DE LA RECHERCHE À L'INDUSTRIE

Fishbone instability and transport of energetic particles

Guillaume Brochard[1,2]

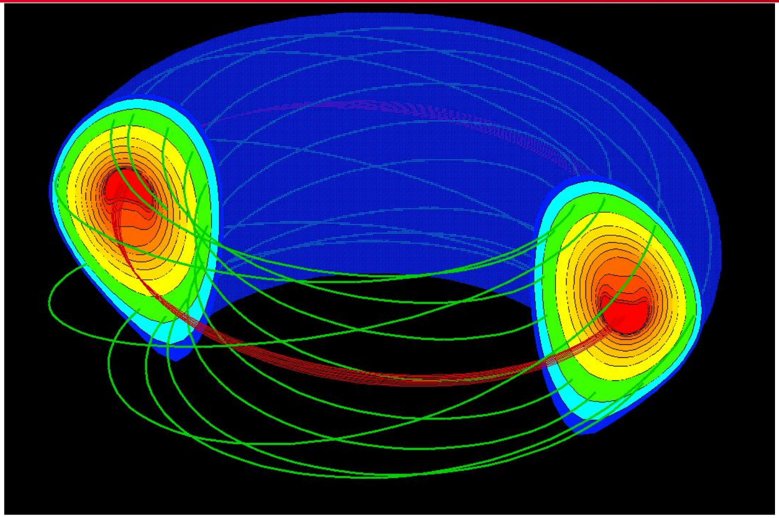
Supervised by Dr. **R.Dumont**[1], Dr. **H.Lütjens**[2], Dr. **X.Garbet**[1]

Acknowledgment to Dr. **T.Nicolas**[2], Dr. **F.Orain**[2]

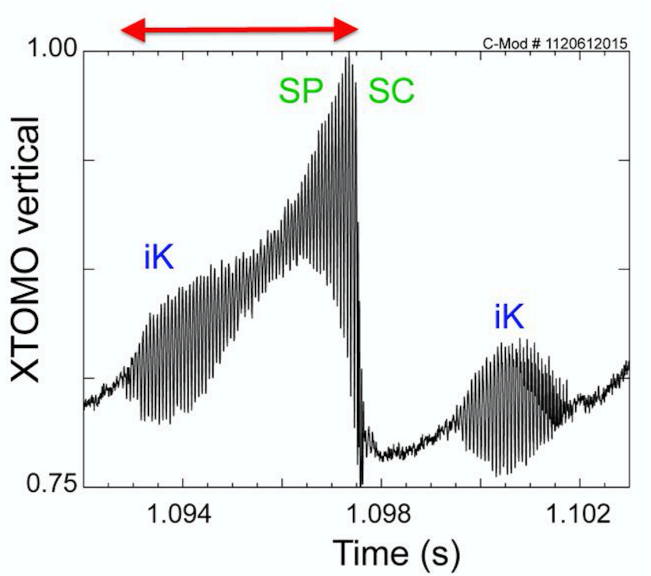
1) *Institut de Recherche sur la Fusion Magnétique*, CEA Cadarache
2) *Centre de Physique Théorique*, Ecole Polytechnique, Paris

www.cea.fr

Motivation



Internal kink mode structure

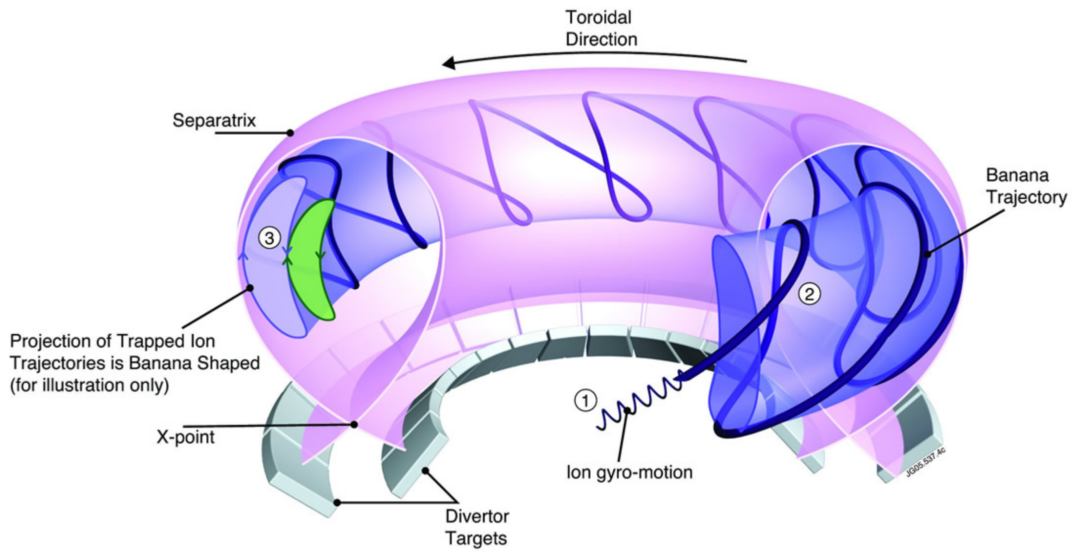


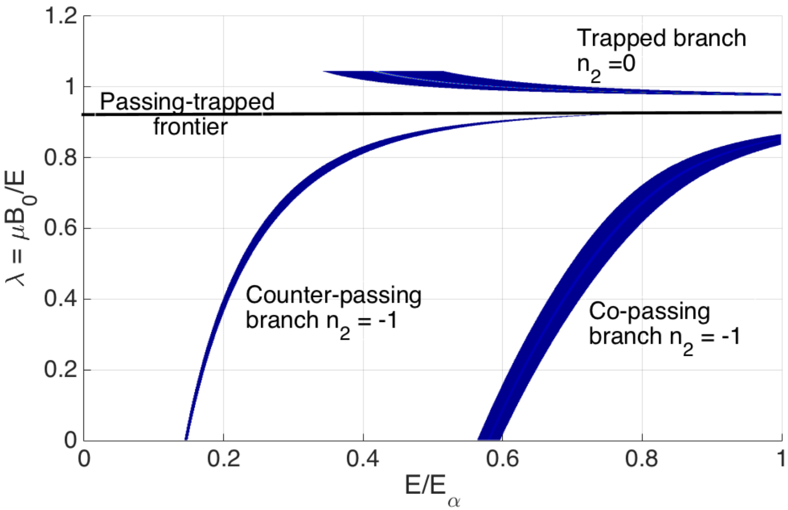
Fishbone oscillations on density

- Energetic particles perturb MHD stability ([1],[2])
- Kinetic-MHD instabilities induce transport of energetic particles (EP) experimentally ([3],[4])
- Energetic particles are needed to sustain the plasma heat
- Nonlinear simulations needed to study transport ([5],[6],[7])

[1]: Chen et al. 1984
[2] White et al. 1989
[3] McGuire et al.1983
[4]Nave et al. 1991
[5] Fu et al. 2005
[5] Vlad et al. 2013
[6] Pei et al. 2017

Kinetic-MHD resonant processes





Resonant curves at fixed P_φ

Resonant condition for trapped particles

$$\omega = \omega_d(E, \mu, P_\varphi)$$

Resonant condition for passing particles

$$\omega = \omega_d(E, \mu, P_\varphi) + (1 - q)\omega_b(E, \mu, P_\varphi)$$

- Resonant processes possible at high energy with precessionnal and bounce frequencies
- Resonances lie on planes in phase space
- At lower energy, the trapped particles are mainly contributing

Outline

- I) Description and linear verification of the hybrid nonlinear code XTOR-K
- II) Determination of the fishbone β_h thresholds for the ITER 15 MA case
- III) Nonlinear study of the fishbone-induced EP transport for a circular ITER-like equilibrium

Kinetic-MHD hybrid code XTOR-K

XTOR-K [1,2]

XTOR-2F

Non-linear MHD solver

Time iteration of

(V,B,p,ρ)

$$\rho\frac{d}{dt}\mathbf{V} = \mathbf{J}\times\mathbf{B} - \nabla p - \nabla\cdot\Pi$$

$$\partial_t\mathbf{B} = \nabla\times(\mathbf{V}\times\mathbf{B}) + \nabla\times(\eta\nabla\times\mathbf{B})$$

$\nabla\cdot\Pi$

$(\mathbf{E},\mathbf{B})$

PIC Module

Full f, full orbit

Particle Pusher

(*r*,*v*)

$$m_i\frac{d}{dt}\mathbf{v} = q_i[\mathbf{E} + \mathbf{v}\times\mathbf{B}]$$

$$\Pi(\mathbf{r},t) = m_i\sum_{n=1}^{N_{part}}\mathbf{v}_n\mathbf{v}_n\delta(\mathbf{r}-\mathbf{r}_n(t))$$

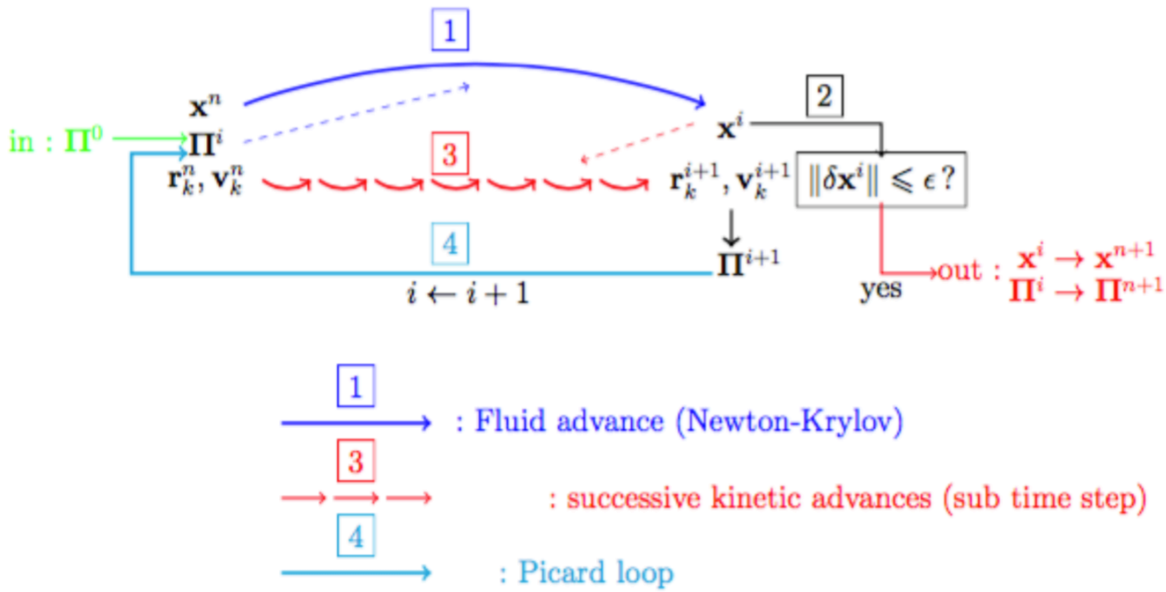
- XTOR-K able to describe self-consistently kinetic-MHD modes during their nonlinear phase

