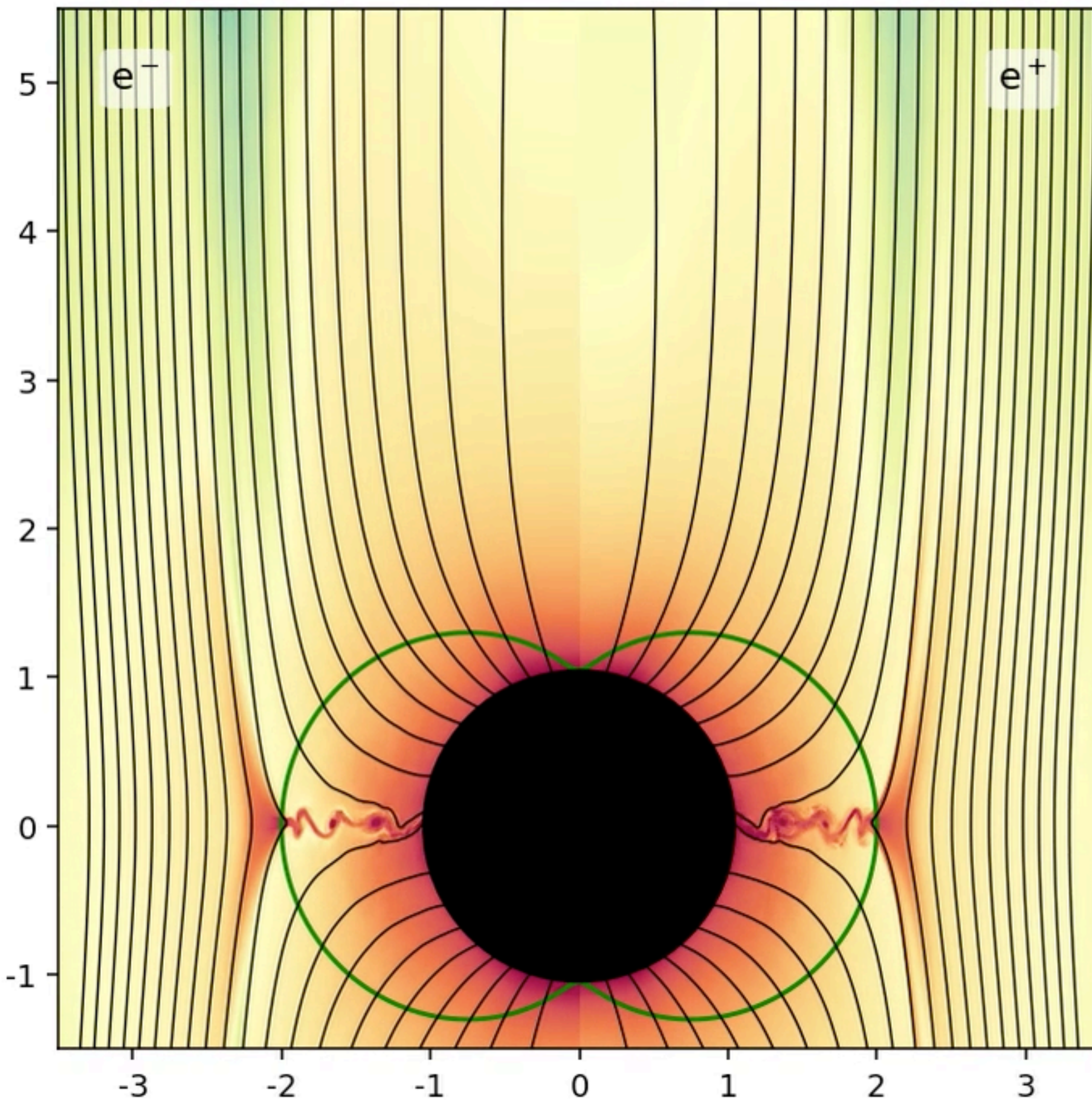
If: $\frac{\vec{D} \cdot \vec{B}}{B^2} > \epsilon_{D \cdot B}$ then: inject particles $n \propto |\vec{D} \cdot \vec{B}|$

Two runs: $\left[\begin{array}{ll} \epsilon_{D \cdot B} = 10^{-3} & \text{“high plasma supply”} \\ \epsilon_{D \cdot B} = 10^{-2} & \text{“low plasma supply”} \end{array} \right.$

