PRINCETON PLASMA PHYSICS LABORATORY

PPPL Theory Seminar

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Collisionless mechanisms of plasma transport in the presence of stochastic open magnetic field lines

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Outline

- **Motivation & Introduction**

  - Thermal quench and open stochastic magnetic fields

  - New 3-D kinetic capabilities for simulating plasma transport in stochastic fields

- **Roles of stochastic open magnetic field lines**

  - 3-D topology of the open stochastic magnetic field lines

  - Magnetically passing and trapped particles

- **Roles of self-consistent electric fields**

  - $E_{\parallel}$: Ambipolarity of plasma transport
    Passing-trapping condition of electrons

  - $E_{\perp}$: ExB mixing effects on the plasma transport

- **Summary**

Thermal quench transport is critical issue in tokamak disruption problem

- **The plasma disruption is a major challenge of tokamak fusion plasma**
  - Thermal Quench (TQ) and Current Quench (CQ)
  - Rapid release of thermal and magnetic energy can damage to PFCs

- **Causes of TQ depend on causes of disruption**
  - Intentional plasma shutdown for machine protection

    - Impurity pellet injection or massive gas puffing
    - Radiative cooling of bulk thermal plasma

  - Disruptive MHD instabilities

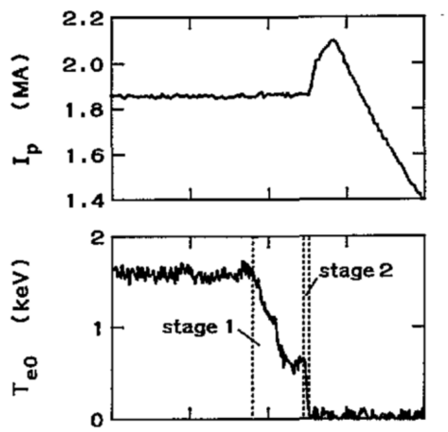
    - Break magnetic surface → magnetic stochasticity

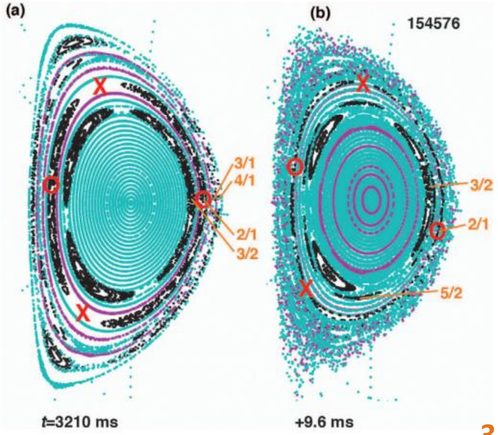
    - Vertical displacement events

  - Duration of TQ ~ a few milliseconds → huge heat load to PFCs

➡ Plasma transport mechanism in the open stochastic magnetic field lines is the key!

F. C. Schuller, PPCF (1995)





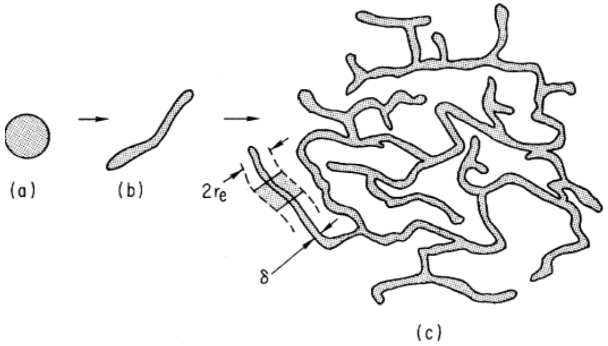
R. Sweeney, NF (2018)

Plasma transport mechanism in stochastic magnetic fields is a long-standing research subject

- **Parallel transport along stochastic field lines with collisional cross-field decorrelation process**

  [Rechester & Rosenbluth (PRL'1978), Krommes (JPP'1983)]

  - Spatial diffusion of stochastic field lines: $\langle(\Delta r)^2\rangle \sim 2LD_m$
  - Particle motion along and decorrelation from a given field line



- **∇B and curvature drift effects due to the toroidal geometry**

  [Mynick & Krommes (1979, 1980)]

  - Trapped electrons should not be stochastic
  - Long confinement of runaway electrons

- **Ambipolar electric fields for quasi-neutrality**

  - Simplified 1-D radial transport model with stochastic magnetic diffusion coefficient [Harvey (PRL'1980)]
  - "Working model" for a gyrokinetic simulation on stochastic heat flux [Wang (POP'2011)]

- **Intensive researches for external Resonant Magnetic Perturbation (RMP)**

  - RMP produces a thin stochastic layer at plasma edge to mitigate/suppress the Edge Localized Modes (ELMs)
  - Plasma transport in stochastic fields + edge physics

  [Evans (Nat.Phys'2006), Park (Nat.Phys' 2018)]

  [Park (POP'2010), Hager (NF' 2019)]

This work focuses on the 3-D topology of open magnetic field lines and ambipolar electric fields

- **Previous studies have mostly focused on:**

  - Infinite length of stochastic magnetic field lines (internal stochastic layer)

  - Characterized by 0-D or 1-D stochastic diffusivity of magnetic fields ($D_{\mathrm{m,st}}$)

  - Dynamics of passing particle along the stochastic field line with collisions

- **Key effects essential for understanding the Thermal Quench physics**

  - 3-D topology of the stochastic ***open*** magnetic field lines

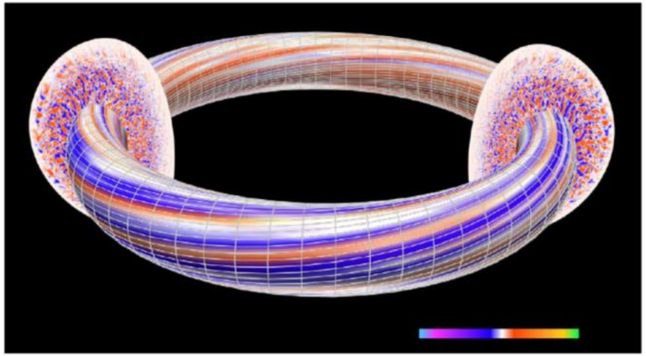
  - *Ambipolarity* of the plasma transport with self-consistent potential in the stochastic layer

  - Dynamics of *trapped particles* (magnetic mirror + electric potential well)

  - Cross-field decorrelation by $E_\perp \times B$ transport and mixing effects

⇨ A comprehensive picture of the relation between the plasma dynamics and the 3-D topology of the stochastic layer is needed

New 3-D kinetic capabilities have been developed for simulating plasma transport in stochastic fields



- GTS (Gyrokinetic Tokamak Simulation)

  - A global gyrokinetic δf particle simulation code to study **micro turbulence physics** of the fusion plasma in tokamaks

- New 3-D kinetic capabilities have been developed to study the plasma transport in the stochastic open magnetic field lines

  - High-resolution Vacuum Field Analysis

  - 3-dimensional Poisson solver

  - Novel delta-f particle method for plasma-wall boundary

  - GPU acceleration using OpenACC (x5 speed-up for total performance)

  - Improved numerical schemes to overcome numerical challenges

Governing equations of system

- **Prescribed magnetic perturbation**

$$\alpha \equiv \frac{\delta A_{\parallel}}{B_0} \qquad \boldsymbol{\delta}\mathbf{B} = \nabla \times (\alpha \mathbf{B_0}) = \nabla\alpha \times \mathbf{B_0} + \alpha(\nabla \times \mathbf{B_0})$$

- **Particle motion in the presence of $\boldsymbol{\delta B}$**

$$\mathcal{L} = q_s\left(\mathbf{A}_0^* + \boldsymbol{\delta}\mathbf{A}\right)\cdot\dot{\mathbf{R}} - (m_s/q_s)\dot{\xi} - \mathcal{H}$$
$$\mathcal{H} = \left(q_s^2\rho_{\parallel}^2 B_0^2/2m_s\right) + \mu B_0 + q_s\bar{\Phi}$$
$$\frac{d\rho_{\parallel}}{dt} = \frac{\mathbf{B}_0^* + \boldsymbol{\delta}\mathbf{B}}{\mathbf{B}_0\cdot(\mathbf{B}_0^* + \boldsymbol{\delta}\mathbf{B})}\cdot\left[-\frac{1}{q_s}\nabla\mathcal{H}\right]$$
$$\frac{d\mathbf{R}}{dt} = \frac{1}{\mathbf{B}_0\cdot(\mathbf{B}_0^* + \boldsymbol{\delta}\mathbf{B})}\left[\frac{1}{q_s}\frac{\partial\mathcal{H}}{\partial\rho_{\parallel}}(\mathbf{B}_0^* + \boldsymbol{\delta}\mathbf{B}) + \frac{1}{q_s}\mathbf{B}_0\times\nabla\mathcal{H}\right]$$

$\boldsymbol{\delta B}$ effects on the particle motion

- particle streaming ($\propto \rho_{\parallel}\boldsymbol{\delta B}$)
- magnetic mirror force ($\propto \boldsymbol{\delta B}\cdot(-\nabla B_0)$)
- electric force ($\propto \boldsymbol{\delta B}\cdot(-\nabla\Phi)$)

- **3-D field equation**

$$-\nabla\cdot\left[\epsilon_0\overleftrightarrow{\mathbf{g}} + \sum_s \frac{n_s m_s}{B_0^2}\left(\overleftrightarrow{\mathbf{g}} - \frac{\boldsymbol{B_0B_0}}{B_0^2}\right)\right]\cdot\nabla\Phi = e(\delta\bar{n}_i - \delta n_e)$$

Prescribed δB applied on "Cyclone base case" Equilibrium

- **Equilibrium magnetic configuration**
  - "Cyclone base case"

- **Circular shape limiter wall at $\sqrt{\psi_t} = 0.9$**
  - Absorbing particle wall
  - Grounded conductor ($\Phi = 0$ at the wall)

- **Magnetic perturbations with multiple harmonics**

$$\alpha = \sum_{m,n} \alpha_{(m,n)} \qquad \alpha_{(m,n)} = \Gamma(\mathrm{r}) \cos(n\phi - m\theta - \omega t + \xi_0)$$