- Nonlinear simulations are mandatory to study the EP transport

- Needs to be verified against linear theory

[1] H. Lütjens et al, JCP 2012
[2] D. Leblond, PhD thesis 2013

XTOR-K Newton-Krylov/Picard algorithm



- Full f hybrid codes require a fluid and a kinetic time step
- Kinetic time step needs to be at least ten times smaller than the ion gyration time
- Algorithm optimized to do few kinetic Picard iterations
- Scheme computable in acceptable time for 10^8-10^9 macro-particles thanks to a massive parallelization of the particle advance

Kinetic-MHD linear theory

[1] Chen et al, PRL 1983
[2] Coppi et al, PFB 1990
[3] White et al, PFB 1990
[4] Porcelli et al, POP 1994
[5] Brochard et al, JPCS 2018

Kinetic-MHD energy principle

$$\delta I = \delta W_{MHD} + \delta W_K$$

Instability kinetic energy — MHD potential energy — EP potential energy

$$\delta W_K = -\frac{1}{2}\int d^3\mathbf{x}d^3\mathbf{v}\ \boldsymbol{\xi}^* \cdot \nabla \cdot (\mathbf{v} \otimes \mathbf{v}\tilde{f}_h)$$

MHD displacement — Perturbed EP distribution function

Vlasov equation

$$\partial_t \tilde{f}_h - \{\tilde{h}, F_{eq,h}\} - \{H_{eq}, \tilde{f}_h\} = 0$$

Equilibrium and perturbed hamiltonians

$$H_{eq} = \frac{1}{2}m\left[v_\parallel^2 + \mu B\right]$$

$$\tilde{h} = Ze\left[\phi - v_\parallel A_\parallel\right]$$

Angle-action formalism

Conjugate set of angle-action variables

$$\dot{\boldsymbol{\alpha}} = \frac{\partial H_{eq}(\mathbf{J})}{\partial \mathbf{J}} = \boldsymbol{\Omega}$$

$$\dot{\mathbf{J}} = -\frac{\partial H_{eq}(\mathbf{J})}{\partial \boldsymbol{\alpha}} = 0$$

- Coordinates linked to the charateristic motion of charged particles in tokamaks
- $(\alpha_1, J_1, \Omega_1)$ describe gyration motion
- $(\alpha_2, J_2, \Omega_2)$ describe bounce motion
- $(\alpha_3, J_3, \Omega_3)$ describe precessional motion

Fourier transform in angle-action coordinates

$$\tilde{f}_h = \sum_{\mathbf{n}} \tilde{f}_{h,\mathbf{n}\omega}(\mathbf{J}) e^{i(\mathbf{n}\cdot\boldsymbol{\alpha} - \omega t)}$$

Resonant Vlasov solution

$$f_{h,\mathbf{n}\omega} = -\frac{\mathbf{n}\cdot\partial F_{eq}(\mathbf{J})/\partial\mathbf{J}}{\omega - \mathbf{n}\cdot\boldsymbol{\Omega}} \tilde{h}_{\mathbf{n}\omega}$$

- Angle-action is the natural set of variables to describe wave-particle resonance

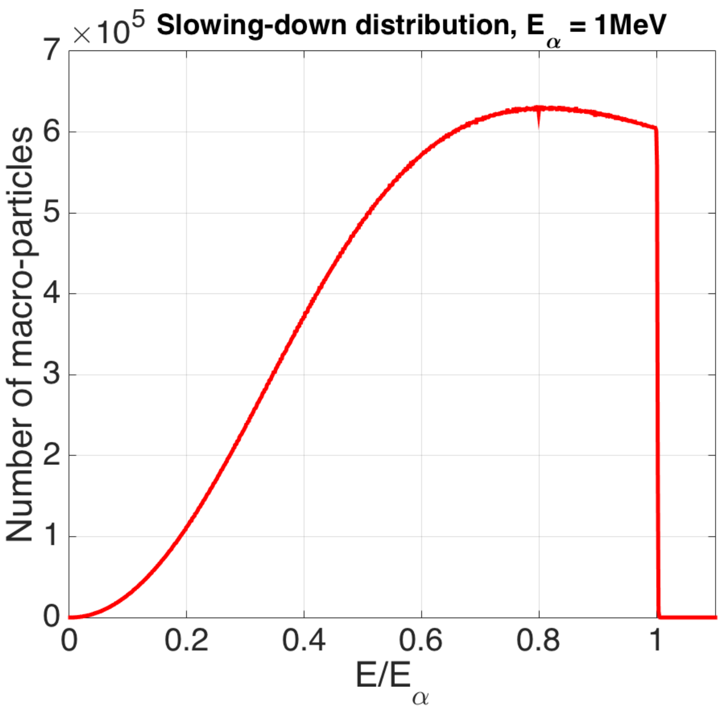
[1] Kaufman, POF 1972
[2] Garbet, PhD thesis 1988

Computation of the kinetic term

$$\lambda_K(\Omega) = \int dP_\varphi d\mu \int_0^{E_\alpha} \frac{Q(P_\varphi, \mu, E)}{(v - v_+)(v - v_-)} dE$$

$$C\delta W_K = \lambda_{K,int} + \lambda_{K,res}(\Omega)$$

$$F_{eq,SD} = \beta\, n_h(r) \frac{\theta(v_\alpha - v)}{v^3 + v_c^3(r)}$$



- Q computed in the thin orbit width limit on circular flux surfaces
- Isotropic Slowing-Down distribution considered
- $v_\pm(P_\varphi, \mu, \Omega) \in \mathbb{C}$, poles are not unique due to passing particles
- Resonant integral can be treated analytically in specific situations

Non-perturbative kinetic dispersion relation

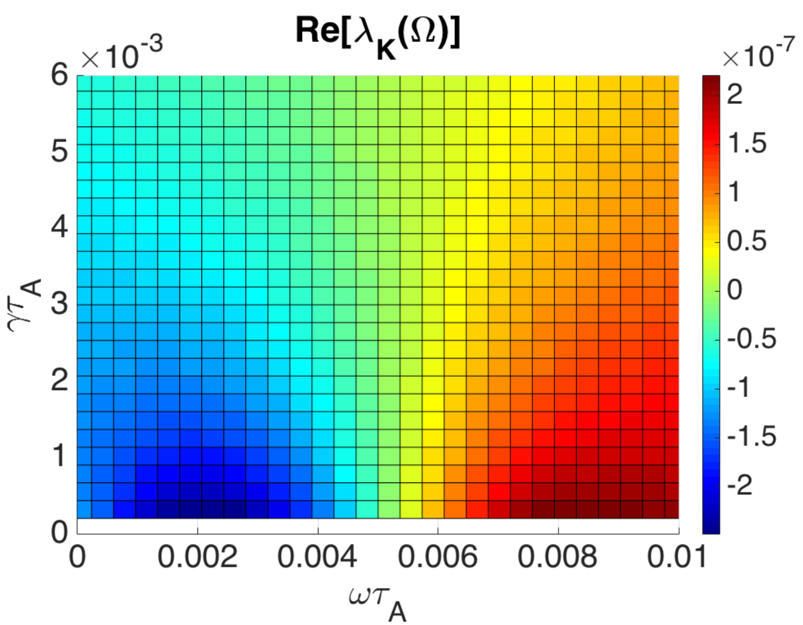
Resistive contribution

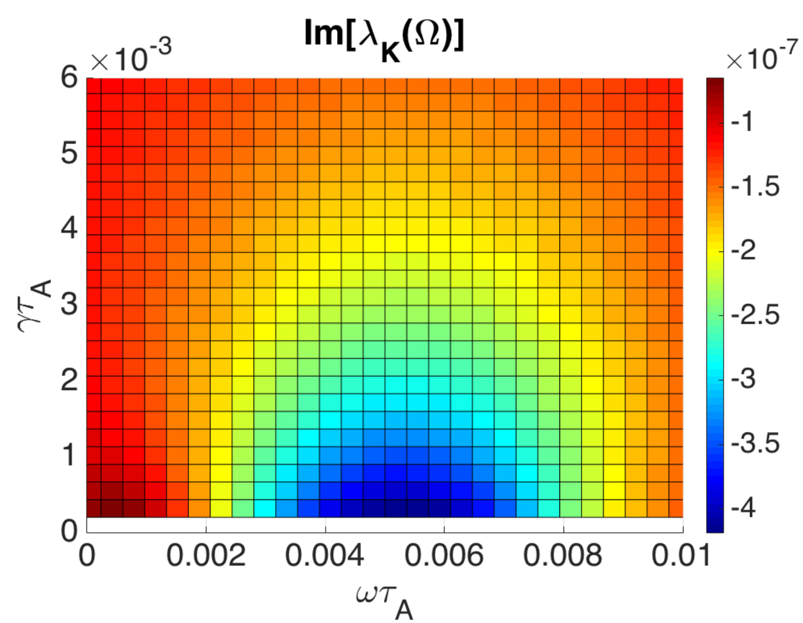
$$I_R(\Omega) = \frac{8\Gamma\left(\frac{\Lambda^{3/2}+5}{4}\right)}{\Lambda^{9/4}\Gamma\left(\frac{\Lambda^{3/2}-1}{4}\right)} \qquad \Lambda = -i\Omega^*\tau_A(S/s_0^2)^{1/3}$$

