Kinetic Effects on Adiabatic Index via Geodesic Acoustic Continuum Calculations

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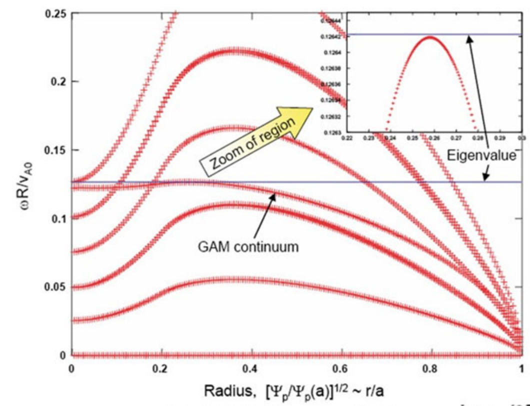
PPPL - Theory Seminar

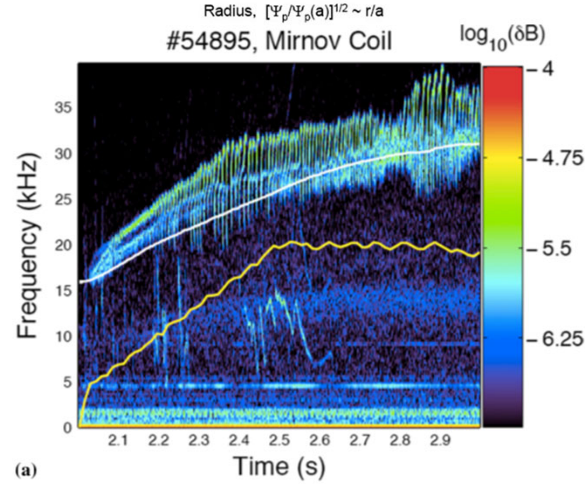
Outline

- Geodesic Acoustic Modes – GAM – brief introduction and relation to energetic particle transport
- MHD and kinetic calculations for the geodesic acoustic continuum.
- Introduction of kinetic effects in the pure MHD code, NOVA.
- Results for the low frequency continuum, with experimental correspondence with global N=0 mode in DIII-D.

GAM – Geodesic Acoustic Mode

- GAM are a low frequency electrostatic oscillation, with toroidal N=0, poloidal M=$\pm$1.
- First predicted by [*N. Winsor, Phys. Fluids (1968)]* in PPPL this mode remained for decades without attention.
- The eigenmode can appear with reverse q profile.
- Was observed in the presence of energetic ions, as in [*H.L. Berk NF (2006)*] that describe the mode in CASTOR, but not match the frequency.
- They can appear in the maximum of the GAM continuum, the accumulation point.





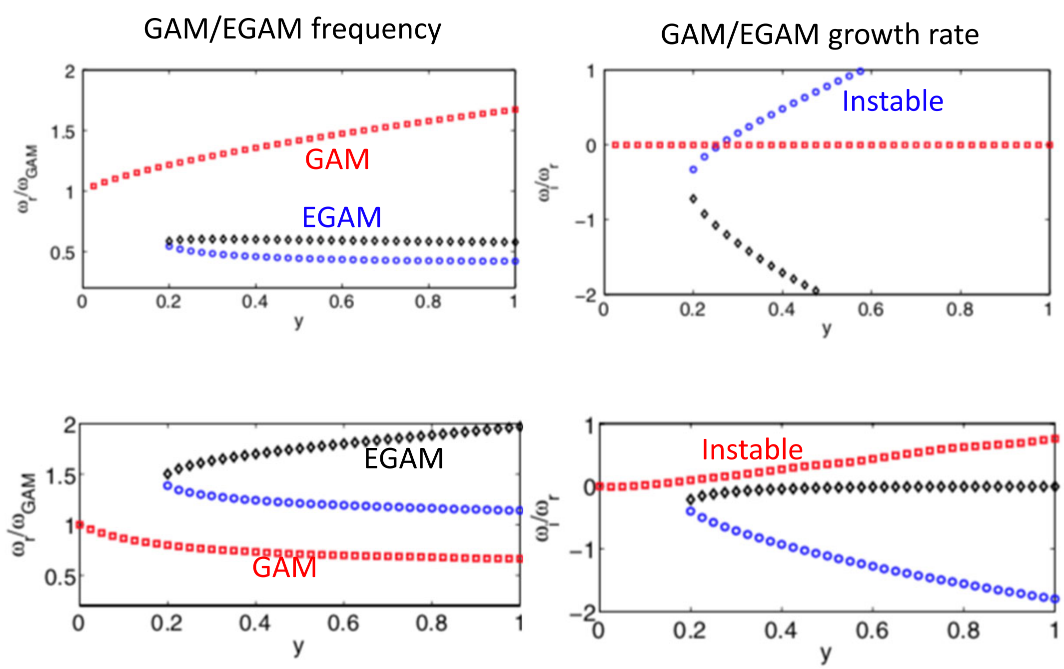
H.L. Berk, NF (2006)

EGAM – Energetic Geodesic Acoustic Mode

- A new Energetic Particle induced GAM, called EGAM was theoretically predicted [*G. Y. Fu, PRL (2008)*] and experimentally observed [*R. Nazikian, PRL (2008)]*.