Initial state: vacuum Wald EM steady-state solution — $a = 0.999$ Wald 1974



$t = 39.46 r_g/c$

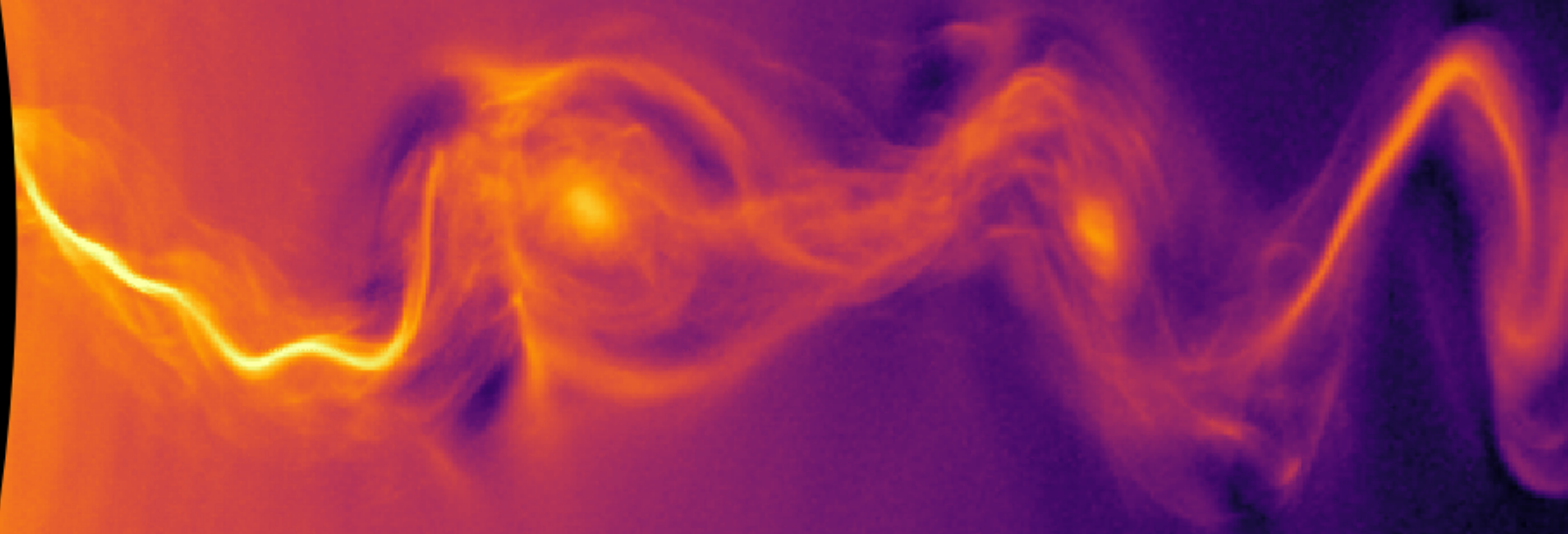


High Plasma
Supply

$$\epsilon_{D.B} = 10^{-3}$$

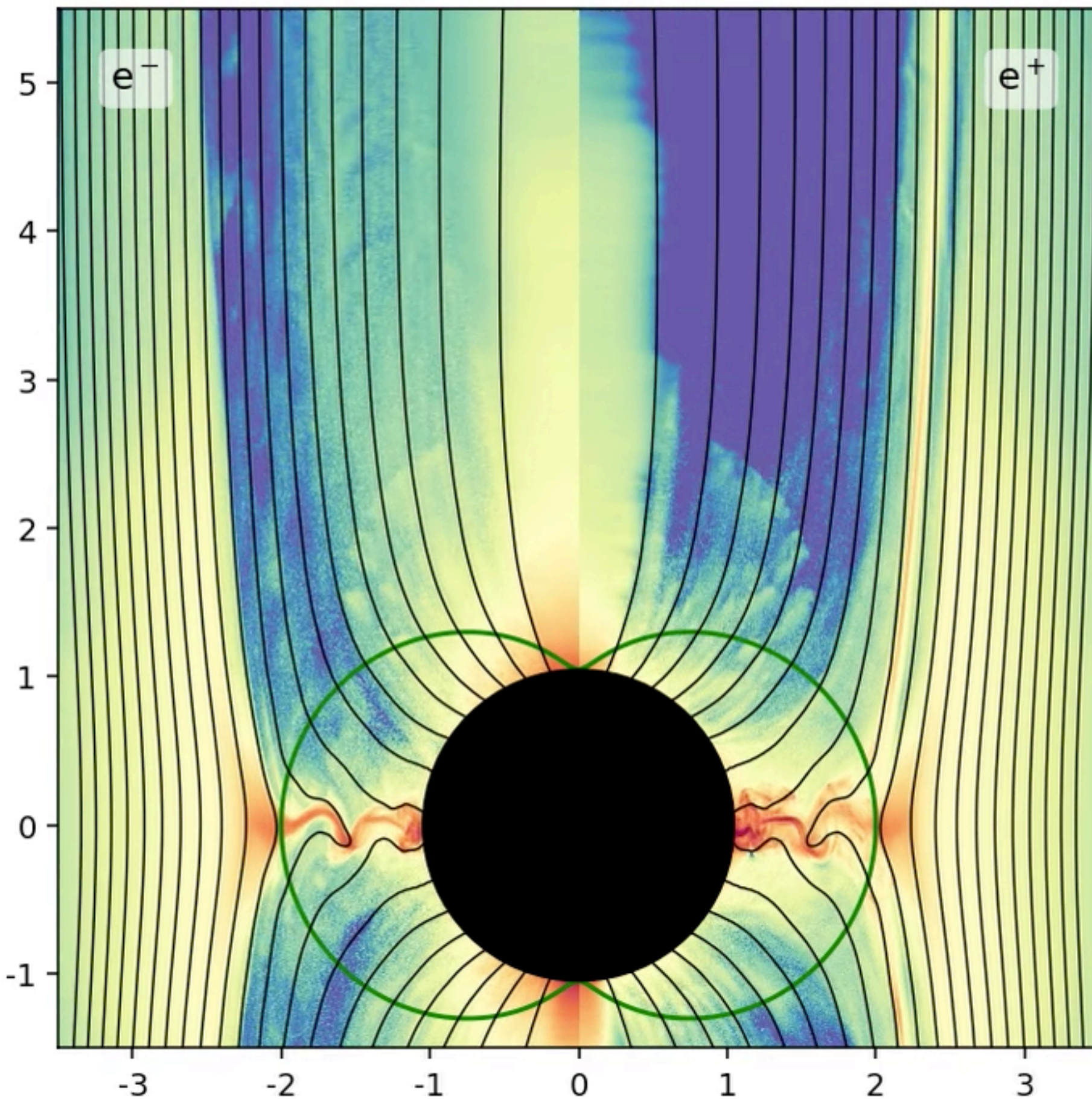
$\log_{10}(n/n_0)$

Collisionless plasma instabilities in full GR



positron density

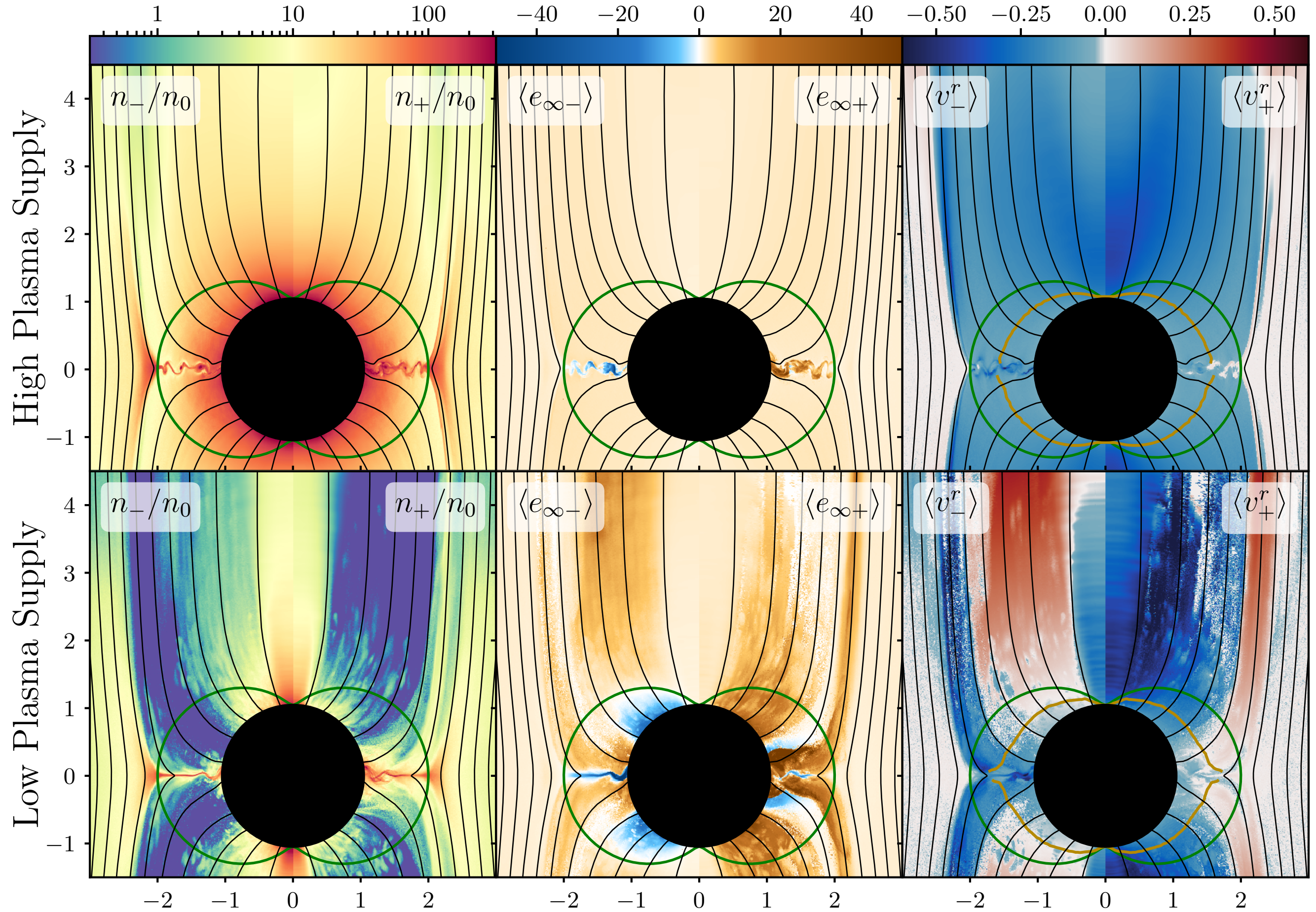
$$t = 37.81 r_g/c$$



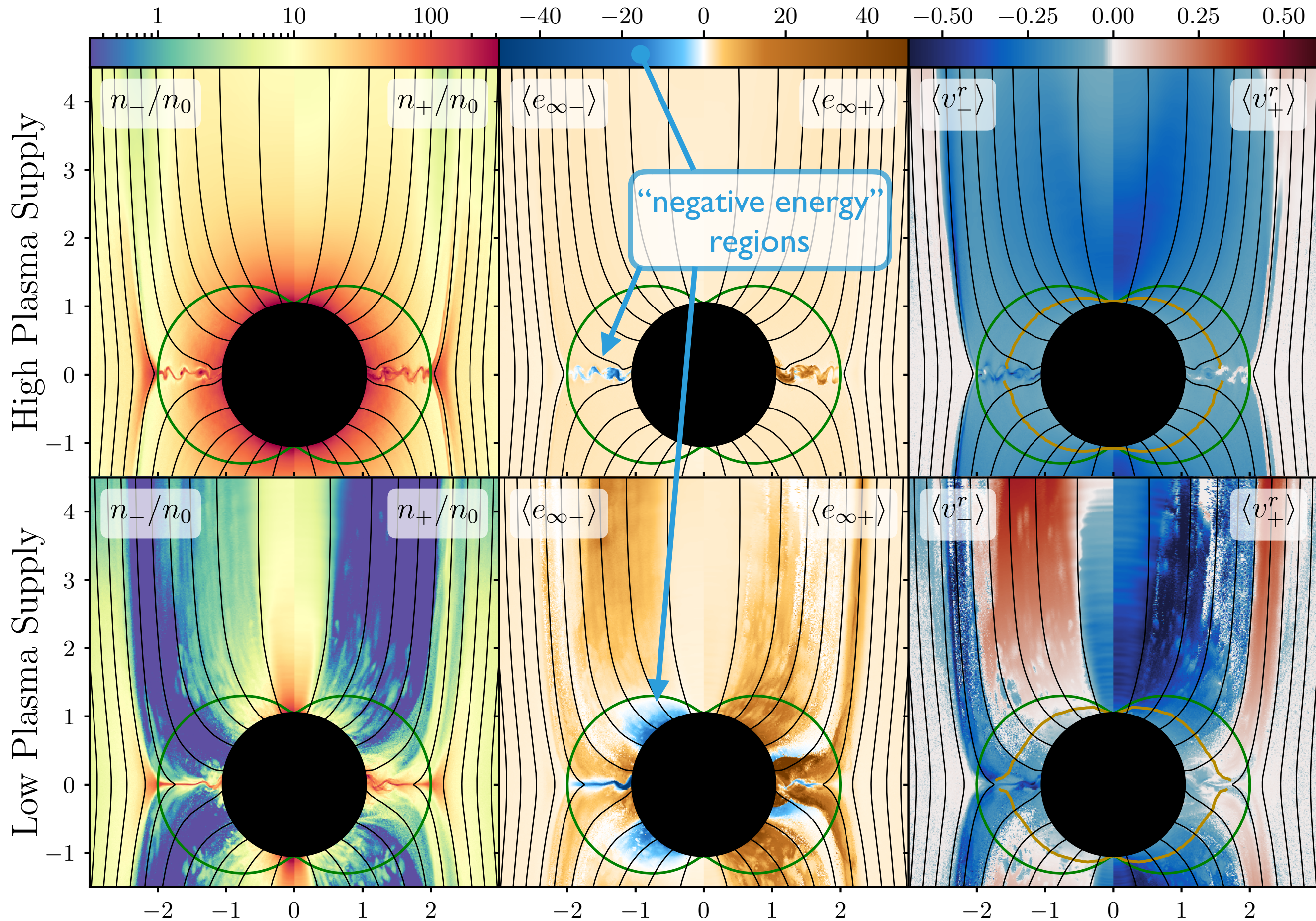
Low Plasma
Supply

$$\epsilon_{D.B} = 10^{-2}$$