Bulk MHD contribution

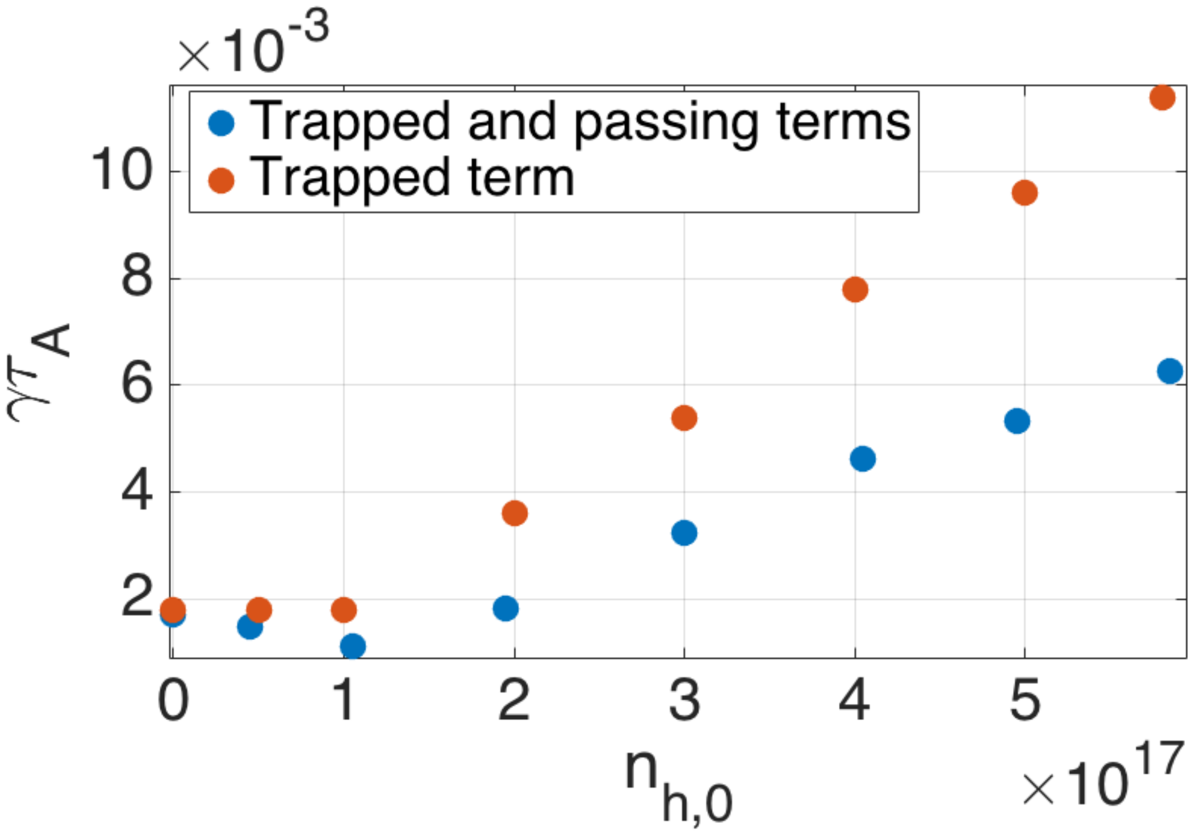
Computed for specific equilibria through the combined codes CHEASE / XTOR-2F

$$\mathcal{D}(\Omega, n_{h,0}) = \Omega I_r(\Omega) - i[\gamma_{MHD} + n_{h,0}\lambda_K(\Omega)] = 0$$



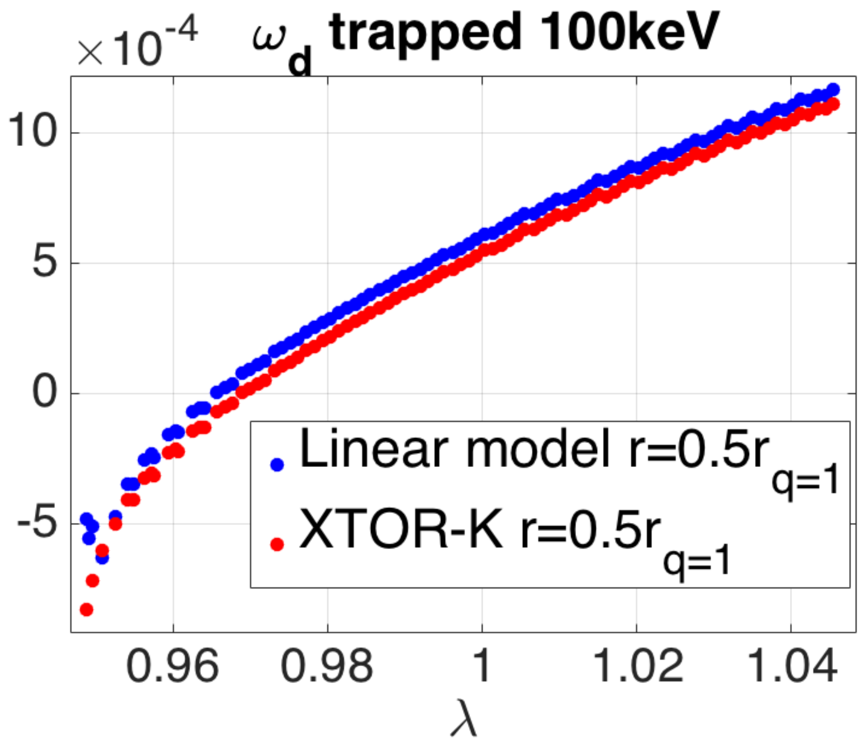


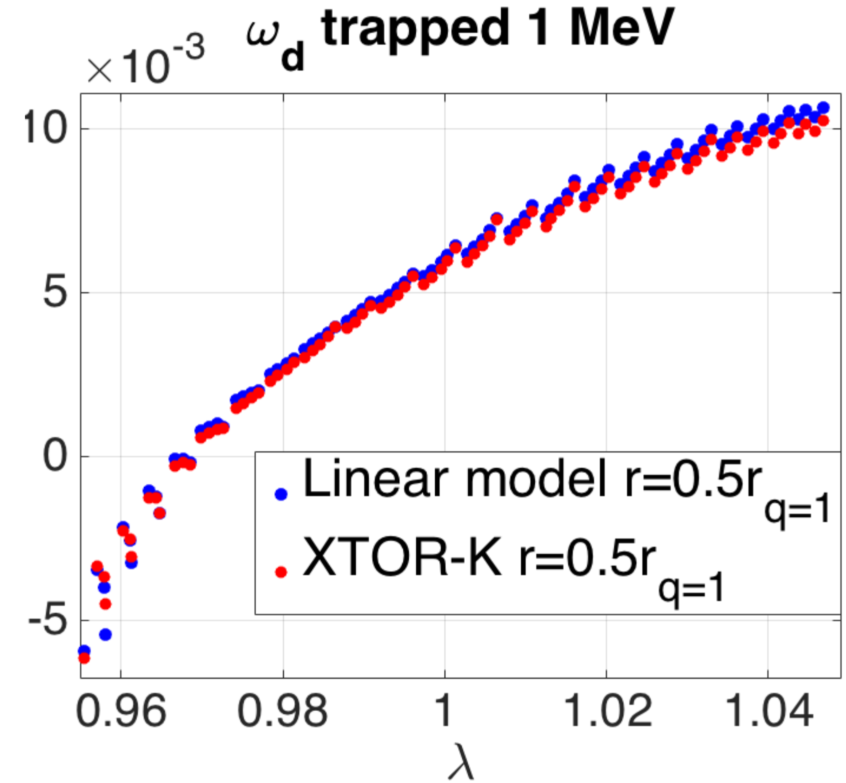
Specificities of the linear model



- Kinetic term takes into account both trapped and passing particles contributions
- Non-resonant kinetic interchange term kept in the computation
- Passing particles contribution is not negligeable

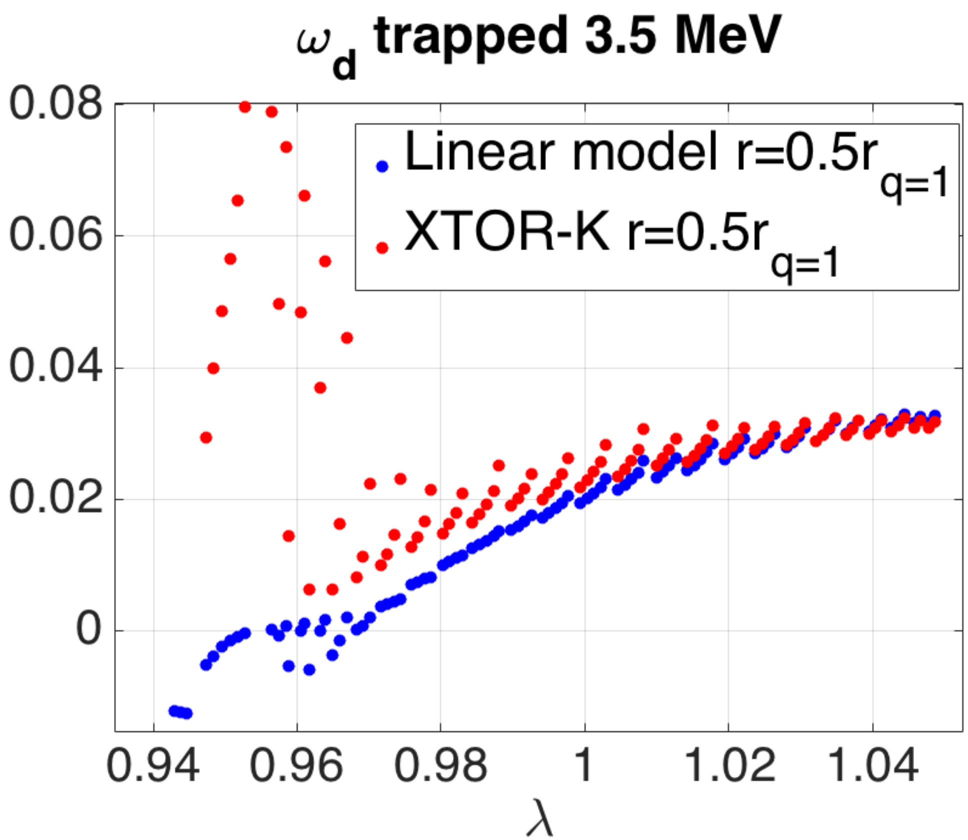
Thin orbit width assumption correct up to E = 1MeV