- This modes are intimately related with zonal flows, that can induce and suppress turbulence and transport.

- Understanding of these modes can improve the control in plasma transport and turbulence.



$$Y = \langle P_{\parallel h} + P_{\perp h} \rangle / 2\gamma P_{\text{th}}$$

G. Y. Fu, PRL (2008)

NOVA Code

- NOVA calculates the continuum and eigenmodes based on the ideal MHD equations [*C.Z. Cheng, Phys. Fluids (1986)*]:

$$\omega^2\rho\frac{|\nabla\psi|^2}{B^2}\xi_s+(\boldsymbol{B}.\nabla)\frac{|\nabla\psi|^2}{B^2}(\boldsymbol{B}.\nabla)\xi_s+\gamma p k_s\nabla.\vec{\xi}=0,$$

$$\left(\frac{\gamma p}{\boldsymbol{B}^2}+1\right)\nabla.\vec{\xi}+\frac{\gamma p}{\omega^2\rho}(\boldsymbol{B}.\nabla)\frac{\boldsymbol{B}.\nabla}{B^2}\nabla.\vec{\xi}+k_s\xi_s=0. \quad (1)$$

where ψ is the poloidal magnetic flux, p the plasma pressure, ρ density, $\xi_s\equiv\frac{\vec{\xi}[\boldsymbol{B}\times\nabla\psi]}{|\nabla\psi|^2}$, $\vec{\xi}$ the plasma displacement, $k_s\equiv\frac{2\boldsymbol{k}.[\boldsymbol{B}\times\nabla\psi]}{|\nabla\psi|^2}$, $\boldsymbol{k}$ the and $\boldsymbol{B}$ are vector of magnetic curvature and field.

- The adiabatic index γ, usually 5/3, is a free parameter.
- The code computes all the continua, eigenmodes and the mode structure, and is largely used to study low frequency branch as in [*N.N. Gorelenkov, PLA (2007)*].

The GAM continuum and the γ_k

- Ideal MHD theory predicts the GAM frequency [*N. Winsor, Phys. Fluids (1968)*] as:

$$\omega^2_{Gmhd} = \frac{\gamma p}{\rho R^2}\left(2 + \frac{1}{q^2}\right) \quad (2)$$

where R the major radius, q the safety factor.

- From kinetic theory [*A.G. Elfimov, PP (2015)*]:

$$\omega^2_{Gkin} = \left[\frac{7}{4} + \tau_e + 4v^2_{0i} + \frac{23+4\tau_e(4+\tau_e)+59v^2_{0i}}{2q^2(7+4\tau_e)}\right]\frac{v^2_{Ti}}{R^2} \quad (3)$$

where $\tau_e = T_e/T_i$ is the ratio between electron and ion temperature, $v_{Ti} = \sqrt{2T_i\sum n_i / \sum n_i m_i}$ the effective thermal ion velocity, $v_{0i} = V_{0i}/v_{Ti}$ the ion rotation velocity.

- So, γ_k is defined were $\omega^2_{Gmhd} = \omega^2_{Gkin}$:

$$\gamma_k \equiv \omega^2_{Gkin}\left[\frac{p}{\rho R^2}\left(2 + \frac{1}{q^2}\right)\right]^{-1} \quad (4)$$

- This γ_k will replace the standard 5/3 in (1), making the MHD continuum matches the kinetic one.

- The eigenmode will appear for strongly reversed q profile, sufficient to overcome the temperature monotonically decreasing profile.

Drift kinetic Equation applied for N=0 GAM continuum

- In the quasitoroidal coordinates, (r, ϑ, ζ), high aspect ratio $(R \gg r)$, and circular surfaces, for small Larmor radius is possible to write the drift kinetic equation, for N=0, as:

$$\frac{\partial f}{\partial \vartheta} - \frac{i\Omega f}{v_T w} = \frac{e}{mw}\left[\frac{wE_\parallel}{k_0 v_T^2} - \frac{(2w^2 - y^2)}{2v_T \omega_c k_0 R} E_r sin\vartheta\right]\frac{1}{u}\frac{\partial F}{\partial u} \qquad (5)$$

where $\Omega = \omega Rq$, $w, y = v_{\parallel,\perp}/v_T$ are the parallel and perpendicular velocity normalized by the thermal velocity, $u = \sqrt{w^2 + y^2}$, ω_c is the cyclotron frequency, the wave vector $k_0 = 1/qR$, and $E_{\parallel,r}$ are the perturbed electric field, F and f the equilibrium and perturbed distribution function.

- Assuming the perturbation as $X = X_s sin\vartheta + X_c cos\vartheta$, for the poloidal number $m = \pm 1$.

