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DREAM

DREAM: a fluid-kinetic framework for tokamak disruption runaway electron simulations

Mathias Hoppe[1]

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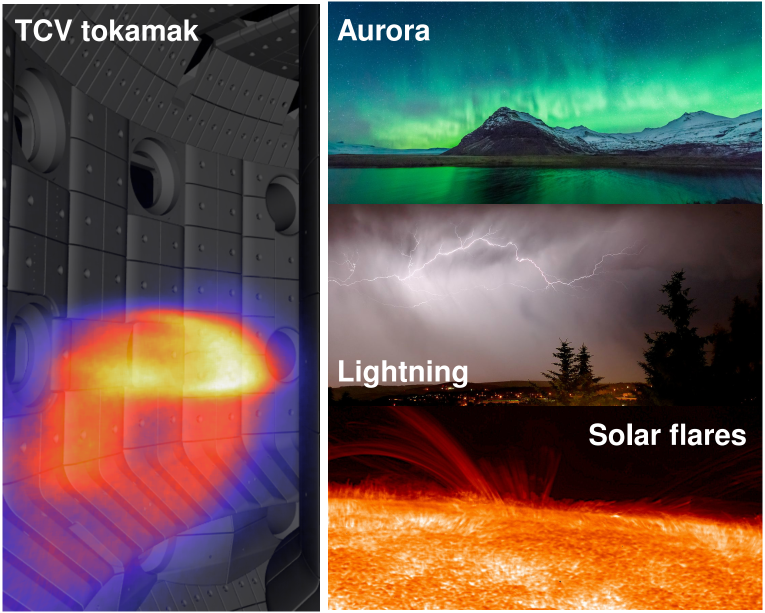
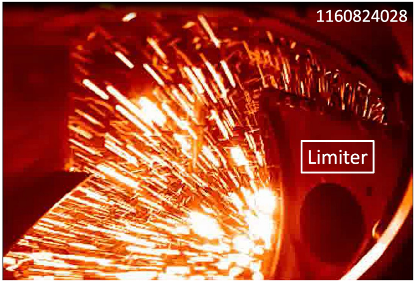
Runaway electrons



- Can be generated during startup or shutdown
- Disruptions can suddenly occur $\Longrightarrow$ significant runaway electron generation
  - Runaways mostly benign in today's devices
  - ...but avalanche multiplication gives

$$n_{\mathrm{RE}} \sim n_{\mathrm{RE},0} \exp\left(I_{\mathrm{p}}/170\,\mathrm{kA}\right)$$

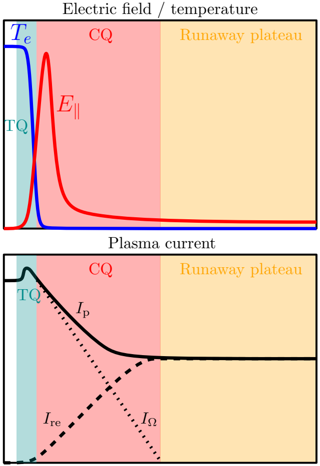
  - Big problem in reactor-scale devices!



Runaway impact in Alcator C-Mod.
(from *R. A. Tinguely et al, Nucl. Fusion* **58** *076019 (2018)*)

Disruptions unfold in three main phases:

1. Thermal quench (TQ): rapid temperature drop
   - Due to turbulent transport and radiation
   - Resistivity increases significantly



Schematic overview of disruption.

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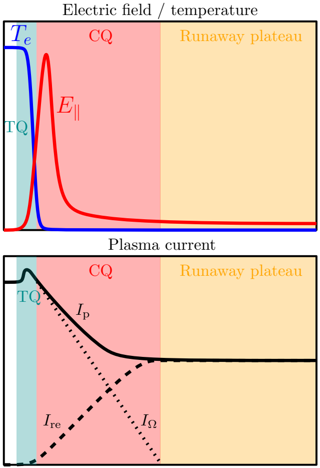
   - Electric field is induced
   - Electrons are accelerated
   - Runaway electrons multiply



Schematic overview of disruption.

Evolution of a tokamak disruption

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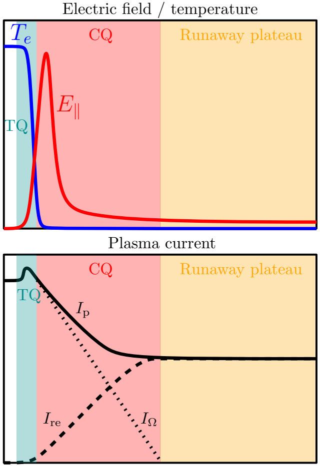
   - Runaway electrons multiply
3. Runaway plateau
   - Remaining current carried by runaway electrons



Schematic overview of disruption.

