Peeling-ballooning modes in low aspect ratio - towards a predictive edge pedestal model for spherical tokamaks

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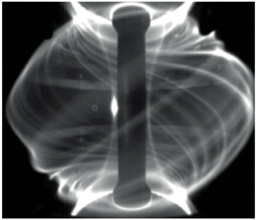
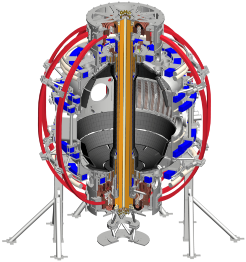
Motivation

- Control of edge-localized modes (ELMs) crucial for future tokamaks
- Predictive pedestal models are useful to determine operational limits and find ELM-free operation scenarios
- Pedestal predictions are difficult in low-aspect ratio machines (NSTX, MAST) with existing ideal-MHD models
  $\rightarrow$ **What is missing?**
  Physical effects? Important mode numbers?
- Resistivity can expand the unstable domain of ballooning modes
  [Sykes et al, PPCF **29**, 1987]



[Ham et al., Nat. Rev. Phys., **2**, 2020]



Goals:

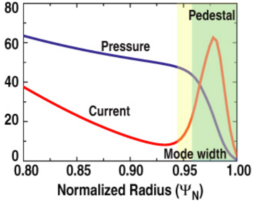
- Extended-MHD PB mode analysis, focus on resistivity
- Ultimately, get a reliable pedestal structure model for STs

Outline

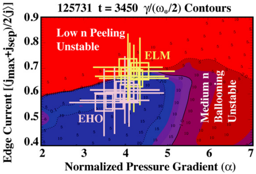
1. Peeling-ballooning modes & pedestal models
2. Numerical approach & equilibrium variation
3. NSTX Peeling-ballooning stability boundaries
4. Effect of resistivity in low vs. large aspect ratio

The peeling-ballooning model

- Large edge pressure gradient → ballooning modes (large n)
- Edge bootstrap current → peeling (external kink) modes (low n)
- Coupled peeling & ballooning modes
- ELITE code
  - Ideal-MHD eigenvalue code
  - Simple model of diamagnetic stabilization: $\gamma_{\text{MHD}} > \omega_{*\text{eff}}/2$



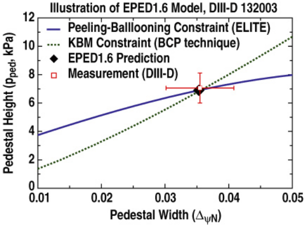
[Snyder et al., NF **51**, 2011]



[Burrell et al., NF **49**, 2009]

Pedestal structure model EPED

- Calculates pedestal width and height based on two constraints: [Snyder et al., PoP **16**, 2009; Snyder et al., NF **51**, 2011]
  - Peeling-ballooning mode onset
  - Local KBM stability threshold
- In conventional aspect ratio EPED predictions are accurate to $\approx 20\%$
- Does not work well in low aspect ratio

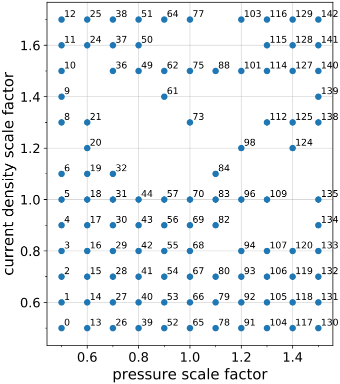


[Snyder et al., NF **51**, 2011]

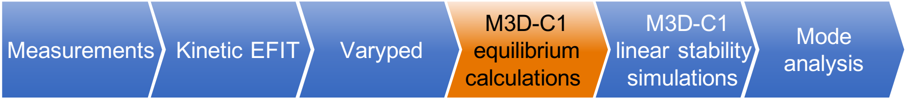
Outline

Equilibrium reconstruction & variation

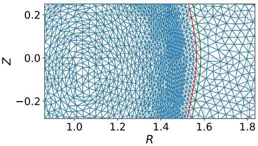
- VARYPED:
  - Variation of pedestal pressure and current
  - Keep total stored energy and collisionality constant
  - Fixed boundary → scale coil currents