- Integrating (5) with the proper distribution function, the quasineutrality and resonance condition as $\sum_\alpha j_{r\alpha} + j_p$, where $j_p = -i\omega c^2 E_r / 4\pi c_A^2$ is the polarization current, with c_A the Alfvén velocity.
- With gaussian distribution shifted by rotation velocity we can have the result of (3)

Slowing down model for energetic particle

- Using slowing down distribution function, similar to analytical in [Z. *Qiu,PPCF (2010)*] for Dirac angular distribution, and numerical in [*G. Y. Fu, PRL (2008)*] for gaussian:

$$F_{0b} = \frac{n_{0b}\sqrt{1-\lambda_0}}{ln(U_b/U_c)\,\pi^{3/2}} \frac{e^{\left(-(\lambda-\lambda_0)^2/\Delta\lambda_0{}^2\right)}}{\Delta\lambda_0(U^3+U_c^3)}$$

where $\lambda_0 = v_\perp^2/v^2$, $\Delta\lambda_0$ is the width of gaussian distribution, U is the velocity, $U_{b,c}$ the injection and critical energy.

- Solving eq (5), in the limit of $u_{b0} > \omega Rq/v_{Ti} > \tau_e$. Expanding around $\lambda_0 \sim 0.6$ and $\Delta\lambda_0 \sim 0.2$, the continuum is the solution of:

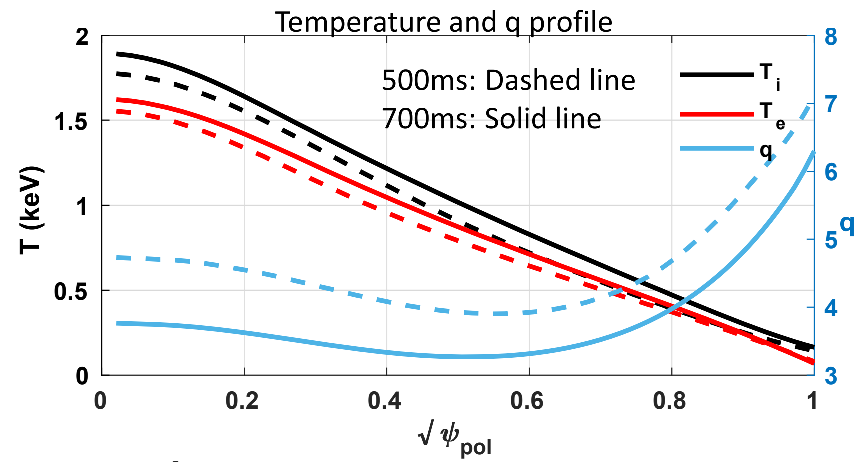
$$\omega^2 = \omega_{G0}^2 + \frac{n_{0b}}{n_0 ln u_b}\left[2\omega^2 q^2(4.7-19\lambda_0+20\lambda_0^2)ln\left(\sqrt{\frac{2}{5}\frac{U_b}{\omega Rq}}\right) - \omega^2 q^2(8.4-31\lambda_0+31\lambda_0^2) + O\left(\frac{\omega^4 q^4 R^2}{U_b^2}\,...\right) - \Delta\lambda_0{}^2\left(2\omega^2 q^2(29-110\lambda_0+111\lambda_0^2)ln\left(\sqrt{\frac{2}{5}\frac{U_b}{\omega Rq}}\right) - \omega^2 q^2(54-202\lambda_0+198\lambda_0^2) + O\left(\frac{\omega^4 q^4 R^2}{U_b^2}\,...\right)\right)\right] \quad (7)$$

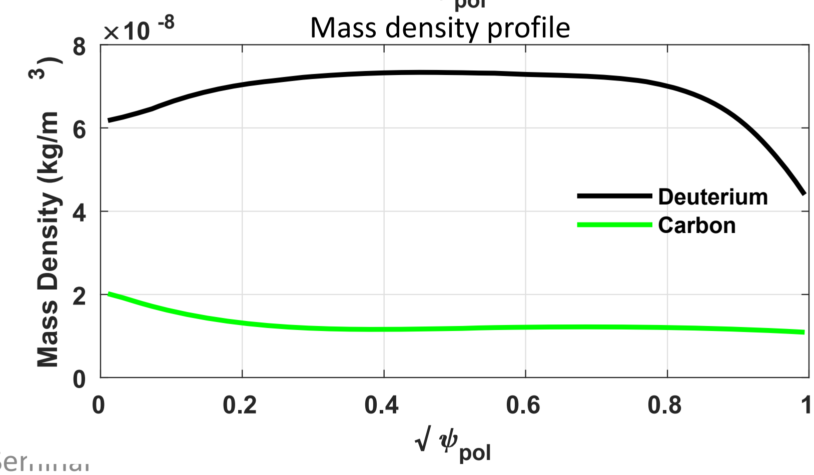
where in ω_{G0} is the thermal GAM frequency.

- The result is similar to GAM branch in [*G. Y. Fu, PRL (2008)*] in analytical form. In the limit $\Delta\lambda_0 \to 0$ is consistent with [Z. *Qiu , PPCF (2010)*]. The necessary condition for instability $\lambda_0 > 0.4$.