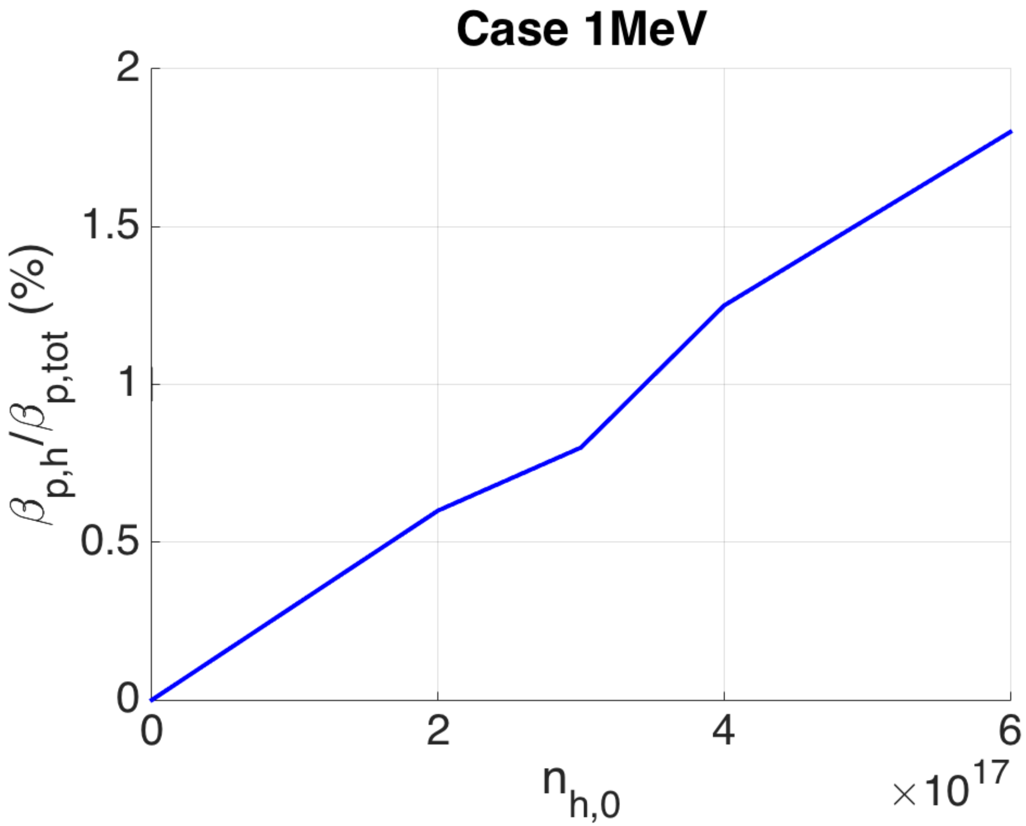
- The linear model use a thin orbit width approximation
- For energies lower than 1 MeV, linear model particle frequencies are correct
- Linear verification possible for particle energy below 1 MeV

Linear model limited at high energies



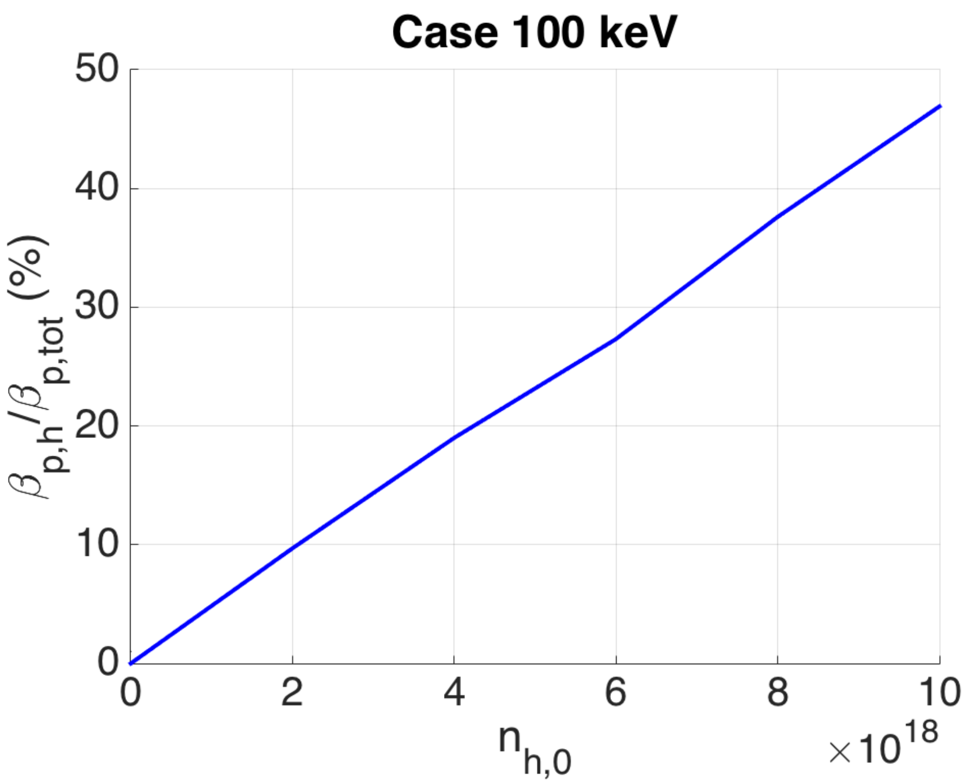
- At high energies (3.5MeV), thin orbit width approximation breaks down
- Linear verification cannot be performed at high energies

Constant MHD contribution assumption valid at 1MeV



- The MHD bulk contribution can only be constant for all $n_{h,0}$ if and only if the metric is weakly affected by EP, with $\beta_{p,h} << \beta_{p,tot}$
- For intermediate energy EP (1MeV), the metric is weakly affected
- EP at peak energy of 1 MeV suitable for linear verification

Linear model limited at low energies

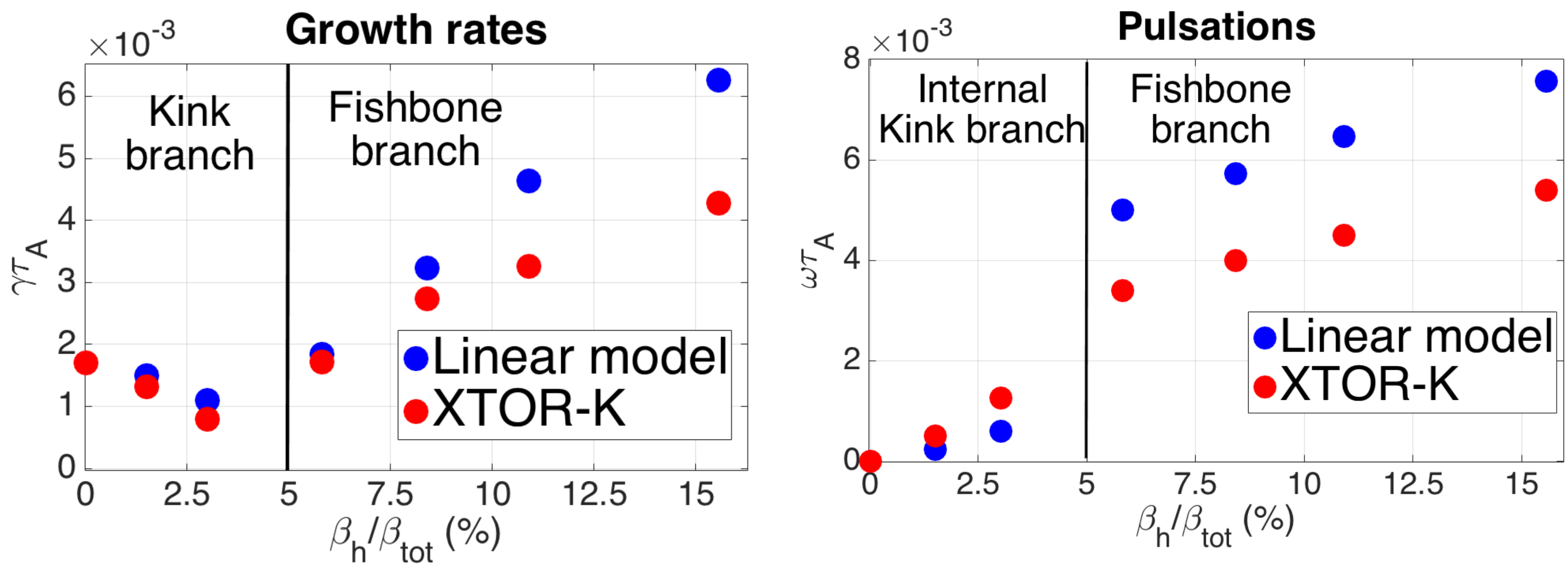


- For low energy EP (100 keV), high density are required to trigger the fishbone instability, which modifies the metric significantly

- δW_{MHD} is then significantly modified as $n_{h,0}$ is increased

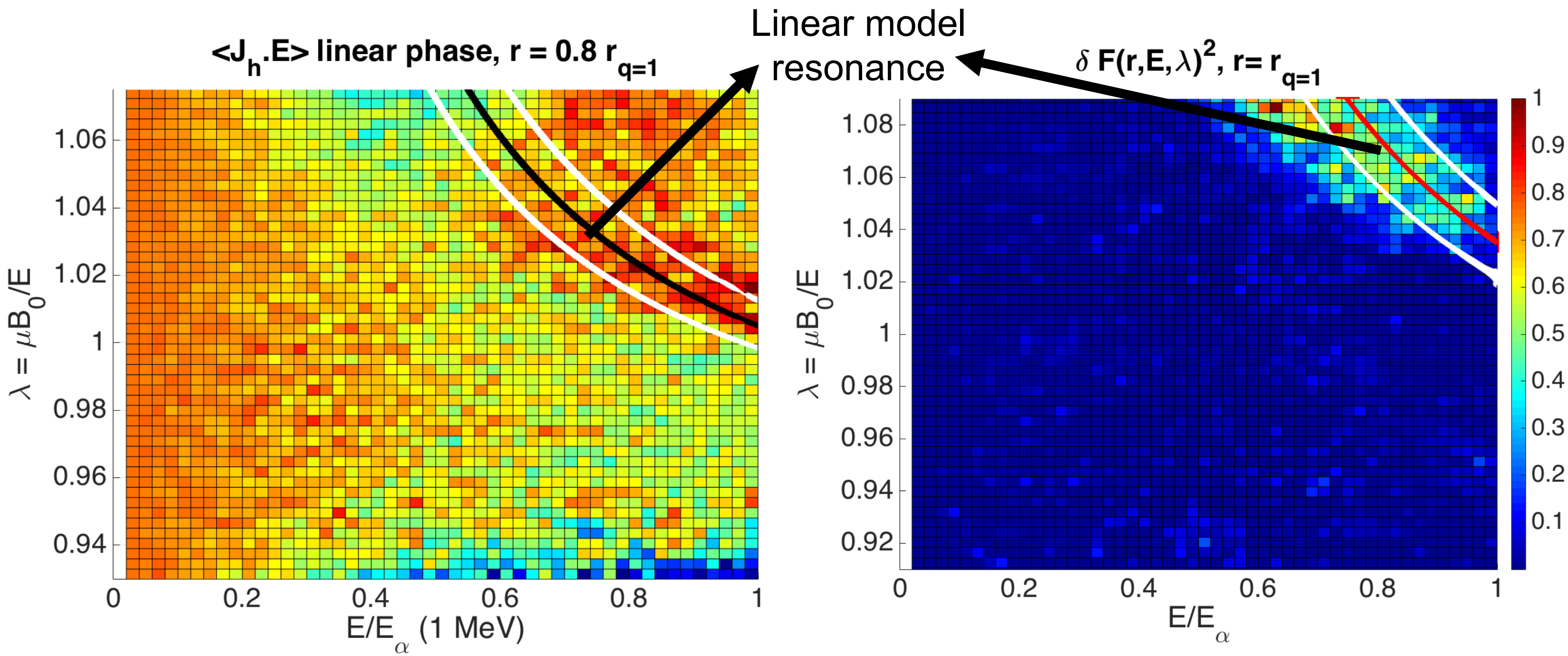
- Linear verification cannot be performed at low energies

