Resonance Broadened Quasi-linear model (RBQ) for fast ion relaxation in the presence of Alfvénic instabilities

N.N.Gorelenkov, *PPPL, Princeton University*

in collaboration with

V.N.Duarte, M.Podesta, R.White, PPPL

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H.L. Berk, IFS, R. Nazikian, PPPL

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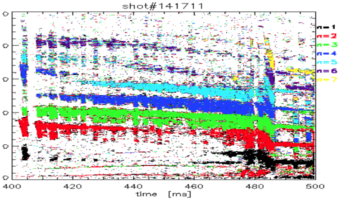
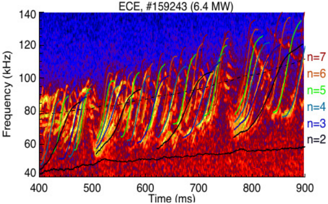
Two regimes of Alfvén mode driven fast ion relaxation are observed

chirping NSTX & *steady state DIII-D*





- Efficient realistic quasi-linear formalism is targeted by RBQ + NOVA-K codes for TRANSP/WDM simulations in steady state regimes.
- Linear perturbative Alfvén eigenmode simulations provide the framework for RBQ: *(Gorelenkov et al., NF'18 and this talk)*
  - Realistic quasi-linear (QL) formalism is adapted: *Berk et al.,'95, Ghantous, PhD'14, Duarte, PhD'17*

Steady-state (QL compatible) together with (marginally expected) **chirping** frequency **regimes** are predicted in BPs, ITER! *(Duarte et al., APS'18 oral talk at ITER BPO session)*

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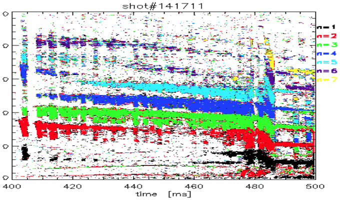

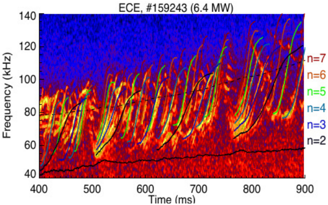

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Outline

1. RBQ formulation
   - Perturbative NOVA-K code provides EP response
   - Quasi-linear equations
2. RBQ1D code structure
   - RBQ and its preparations by NOVA/NOVA-K
   - Rigorous verifications are undertaken
3. RBQ1D application to DIII-D shot #159243
   - Multiple mode induced transport
4. Summary and Plans

Outline

Flux coordinate NOVA-K formalism is consistent with Hamiltonian approach

3 constants of motion (COM): Canonical toroidal momentum

$$P_\varphi = \frac{e\psi}{mc} - \sigma_\parallel v\sqrt{1-\lambda B/B_0}R\frac{B_\varphi}{B},$$

magnetic moment

$$\mu \equiv \frac{\mathscr{E}_\perp}{B} \equiv \frac{\lambda \mathscr{E}}{B_0},$$

kinetic energy

$$\mathscr{E} = \frac{v^2}{2}.$$

Wave particle interaction (WPI) results from energy exchange due to AE electric field ($\boldsymbol{E} = -\nabla\phi$):

$$\boldsymbol{v}\cdot\boldsymbol{E} \simeq \boldsymbol{v}_{dr}\cdot\boldsymbol{E}_\perp \sim \boldsymbol{v}_{dr}\cdot\sum_m \frac{m\phi_m}{r} \sim \sum_m \left\langle (2\mathscr{E} - 3\mu B) J_0\left(\frac{m}{r}\rho_\perp\right)\frac{m\phi_m}{r}\right\rangle_{orbit},$$

supplemented by wave-particle-interaction resonances:

$$\Omega = \omega - n\dot{\varphi} - m\dot{\theta} - l\omega_b = 0.$$

NOVA-K framework allows:

Realistic representation of EP drift orbits.
Orbit averaging is performed for WPI matrices: $\langle \boldsymbol{E}\cdot\boldsymbol{v}_d\rangle_{drift}$.

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Outline

RBQ1D diffusion operator through pitch angle scattering

Action-angle formalism through flux variables results in a set of equations for fast ion DF: *(H.L.Berk, B.N.Breizman, NF'95)* and adapted for RBQ *(Duarte, PhD'17, Gorelenkov, NF'17)*

$$\frac{\partial}{\partial t} f = \pi \sum_{\mathbf{l},k} \frac{\partial}{\partial P_\varphi} C_k^2 \mathscr{E}^2 \frac{G_{m'p}^* G_{mp}}{\left|\partial \Omega_{\mathbf{l}} / \partial P_\varphi\right|_{res}} \mathscr{F}_{\mathbf{l}} \frac{\partial}{\partial P_\varphi} f + \nu_{eff}^3 \left|\frac{\partial \Omega_{\mathbf{l}}}{\partial \bar{P}_\varphi}\right|^{-2} \frac{\partial^2}{\partial P_\varphi^2} (f - f_0),$$

where EP distribution is evolved due to scattering terms on RHS amended by the "source" operator neglecting velocity drag.