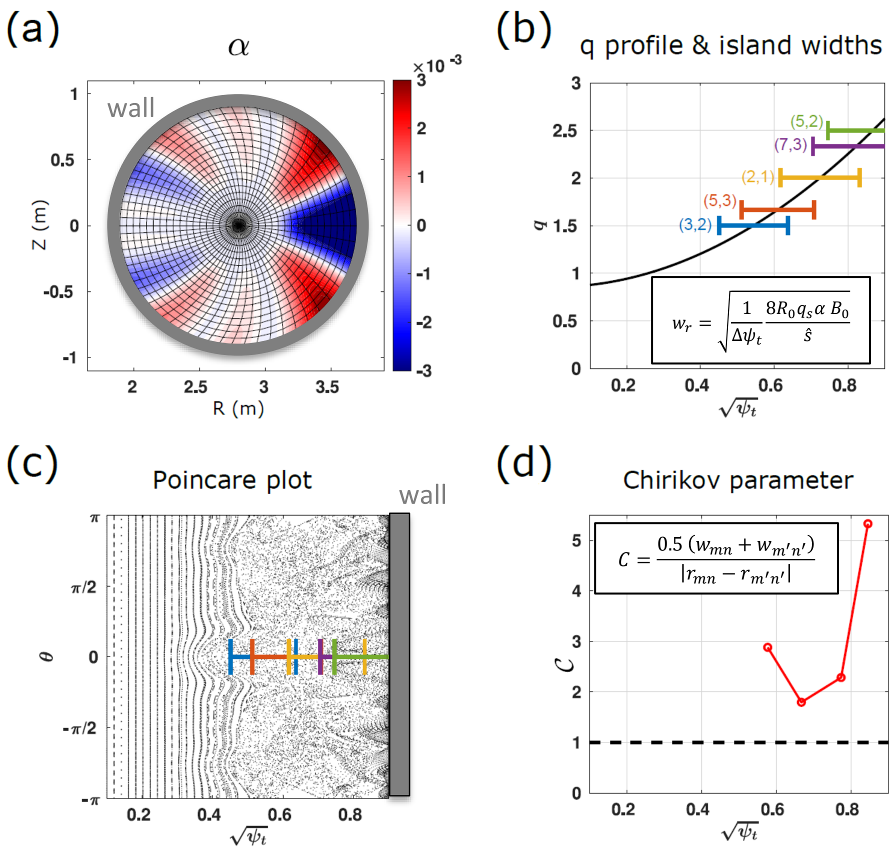
$$(m, n) = [(2,1), (3,2), (4,2), (5,2), (5,3), (6,3), (7,3), (8,3)]$$

$$|\delta B / B_0| \leq 10^{-2}$$

➡ Stochastic layer is produced from $\sqrt{\psi_t} \sim 0.45$

Auto-correlation length of stochastic fields

$$L_{\text{auto}} = \pi R_0 / \ln(0.5\pi\mathcal{C}) \approx 5.6 \text{ m when } C = 3$$



Roles of stochastic open magnetic field lines

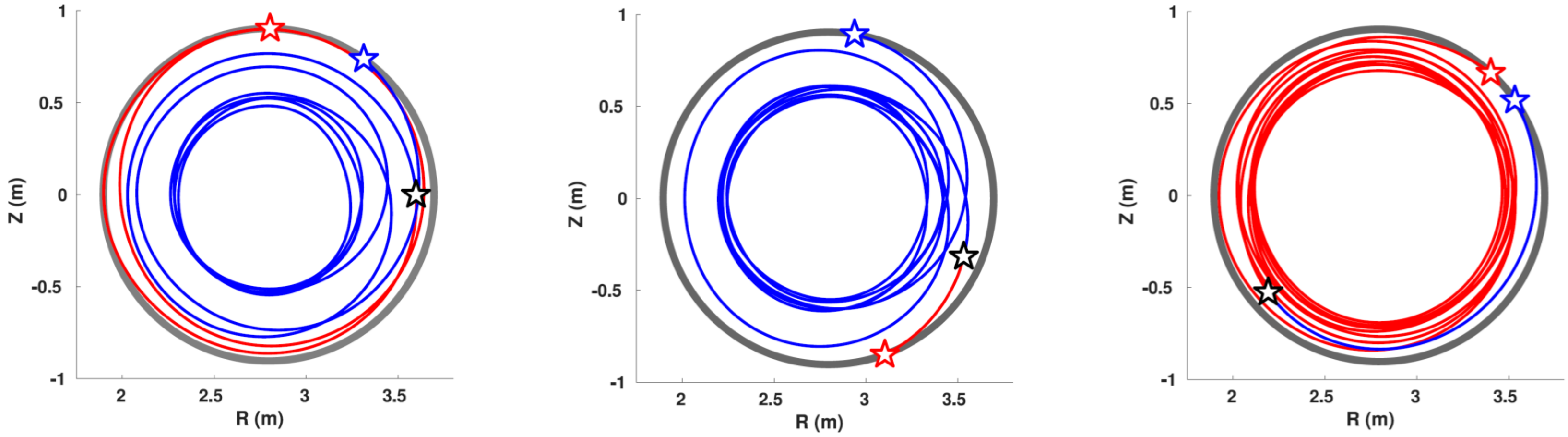
- **Vacuum Field Analysis in high-resolution**
  - Connection length of open magnetic field lines
    - ⇨ Passing particle dynamics
  - Effective magnetic mirror ratio
    - ⇨ Trapped particle dynamics

- **Test particle simulation without electric fields**
  - Temporal evolution of plasma profile in the stochastic layer
  - Characteristics of magnetically passing and trapped particles

Stochastic open magnetic field lines

- 3 stochastic open magnetic field lines started from the same $\sqrt{\psi_t}$ but different θ position

☆ Starting point
☆ Wall endpoint ($+\zeta$ direction)
☆ Wall endpoint ($-\zeta$ direction)
Field line trajectory ($+\zeta$ direction)
Field line trajectory ($-\zeta$ direction)

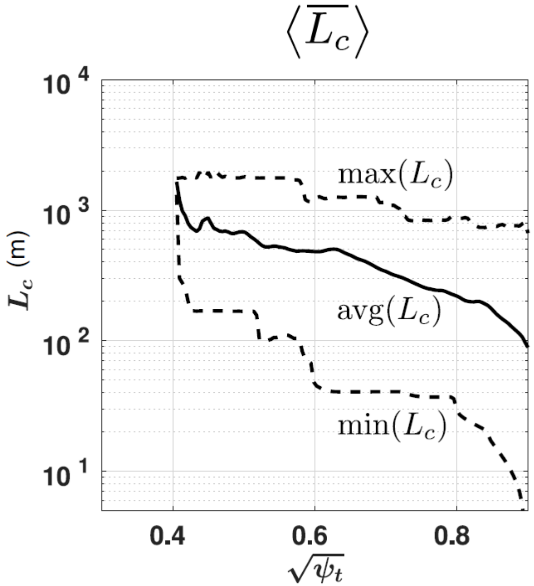


⇨ Each field line can have a different connection length between two wall endpoints

Connection length of open field lines

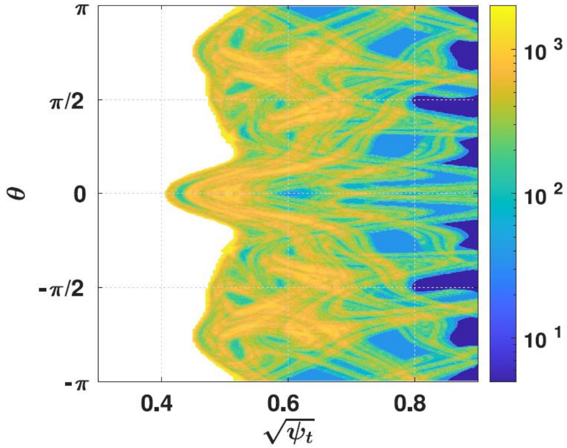
❑ **“High-resolution vacuum field analysis” enables understanding 3-D magnetic topology**