XTOR-K verified by Linear theory



- Linear theory agrees reasonably well with XTOR-K
- Discrepancies arise at higher EP density
- At these densities, differences between XTOR-K and the linear model are enlarged, which explains discrepancies

Matching phase-space resonant zones



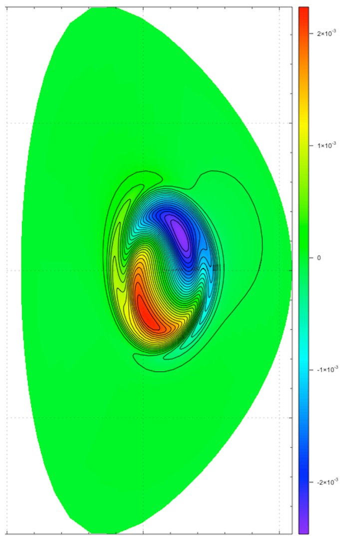
- Similar zones of precessionnal resonance are found with nonlinear simulations with XTOR-K
- Energy exchange noisy because the end of the linear phase is only a couple of kink rotation periods
- Discrepancies in phase space positions can be due to the thin orbit width assumption

Outline

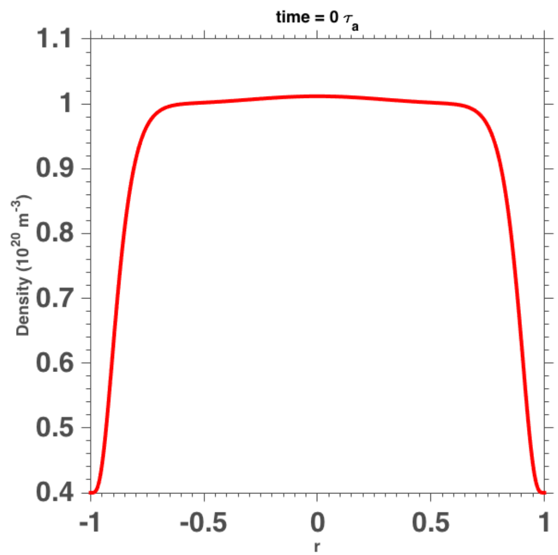
- I) Description and linear verification of the hybrid nonlinear code XTOR-K
- II) Determination of the fishbone β_h thresholds for the ITER 15 MA case
- III) Nonlinear study of the fishbone-induced EP transport for a circular ITER-like equilibrium

Linear simulation of the ITER 15 MA case with XTOR-K

Up-down symmetric geometry

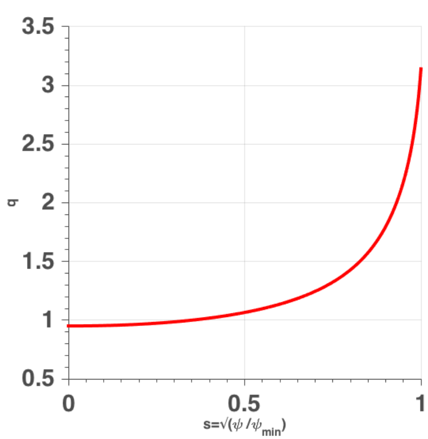


Flat bulk density profile

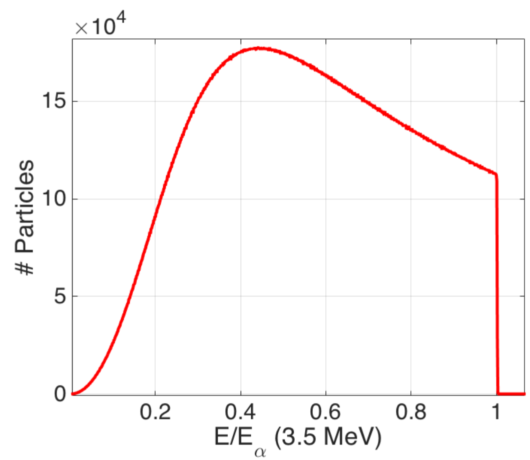


- Parametric study of the ITER 15 MA case
- Realistic geometry and profiles similar to those from integrated modelling codes

Flat q profile



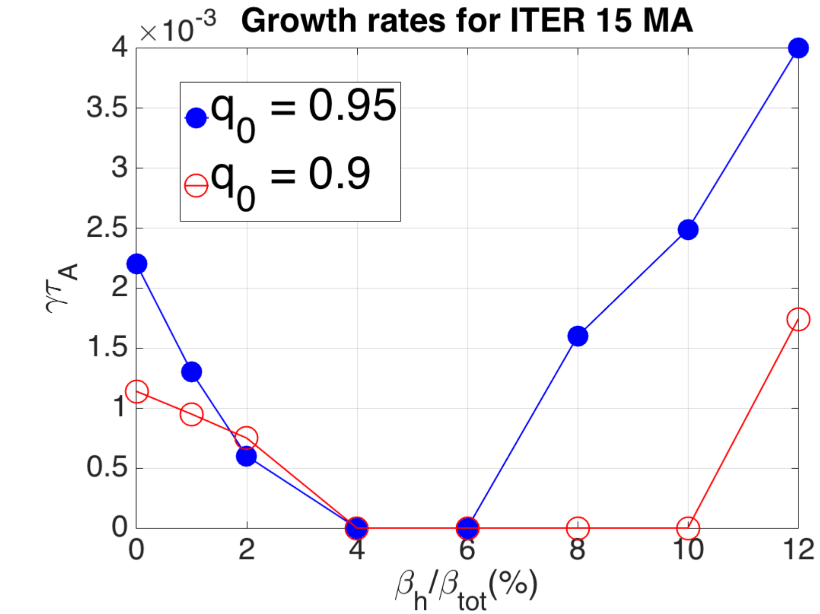
Slowing-Down distribution

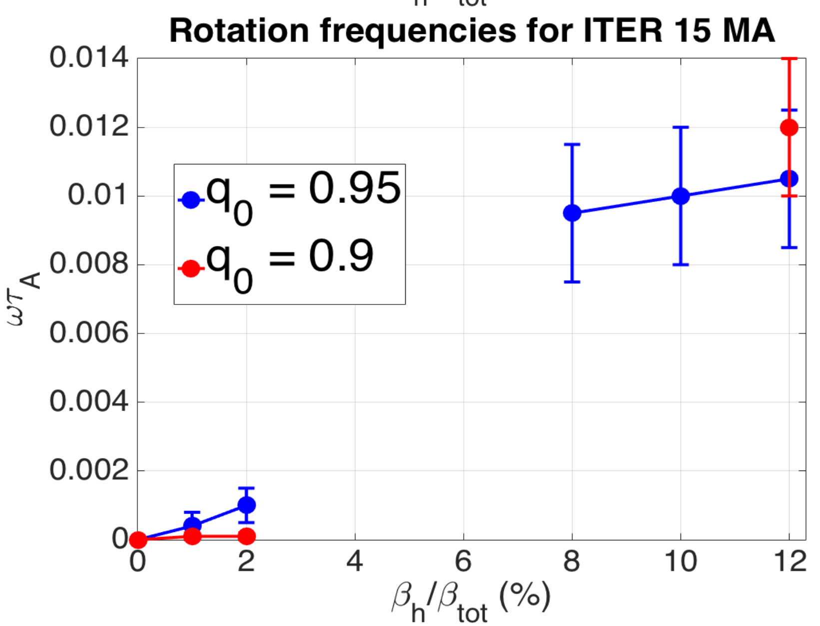


- $n_{i0} = 10^{20}\ m^{-3}$
- $T_{i0} = T_{e0} = 23$ keV
- Peaked EP density profile

$$n_\alpha(r) = n_{\alpha,0}(1 - r^2)^6$$

ITER 15 MA could be fishbone-unstable





- For $q_0 = 0.95, 0.9$, threshold for the fishbone instability is around p_h / p_{tot} = 5-8%
- ITER is unstable against fishbone instability for specific equilibria
- Differences with [1] can be due to different EP density profiles and q profiles
- Several equilibria need to be tested to complete this study, with different q_0