TRANSP profile of #159243 – Thermal Plasma

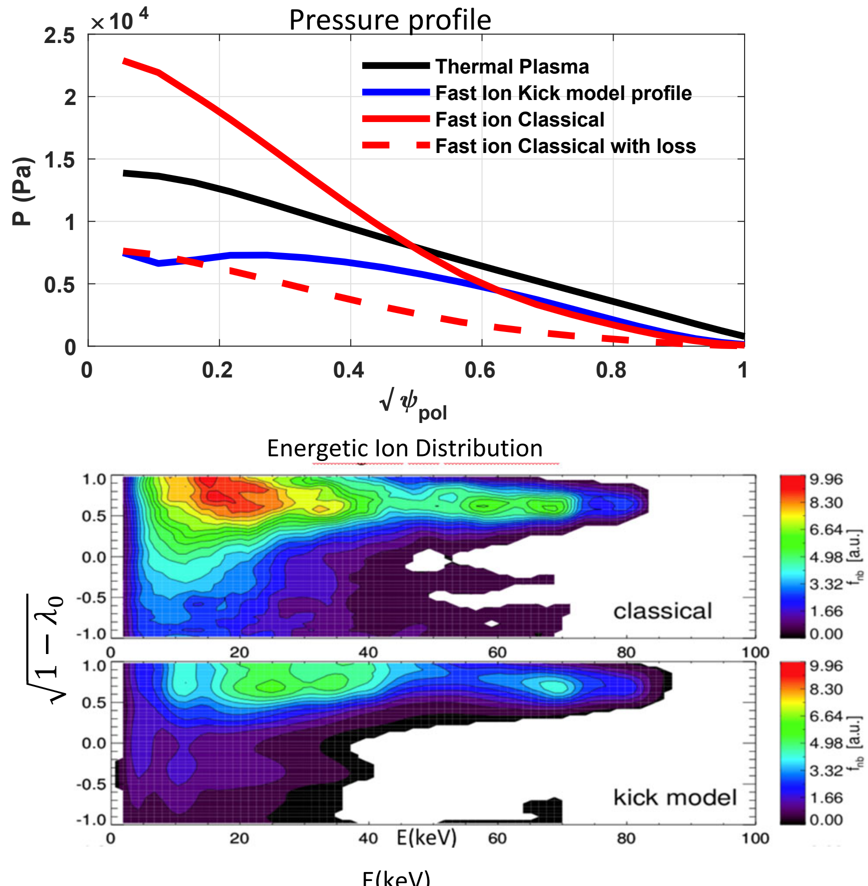
- Time evolution:
  500ms - Dashed line
  700ms - Solid line
- No strong radial variation in temperature, and the ratio remains around $\tau \sim 0.86$.
- The reverse q profile is not strong enough to create a maximum in continuum as analyzed in [*H.L. Berk, NF (2006)*].
- The time variation is important to analyze the time dependence of the GAM frequency.
- Carbon impurities correction are important especially near the center.





TRANSP profile of #159243 – Energetic Ions

- Correction of anomalous losses in the pressure for classical profile is needed to match neutron data.
- The reduced profile are similar to kick model profile.
- Injection angle can be estimated between, $\lambda_0 \sim 0.5$ to 0.65 and $\Delta\lambda_0 \sim 0.2$.
- Energy of injection is 70 to 80keV.
- Critical velocity was used as $5.5. A_b^{1/3}\sqrt{T_e/m_b}$, where A_b and m_b are the mass number and mass of beam particles.



N. Gorelenkov, NF (2018)

GAM continuum calculated by NOVA

- The GAM continuum for 700ms, $\lambda_0 = 0.6$.