Runaway electron generation during disruptions has typically been treated using fluid models:

- T_e (temperature): $P_\Omega - P_{\mathrm{rad}} - P_{\mathrm{transp}}$
- n_i (ion density): $I - R$ (ionized – recombined)
- $E_\parallel$ (electric field): Ampère-Faraday equation
- j (current density): $j_\Omega + j_{\mathrm{re}}$
- n_{re} (runaway density): $\gamma_{\mathrm{Dreicer}} + \Gamma_{\mathrm{ava}} n_{\mathrm{re}} + \ldots$

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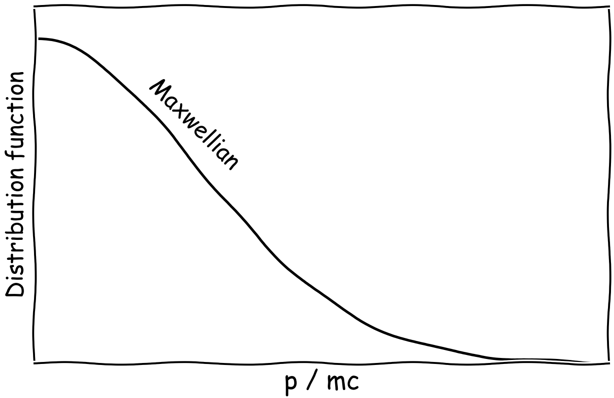
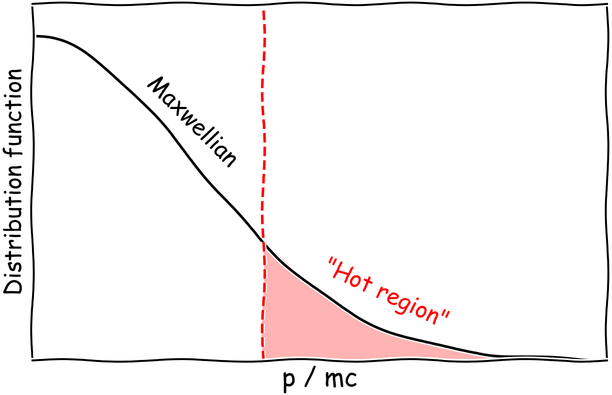
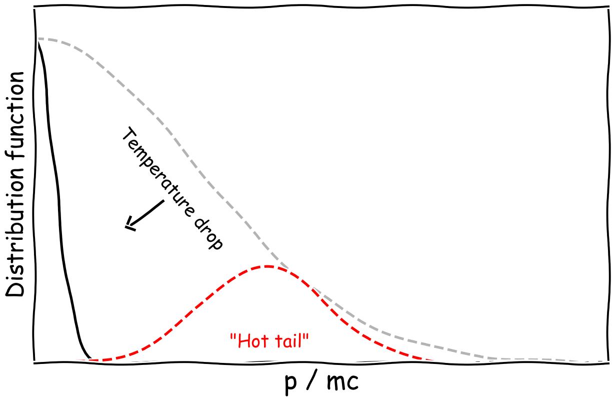
However, some aspects of RE are difficult to capture with a fluid model…


Maxwellian
Distribution function
p / mc


Maxwellian
"Hot region"
Distribution function
p / mc

Hot-tail mechanism



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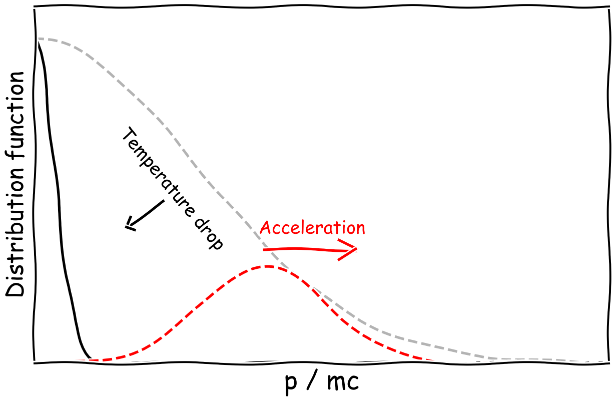

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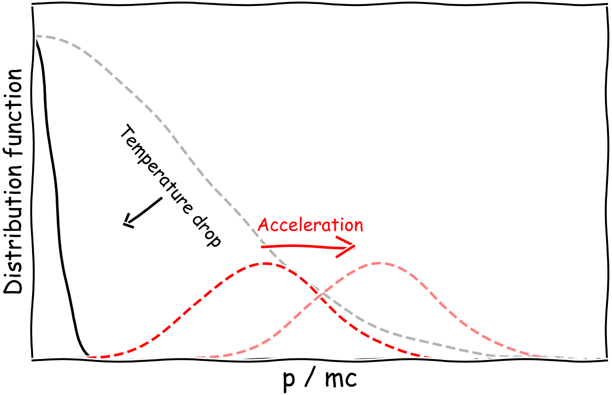