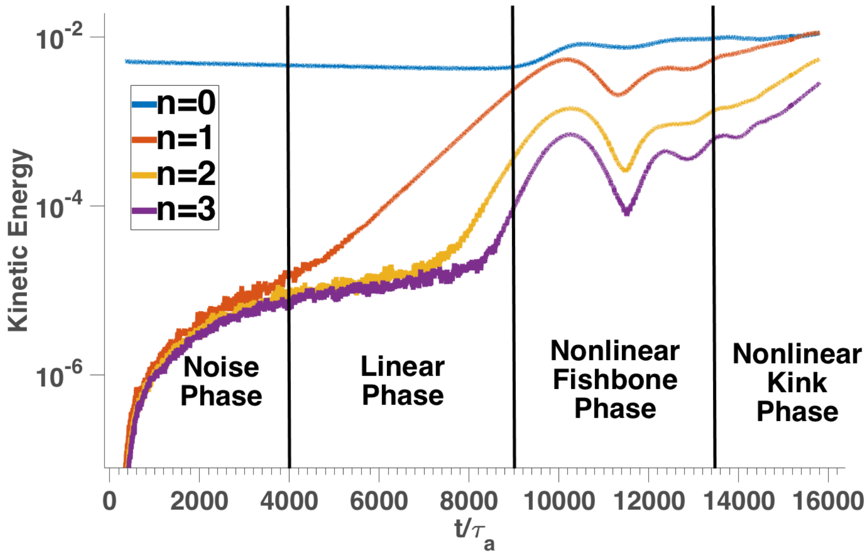
[1] G. Fu et al, POP 2006

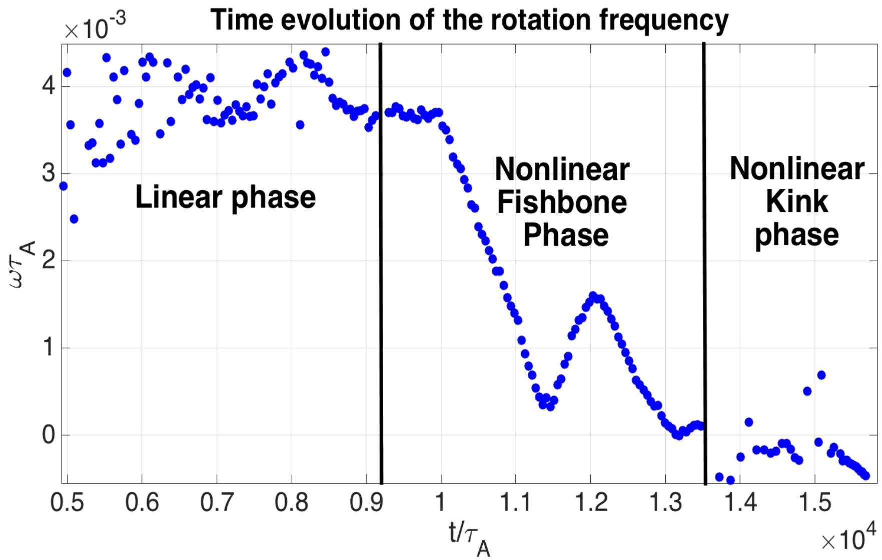
Outline

- I) Description and linear verification of the hybrid nonlinear code XTOR-K
- II) Determination of the fishbone β_h thresholds for the ITER 15 MA case
- III) Nonlinear study of the fishbone-induced EP transport for a circular ITER-like equilibrium

Nonlinear simulation of the fishbone instability

Time evolution of the kinetic energy

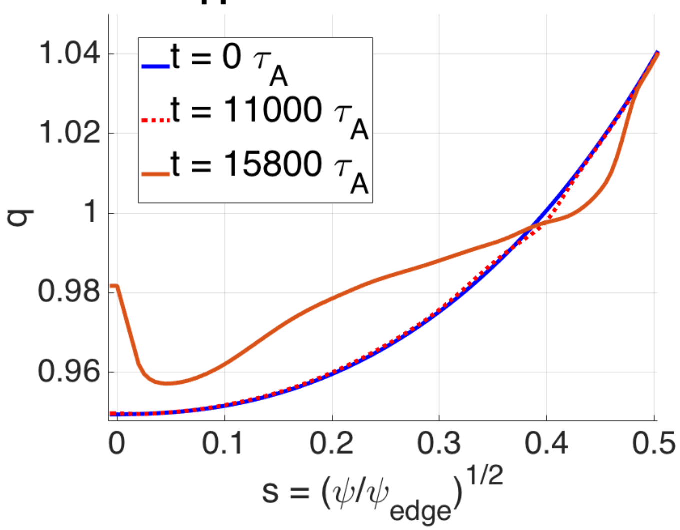


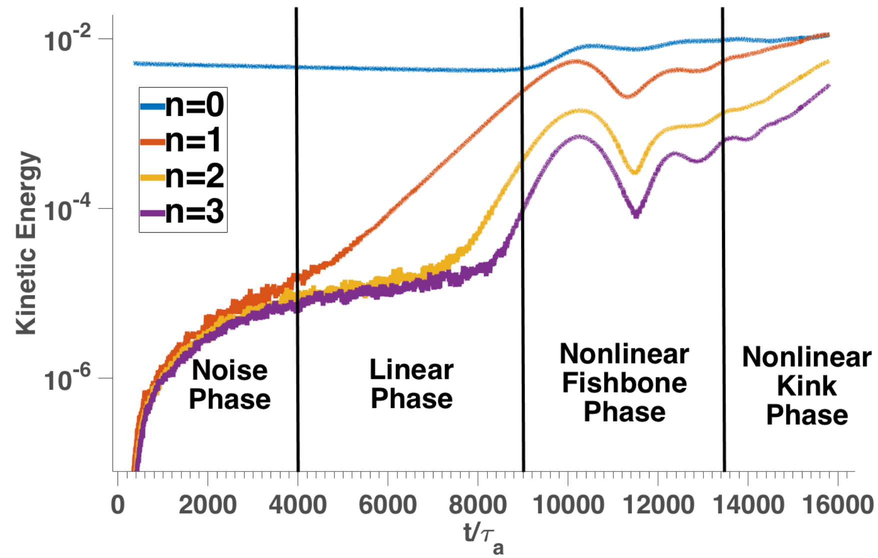


- A first non-linear simulation has been performed for a circular equilibrium, peak energy at 1 MeV
- Fishbone oscillations are observed before reconnection due to the kink instability
- Strong chirping is associated with the fishbone oscillations, as well as mode saturation
- Instability rotation goes to zero in the kink phase

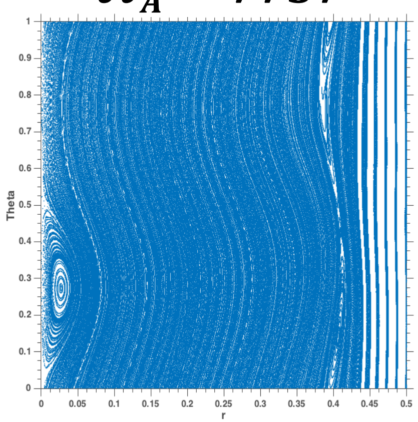
Evidences for fishbone oscillations

q profile time evolution

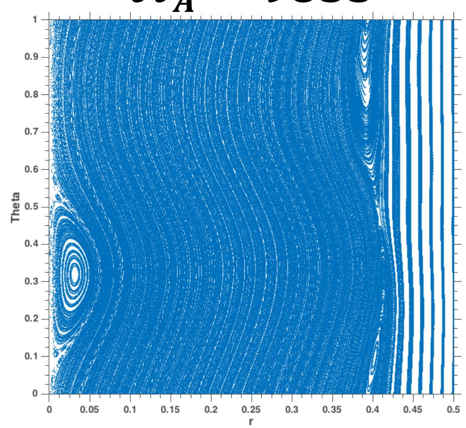




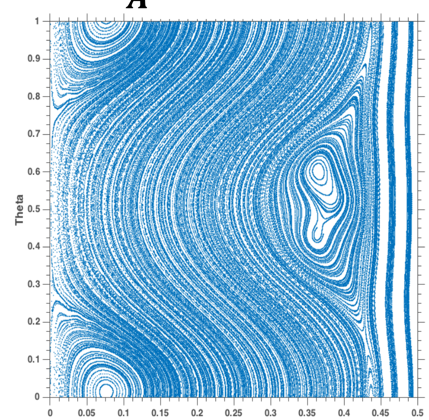
Beginning fishbone phase
$t\tau_A = 7757$



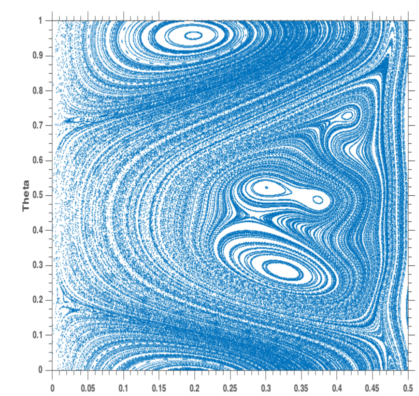
Saturation fishbone phase
$t\tau_A = 9853$