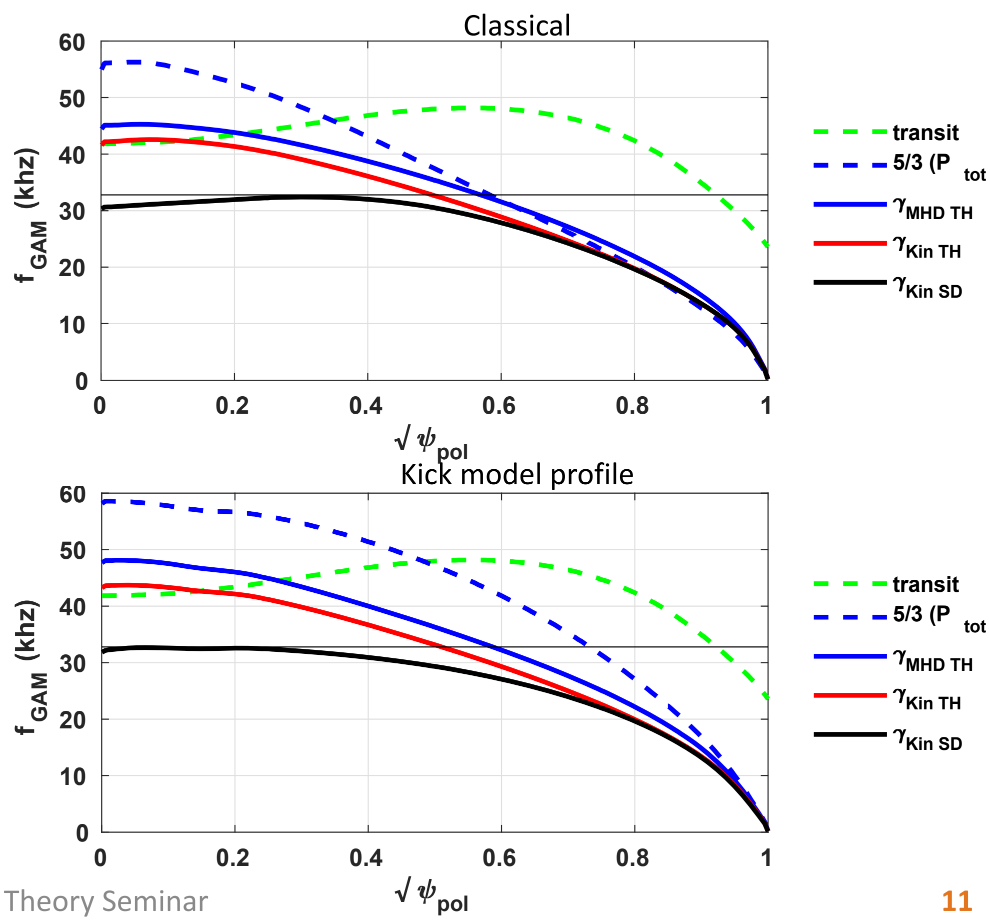
- The "coupling" with energetic ions transit frequency $f_{tr} \equiv U_b\sqrt{1-\lambda_0}/2\pi qR$ produce a maximum in the continuum, for:

  $\sqrt{\psi_{pol}} \approx$ 0.32 for classical

  0.24 for kick model profile.

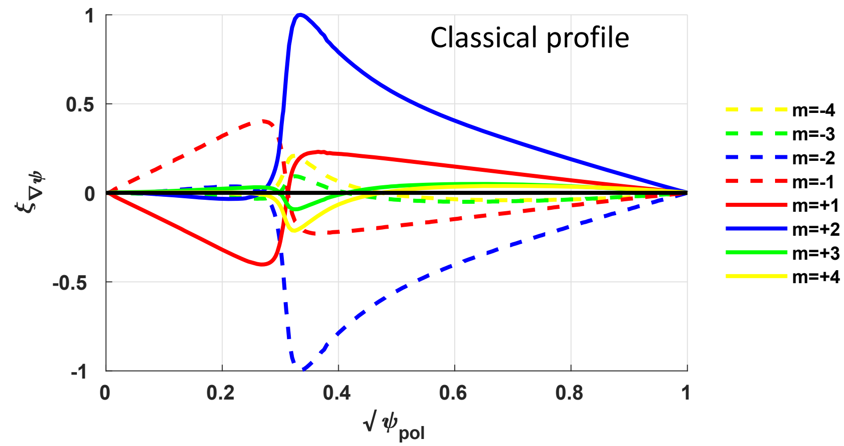
- The eigenmode can be found on the top of the continuum, that is the accumulation point.

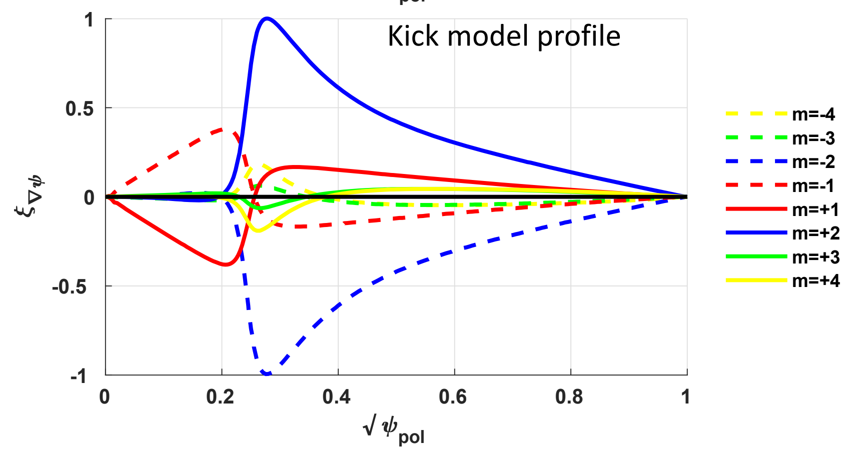
- The frequency is similar for both energetic ion profiles.



GAM Eigenmode calculated by NOVA

- The mode structure is plotted, similar to [*Berk H.L. NF (2006)*]. Calculated by CASTOR code for strongly reversed q profile in JET, with no frequency correspondence due limitations in MHD theory.
- For $m = \pm 1$, responsible for the acoustic oscillation, dominates between the center and the maximum of continuum, its phase is inversed to the edge.
- For $m = \pm 2$, has its maximum at the maximum of continuum. This component correspond to magnetic oscillations [*Lakhin, PLA (2014)*] often observed by Mirnov coils.
- γ_k does not affect the mode structure, that can be modified due to passing and trapped ions, so the mode structure may change.