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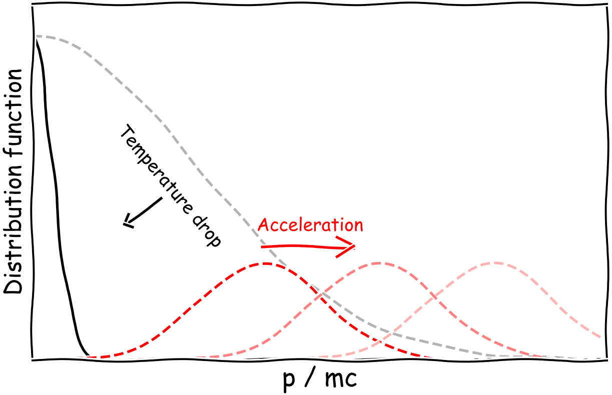
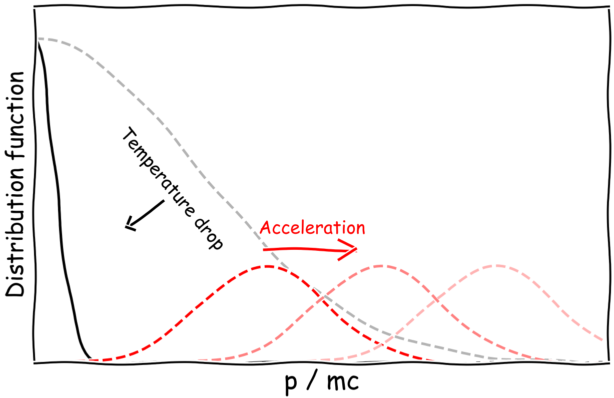
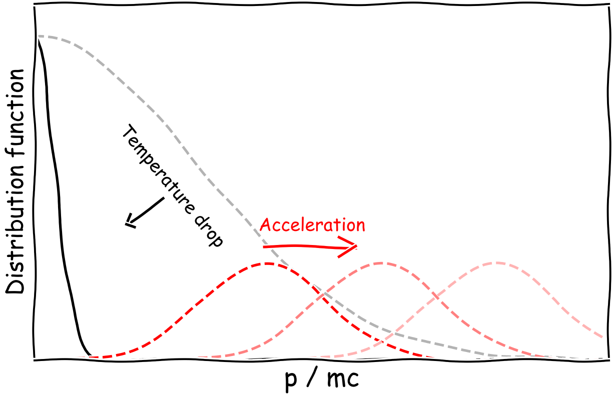




$\Longrightarrow$ Difficult to model because of its inherently transient nature



$\Longrightarrow$ Difficult to model because of its inherently transient nature

$\Longrightarrow$ Requires kinetic equation to be solved

DREAM

Disruption Runaway Electron Analysis Model

Hoppe, Embreus and Fülöp, Comp. Phys. Comm. **268** 108098 (2021)

- **Goal:** Flexible and lightweight tokamak disruption simulator
- Axisymmetric geometry
  - 1D space
  - 0D/1D/2D momentum space
- Emphasis on advanced RE models
  - Many possible **hot-tail** models[1,2]
  - Accounting for **partial screening**[3,4,5]
  - Fast electron impact ionization[6]
  - RE radial **transport**[7,8]

https://github.com/chalmersplasmatheory/DREAM

[1] Smith and Verwichte, Phys. Plasmas **15** (6) 072502 (2008) doi:10.1063/1.2949692
[2] Aleynikov and Breizman, Nucl. Fusion **57** (4) 046009 (2017) doi:10.1088/1741-4326/aa5895
[3] Hesslow *et al.*, ($E_{c,\text{eff}}$) Plasma Phys. Control. Fusion **60** (7) 074010 (2018) doi:10.1088/1361-6587/aac33e
[4] Hesslow *et al.*, (Avalanche growth rate) Nucl. Fusion **59** (8) 084004 (2019) doi:10.1088/1741-4326/ab26c2
[5] Hesslow *et al.*, (Dreicer rate) J. Plasma Phys. **85** (4) 475850601 (2019) doi:10.1017/S0022377819000874
[6] Garland *et al.*, Phys. Plasmas **27** (4) 040702 (2020) doi:10.1063/5.0003638
[7] Rechester and Rosenbluth, Phys. Rev. Lett. **40** (1) 38 (1978) doi:10.1103/PhysRevLett.40.38
[8] Svensson *et al.*, J. Plasma Phys. **87** (2) 905870207 (2021) doi:10.1017/S0022377820001592

Ampère's & Faraday's laws — $E_\|$, ψ, $j_{\rm tot}$

$$2\pi\mu_0 \left\langle \boldsymbol{B}\cdot\nabla\phi \right\rangle \frac{j_{\rm tot}}{B} = \frac{1}{V'}\frac{\partial}{\partial r}\left[V'\left\langle\frac{\left|\nabla r\right|^2}{R^2}\right\rangle\frac{\partial\psi}{\partial r}\right], \tag{1a}$$

$$\frac{\partial\psi}{\partial t} = -V_{\rm loop} + \frac{\partial}{\partial\psi_{\rm t}}\left(\psi_{\rm t}\mu_0\Lambda\frac{\partial}{\partial\psi_{\rm t}}\frac{j_{\rm tot}}{B}\right). \tag{1b}$$

Boundary conditions accounting for conducting structures.

Ion charge state densities — $n_i^{(Z_0)}$, $Z_0 = 0, 1, \ldots, Z$

$$\frac{\partial n_i^{(Z_0)}}{\partial t} = \left(I_i^{(Z_0-1)} \langle n_e \rangle + \left\langle \sigma_{\mathrm{ion},i}^{(Z_0-1)} v \right\rangle \right) n_i^{(Z_0-1)} - \left(I_i^{(Z_0)} \langle n_e \rangle + \langle \sigma_{\mathrm{ion},i} v \rangle \right) n_i^{(Z_0)} + R_i^{(Z_0+1)} \langle n_e \rangle n_i^{(Z_0+1)} - R_i^{(Z_0)} \langle n_e \rangle n_i^{(Z_0)}, \tag{2}$$

with

$$\left\langle \sigma_{\mathrm{ion},i}^{Z_0} v \right\rangle = \int \mathrm{d}p \int_{-1}^{1} \mathrm{d}\xi_0 \frac{\mathcal{V}'}{V'} v \sigma_{\mathrm{ion},i}^{(Z_0)} f_e(t, r, p, \xi_0). \tag{3}$$

Rate coefficients $I_i^{(Z_0)}$, $R_i^{(Z_0)}$ taken from ADAS[9].
Ionization cross-section $\sigma_{\mathrm{ion},i}^{(Z_0)}$ taken as in Ref. 6.