End fishbone phase
$t\tau_A = 13614$

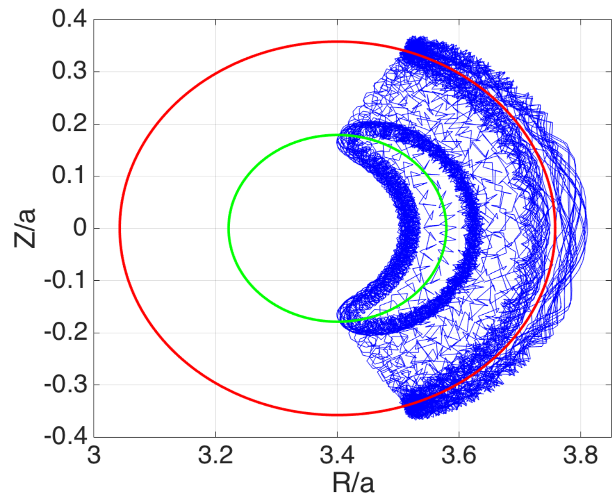


Kink phase
$t\tau_A = 15801$

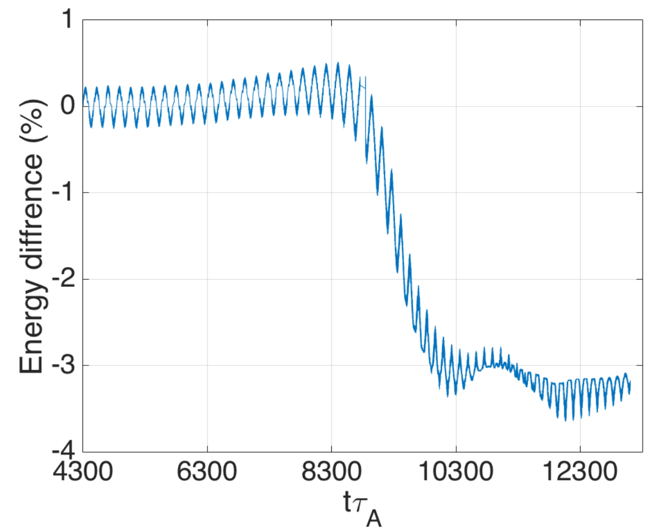


Typical evolution of a resonant particle

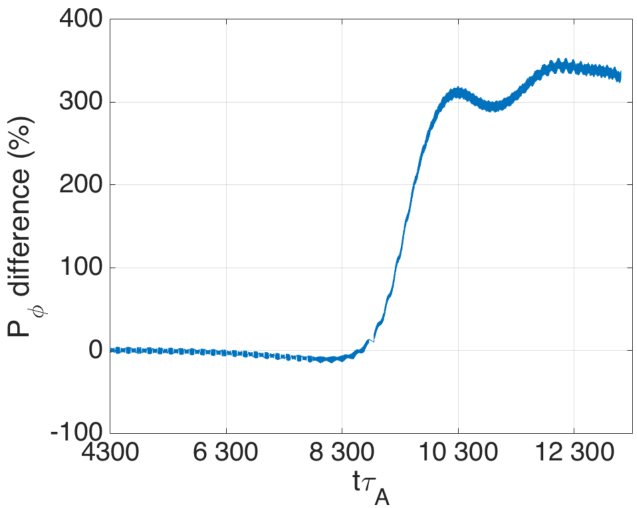
Trajectory in the poloidal plane



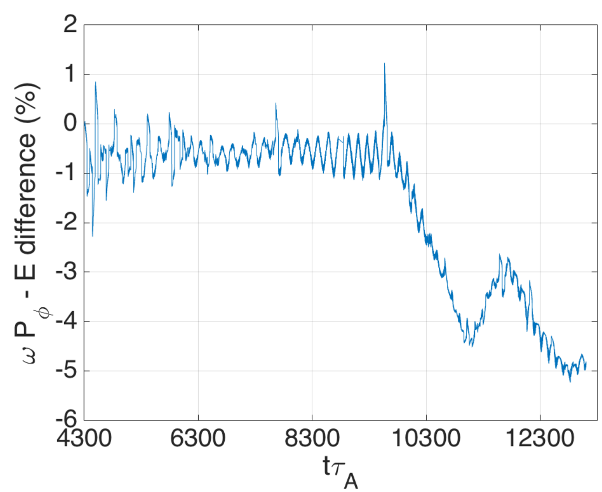
Variation of energy



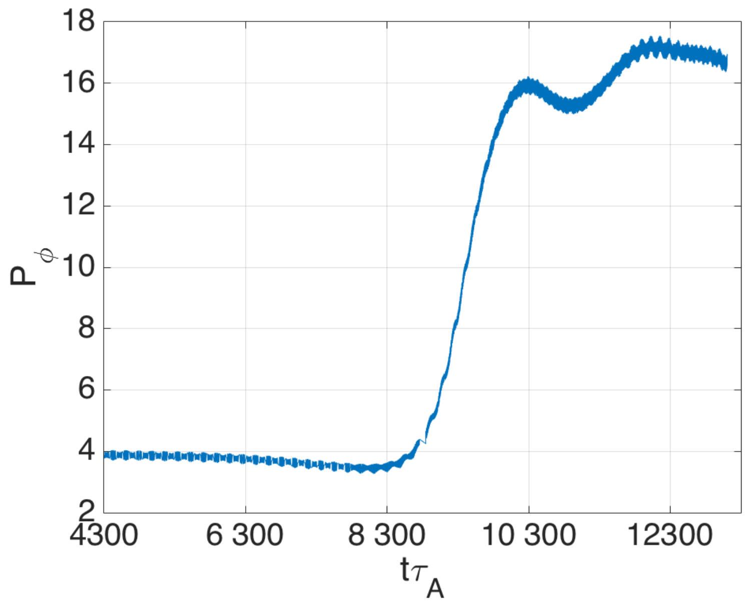
Variation of the canonical toroidal momentum

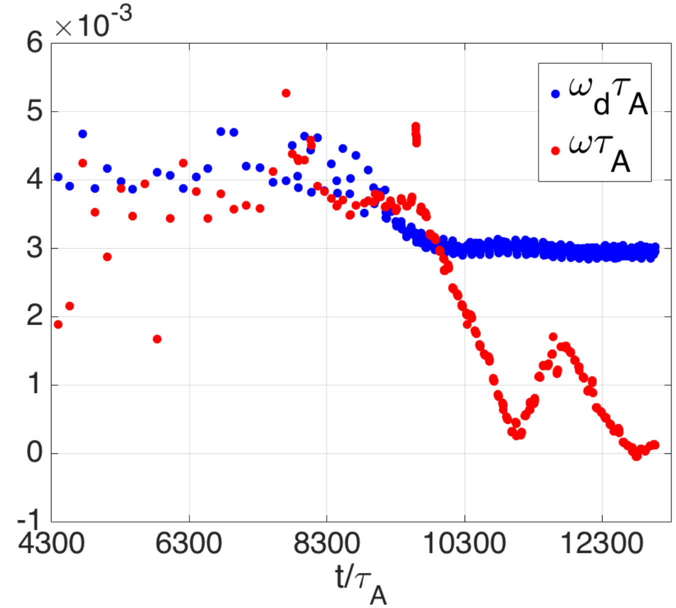


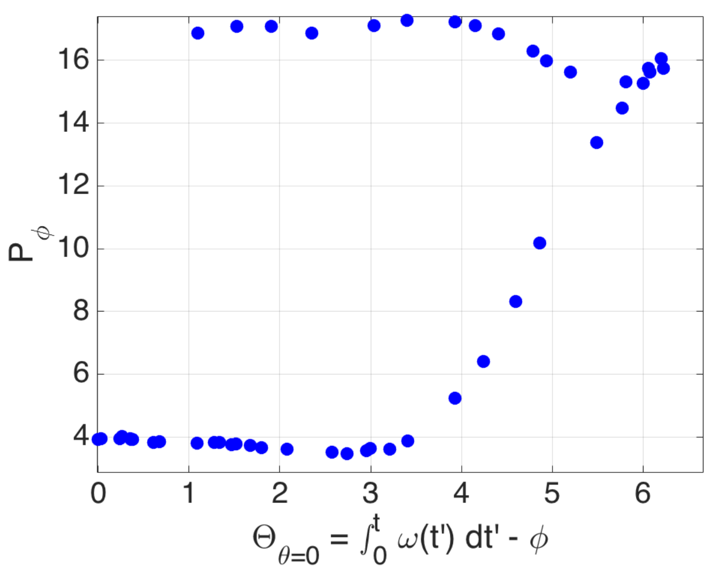
Variation of the perturbed invariant



Interaction of a resonant particle with the kinetic-MHD mode

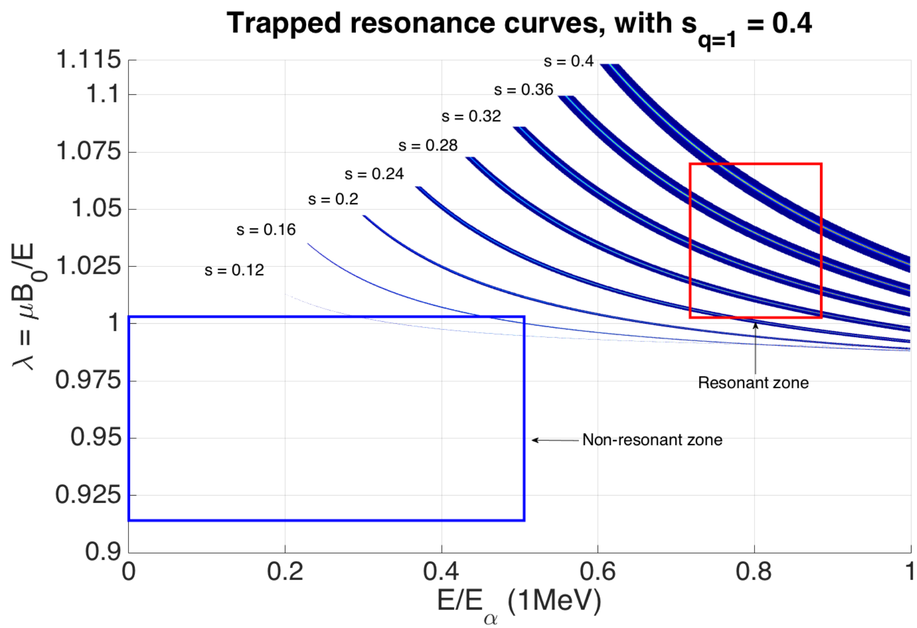


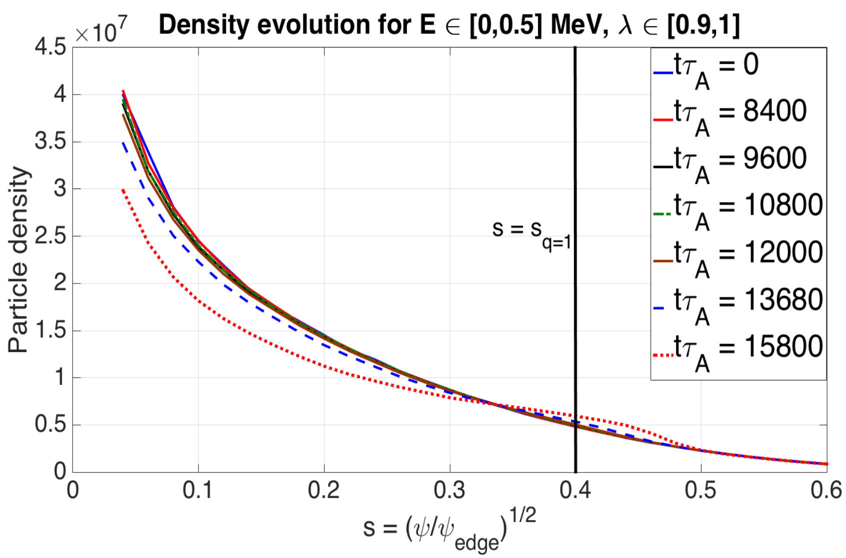


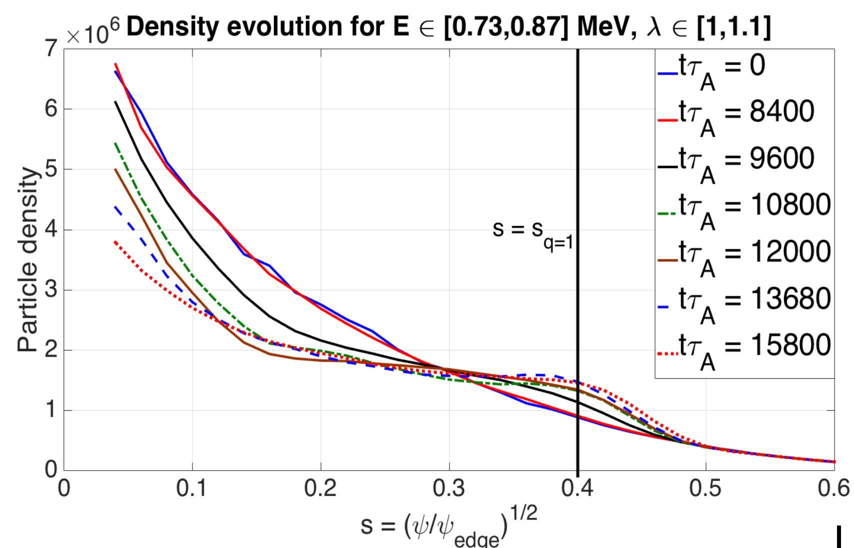


- Resonant particles move radially outward
- Transport induced by particle mode detrapping, due to wave-particle detuning
- Relationship between detrapping and chirping to be investigated