1-D averaged Connection Length

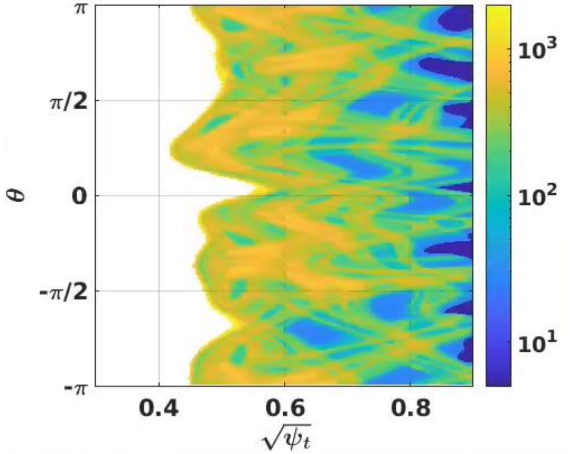


3-D Connection Length

L_c @ $\zeta = 0$



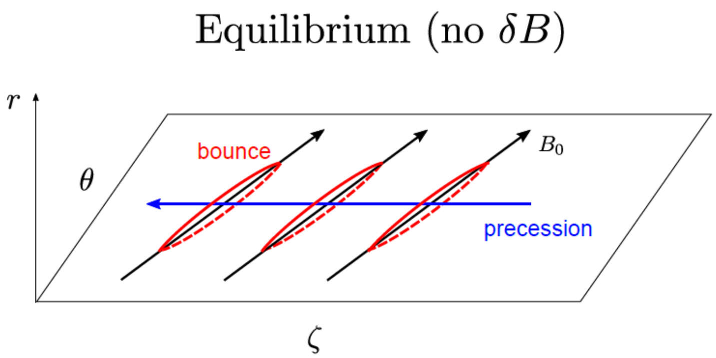
L_c @ $\zeta = \frac{\pi}{2}$

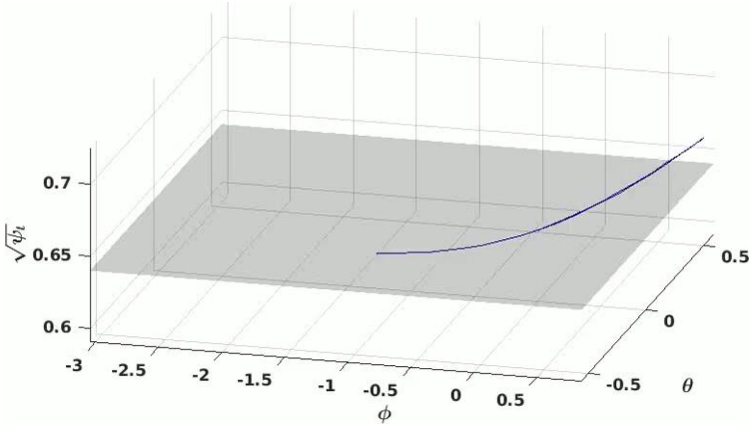


➡ Confinement time of ***passing particles*** is proportional to L_c

$$\tau_{\parallel} \sim \frac{0.5\, L_c}{v_{th}}$$

Trapped particle motion under strong 3-D δB





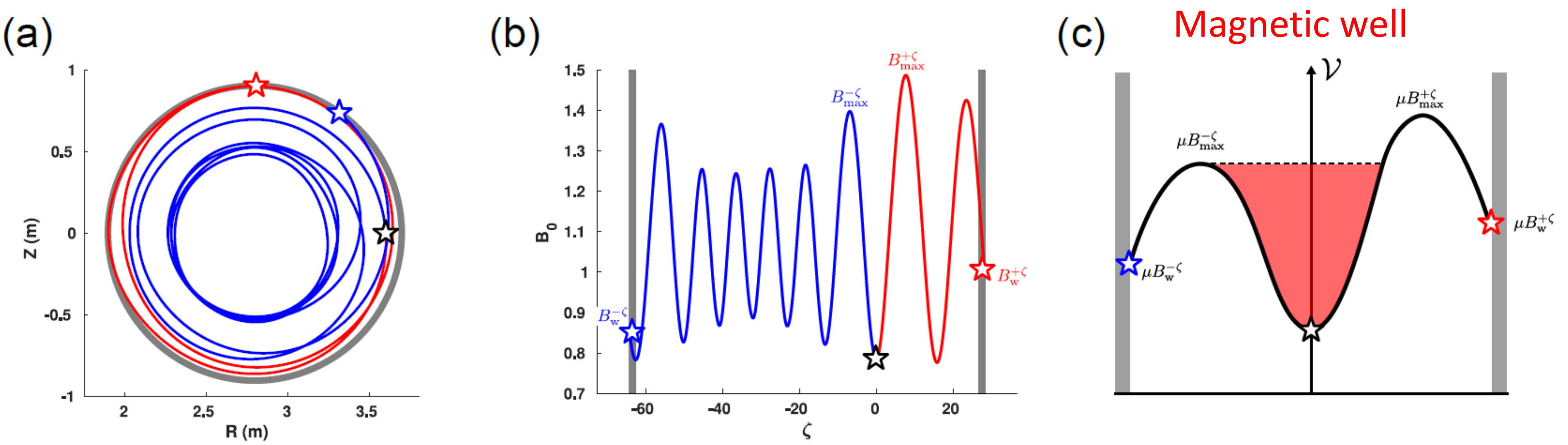
Δr = electron banana width $\ll$ Δr = determined by stochastic field structure

- Δr of trapped particle trajectory is much larger than the electron banana width
- The toroidal precession is one of the ***cross-field decorrelation mechanism***

  - Electrons can slowly move to different magnetic field lines and positions

  - Collisionless detrapping by moving electrons from magnetic uphill ($M_{\mathrm{eff}} > 1$) to downhill ($M_{\mathrm{eff}} < 1$)

Magnetic mirror effect along field line trajectory

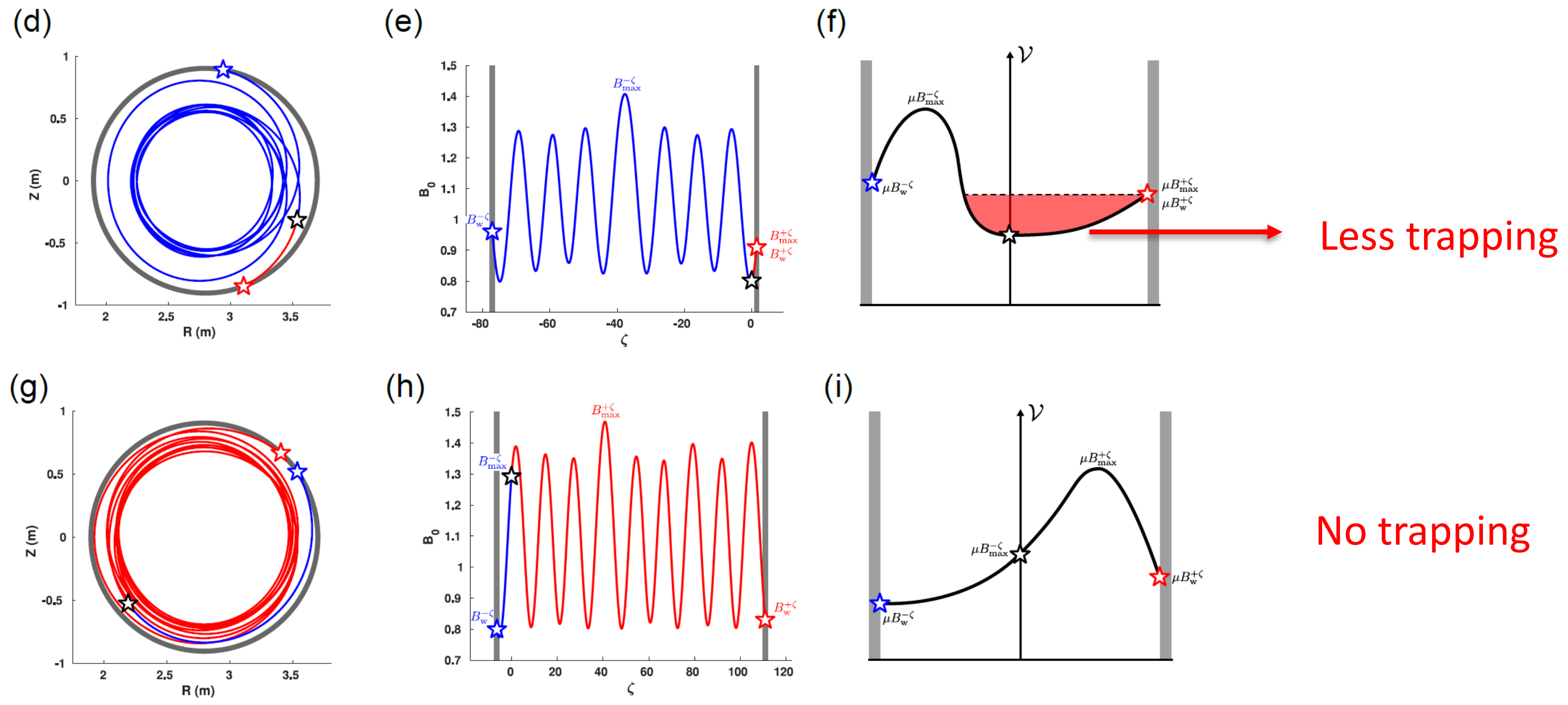


- $|B|$ is toroidally asymmetric along field line trajectory

$$B_{max}^{+\zeta}(x) \neq B_{max}^{-\zeta}(x) \longrightarrow B_{max}^{eff}(x) = \min\left(B_{max}^{+\zeta}(x), B_{max}^{-\zeta}(x)\right)$$

- $B_{max}^{eff}(x)$ determines the passing-trapping condition
- $B_{max}^{eff}(x)$ depends on the position x even in the same field line

Magnetic mirror effect along field line trajectory



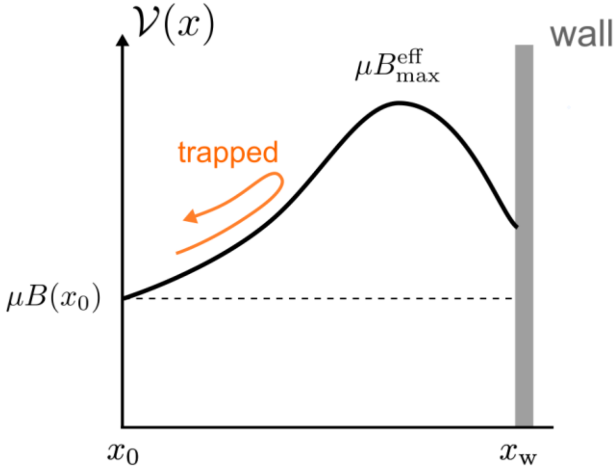
- If one of the trajectories in $\pm\zeta$ directions has a very short connection length, the effective magnetic mirror becomes weaker, and particle can more easily exit to the wall

Effective magnetic uphill and downhill

- Effective magnetic mirror ratio: $M_{\text{eff}} = \min\left(M^{+\zeta}, M^{-\zeta}\right)$

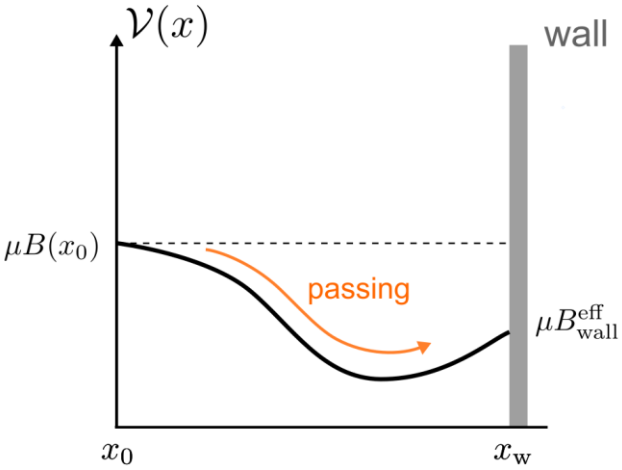
$$M^{\pm\zeta}(x) = \begin{cases} B^{\pm\zeta}_{\text{max}}/B(x), & \text{if } B^{\pm\zeta}_{\text{max}} > B(x) \\ B^{\pm\zeta}_{\text{w}}/B(x), & \text{if } B^{\pm\zeta}_{\text{max}} = B(x) \end{cases}$$

Effective magnetic uphill
($M_{\text{eff}} = B^{\text{eff}}_{\text{max}}/B_0 > 1$)



Trapped by the magnetic mirror force
if $\left|v_{\parallel}(x_0)/v_{\perp}(x_0)\right| < \sqrt{M_{\text{eff}}(x) - 1}$

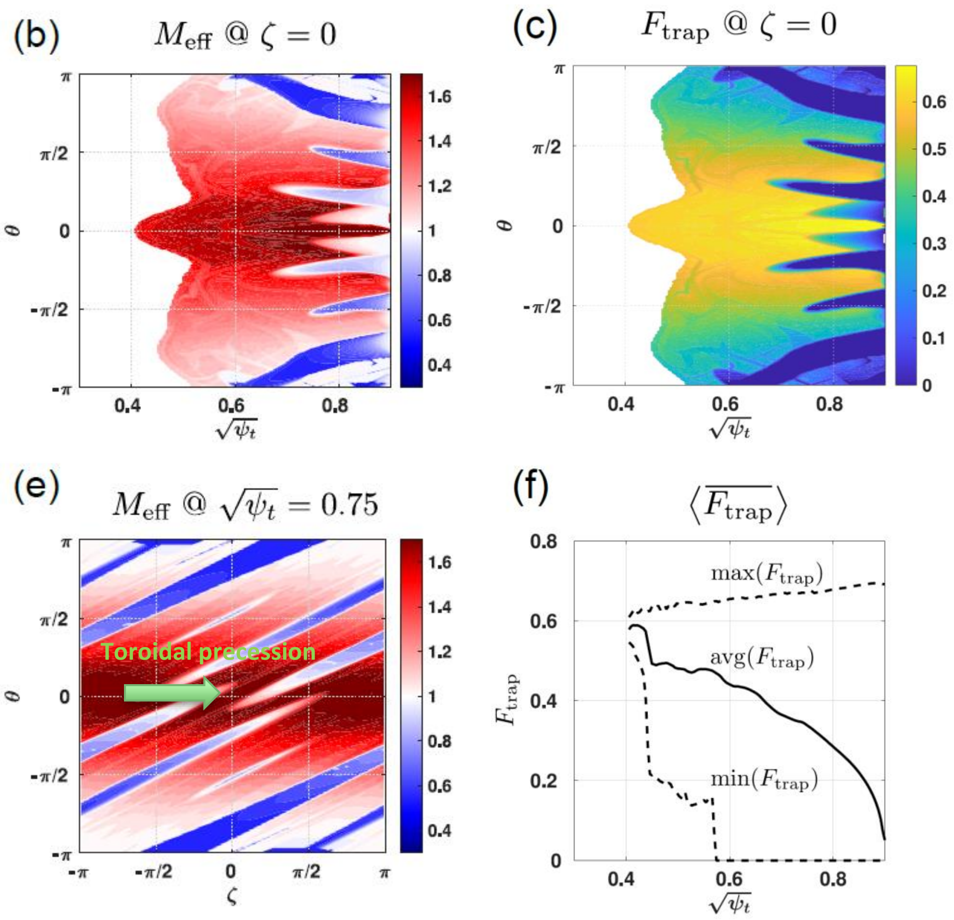
Effective magnetic downhill
($M_{\text{eff}} = B^{\text{eff}}_{\text{wall}}/B_0 < 1$)