[6] Garland *et al.*, Phys. Plasmas **27** (4) 040702 (2020) doi:10.1063/5.0003638
[9] Summers, *The ADAS user manual, version 2.6* http://www.adas.ac.uk (2004)

Energy balance — T_e, T_i

$W_k = 3n_k T_k/2 =$ thermal energy of species k.

Electron balance:

$$\frac{\partial W_e}{\partial t} = \frac{j\Omega}{B}\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle \qquad \text{(Ohmic heating)}$$

$$- \langle n_e\rangle \sum_i \sum_{Z_0=0}^{Z_i-1} n_i^{(Z_0)} L_i^{(Z_0)} \qquad \text{(Ionization/radiation)}$$

$$- \langle Q_c\rangle \qquad \text{(Collisions)}$$

$$+ \frac{1}{V'}\frac{\partial}{\partial r}\left[V'\frac{3\langle n_e\rangle}{2}\left(A_W T_e + D_W\frac{\partial T_e}{\partial r}\right)\right], \qquad \text{(Transport)}$$

and ion balance

$$\frac{\partial W_i}{\partial t} = \sum_j Q_{ij} + Q_{ie} \qquad \text{(Collisions)}$$

Runaways treated as a fluid in “conventional” simulation mode:

$$\begin{aligned}\frac{\partial n_\mathrm{re}}{\partial t} &= \gamma_\mathrm{Dreicer} + \gamma_\mathrm{hottail} + \Gamma_\mathrm{ava} n_\mathrm{re} + \gamma_\mathrm{tritium} + \gamma_\mathrm{Compton} \\ &+ \frac{1}{V'}\frac{\partial}{\partial r}\left[V'\left(A\left\langle n_\mathrm{re}\right\rangle + D\frac{\partial\left\langle n_\mathrm{re}\right\rangle}{\partial r}\right)\right]\end{aligned} \tag{4}$$

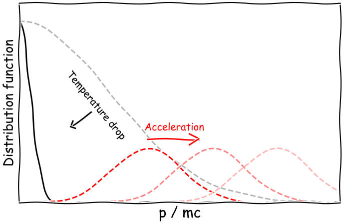
(can also be evolved fully kinetically)

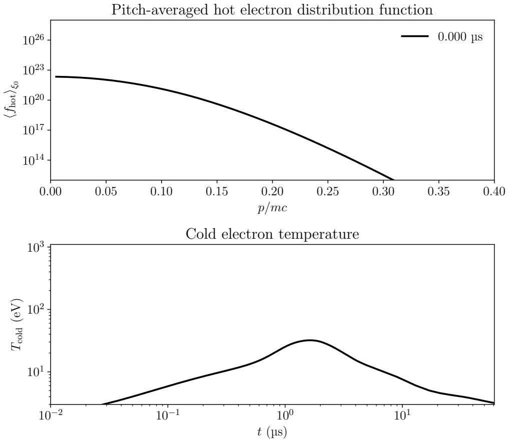
Runaways treated as a fluid in “conventional” simulation mode:

$$\frac{\partial n_{\mathrm{re}}}{\partial t} = \gamma_{\mathrm{Dreicer}} + \gamma_{\mathrm{hottail}} \rightarrow \color{red}{F_{\mathrm{kinetic}}} + \Gamma_{\mathrm{ava}} n_{\mathrm{re}} + \gamma_{\mathrm{tritium}} + \gamma_{\mathrm{Compton}} + \frac{1}{V'}\frac{\partial}{\partial r}\left[V'\left(A\langle n_{\mathrm{re}}\rangle + D\frac{\partial\langle n_{\mathrm{re}}\rangle}{\partial r}\right)\right] \tag{4}$$

(can also be evolved fully kinetically)

$$F_{\mathrm{kinetic}} = \int \mathcal{V}'\left(\{A^p\} f_e + \{D^{pp}\}\frac{\partial f_e}{\partial p}\right)\mathrm{d}\xi_0\bigg|_{p=p_{\mathrm{re}}}$$





- Injection of cold impurities

$\Longrightarrow$ Two electron populations:
  - Injected, **cold** electrons: n_{cold} (fluid)
  - Original, **hot** electrons: f_{hot} (kinetic)
- Only account for hot-cold collisions
- Superthermal limit of FP equation
  - $v \gg v_{th}$
  - Sink in $p = 0$

[2] Aleynikov and Breizman, Nucl. Fusion **57** (4) 046009 (2017) doi:10.1088/1741-4326/aa5895

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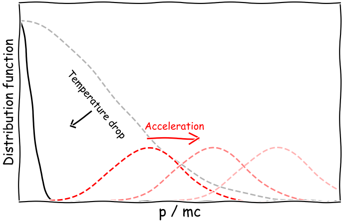