$\log_{10}(n/n_0)$

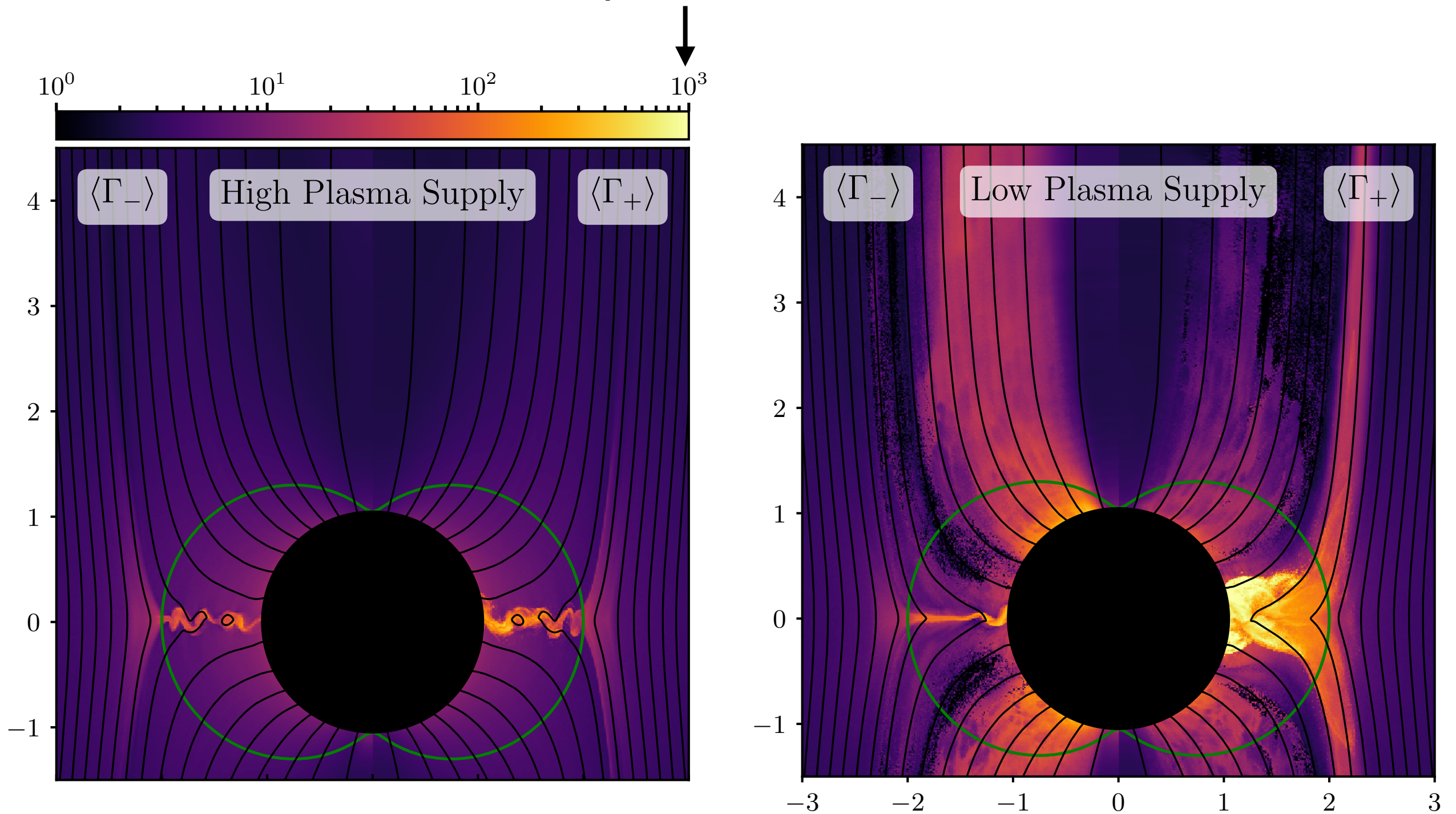


“energy at infinity”: $e_\infty = -u_t$



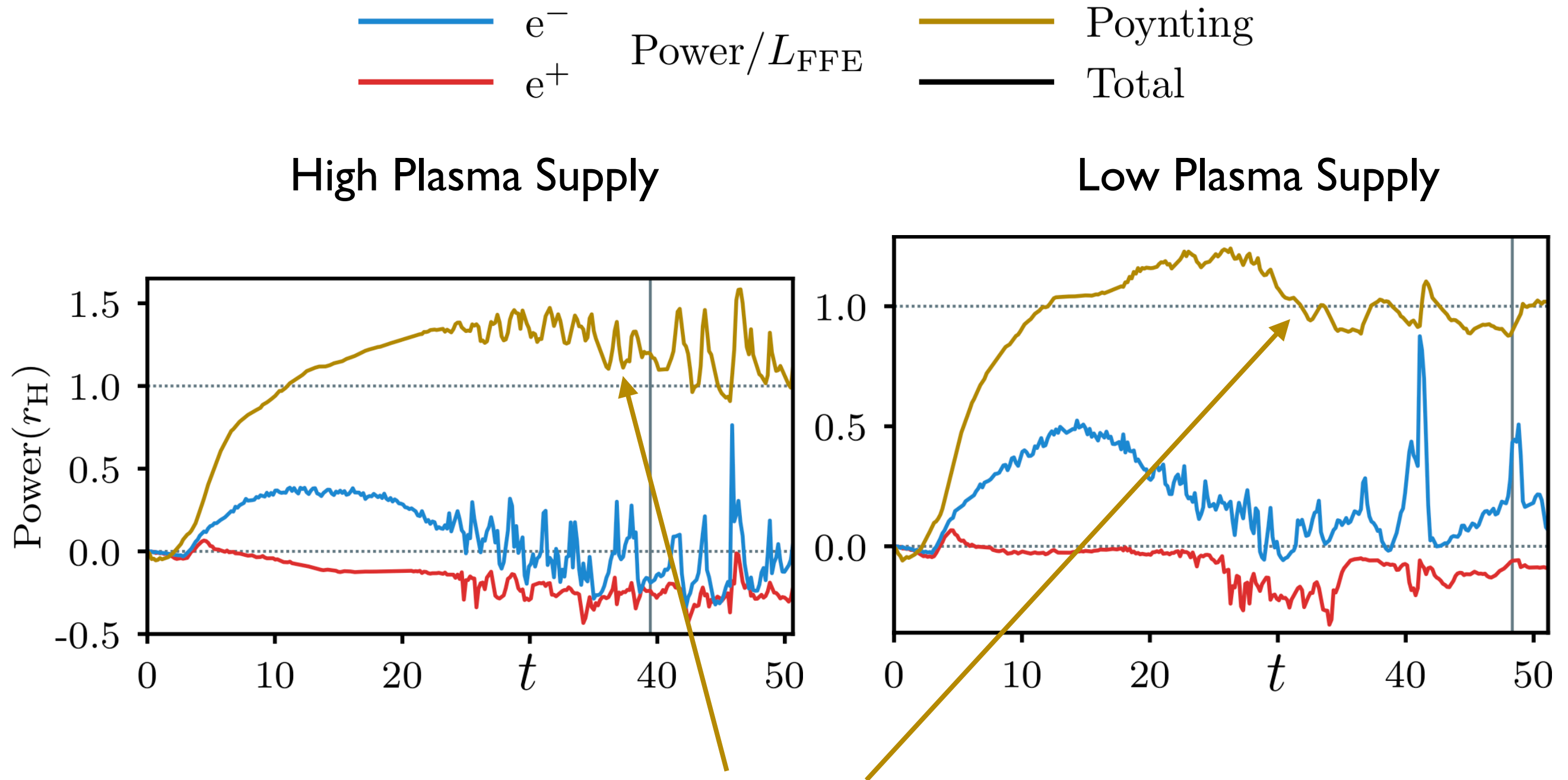
Particle acceleration

$\sim$ full potential $\Gamma_{\max}$



Average particle Lorentz factor

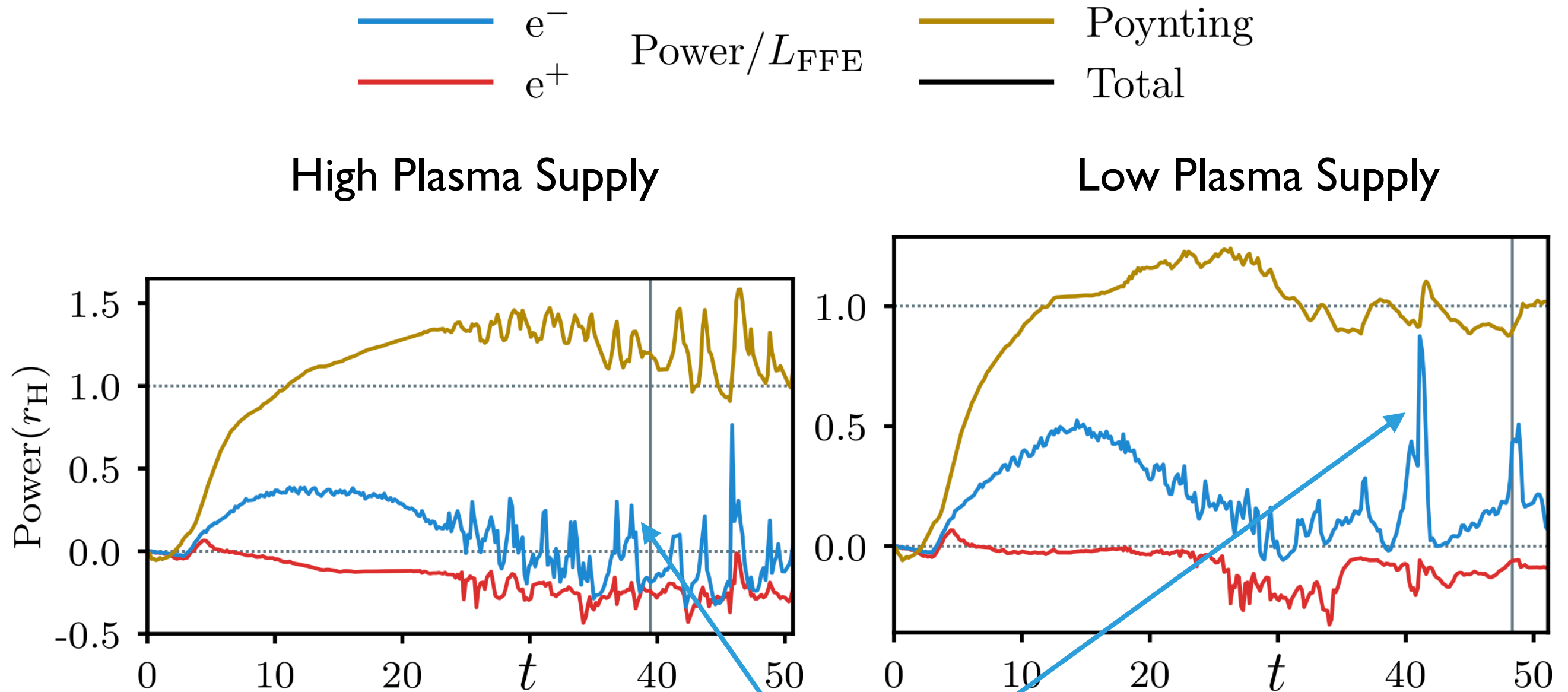
Energy at infinity flux: integrated over horizon



electromagnetic extraction of
black hole energy

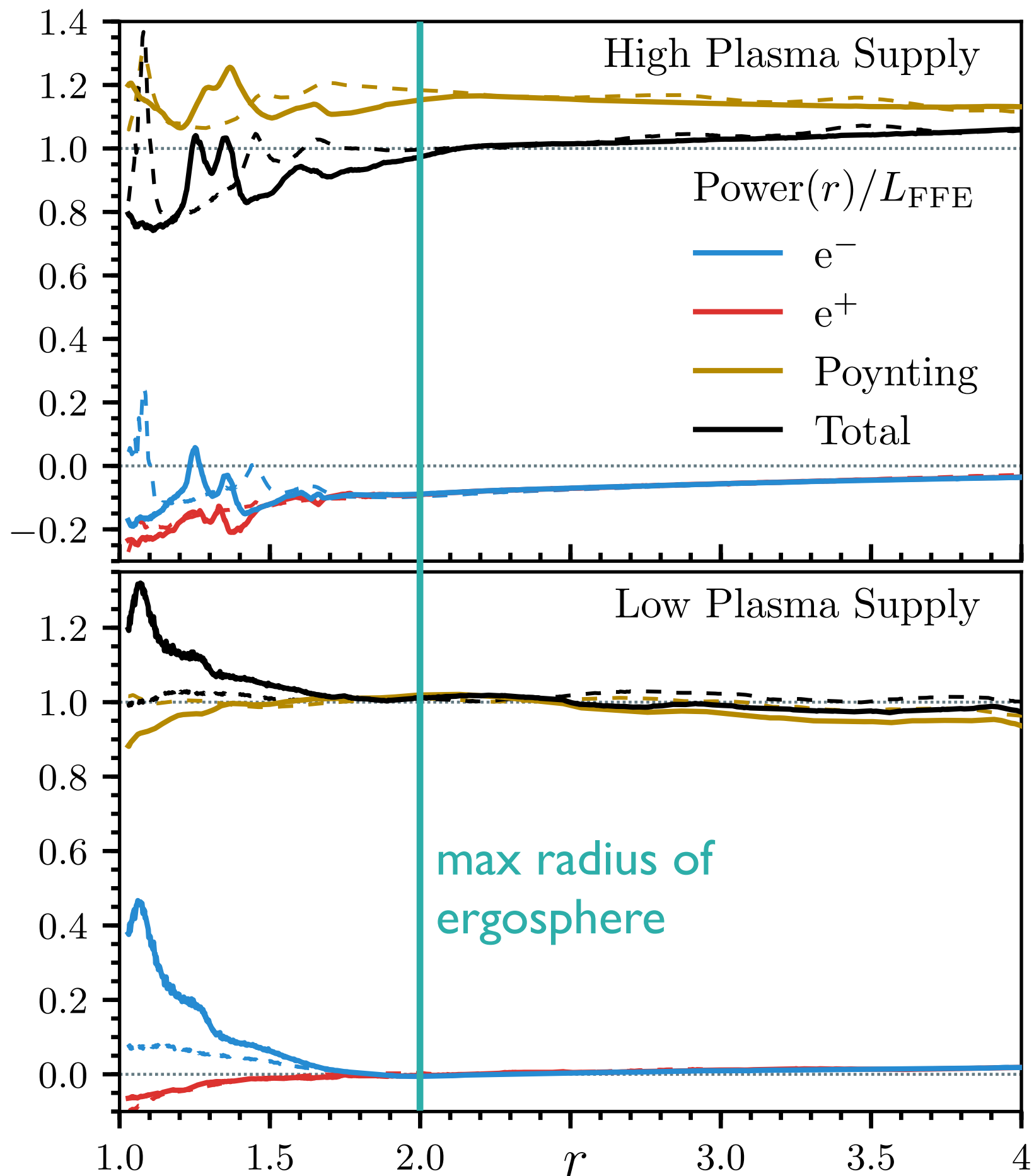
*Blandford-Znajek (1977) mechanism
for relativistic jets*