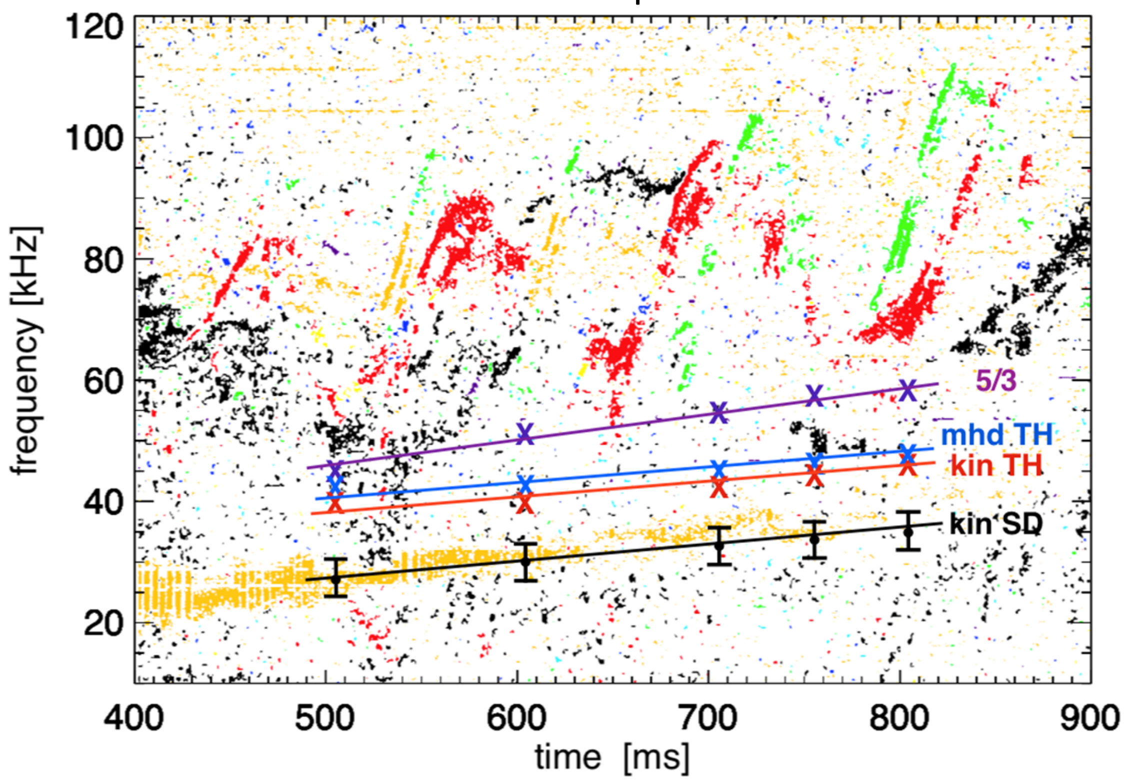




Consistency with N=0 oscilations observed in DIII-D

- The results match the observed N=0 mode by Mirnov coils on #159243 in DIII-D.
- The evolution in time is related mainly due to decreasing of q profile, that directly affected the GAM continuum and transit frequency.
- X means that there is no maximum in the continuum and no eigenmode is formed.
- For the kick model just the time window between 700 – 800 was available.