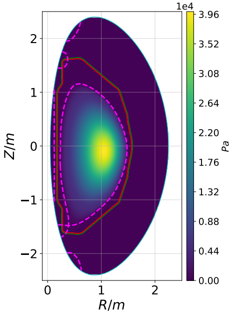


M3D-C1 equilibrium

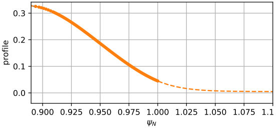


- Non-linear extended-MHD initial value stability code
  [Jardin et al., J. Comput. Phys. **226**, 2007]
- High resolution equilibrium based on kinetic EFIT
- C^1 finite elements
  [Jardin, J. Comput. Phys. **200**, 2004]
  - Mesh is adapted to each equilibrium





- Extended-MHD region beyond LCFS, resistive wall, vacuum region
  - Kinetic profiles are expanded across LCFS



M3D-C1 stability simulations

Measurements → Kinetic EFIT → Varyped → M3D-C1 equilibrium calculations → **M3D-C1 linear stability simulations** → Mode analysis

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{u}) = 0 \quad (1)$$

$$nm_i \left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = \vec{J} \times \vec{B} - \nabla p - \nabla \cdot \Pi + \vec{F} \quad (2)$$

$$\frac{\partial p}{\partial t} + \vec{u} \cdot \nabla p + \Gamma p \nabla \cdot \vec{u} = (\Gamma - 1) \left[Q - \nabla \cdot \vec{q} + \eta J^2 - \vec{u} \cdot \vec{F} - \Pi : \nabla u \right] \quad (3)$$

$$+ \frac{1}{ne} \vec{J} \cdot \left(\frac{\nabla n}{n} p_e - \nabla p_e \right) + (\Gamma - 1) \Pi_e : \nabla \left(\frac{1}{ne} \vec{J} \right) \quad (4)$$

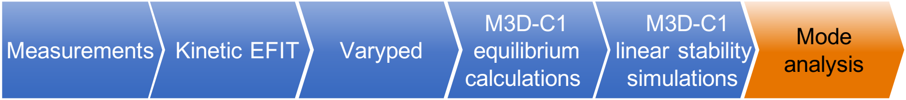
$$\frac{\partial p_e}{\partial t} + \vec{u} \cdot \nabla p_e + \Gamma p_e \nabla \cdot \vec{u} = (\Gamma - 1) \left[Q_e - \vec{q}_e + \eta J^2 - \vec{u} \cdot \vec{F}_e - \Pi_e : \nabla u \right] \quad (5)$$

$$+ \frac{1}{ne} \vec{J} \cdot \left(\frac{\nabla n}{n} p_e - \nabla p_e \right) + (\Gamma - 1) \left[\Pi_e : \nabla \left(\frac{1}{ne} \vec{J} \right) + \frac{1}{ne} \vec{J} \cdot \vec{F}_e \right] \quad (6)$$

$$\vec{E} = -\vec{u} \times \vec{B} + \eta \vec{J} + \frac{1}{ne} \left(\vec{J} \times \vec{B} - \nabla p_e - \nabla \cdot \Pi_e + \vec{F}_e \right) \quad (7)$$

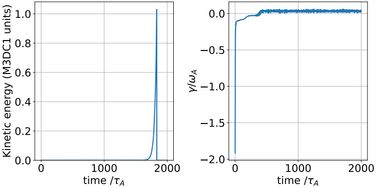
$$\vec{J} = \frac{1}{\mu_0} \nabla \times \vec{B} \quad , \quad \frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \quad (8)$$