Energy at infinity flux: integrated over horizon



extraction of black hole
energy by **particles**

variant of *Penrose (1969) process*

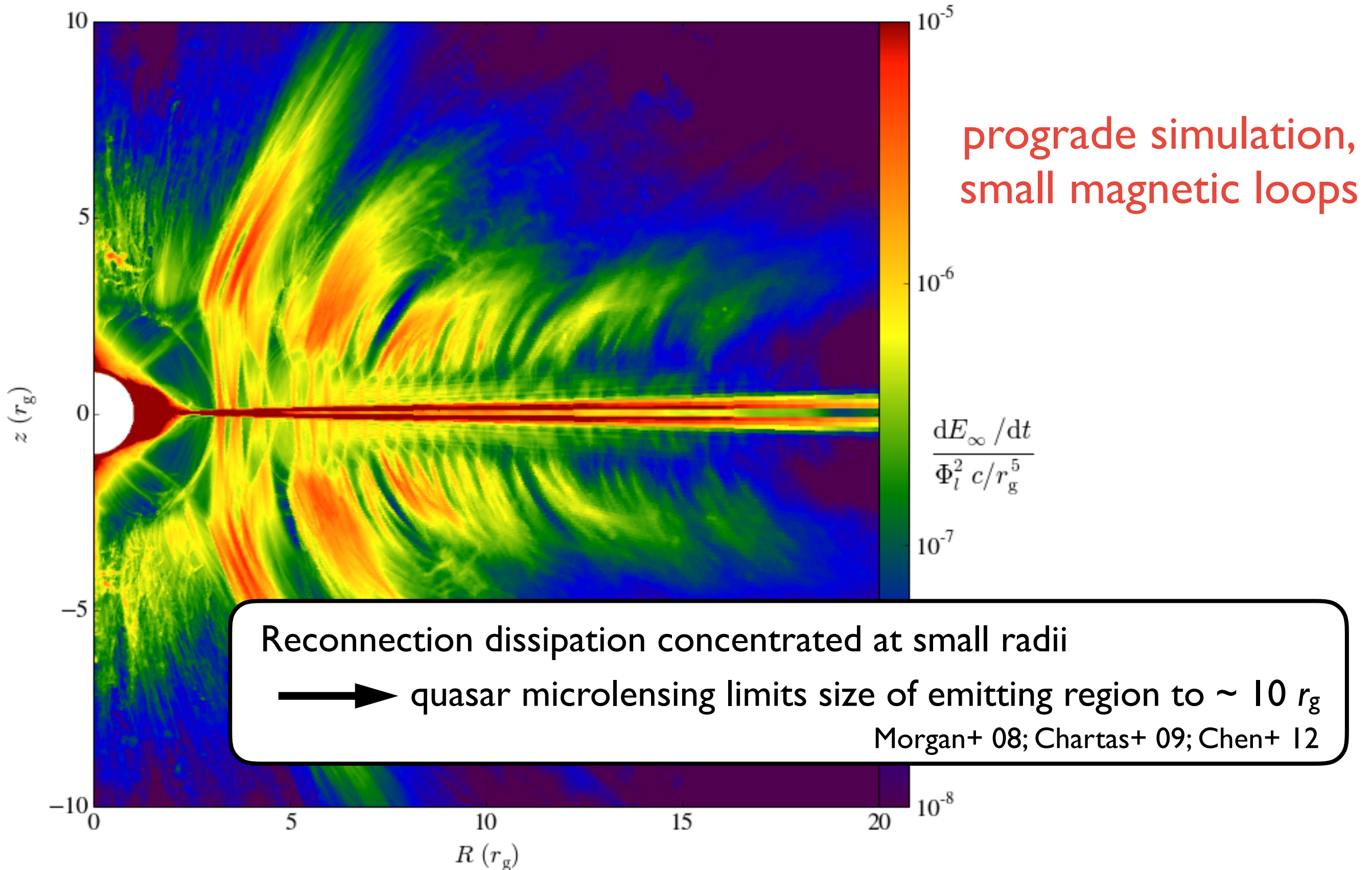


Energy at infinity flux
vs radius

time indicated by grey
line in previous plot

$t = 50 r_g/c$

Dissipation map – reconnection-heated X-ray corona?



How important are the particles' curvature terms?

Maxwell's
equations

$$\partial_t \mathbf{D} = \nabla \times \mathbf{H} - \mathbf{J}$$

$$\nabla \cdot \mathbf{D} = \rho_e$$

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E}$$

$$\nabla \cdot \mathbf{B} = 0$$

Constitutive relations:

$$\mathbf{E} = \alpha \mathbf{D} + \boldsymbol{\beta} \times \mathbf{B}$$

$$\mathbf{H} = \alpha \mathbf{B} - \boldsymbol{\beta} \times \mathbf{D}$$

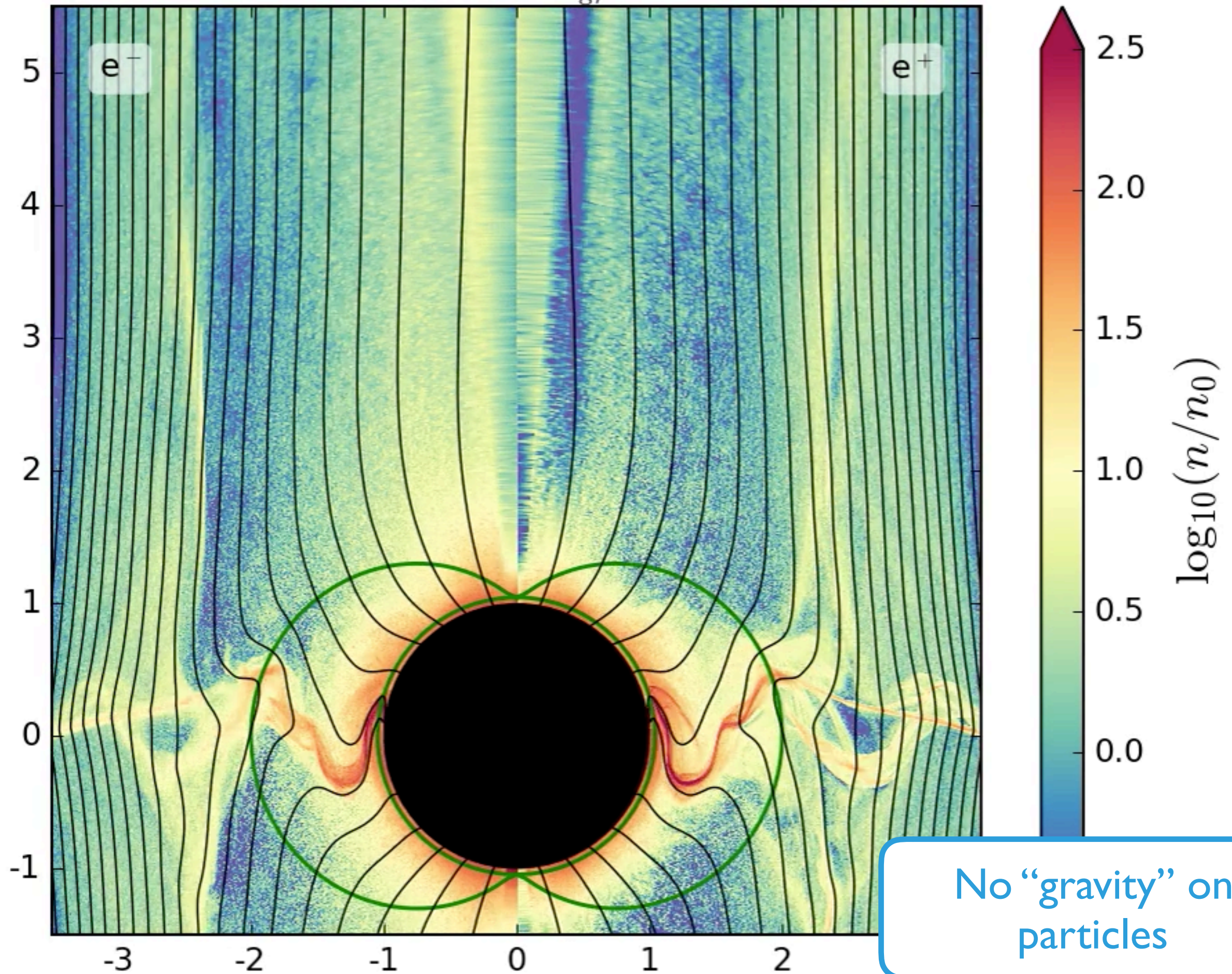
$$\frac{dx^i}{dt} = \frac{\alpha}{m\Gamma} p^i - \beta^i$$