Classical profile



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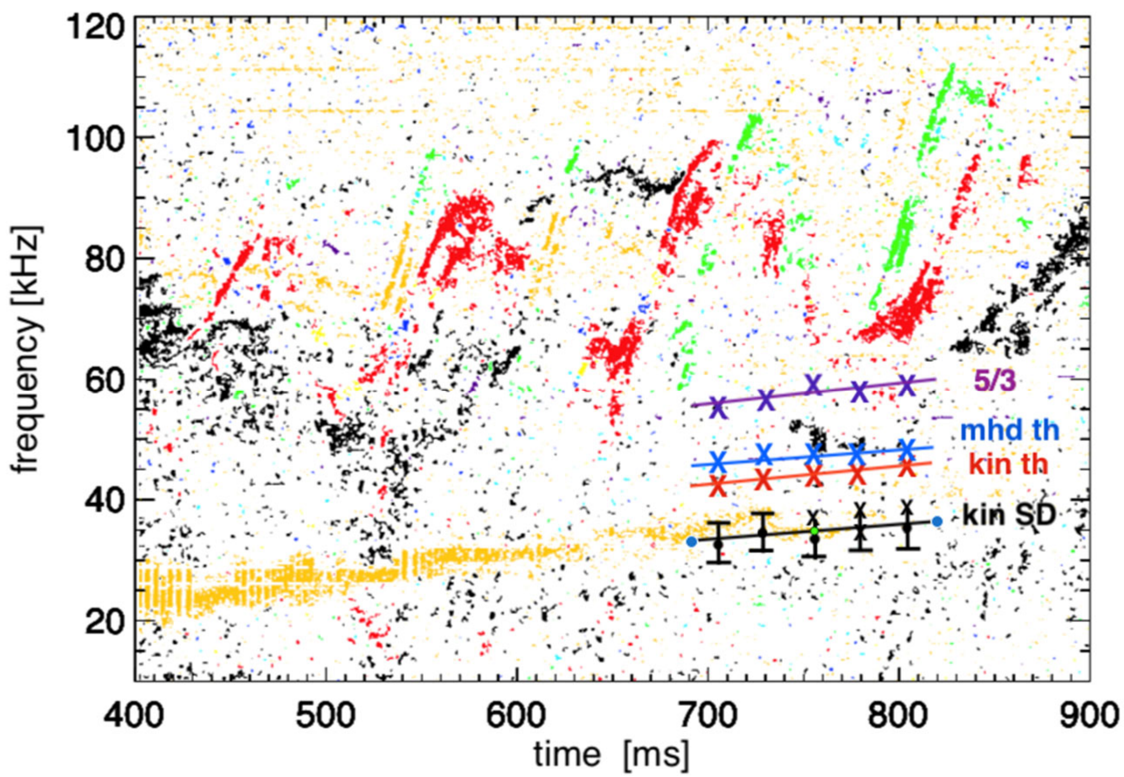
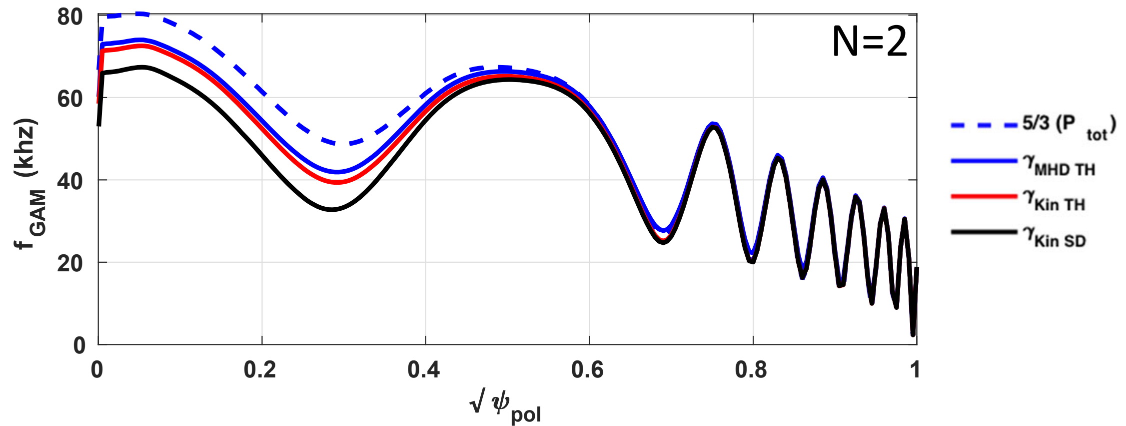
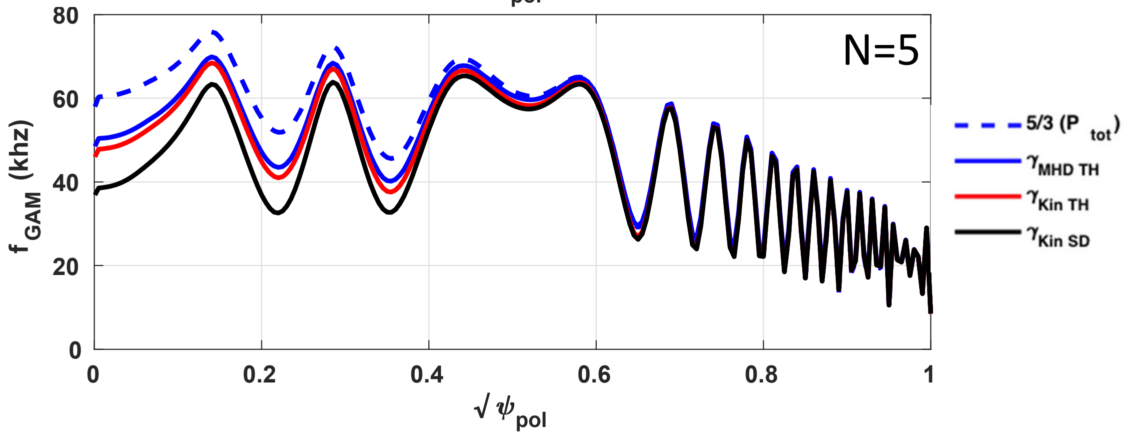
Kick model profile