Mode analysis



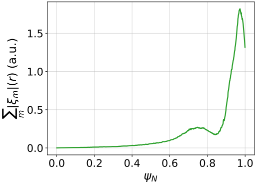
Determine mode type based on

- growth rate $\gamma = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t} \ln E_{kin}$



  - γ is based on whole plasma E_{kin}
  - Linear growth: γ should be constant
  - Detect numerical instability (noise)

- eigenfunction



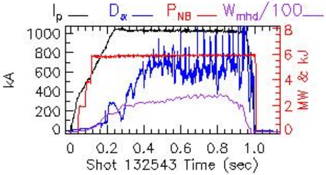
  - Superposition of multiple modes possible
  - Identify PB mode

Outline

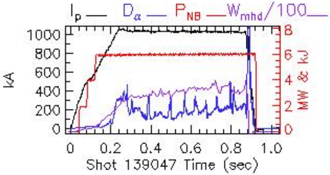
NSTX discharges

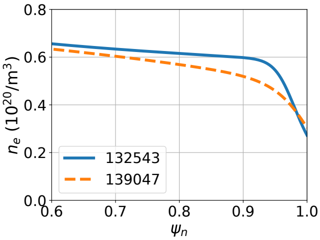
- Kinetic EFIT equilibrium reconstruction of ELMing NSTX discharges

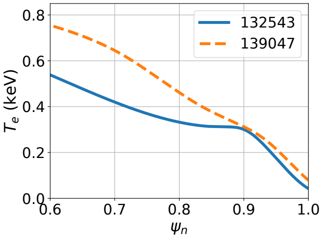
#132543 at $t = 700\,\text{ms}$



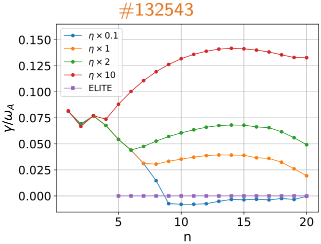
#139047 at $t = 665\,\text{ms}$

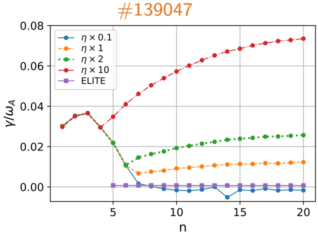




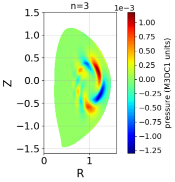


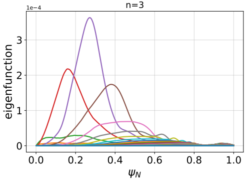
Effect of resistivity on linear stability (1-fluid model, Spitzer resistivity)

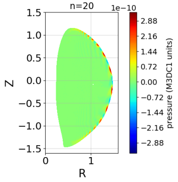


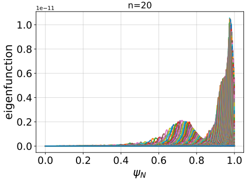


- Ideal core (ballooning) modes at low n
- PB modes at larger $n \gtrapprox 7$, sensitive to resistivity
- PB modes are stable in the ideal-MHD limit (M3D-C1 and ELITE)