Particle equations
of motion

Set metric derivatives to **flat spacetime** values

$$\frac{dp_i}{dt} = \left[-m\Gamma \partial_i \alpha + p_j \partial_i \beta^j - \frac{\alpha}{2\Gamma m} \partial_i (\gamma^{lm}) p_l p_m \right] + q \left\{ \alpha D_i + \epsilon_{ijk} (v^j + \beta^j) B^k \right\}$$

$$t = 30.60 r_g/c$$



Future Directions

Neutron star accretion

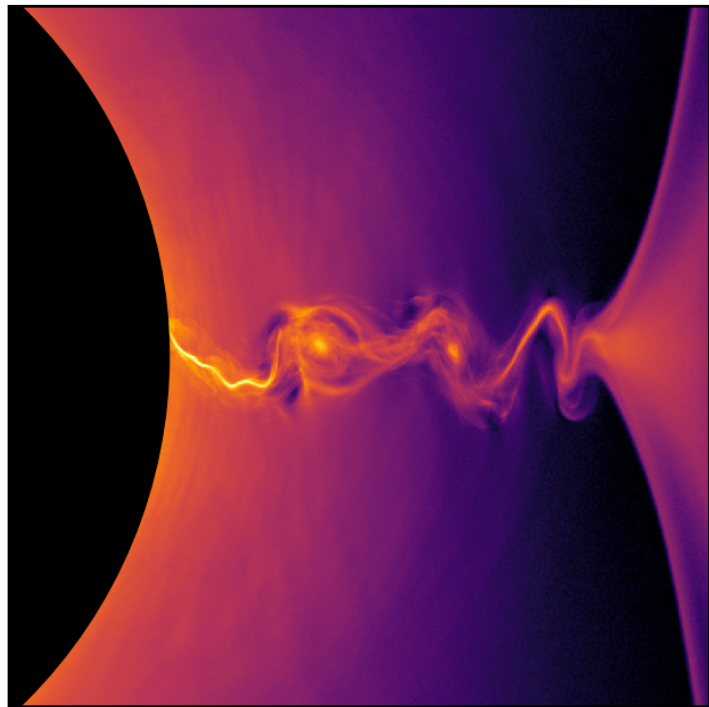
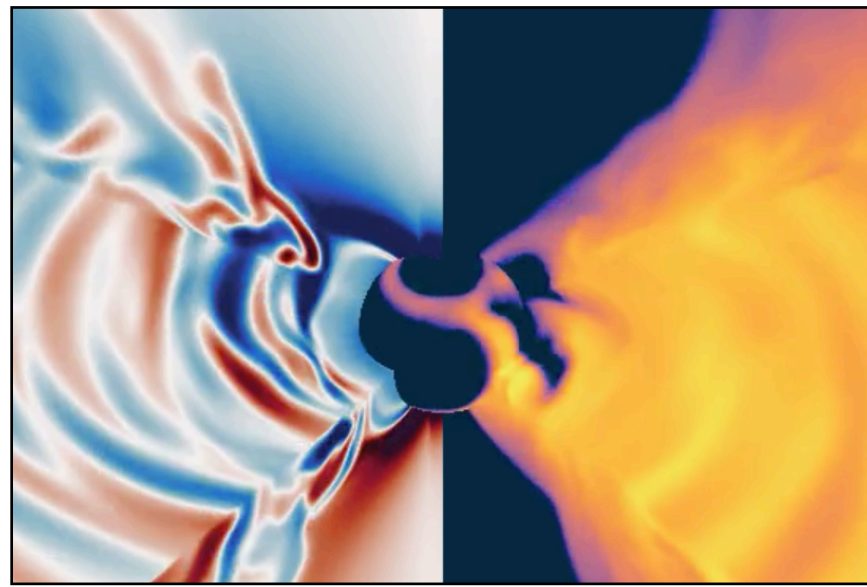
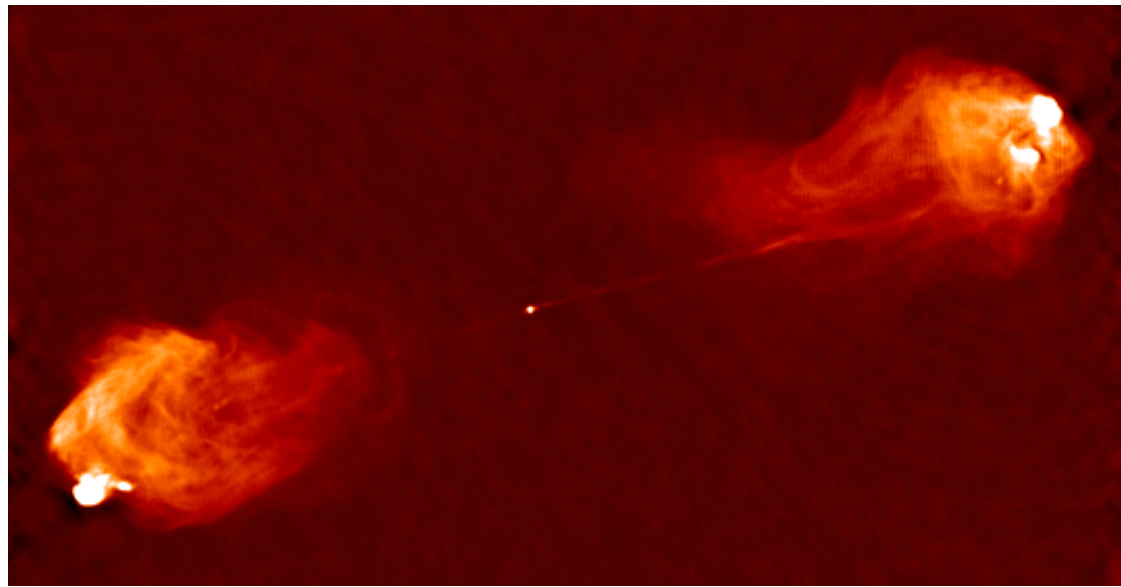
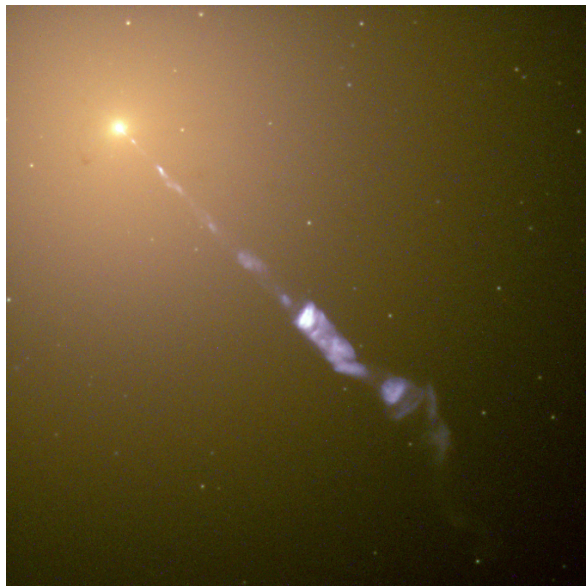
- Add radiation field to model pulsing ultra-luminous X-ray sources
- Directly compare black hole and neutron star accretors

Black hole coronae and jets

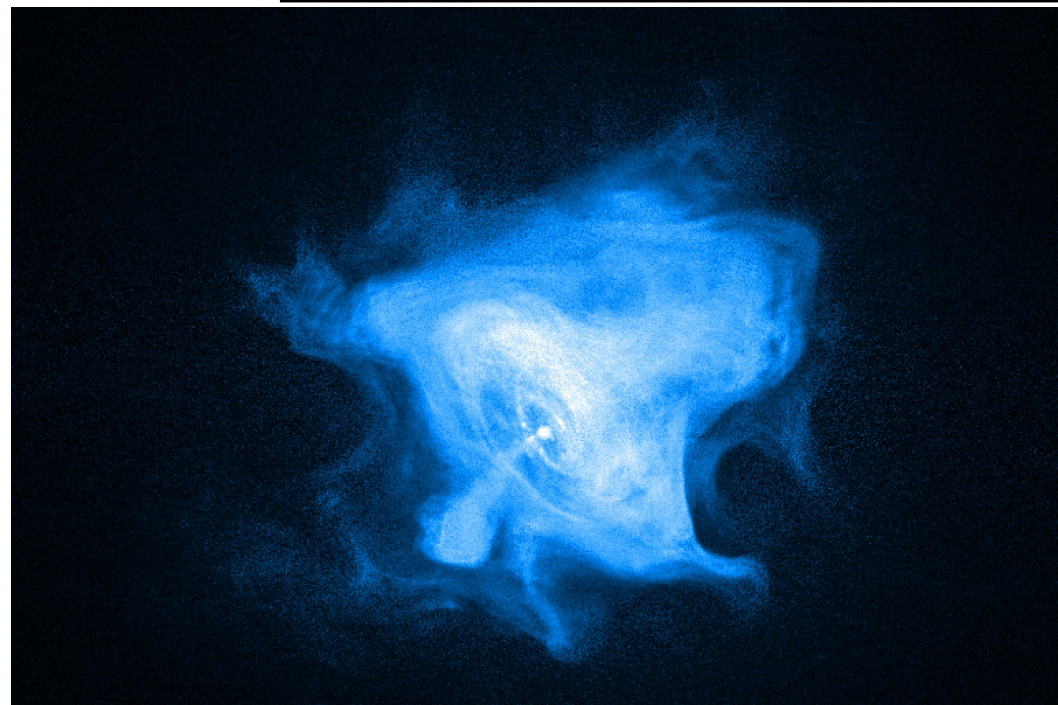
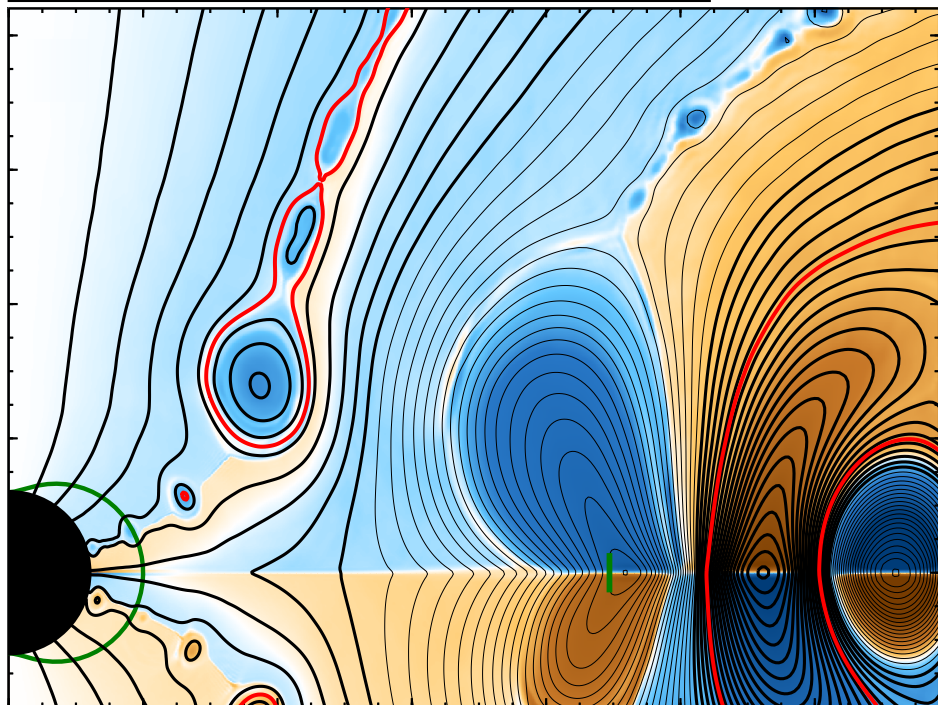
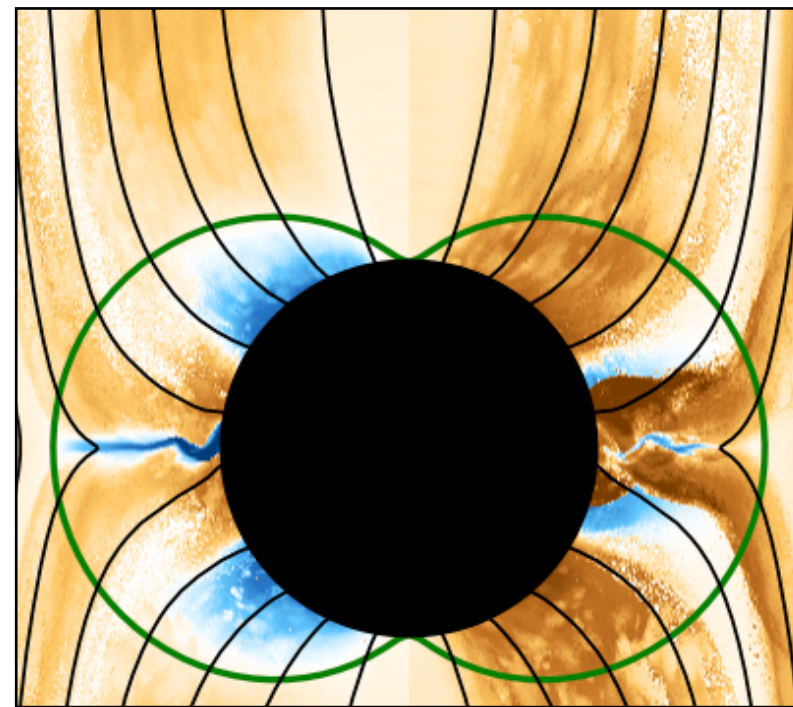
- 3D simulations: jets from general non-axisymmetric magnetic fields?
- Where does coronal magnetic dissipation occur?