No magnetic trapping

Particle is accelerated to the wall

Magnetic mirror ratio and Trapped Particle Fraction



- Effective magnetic uphills ($M_{\text{eff}} > 1$)
  Effective magnetic downhills ($M_{\text{eff}} < 1$)

- Trapped particle fraction of Maxwellian distribution

$$F_{\text{trap}}(x) = \begin{cases} \sqrt{1 - M_{\text{eff}}(x)^{-1}}, & \text{if } M_{\text{eff}}(x) \geq 1 \\ 0, & \text{if } M_{\text{eff}}(x) < 1 \end{cases}$$

- A considerable amount of electrons (≤ 60 %) can be trapped in the device

- The electron trapped at the uphill can move to the downhill by ***the toroidal precession*** and exit to the wall

  → Collisionless detrapping

➡ ***Trapped electron dynamics*** is critical to understand the ***electron thermal transport***

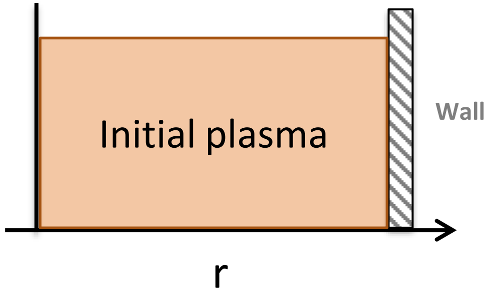
Particle simulation setup

- **Uniform density & temperature with Maxwellian distribution**

$$T_e = T_i = 5 \text{ keV}$$

$$n = 1.6 \times 10^{19}\ m^{-3}$$

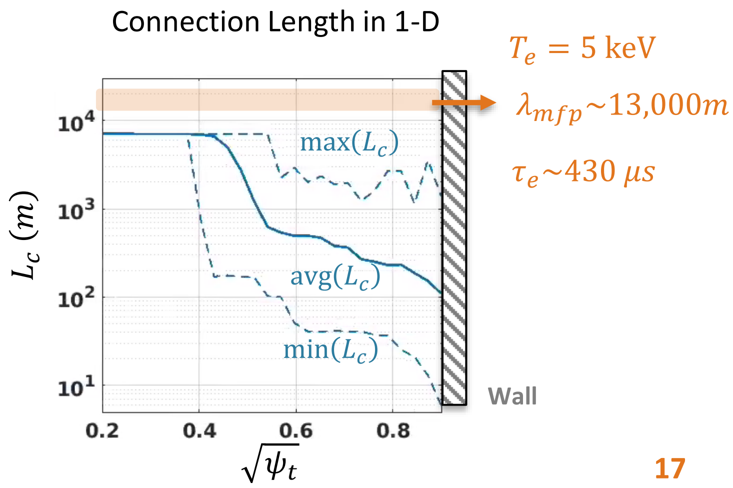
⇨ Focusing on how the plasma collapses to the wall



- **Collisionless plasma due to long mean-free-path**

$$\lambda_{mfp} \gg L_c \gg L_{auto}$$

10 km   1 km   10 m

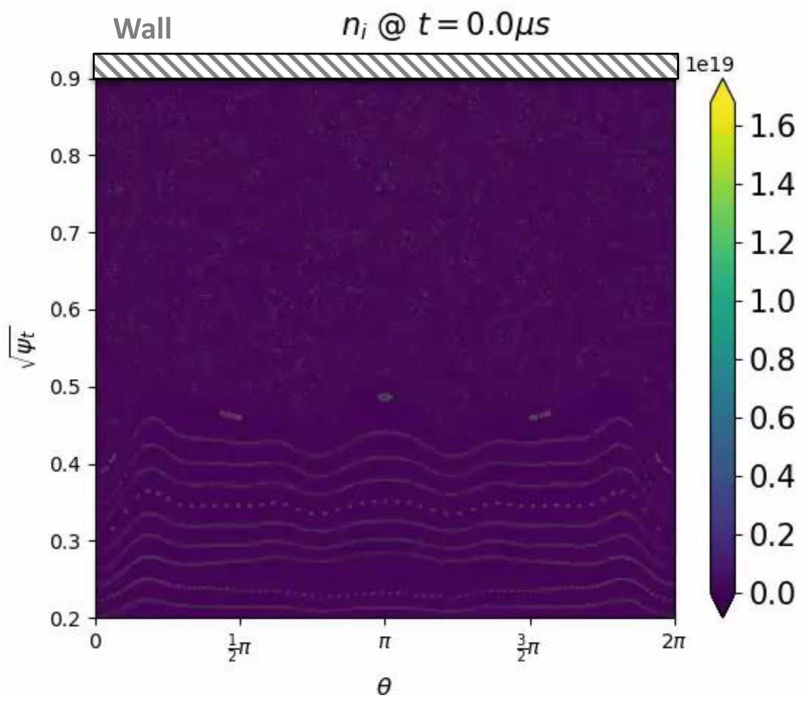


- **Three types of particle simulations**

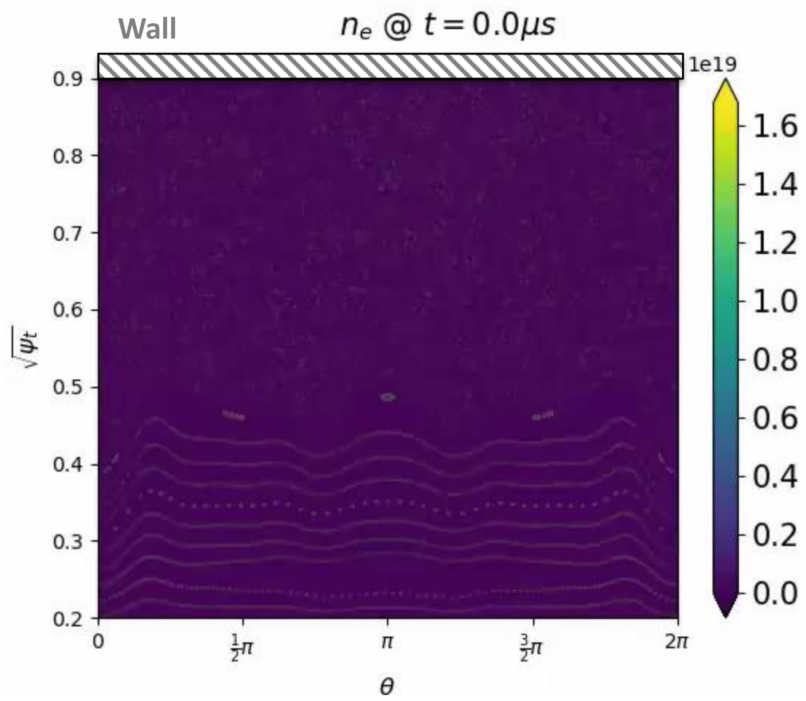
1. Test particle simulation (without E fields)
2. Ambipolar transport simulation ($E_\parallel$ only; ignoring ExB)
3. Full simulation with consistent potential and ExB ($E_\parallel + E_\perp \times B$)

2D Evolution of Electron and Ion Density (without E fields)