Generalizing for higher N – Lowest branch

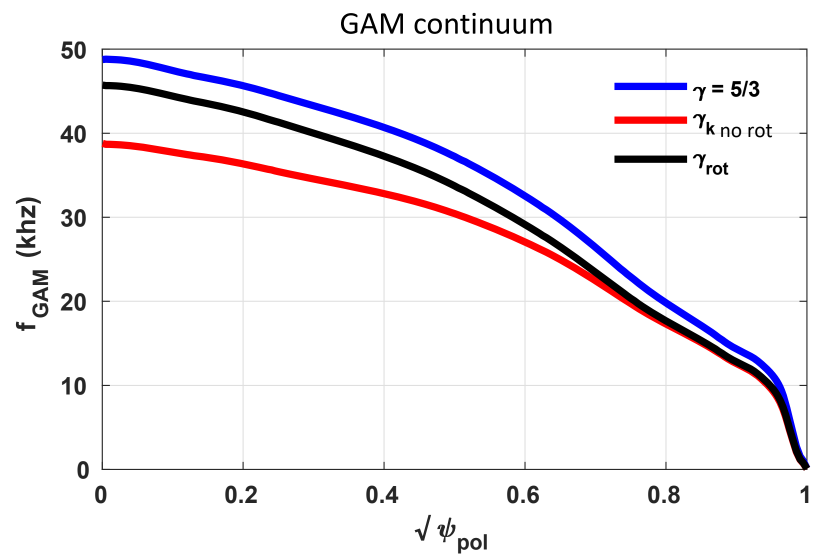
- In case of generalization of the previous results for N=2 and N=5.
- Effects can be important in the center.
- More applications of these modifications are to be analyzed.
- Other models for the energetic ions can also be used to match different situations.

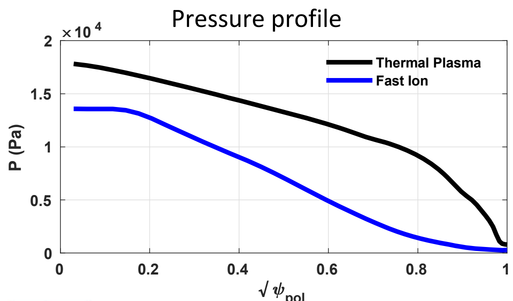


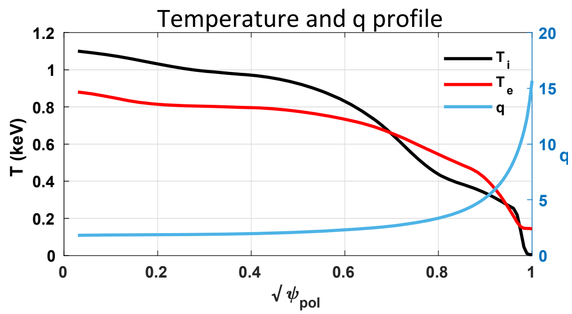


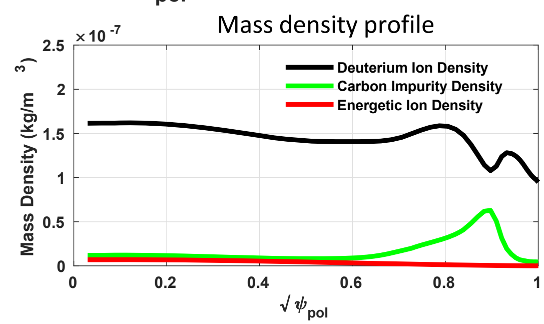
Aplication to NSTX – rotation correction

- Here is plotted the influence in the continuum created by the rotation in NSTX
- Small value of *q* and high energy of injection decrease the coupling between the GAM continuum and the transit frequency.









Analysis of results

- The eigenmodes found can be identified as a GAM excited just above the maximum in the m continuum modified by EPs, which is strongly coupled with energetic particles transit frequency.
- In the case of #159243, this results are consistent with [Z. *Qiu, PPCF (2010)*] and [*G. Y. Fu, PRL (2008)*], which predict that energetic ions can modify the GAM continuum.
- NOVA sideband corrections have substantial influence on the mode, and the code modification is important to correct the frequency and find the eigenmode.
- Effects of trapped energetic ions, more accurate energetic ions distribution function, FLR, FOW and others can be included for better agreement and more complete analyses.

References

[1] C.Z. Cheng, M.S. Chance, Phys. Fluids 29 3695 (1986)

[2] N. Winsor, J. Johnson and J.M. Dawson, *Phys. Fluids* **11** 2448 (1968)

[3] N.N. Gorelenkov et al, Physics Letters A 370 (2007)

[4] P.H. Diamond, et al, Plasma Phys. Control. Fusion 47 (2005) R35.

[5] R. Nazikian, et al, Phys. Rev. Lett. 101, 185001 (2008).

[6] G. Y. Fu, Phys. Rev. Lett. 101, 185002 (2008)

[7] H.L. Berk. *et al, Nucl. Fusion* **46** S888 (2006)

[8] A.G. Elfimov, F. Camilo de Souza, R.M.O. Galvão, Phys. Plasmas 22 114503 (2015)

[9] W. Guo, S. Wang and J. Li **17**, 112510 (2010)

[10] Z. Qiu , F. Zonca and Chen Liu Plasma Phys. Control. Fusion 52 095003 (2010)

[11] Lakhin a V.P., Sorokina E.A., Physics Letters A 378 535–538 (2014)

[12] N.N. Gorelenkov et al, Nuclear Fusion (2018)

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