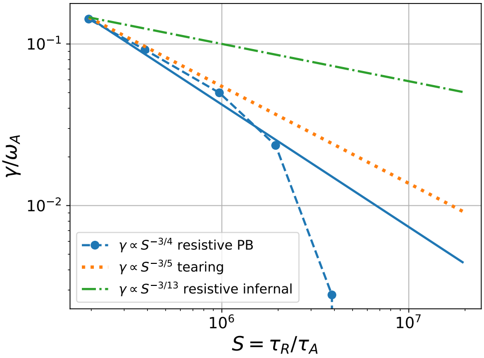






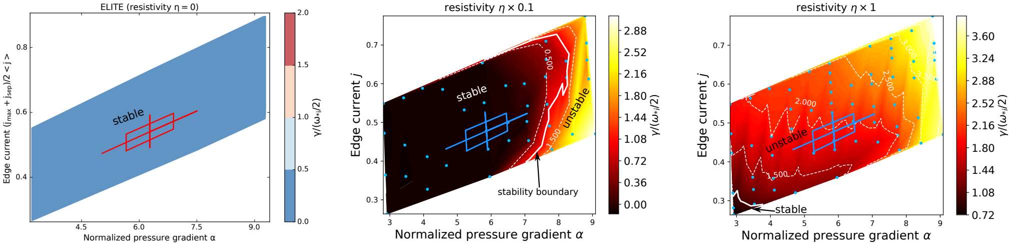
Resistive scaling

How does the resistive scaling compare to other resistive modes?



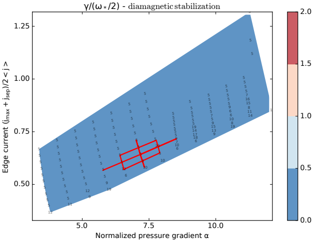
Resistive time $\tau_R = \mu_0 a^2/\eta$

Stability boundary for NSTX #132543

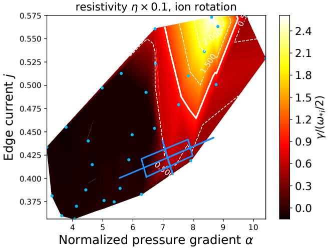


- ELITE predicts stability in whole domain
  - inconsistent with experiment
- Ideal limit in M3D-C1: discharge is located on stable side
- Ideal mode at large α
- Resistivity extends unstable region
- Discharge is correctly located on unstable side

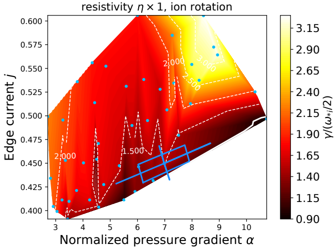
Stability boundary for NSTX #139047



- ELITE predicts stability in whole domain
  - ⚡ inconsistent with experiment

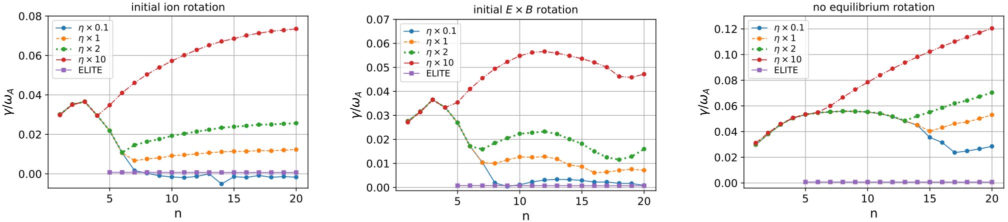


- Ideal limit in M3D-C1: discharge is located on stable side
- Ideal mode at large α

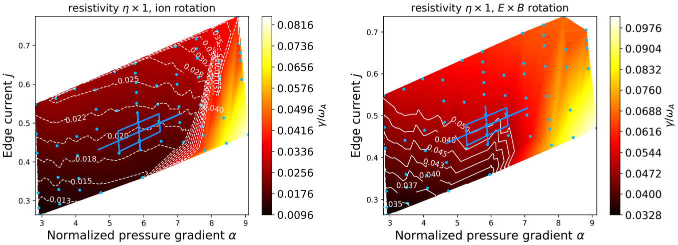


- Resistivity extends unstable region
- Discharge is correctly located on unstable side