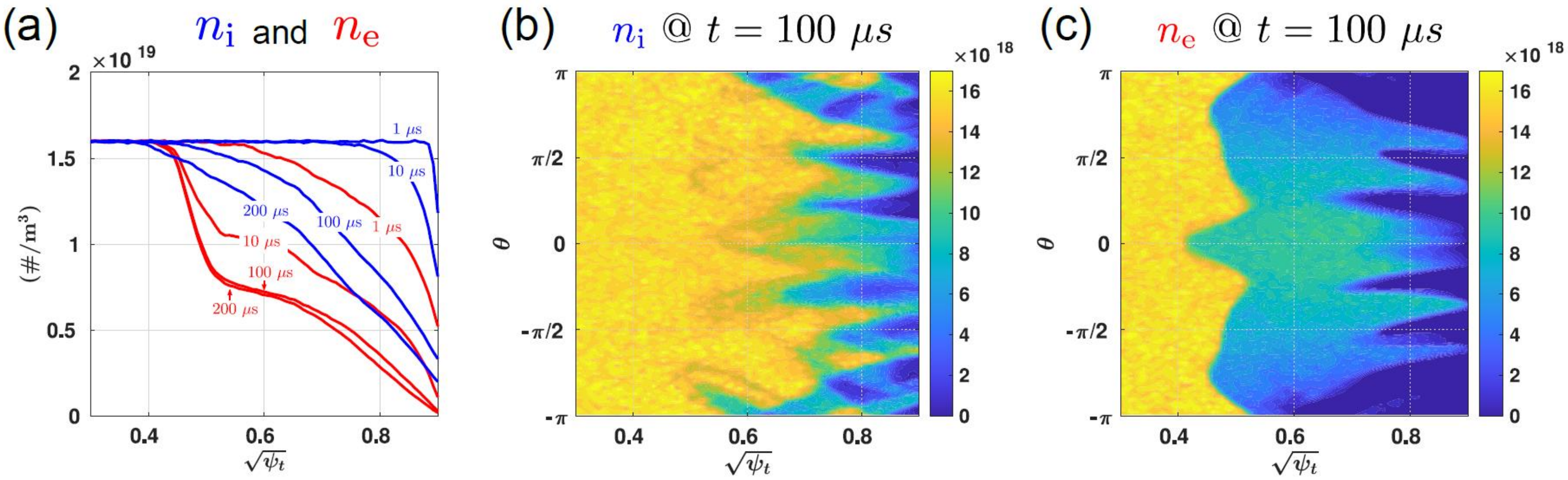
n_i



n_e

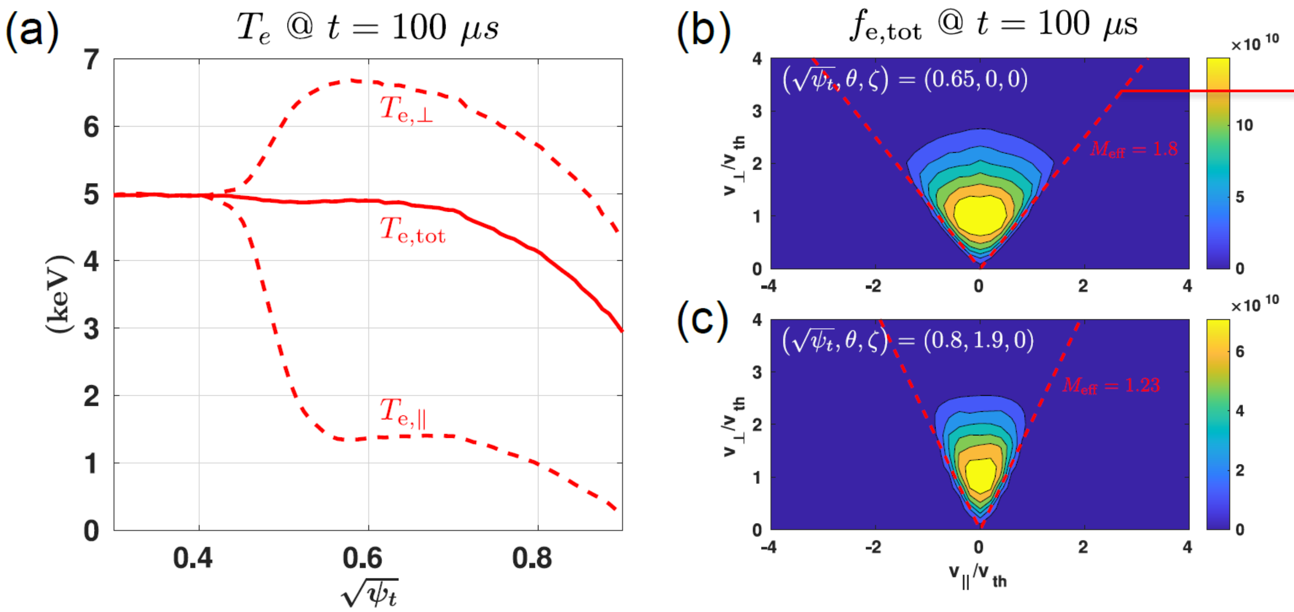


Test Particle Simulation: Density Evolution



- Electron density collapse is 60 times faster than ion collapse ($v_{th}^e/v_{th}^i = \sqrt{m_i/m_e} \sim 60$)
- Electron density quickly saturates to the level of the trapped particles

Test Particle Simulation: Electron Temperature

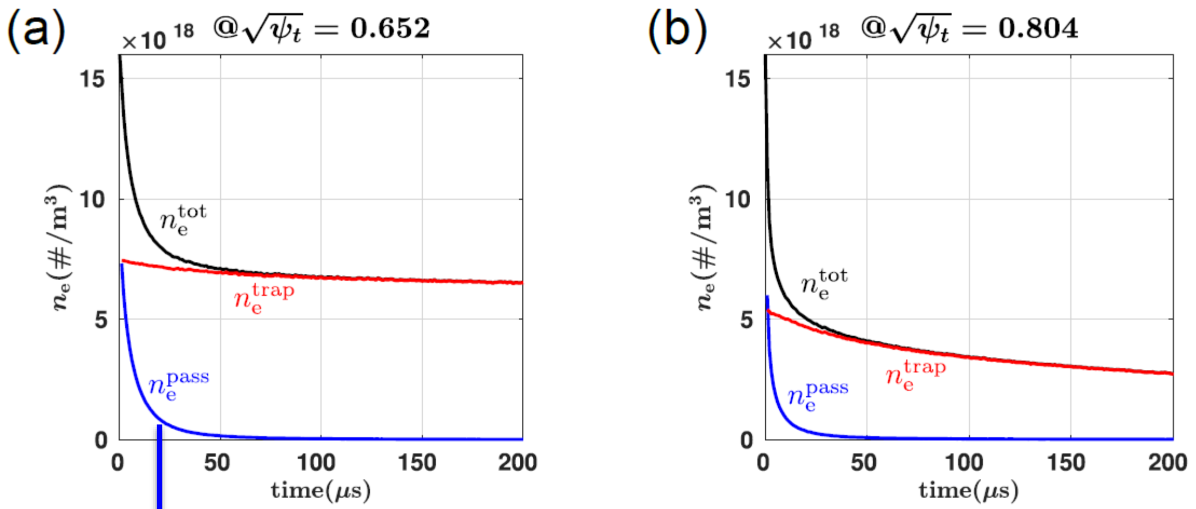


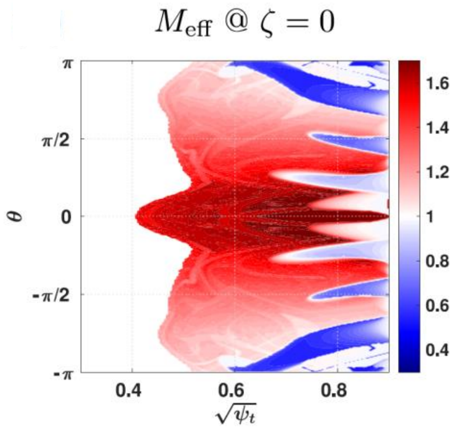
Passing-trapping boundary

$$|v_\parallel(x)/v_\perp(x)| < \sqrt{M_{eff}(x) - 1}$$

- Electron temperature is anisotropic ($T_{e,\perp} \gg T_{e,\parallel}$) due to the remained trapped electrons
- Edge temperature is slowly decreasing because of the detrapping by toroidal precession

Different confinement times of Passing and Trapped particles





Fitted $\tau_{pass} \sim 7.8\ \mu s$

$0.5\ \langle L_c \rangle / v_{th} = 7.5\ \mu s$

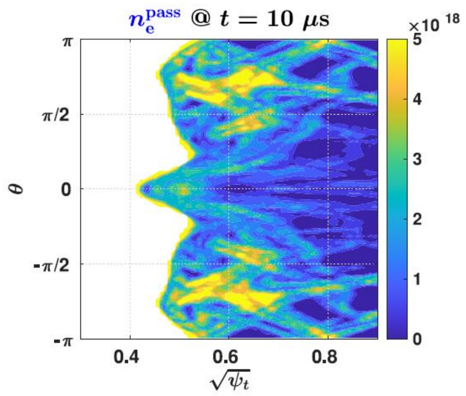
$\tau_{\parallel,pass} \sim 0.5\ L_c / v_{th}$

- Passing electron density quickly decays with a short confinement time
- Trapped electron density very slowly decays
  due to the collisionless detrapping by the toroidal precession

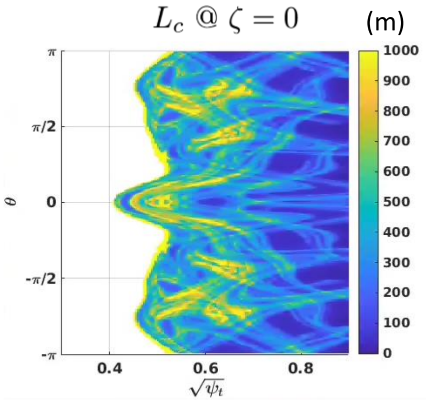
  - The outer radial surface has more magnetic downhill regions
    → faster detrapping

"Vacuum Field Analysis" well predicts the dynamics of test particles

Passing electron density



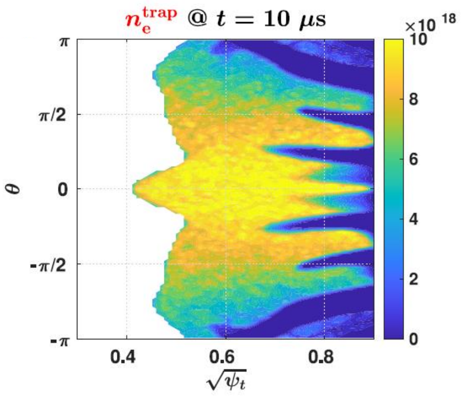
Connection length



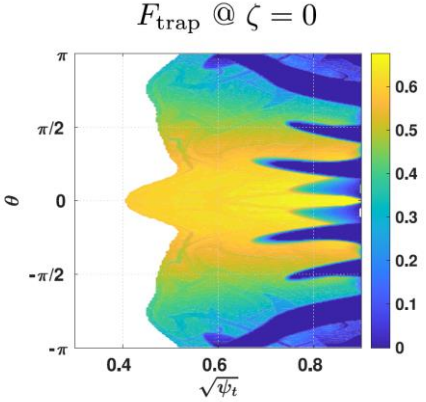
- The passing particle density is higher at longer connection length regions

$$\tau_{\parallel,\text{pass}} \sim 0.5\ L_c / v_{th}$$

Trapped electron density



Trapped particle fraction



- The trapped particle density ratio is the same as the trapped particle fraction

$$F_{\text{trap}}(x) = \begin{cases} \sqrt{1 - M_{\text{eff}}(x)^{-1}}, & \text{if } M_{\text{eff}}(x) \geq 1 \\ 0, & \text{if } M_{\text{eff}}(x) < 1 \end{cases}$$

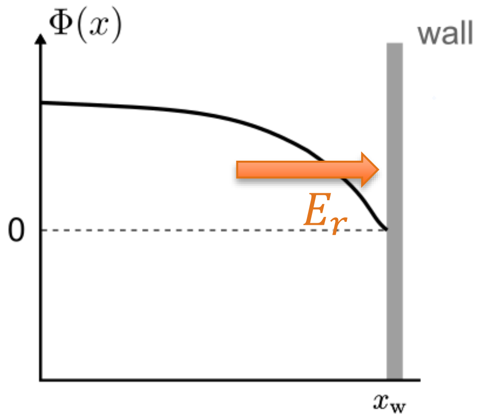
Roles of the electric fields

- **$E_{\parallel}$: acceleration of charged particles**
  - Impedes the fast electron loss
    - ⇨ ambipolar plasma transport (quasi-neutrality)
  - Determines the passing-trapping condition of electrons (combined with the magnetic potential)