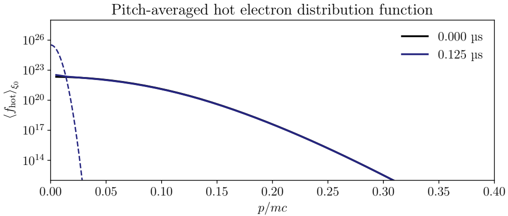

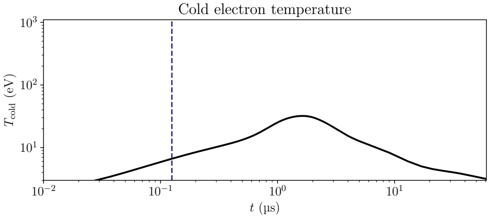

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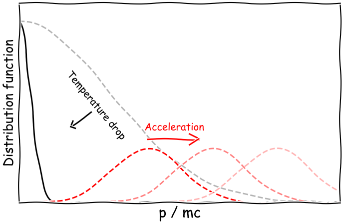

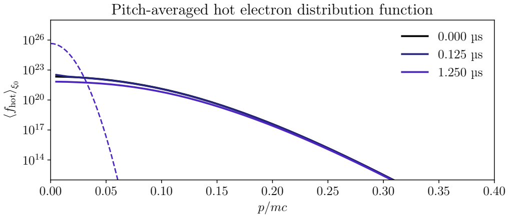

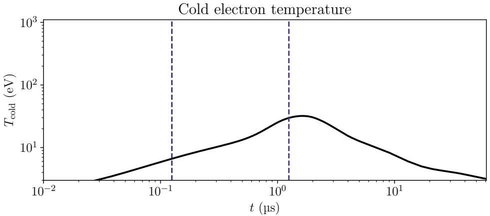

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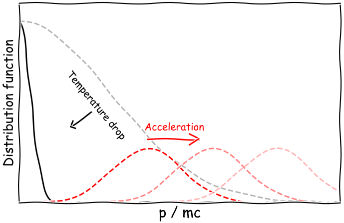
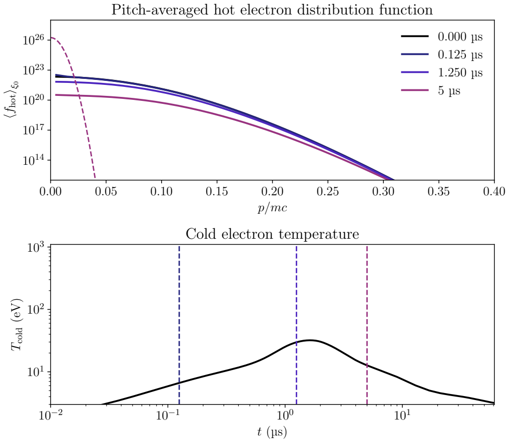
Hot-tail generation: two-temperature model[2]





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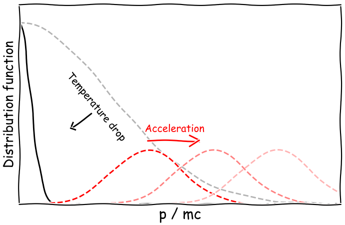

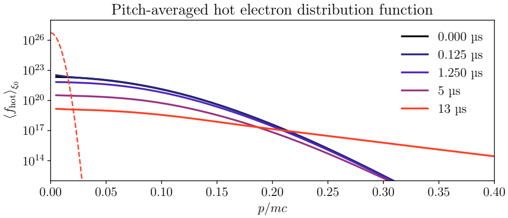

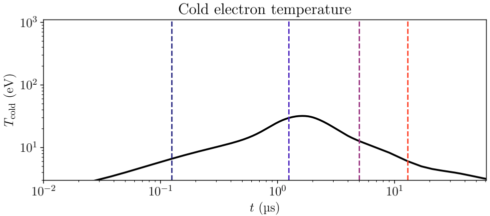

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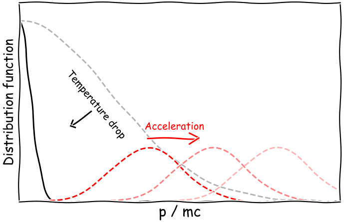

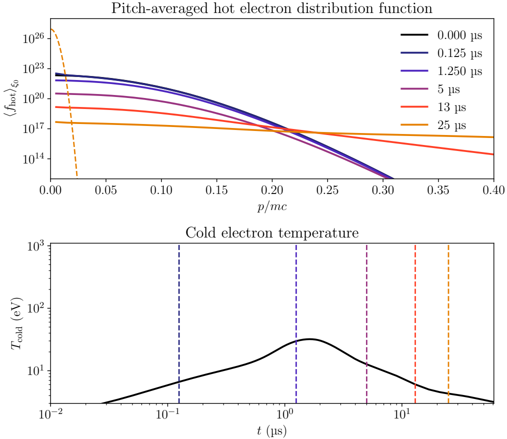

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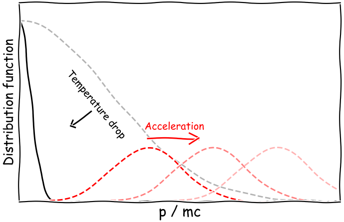