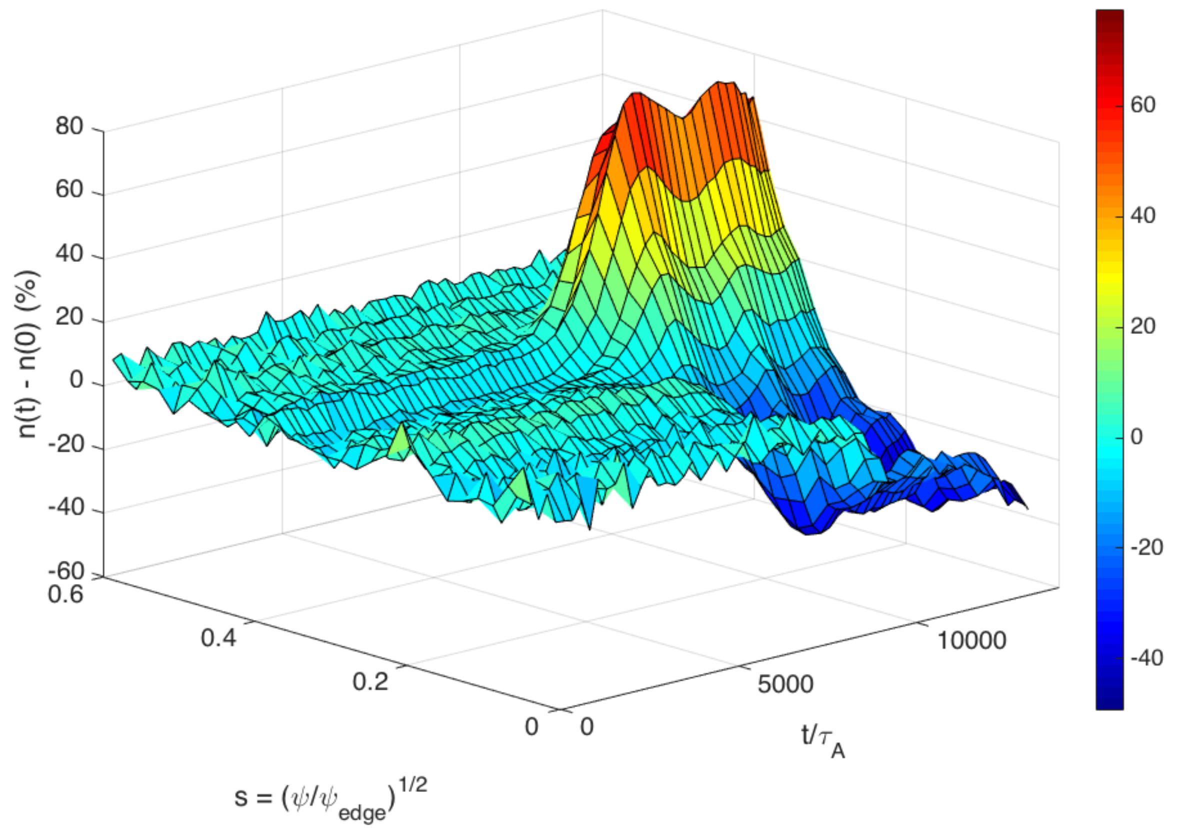
Resonant EP density profile flattens





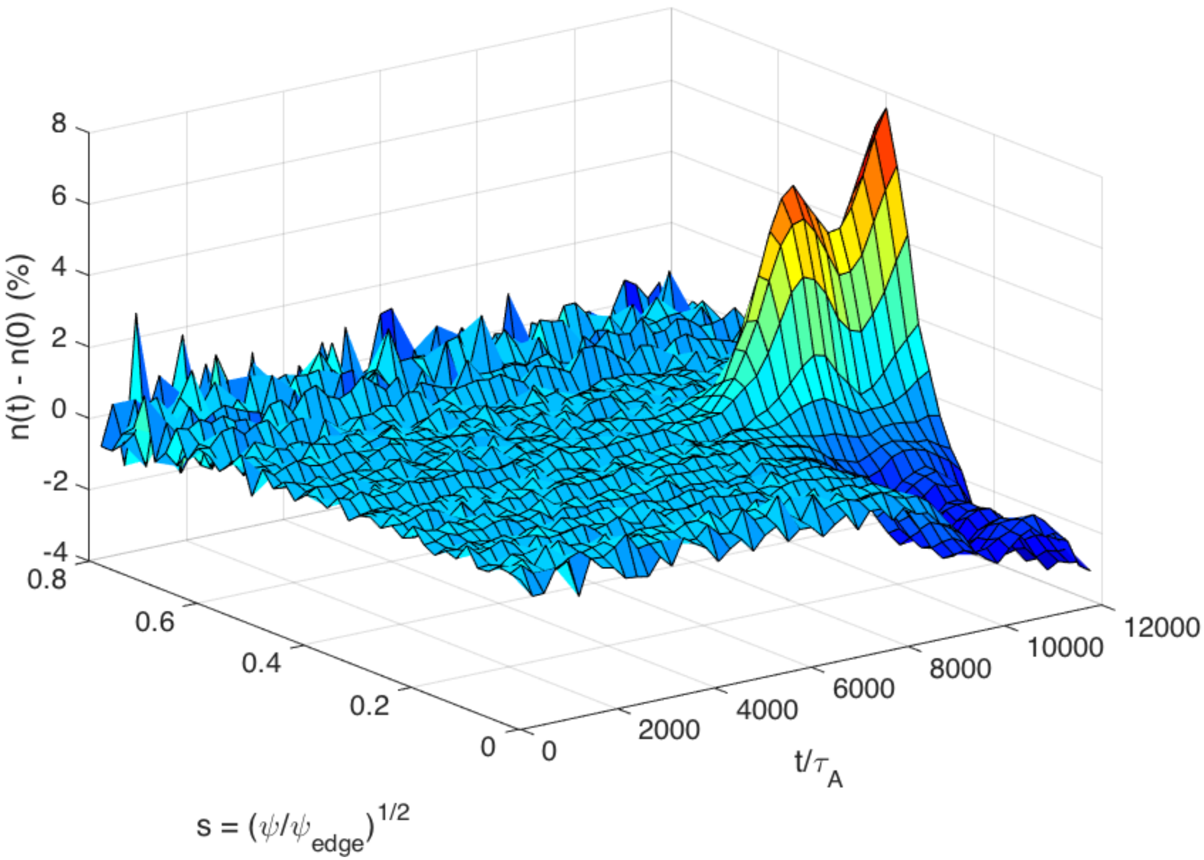


Significant radial transport of resonant EP



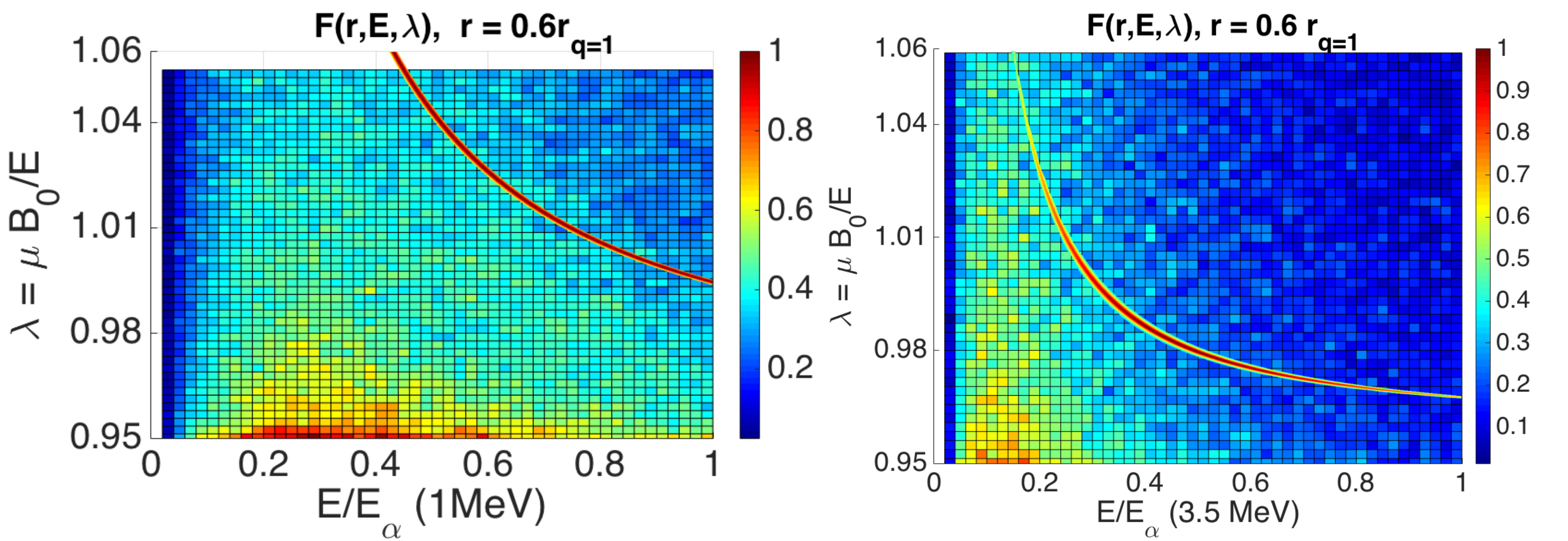
- In resonant regions, transport of EP is substancial (50%)
- Resonant region are very dependent of the imposed EP distribution

Weak transport of all particles



- Overall EP transport in core plasma around 5% in the fishbone phase
- EP are transported to q=1
- Fast mode chirping may prevent to transport large amount of EP

What transport for more ITER realistic equilibria ?



- The magnitude of the EP transport is directly related to position of resonances onto EP distribution
- For more ITER realistic equilibria, precessional resonance spans wider portions of the EP distribution
- Nonlinear simulation with more realistic ITER equilibrium needed

Conclusion

- Nonlinear hybrid code XTOR-K verified against analytical theory

- ITER found to be unstable against fishbone instability for specific equilibria

- Fishbone induced transport of EP in nonlinear phase found to be weak for a specific equilibrium

Perspectives

- Linear analysis of fishbone thresholds on ITER to be extended for $q_0 > 1$

- More complete Kinetic Poincaré diagnostics are implemented to understand the nonlinear interplay between mode chirping and EP transport

- Nonlinear results need to be generalized for more ITER realistic equilibria, closer to the fishbone threshold