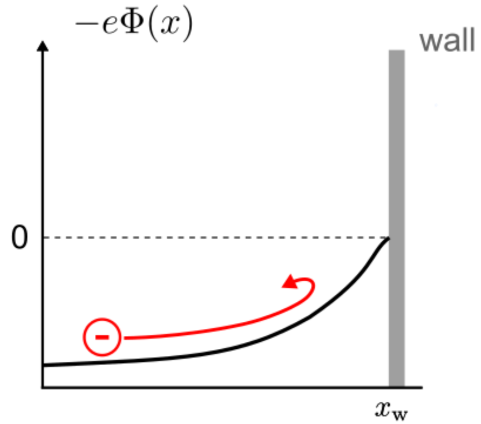
- **$E_{\perp}$: ExB drift motion across the magnetic field lines**
  - Deforms the plasma structure (mixing effect)
  - Direct cross-field transport in the radial direction
  - Enhances the collisionless detrapping of high-$v_{\perp}$ trapped particle
    - ⇨ The steady decrease of the electron temperature

The ambipolar electric field impedes the fast electron loss

Electrostatic Potential



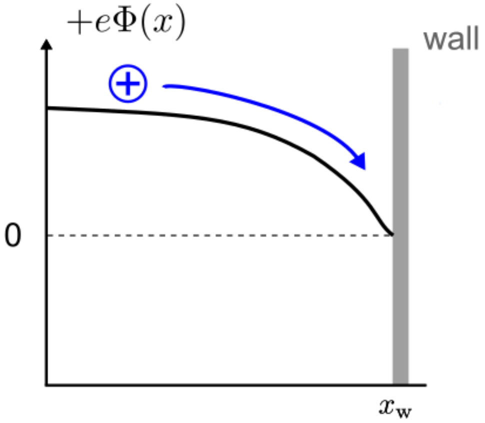
Electron potential energy



Deceleration of electrons

$\Gamma_e^r \downarrow$

Ion potential energy



Acceleration of ions

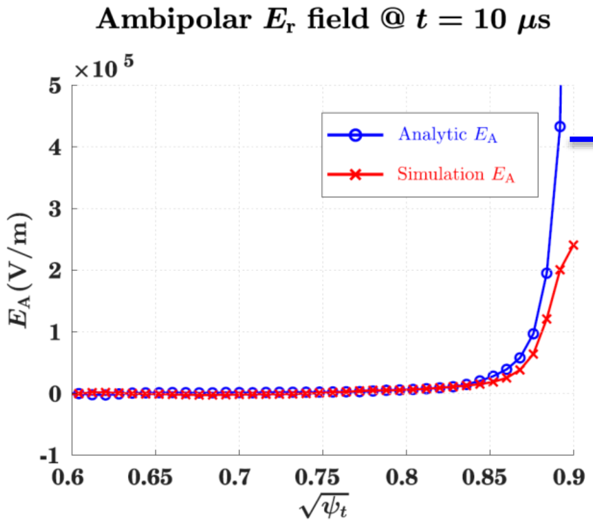
$\Gamma_i^r \uparrow$

$\left(v_{th}^e / v_{th}^i = \sqrt{m_i/m_e} \sim 60\right)$

- The electron loss is 60 times faster than the ion loss without the ambipolar electric field
- The ***positive ambipolar potential*** ($\mathrm{e}\Phi \sim \mathrm{T_e}$) builds up for the ***ambipolar transports*** (***$\Gamma_i \approx \Gamma_e$***)

  → Impedes the fast electron loss to match with the ion loss *(for quasi-neutrality)*

1-D model of ambipolar electric fields



1-D ambipolar radial electric fields (Harvey PRL' 1981)

$$E_r^A(r) = -\left(\frac{\langle T_e \rangle}{e}\right)\frac{\partial}{\partial r}\left[\ln(\langle n_e \rangle \langle T_e \rangle^{1/2})\right]$$

- Maxwellian Distribution
- Zero flux assumption ($\Gamma_i = \Gamma_e = 0$)

- Radial electric fields from the simulation agree with the analytic model except the edge
- At the edge, the ion flux is not negligible, and the distribution function is deformed from Maxwellian

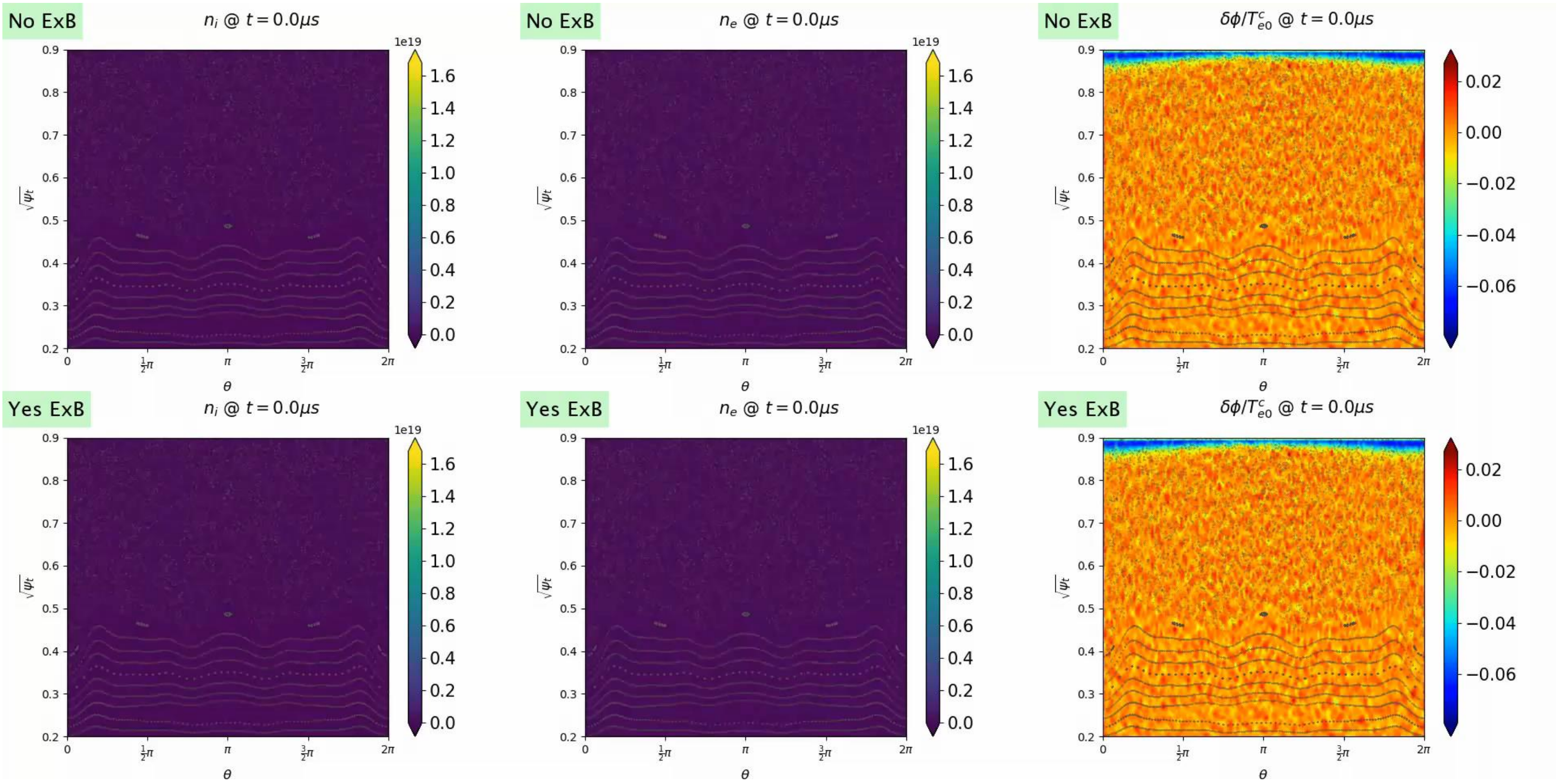
⇨ ***The ambipolar potential has 3-D structure*** associated with the topology of the stochastic layer

Plasma ambipolar transports by electric fields

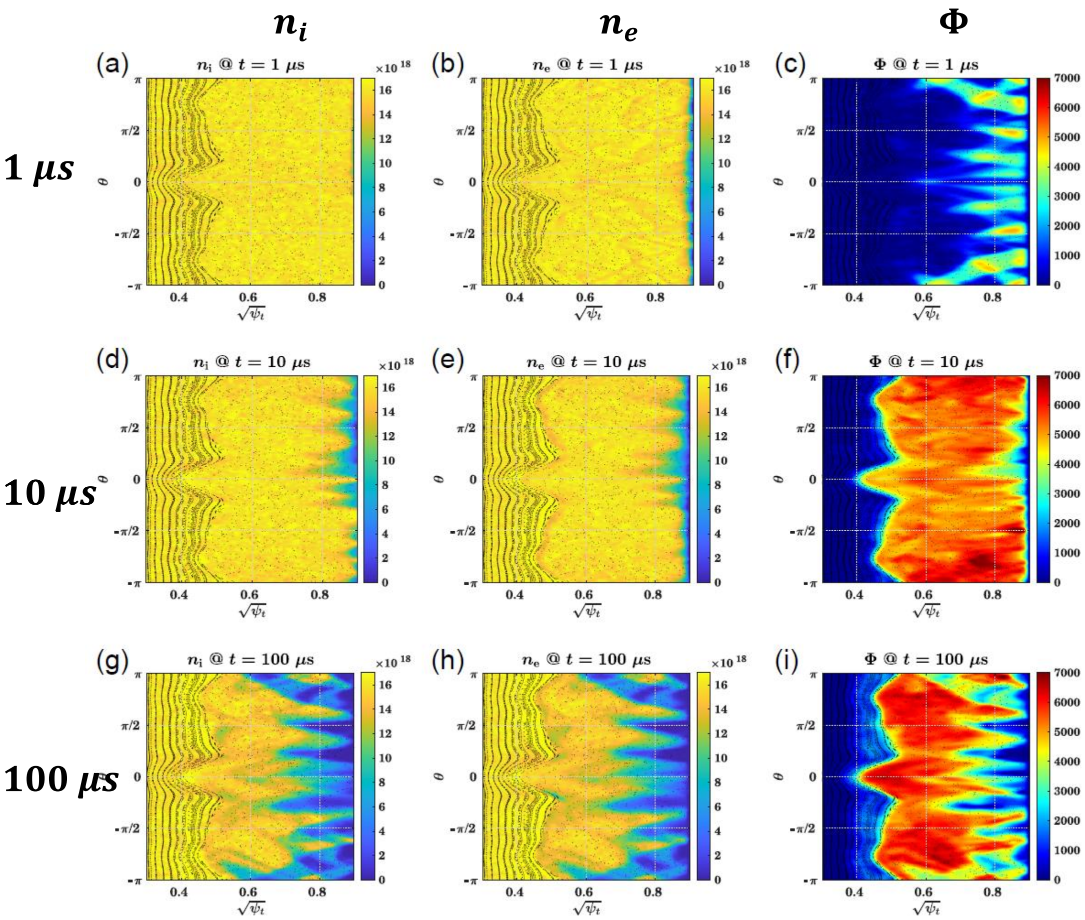
n_i n_e ϕ

$E_\parallel$ only

$(E_\parallel + E_\perp \times B)$



Temporal evolution of the plasma (only $E_{\parallel}$ case)