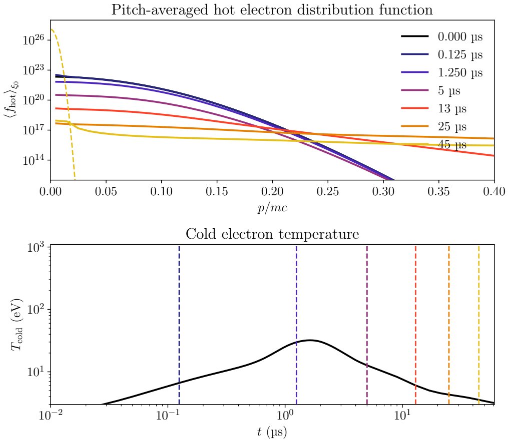

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RE seed transport in ITER

from '*Hot-Tail Runaway Seed Landscape during the Thermal Quench in Tokamaks*'
Svenningsson *et al*, Phys. Rev. Lett. **127** (3) 035001 (2021)

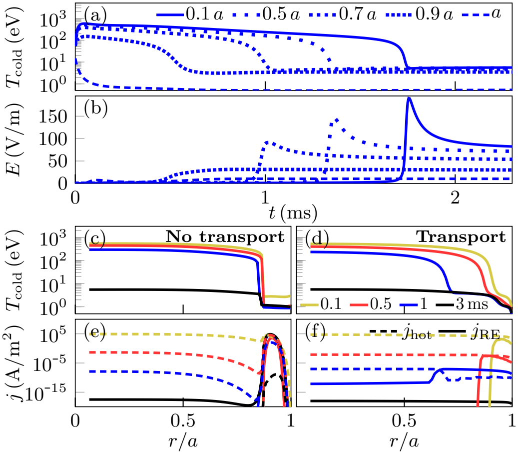
doi:10.1103/PhysRevLett.127.035001

- MMI-triggered ITER disruption: **Deuterium + Neon**
- Evolve hot-tail with ***two-temperature*** model
- Include **radial transport** of hot electrons[7] and heat
- Scan in:
  - Amount of injected Ne
  - Magnetic field perturbation amplitude ($\delta B/B$)
- Consider mainly TQ (no avalanche generation)

We optimize for

- Minimal seed RE current $I_{\mathrm{RE}}^{\mathrm{seed}}$
- Maximal cooling via radiation

[7] Rechester and Rosenbluth, Phys. Rev. Lett. **40** (1) 38 (1978) doi:10.1103/PhysRevLett.40.38



$I_{\rm p} = 15$ MA, $T_0 = 15$ keV

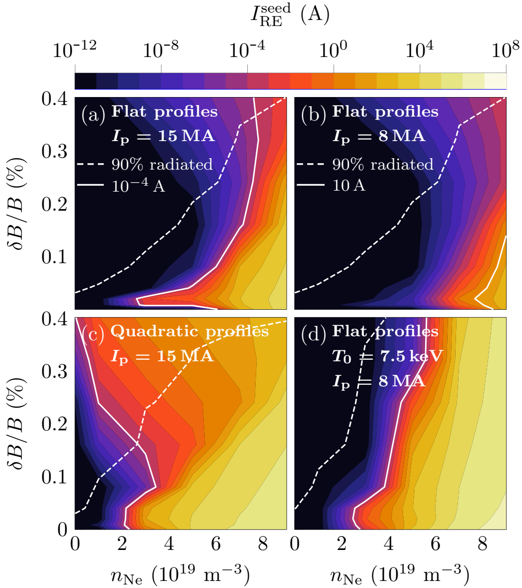
$n_{\rm D} = 1 \times 10^{21}$ m^{-3}, $n_{\rm Ne} = 9 \times 10^{19}$ m^{-3} (uniform)

$\delta B/B = 0.16\%$ (except (c) and (e))

- TQ evolves edge $\rightarrow$ core
  - Cold front propagates inwards
- Rapid TQ near edge $\rightarrow$ RE generation **near edge**
  - More easily deconfined via external perturbations

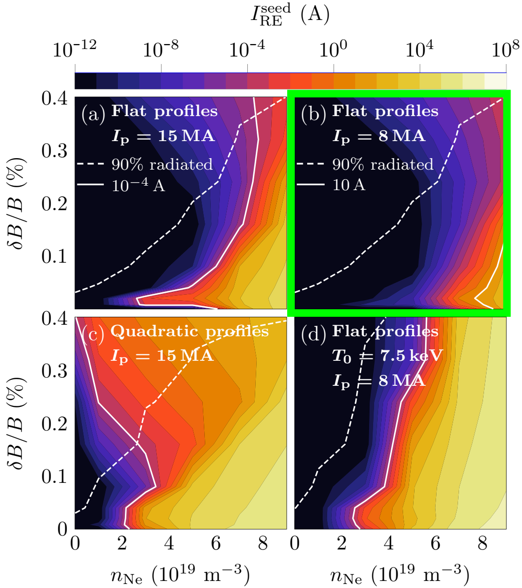
- Suppression of RE seed with transport is possible!
- Care must be taken to avoid
  - Too large runaway seed (solid white)
  - Too high heat flux to PFCs (dashed white)