Importance of rotation

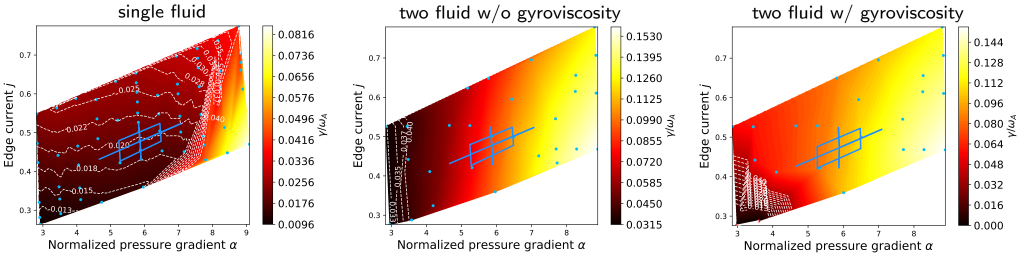


- Similarity between both rotation models
- Rotation stabilizes core modes at intermediate n



Stability boundary with two-fluid model

#132543

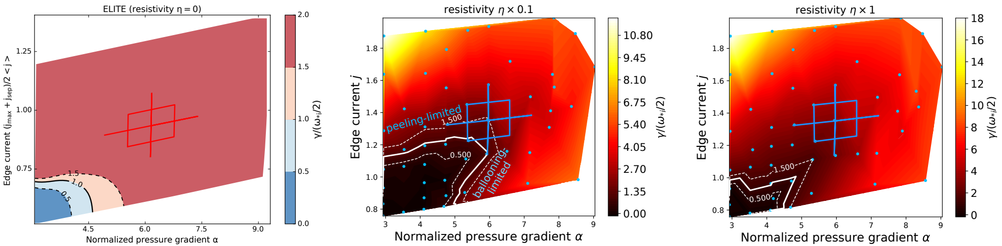


- Two fluid (w/ gyroviscosity) model stability threshold consistent with that of single fluid model

Outline

What about resistivity in large aspect ratio?

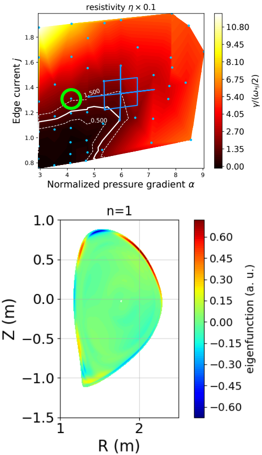
- DIII-D discharge #147105 at 3750 ms [Wade et al., NF **55**, 2015]



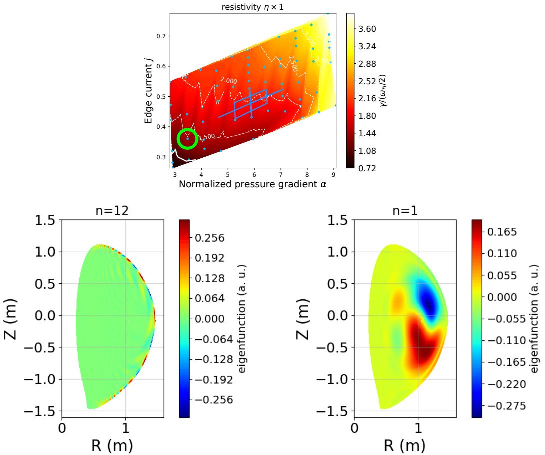
- Consistency among ELITE and M3D-C1 in the ideal limit
- Resistivity has a lower impact in DIII-D compared to NSTX

Mode character on the peeling side of the stability boundary

DIII-D #147105



NSTX #132543



- **DIII-D**: Low *n*, peeling-like
- **NSTX**: Higher *n*, more ballooning-like in NSTX

Summary & Outlook

- Edge stability thresholds have been calculated in NSTX discharges
- Ideal-MHD models predict stability (ELITE and M3D-C1) in ELMing NSTX discharges
- Resistivity expands unstable region of ideal modes towards lower values of α and j
- New resistive model shows stability boundaries consistent with experiments

Future work:

- Establish stability boundaries in other scenarios (NSTX lithium coated discharges, NSTX enhanced pedestal H-mode, QH-mode, MAST discharges, RMP suppressed cases)
- Calculate stability boundary with two-fluid model
- Gyrokinetic studies to obtain a first predictive pedestal structure model