1. Ambipolar potential builds-up with electron thermal speeds

2. Plasma collapses with ion sound speeds
   - Ambipolar transport
   - Lower density at shorter L_c regions

3. The potential becomes smaller again at the edge due to lower pressure gradient

$$E_r^A(r) = -\left(\frac{\langle T_e \rangle}{e}\right)\frac{\partial}{\partial r}\left[\ln(\langle n_e \rangle \langle T_e \rangle^{1/2})\right]$$

$$\nabla_{\parallel}(n_e T_e^{1/2}) \Downarrow \rightarrow E_{\parallel}^A \Downarrow \rightarrow \Phi \Downarrow$$

➡ The plasma has 3-D structures correlated to L_c structure

Particle passing-trapping condition along the field line

- **Total potential energy for charged particles**

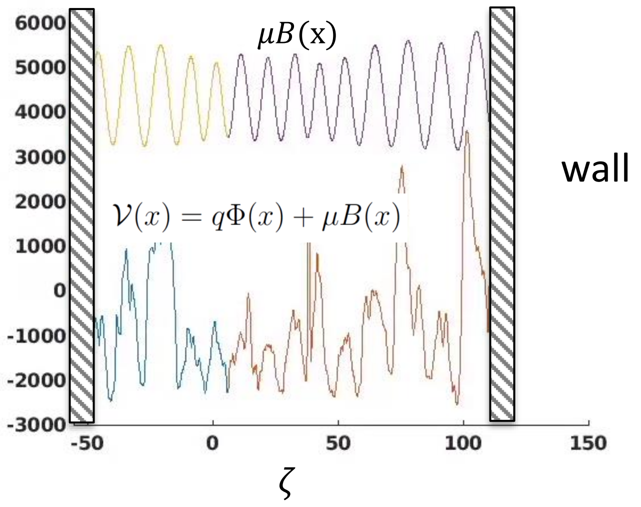
$$\mathcal{V}(x) = q\Phi(x) + \mu B(x)$$

- **Exact Trapping condition**

$$\varepsilon_{\|}(x) + \mathcal{V}(x) < \mathcal{V}^{\text{eff}}_{\text{max}}(x)$$

- It is hard to find exact $V^{\text{eff}}_{\text{max}}(x)$ due to fluctuating Φ

Potential energies along field line trajectory



- **Simplified trapping conditions**

Trapping for high μB particles ($\mu B \gg q\Phi$)

$$\varepsilon_{\|}(x) < \mathcal{V}^{\text{eff}}_{\text{max}}(x) - \mathcal{V}(x) \approx \mu\left(B^{\text{eff}}_{\text{max}}(x) - B(x)\right)$$

$$v_{\|}^2(x) < \left(B^{\text{eff}}_{\text{max}}(x)/B(x) - 1\right) v_{\perp}^2$$

The same as the pure magnetic mirror condition

+

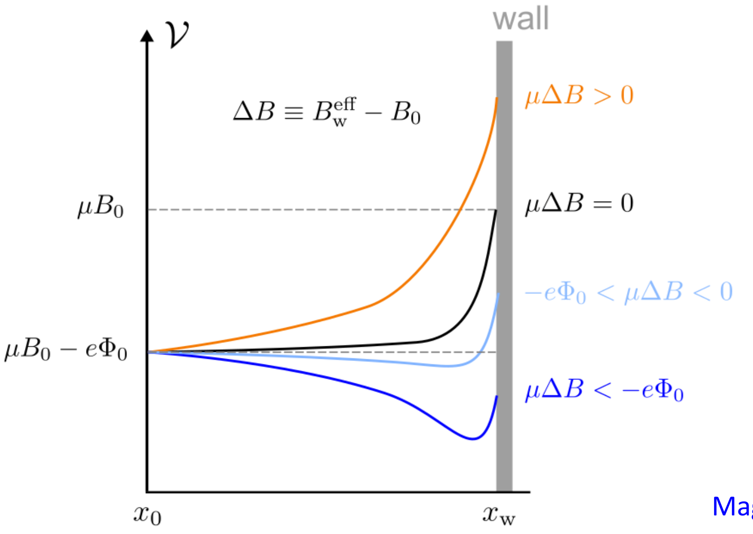
Global trapping with respect to wall endpoints

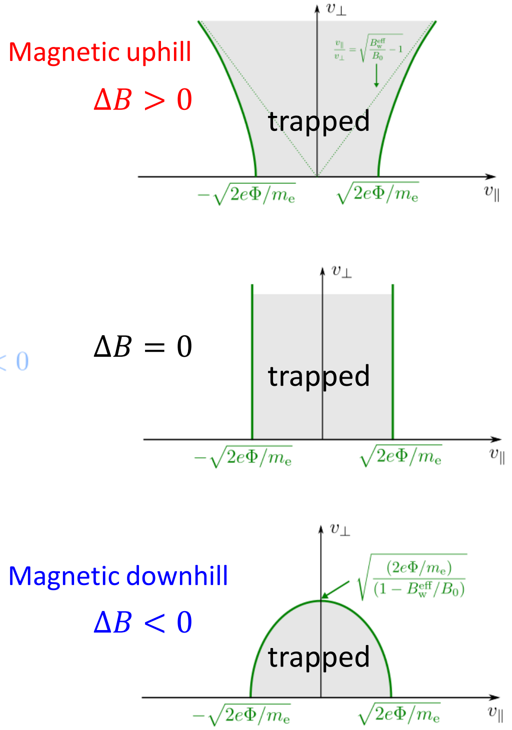
($\Phi_{\text{wall}} = 0$)

$$\varepsilon_{\|}(x) + \mathcal{V}(x) < \mu B^{\text{eff}}_{\text{w}}$$

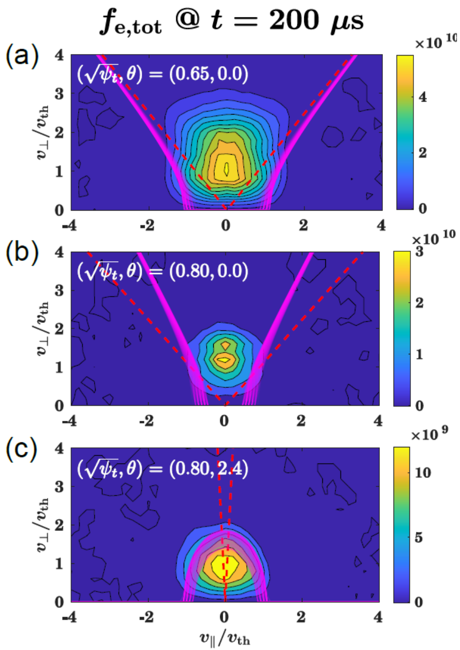
$$v_{\|}^2(x) + \left(1 - \frac{B^{\text{eff}}_{\text{w}}}{B(x)}\right) v_{\perp}^2(x) < -\frac{2q}{m}\Phi(x)$$

Global trapping with respect to wall endpoints





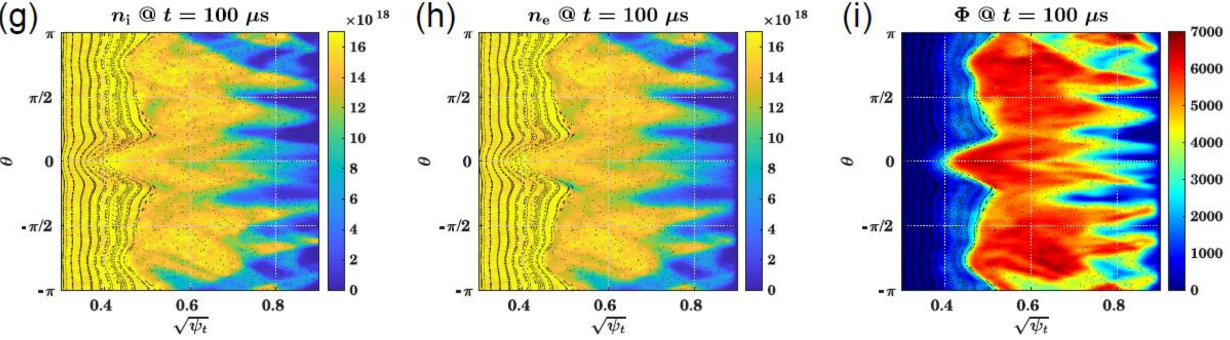
Simulation results



- At the magnetic uphill ($\Delta B > 0$), more particles are trapped due to additional magnetic mirror effects
- At the magnetic downhill ($\Delta B < 0$), ***the high $v_\perp$ particles can be passing particles***