Flat profiles: impurities uniformly distributed



(b) Lower I_p

- Higher I_{RE}^{seed} allowed
- Weaker electric fields induced



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- Weaker electric fields induced

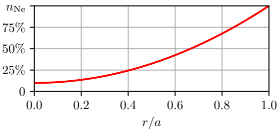
(c) Quadratic impurity profile

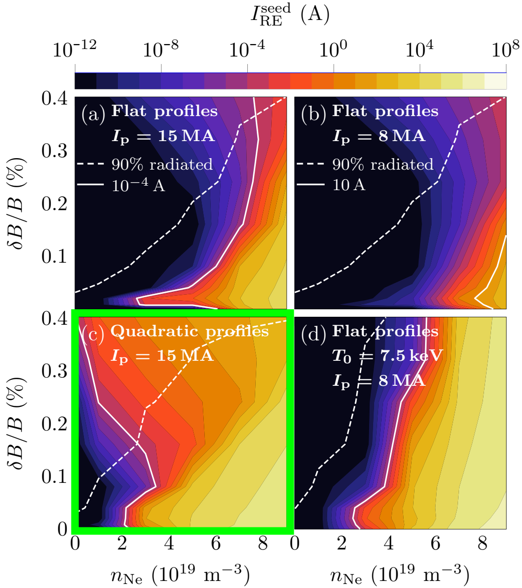
- Radiated power $\propto n_i^2$
- Transported heat $\propto n_i$

$\Longrightarrow$ More heat **radiated away**

- Low core density

$\Longrightarrow$ Lower collisionality, **survival** of fast electrons





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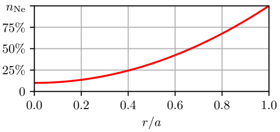
- Transported heat $\propto n_i$

$\Longrightarrow$ More heat **radiated away**

- Low core density

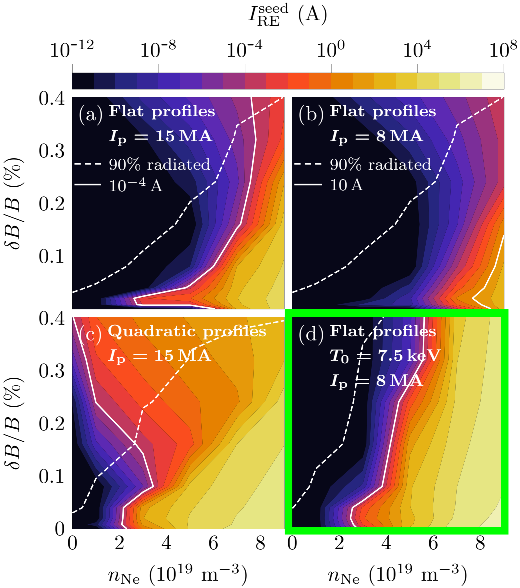
$\Longrightarrow$ Lower collisionality, **survival** of fast electrons



(d) Lower temperature

- TQ time decreases faster than the collision time with T_e

$\Longrightarrow$ Hot-tail generation *more* efficient



Outlook

Some completed features:

- Everything shown in this presentation
- Fluid runaway transport model[8]
- Shattered Pellet Injection (SPI) module[10]

https://github.com/chalmersplasmatheory/DREAM

Currently under development:

- Ion transport
- Kinetic + synthetic diagnostic simulations: synchrotron radiation in JET[11] and TCV
- Prevention of RE formation in SPARC with passive coils[12]
- Tokamak burn-through model

[8]Svensson *et al.*, J. Plasma Phys. **87** (2) 905870207 (2021) doi:10.1017/S0022377820001592
[10]Vallhagen, MSc Thesis (2021) https://hdl.handle.net/20.500.12380/302296
[11]Brandström, MSc Thesis (2021) https://hdl.handle.net/20.500.12380/302636
[12]Tinguely *et al*, TSDW Princeton (2021) https://tsdw.pppl.gov/Talks/2021/Tinguely.pdf

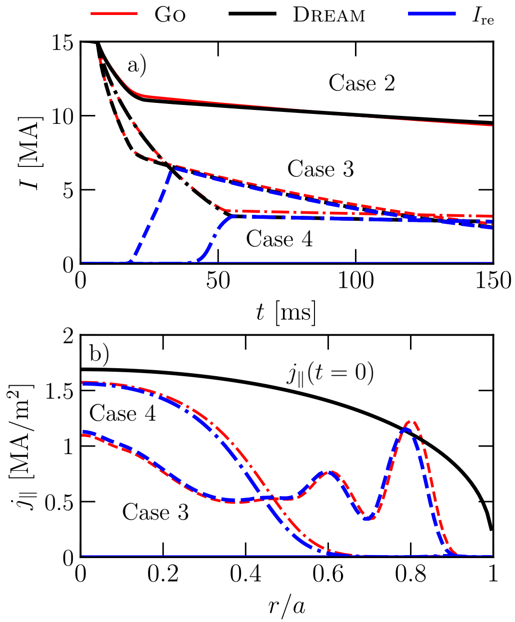
Summary

https://github.com/chalmersplasmatheory/DREAM

- **DREAM**: a new framework for tokamak **disruption simulations**, with focus on the **runaway generation**
- Many options for simulation runaway electrons:
  - As a fluid (density n_{re})
  - Kinetic hot-tail (e.g. two-temperature model)
  - Kinetic hot-tail and runaway
- **Magnetic perturbations** (naturally induced or externally applied) may provide safe **suppression of seed electrons** in ITER