Black hole plasma simulations

- Add realistic pair-creation physics: how do vacuum gaps behave?
- Full simulation of a collisionless accretion flow

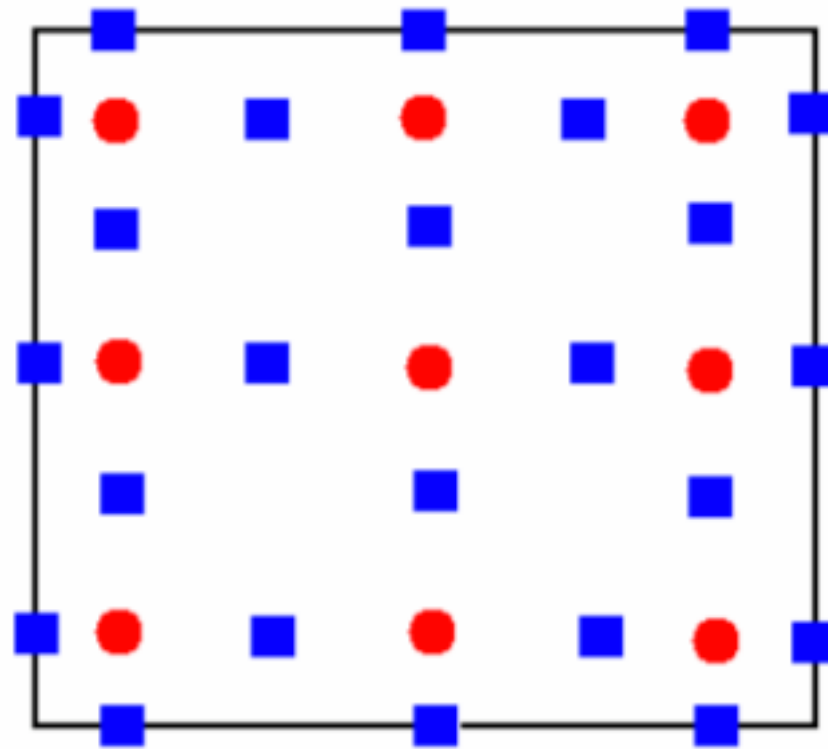


Questions?



Staggered Chebyshev method

Sun+ 2007



● Solution points

Gauss points

$$X_s = \frac{1}{2} \left[1 - \cos \left(\frac{2s-1}{2N} \cdot \pi \right) \right], \quad s=1,2,\dots,N.$$

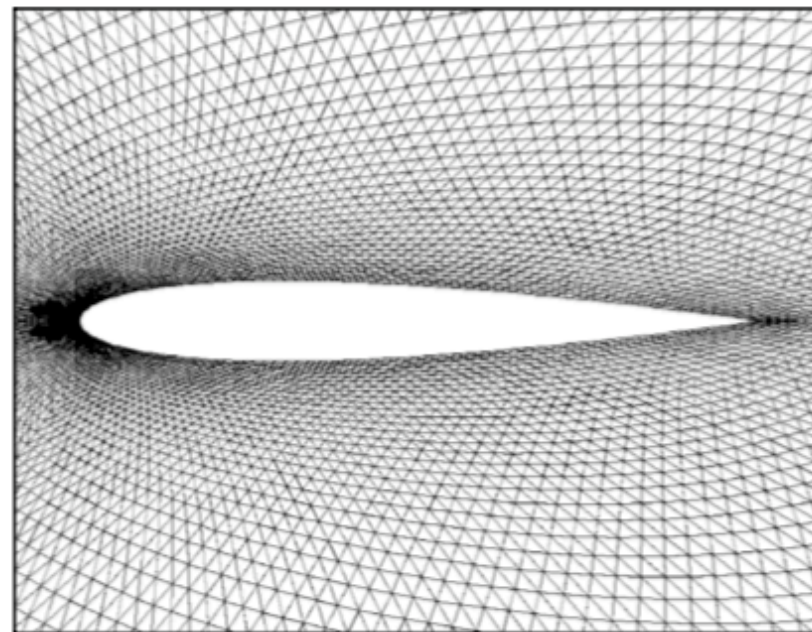
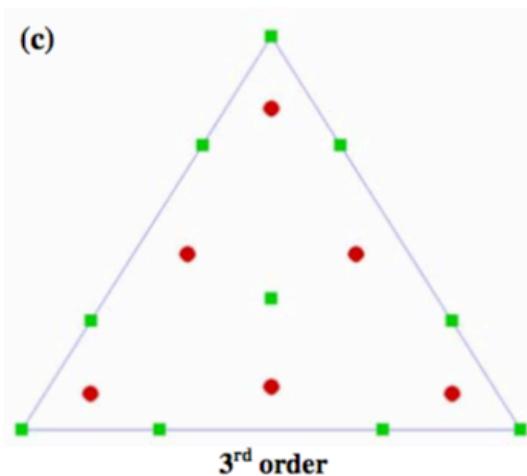
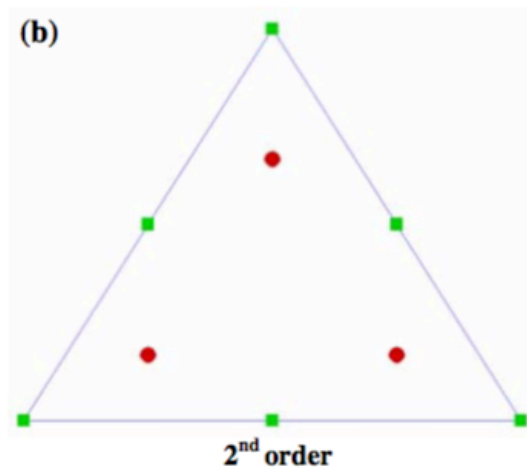
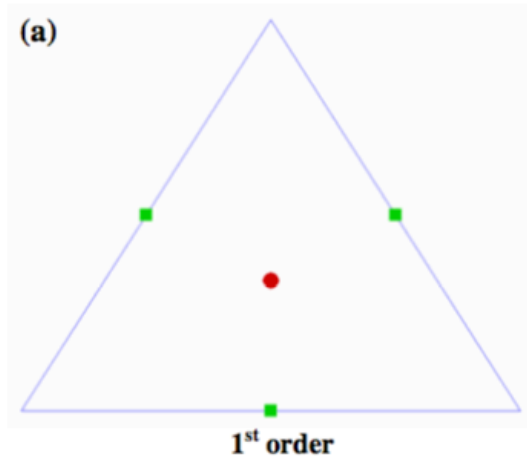
↕ spectral transforms

■ Flux points

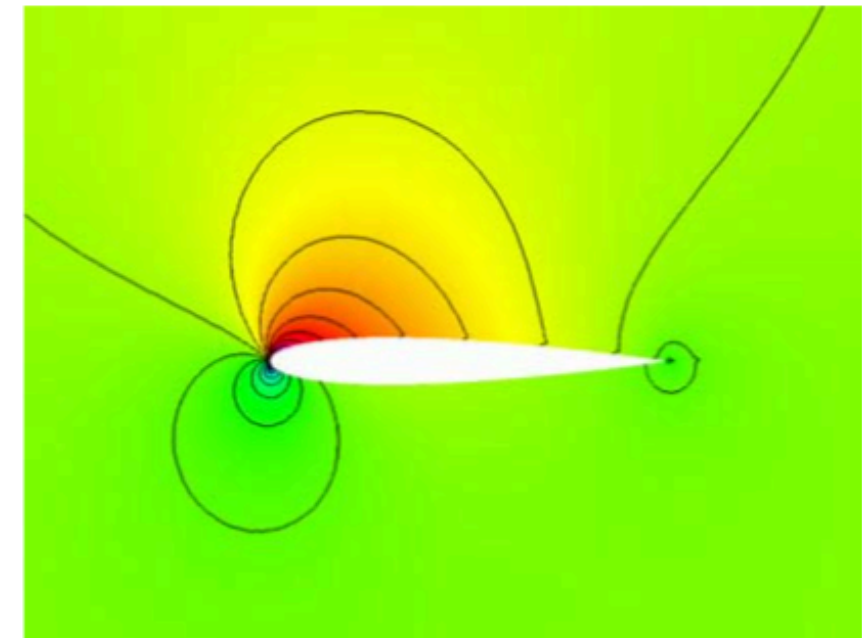
Gauss-Lobatto points — $X_{s+1/2} = \frac{1}{2} \left[1 - \cos \left(\frac{s}{N} \cdot \pi \right) \right], \quad s=0,1,\dots,N.$