ExB mix the plasma across the field line

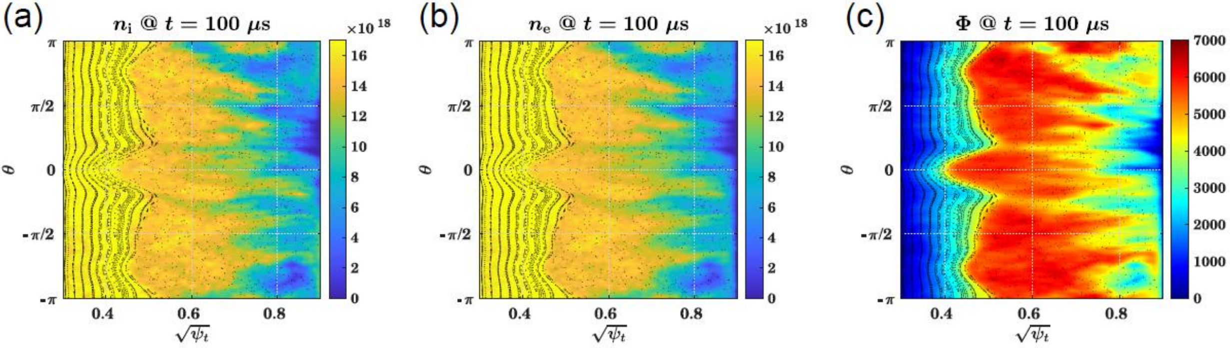
Simulation including only $E_{||}$



- Strong $E_{\perp}$ across different L_c regions

  →Fast ExB transport and mixing

Full Simulation including both $E_{||}$ and $E_{\perp}$

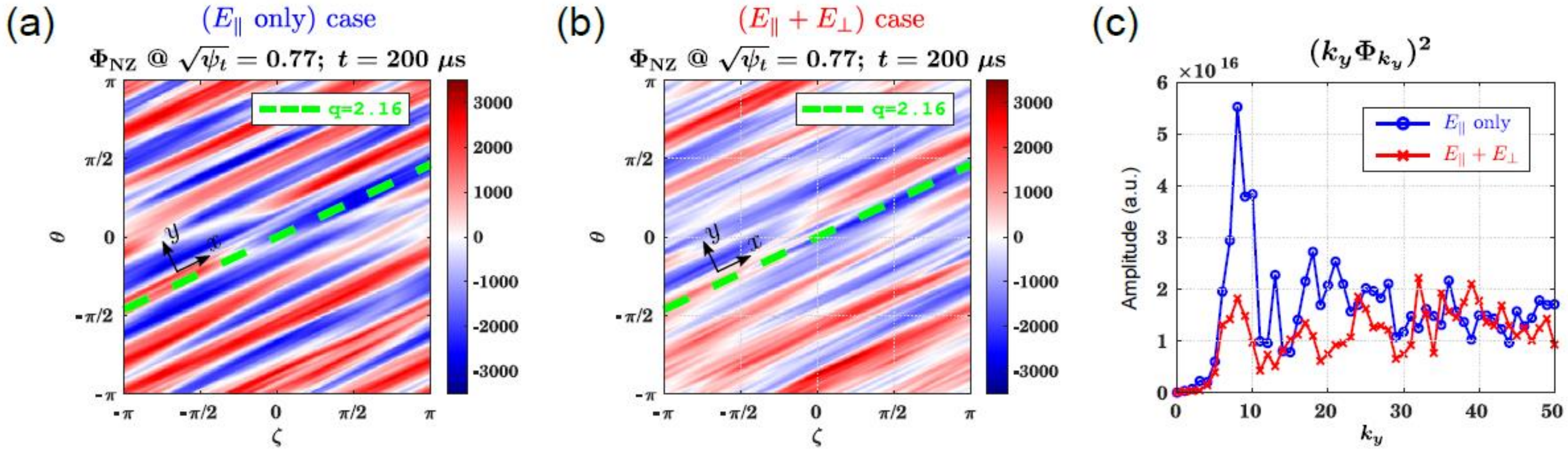


- ExB mixing deforms the plasma structure

  - Radial eddies + poloidal flow

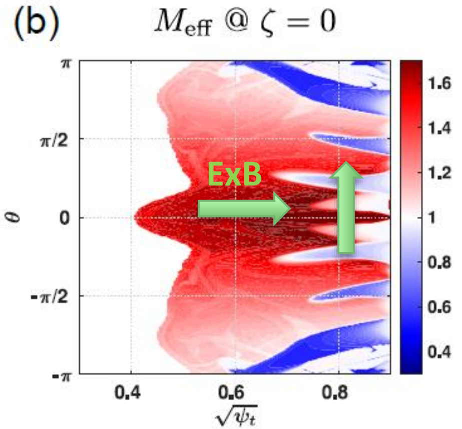
ExB mix the plasma across the field line

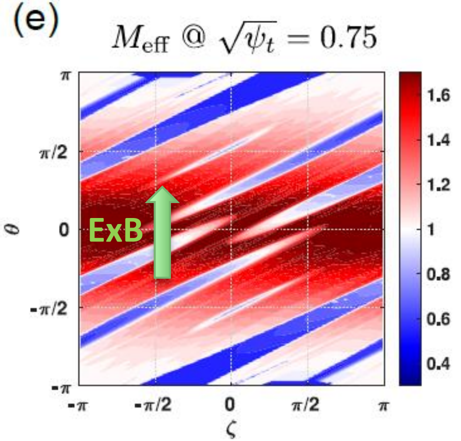
- Non-zonal potential $\delta\Phi$ at a specific radial surface ($\sqrt{\psi_t}$=0.77)



- $E_{\parallel}$ only case
  - Field-aligned structures
  - A sharp difference across field line
  - Peak energy spectrum at $k_y \sim 8$
- $(E_{\parallel} + E_{\perp} \times B)$ case
  - Field-aligned structures
  - **Higher k_y modes** become more important
  - Dynamic finer structures by ExB mixing

ExB mixing enhances the collisionless detrapping of high-$v_\perp$ particle





- **Confinement of high $v_\perp$ particles (parallel dynamics)**
  - They are well-trapped at magnetic uphill regions
  - They can exit to the wall at magnetic downhill regions by overcoming the electric reflective force

- **ExB transport carries and mixes the electrons radially and poloidally**

high $v_\perp$ particles

trapping

ExB mixing

(magnetic uphill) ⟷ (magnetic downhill)

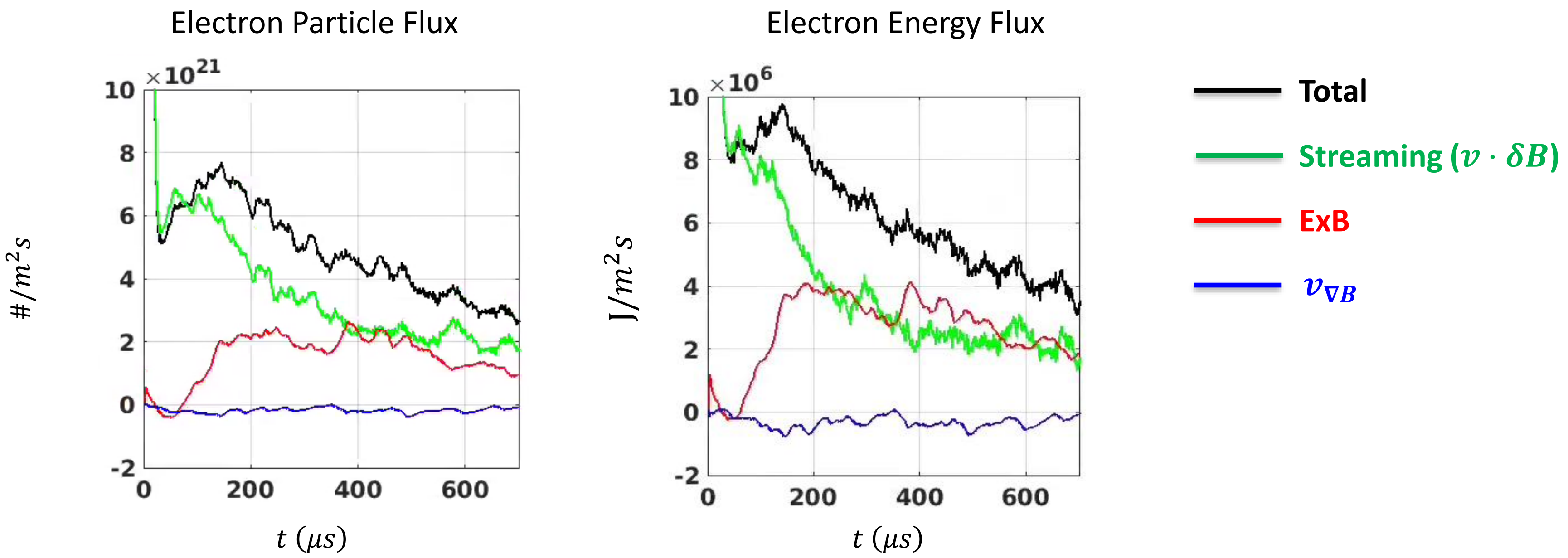
trapped — passing

collisionless detrapping

➡ ExB mixing enhances the collisionless detrapping in average

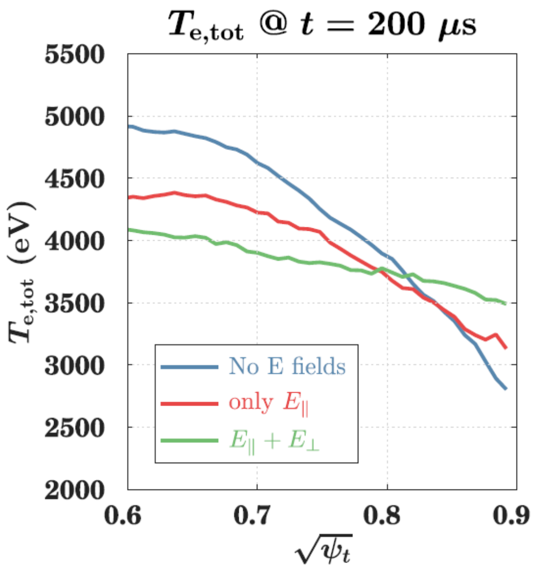
ExB contributes considerable amount of electron fluxes

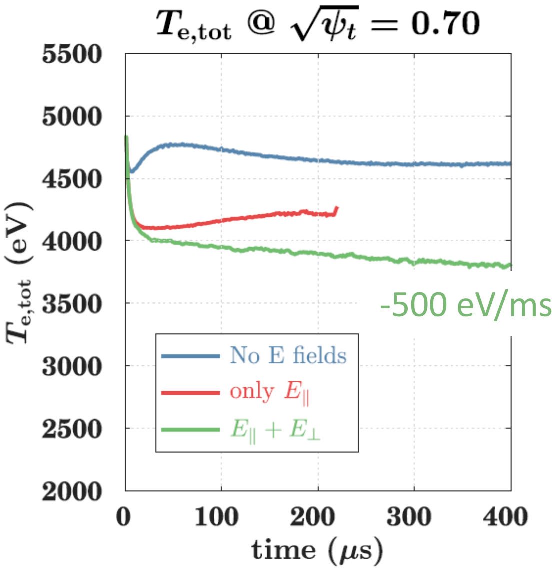
Electron Radial Fluxes @ $\sqrt{\psi_t} = 0.65$



- **ExB transport** contributes about **(30~40)% of particle flux** and (**50~60)% of heat flux**

Comparison of Electron temperatures with 3 different physical models





- ***Trapped particle transport*** is critical for determining the electron thermal transport and temperature
  - **(No E field case)** and **(only $E_\parallel$)** cases have saturated electron temperature with higher gradients
  - **($E_\parallel + E_\perp$)** case shows that the electron temperature steadily decrease in the time scale of milliseconds
    - *High-$v_\perp$ trapped electrons* can be detrapped by ExB mixing and toroidal precession
    - At $\sqrt{\psi_t} = 0.7$ surface, the temperature decreasing rate is about **(-500 eV/ms)**

Summary

- First-principles-based calculation of plasma transport in stochastic magnetic fields has been developed for a global gyrokinetic code GTS

- We found that ***self-consistent electric fields*** for **plasma transport** **ambipolarity** play critical roles in determining plasma transport associated with the ***3-D topology of stochastic layer***

  - $E_{\parallel}$ makes ambipolar plasma transport that propagates along stochastic fields with ion sound speed
  - $E_{\parallel}$ and *3-D magnetic mirror ratio* determines the passing-trapping condition of the particles
  - $E_{\perp} \times B$ radial transport is considerable (particularly for the trapped particles)
  - $E_{\perp} \times B$ mixing across the stochastic fields enhances the collisionless detrapping of high-$v_{\perp}$ trapped particle

- We observed a considerable degradation of the global plasma profile and electron temperature within the timescale of milliseconds that agrees with the typical time scale of the thermal quench

- Future works

  - Collisional transports
  - Recycling particles
  - More realistic plasma profile and magnetic perturbations