Hoppe, Embreus and Fülöp, Comp. Phys. Comm. **268** 108098 (2021)
Svenningsson *et al*, Phys. Rev. Lett. **127** (3) 035001 (2021)

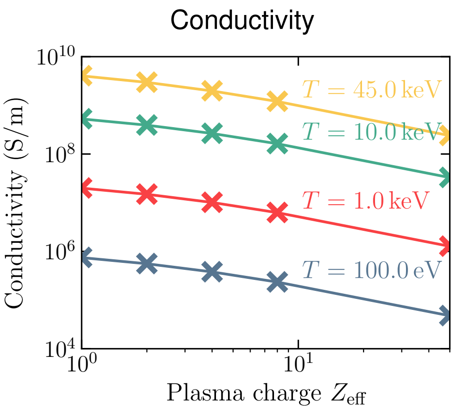
Benchmark



Comparison with figure 4 of 'Vallhagen *et al*, J. Plasma Phys. **86** (4) 475860401 (2020)'

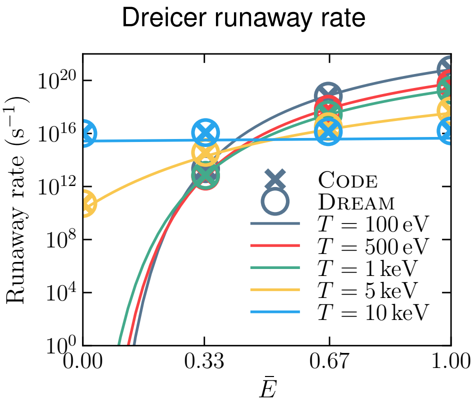
- Good quantitative agreement
- Differences attributed to improved model for $E_{c,\text{eff}}$ used in DREAM

Benchmark: DREAM and CODE



CODE = lines
DREAM = crosses

(agreement to within 0.3%)



$\bar{E} = (E - 2E_c)/(0.04E_D - 2E_c)$

(agreement to within 3.5%)

Energy balance — T_e, T_i

$W_e, W_i =$ thermal energy of electrons/ions

$$\frac{\partial W_e}{\partial t} = \frac{j_\Omega}{B}\left\langle \boldsymbol{E}\cdot\boldsymbol{B}\right\rangle - \left\langle n_e\right\rangle \sum_i \sum_{Z_0=0}^{Z_i-1} n_i^{(Z_0)} L_i^{(Z_0)} - \left\langle Q_c\right\rangle + \frac{1}{V'}\frac{\partial}{\partial r}\left[V'\frac{3\left\langle n_e\right\rangle}{2}\left(A_W T_e + D_W\frac{\partial T_e}{\partial r}\right)\right] \tag{5a}$$

$$\frac{\partial W_i}{\partial t} = \sum_j Q_{ij} + Q_{ie}, \tag{5b}$$

with

$$\left\langle Q_c\right\rangle = \int \mathrm{d}p \int_{-1}^{1} \mathrm{d}\xi_0 \frac{\mathcal{V}'}{V'}\Delta\dot{E}_{ee} f_e(t,r,p,\xi_0) + \sum_i Q_{ei}$$

$$Q_{kl} = \frac{\left\langle nZ^2\right\rangle_k \left\langle nZ^2\right\rangle_l e^4 \ln\Lambda_{kl}}{\left(2\pi\right)^{3/2}\epsilon_0^2 m_k m_l}\frac{T_k - T_l}{\left(\frac{T_k}{m_k}+\frac{T_l}{m_l}\right)^{3/2}},$$

$$L_i^{(Z_0)} = L_{\mathrm{line}} + L_{\mathrm{free}} + \Delta W_i^{(Z_0)}\left(I_i^{(Z_0)} - R_i^{(Z_0)}\right).$$

$$\frac{\partial f}{\partial t} = \sum_{m,n} \frac{1}{\mathcal{V}'} \frac{\partial}{\partial z^m} \left[\mathcal{V}' \left(-\{A^m\} f + \{D^{mn}\} \frac{\partial f}{\partial z^n} \right) \right] + \{S_A\}$$

$$\boldsymbol{A} = \boldsymbol{A}_E + \boldsymbol{A}_C + \boldsymbol{A}_B + \boldsymbol{A}_S + \boldsymbol{A}_T$$

$$\mathsf{D} = \mathsf{D}_C + \mathsf{D}_T$$

with

$$\boldsymbol{A}_E = \text{Electric field acceleration,}$$
$$\boldsymbol{A}_C = \text{Collisional friction,}$$
$$\boldsymbol{A}_B = \text{Bremsstrahlung radiation reaction,}$$
$$\boldsymbol{A}_S = \text{Synchrotron radiation reaction,}$$
$$\boldsymbol{A}_T = \text{Radial transport,}$$
$$\mathsf{D}_C = \text{Collisional momentum-space diffusion,}$$
$$\mathsf{D}_T = \text{Radial diffusion,}$$
$$S_A = \text{Avalanche source.}$$