Staggered Chebyshev method

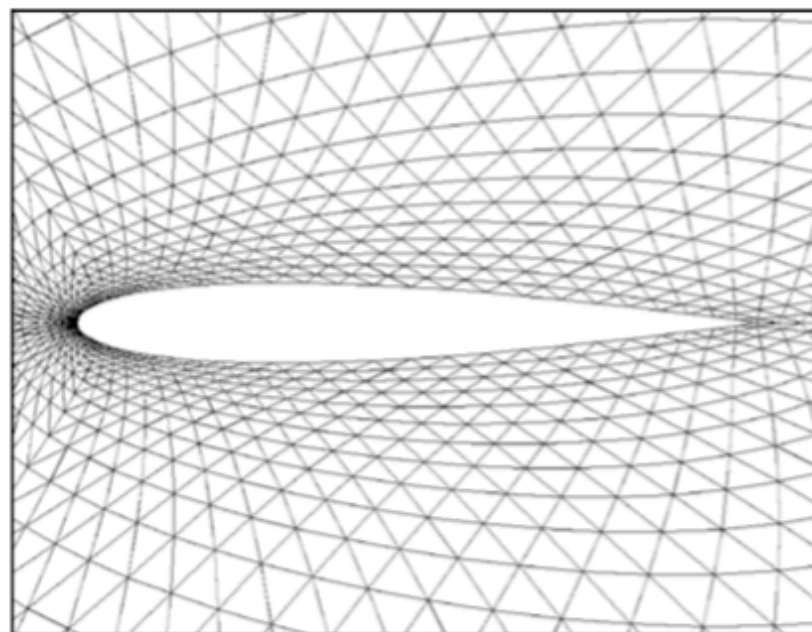
Equivalent to the *spectral difference* method



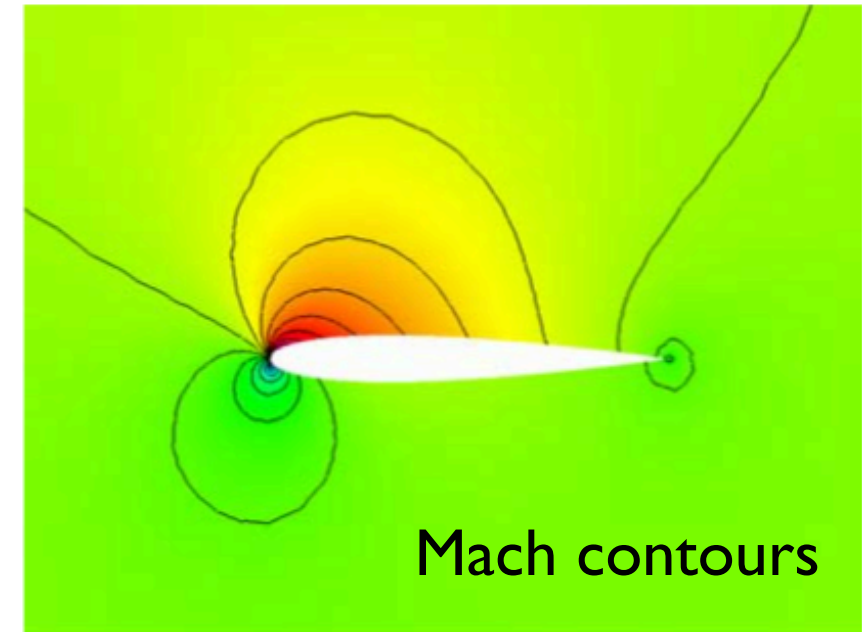
(a) 2nd FV grid



(a) 2nd order FV

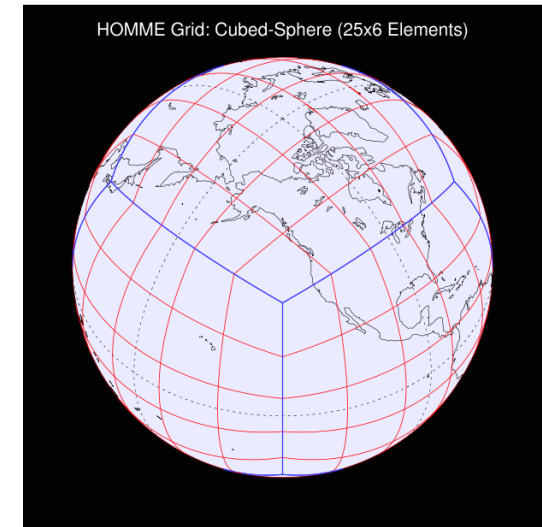
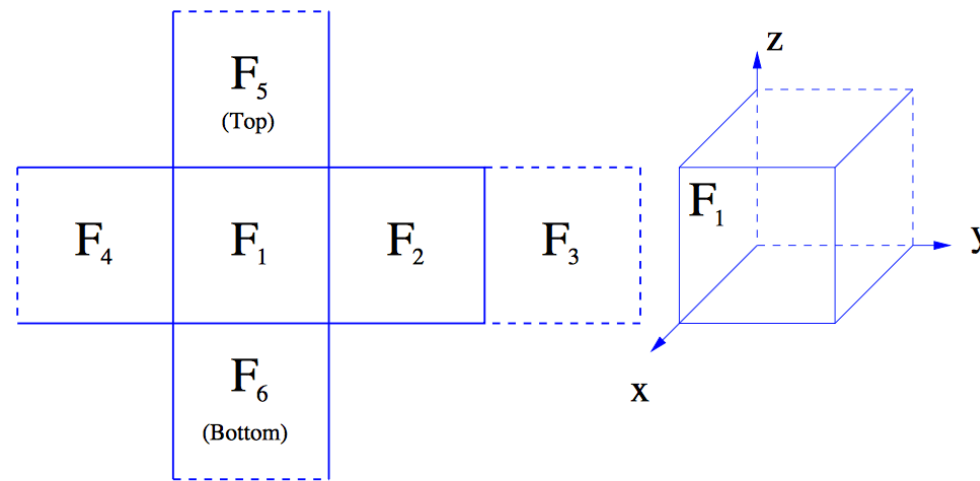
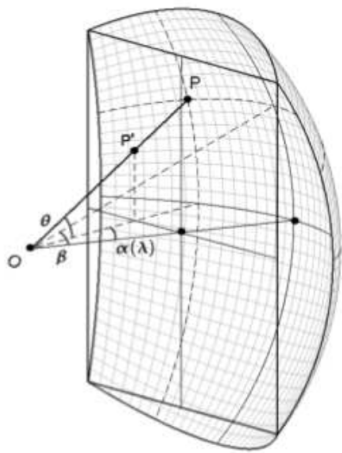


(b) 3rd SD grid



(b) 3rd order SD

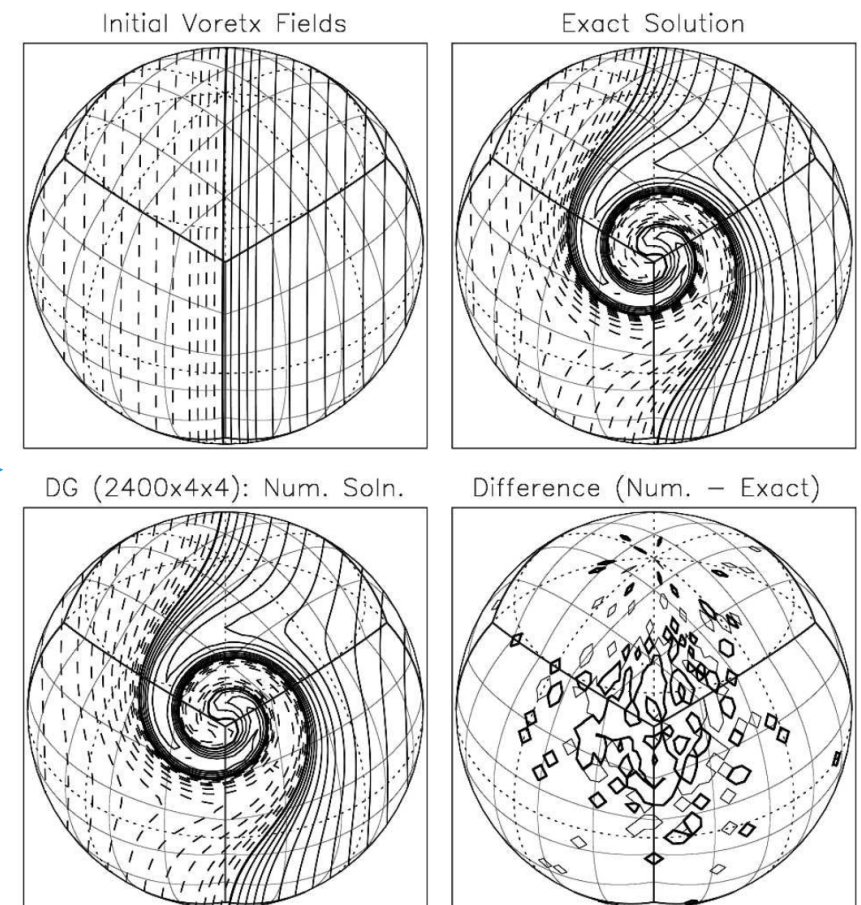
Well suited to cubed sphere geometry



3D spherical mesh via **gnomonic cubed sphere**

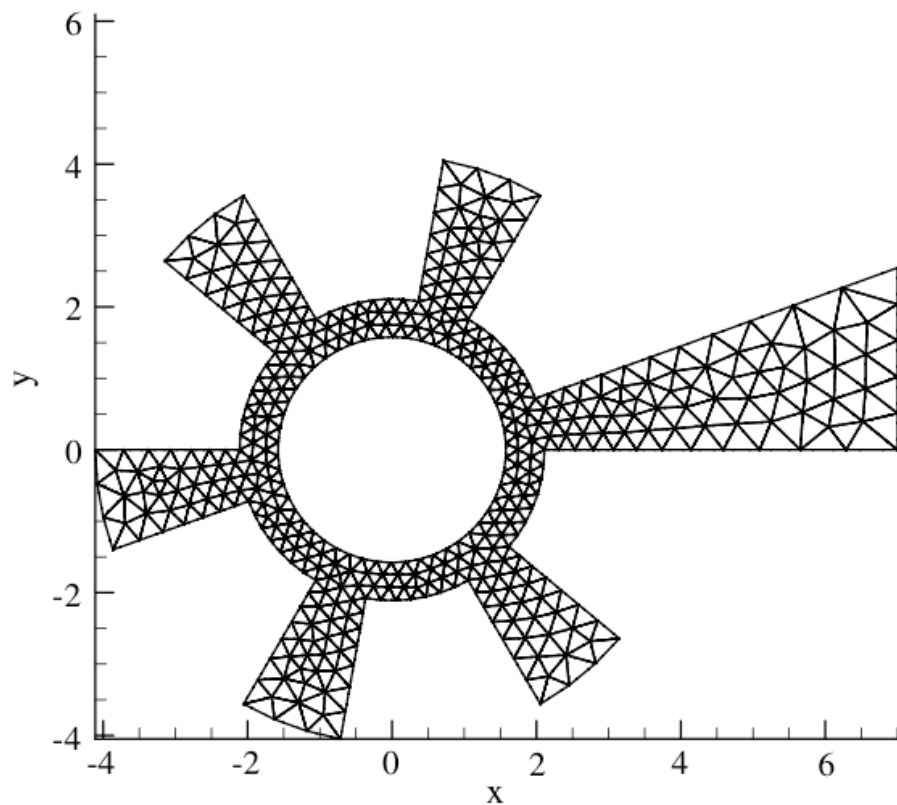
good results with closely related
discontinuous Galerkin method

No solution or flux points at corners

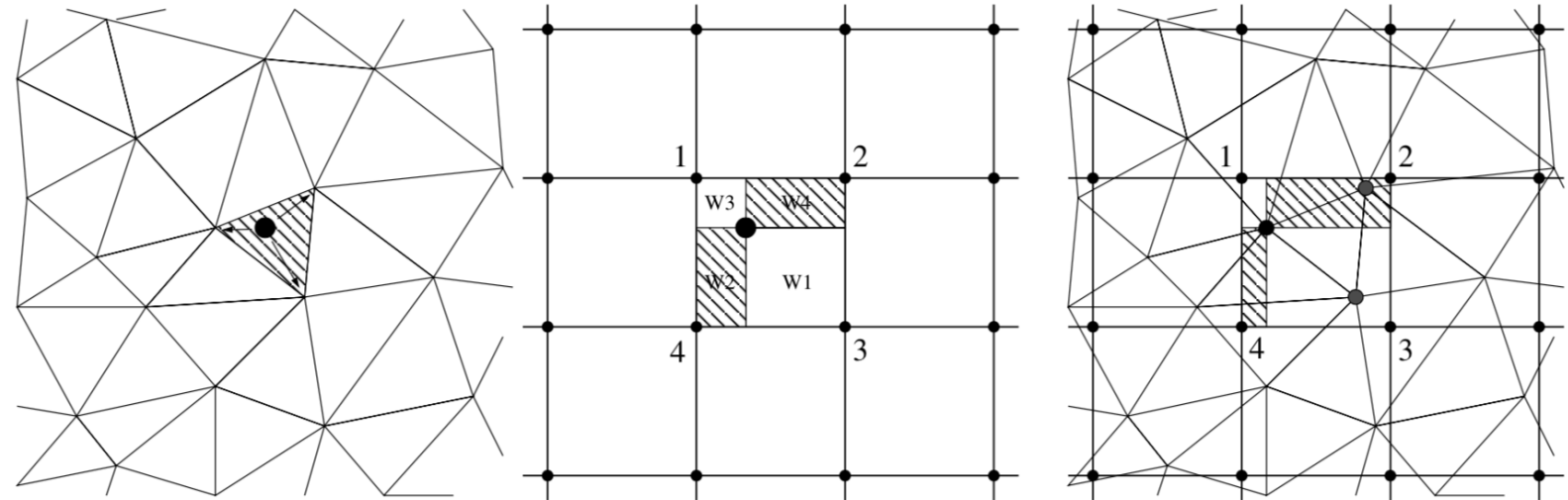


Wider plasma simulation community

Discontinuous Galerkin-based PIC codes



Jacobs & Hesthaven 2006



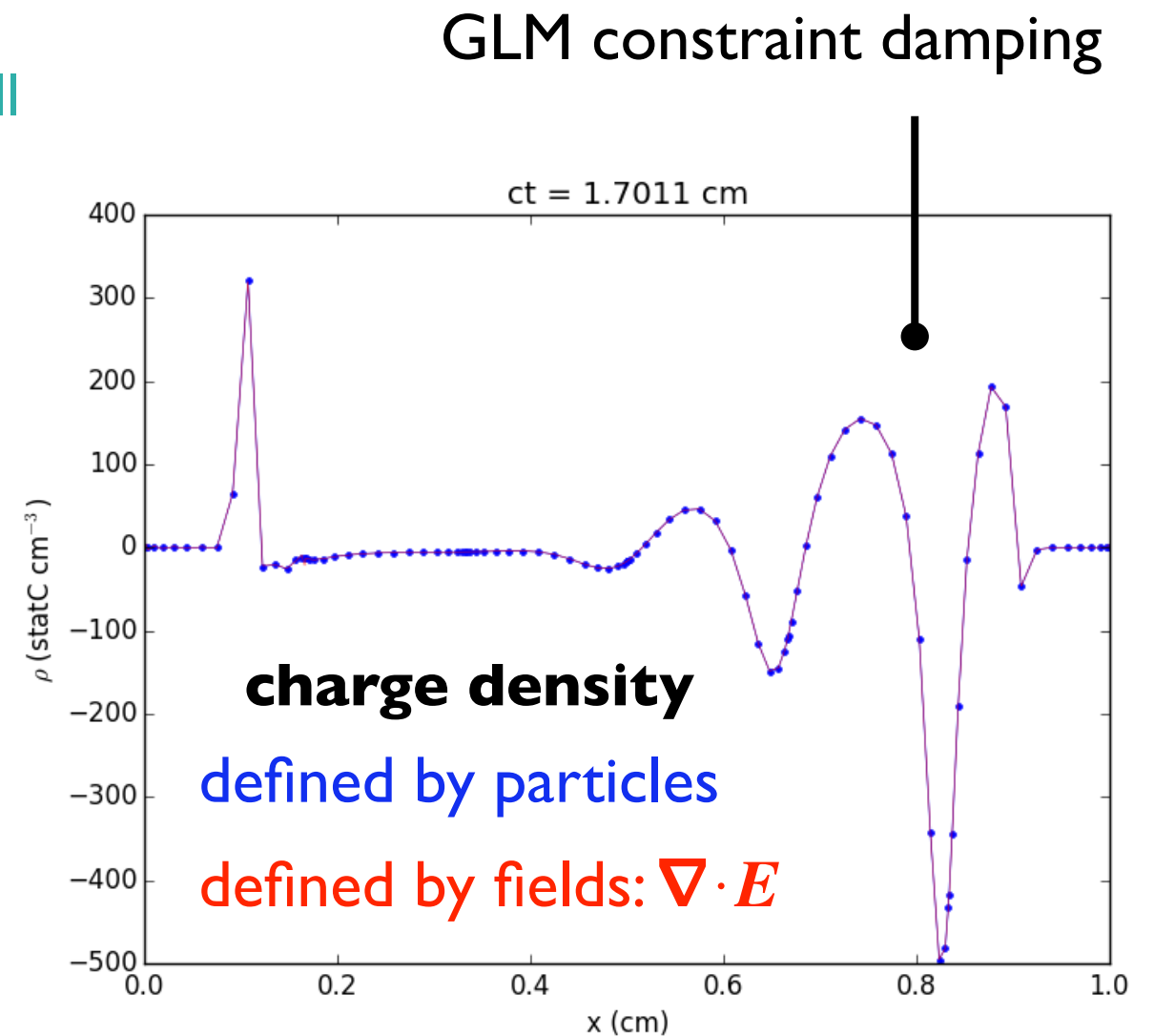
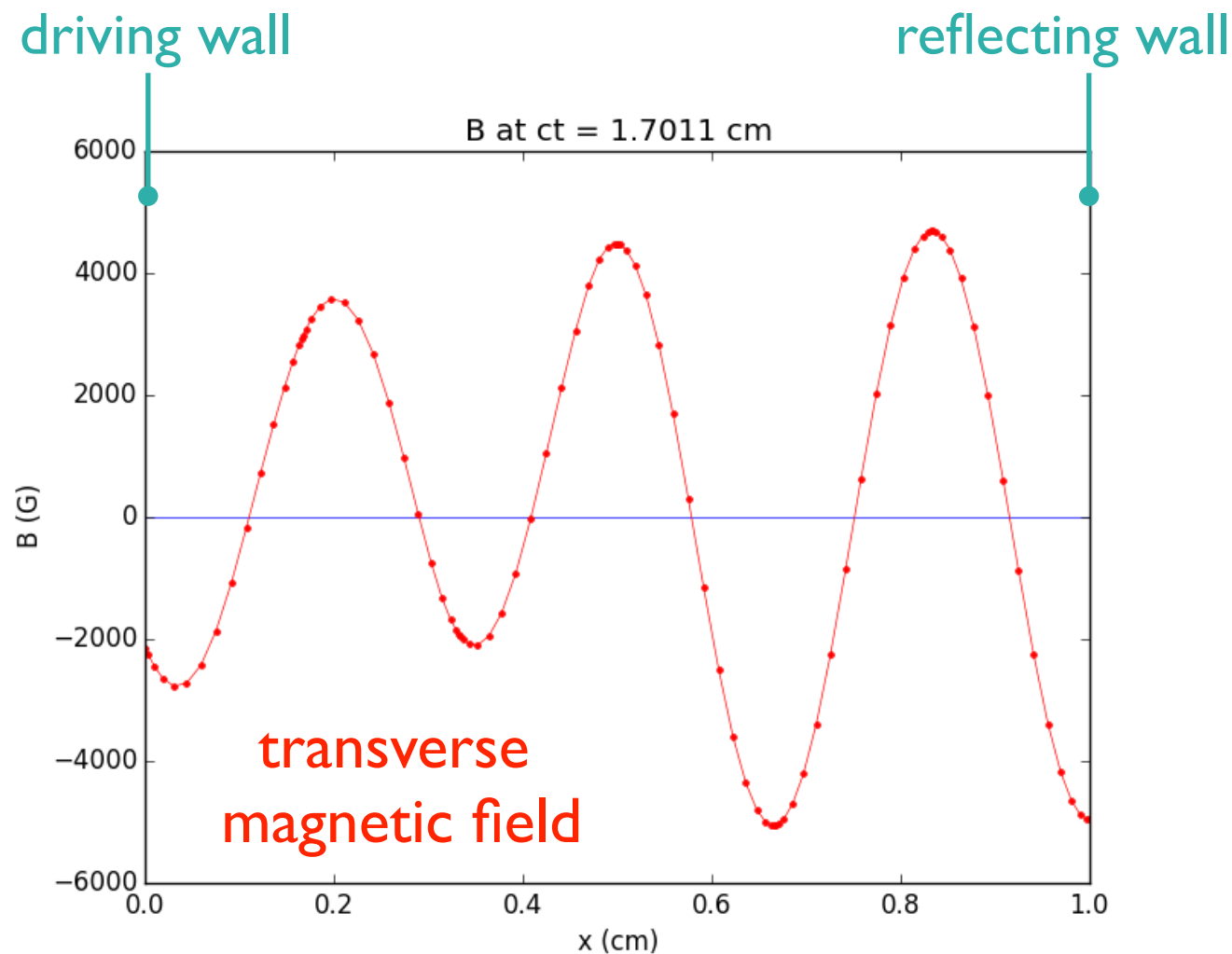
Stindl+ 2011

1D pathfinder code — PIC & FFE

Example: transverse EM wave driven through plasma

6 x 16-point elements

flat spacetime



Constraint equations: hyperbolic-parabolic generalised Lagrange multipliers

Munz+ 2000, Dedner+ 2002

$$t/P = 0.00$$

$$\omega_0 t = 0.00 \text{ rads}$$

Twist ring
between 9 & 12
times rot. flux