AE amplitude satisfies

$$C_k(t) \sim e^{(\gamma_L + \gamma_d) t} \Rightarrow \frac{dC_k^2}{dt} = 2(\gamma_L + \gamma_d) C_k^2.$$

RBQ does not evolve *AE mode structure, only amplitudes.

Critical for RBQ platform *(Dupree'66, Berk'95)* is the resonant frequency and its broadening by nonlinear bounce ω_{bNL} and effective scattering ν_{eff}:

$$\delta\left(\Omega = \omega + n\dot{\varphi} - m\dot{\theta} - l\omega_b\right) \to \mathscr{F}\left(\Delta P_\varphi\right) - window\ \ function.$$

RBQ1D benefits are:

- Time efficient.
- Realistic computations of CD, loss distribution over the first wall, intermittency.

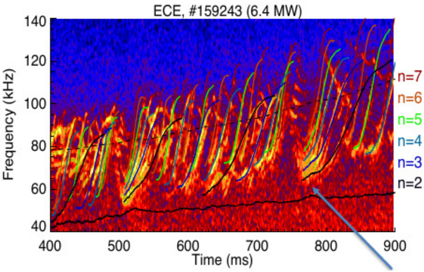
Outline

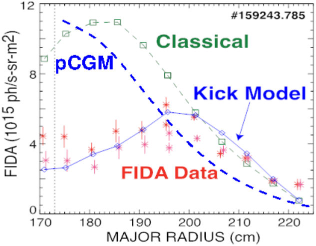
Building blocks of RBQ1D

- Preprocessing: ideal MHD (NOVA) + kinetic (NOVA-K) to compute WPI matrices
  - Use built-in **chirping criterion** to determine the QL model (non-)applicability. ITER is marginally QL-compatible in steady state regime.
- & simultaneously solves AE amplitude evolution system of equations.
  - RBQ in predictive runs finds self-consistent AE amplitudes and evolves them.
- RBQ evolves EP distribution function.
  - Employ diffusion solver for EP distribution advance.
- RBQ postprocess EP diffusion for WDM.
  - Employ PDF (probability distribution function) - make use of the kick model PDFs for TRANSP interface.
- NUBEAM employs the EP diffusion within WDM (TRANSP).

Apply to DIII-D case studied in CGM experiments

Detailed analysis of DIII-D experiment #159243 & 6.4MW *(Collins et al, PRL'16, Heidbrink et al., PoP'17).*





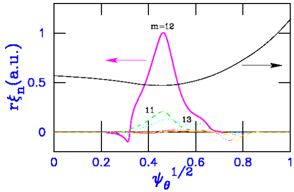
805msec is chosen near rational q_{min} for detailed study

Earlier developed critical gradient model (pCGM) does not reproduce hollow EP profiles and underestimates neutron deficit by two times.

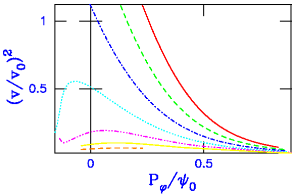
Need velocity space resolution such as in the Kick Model *(Podesta et al., PPCF'14)*

RBQ workflow illustration for a single $n = 4$ RSAE (at q_{min})

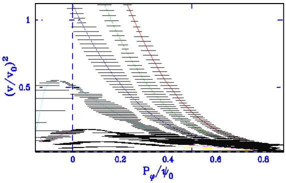
NOVA preprocessing: mode structure



resonances from NOVA-K



RBQ broadening



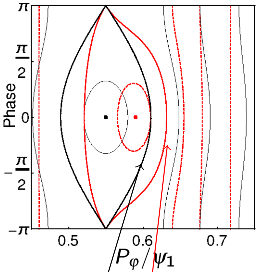
- Broadening is shown for measured RSAE amplitude $\delta B_\theta/B = 7 \times 10^{-3}$, $f = 84kHz$ *(Collins, PRL'16)*.
- RSAE provides a channel to form a particle diffusion and an inversed fast ion pressure profile: resonant particles are close to the injected pitch angle near the axis.
- As a result RBQ computes Probability Density Function (PDF) for futher processing by NUBEAM (within TRANSP WDM).
  - Three versions of RBQ1D were developed:
    - Interpretive, predictive with single mode saturattion, and predictive with multiple mode amplitude computed selfconsistently.

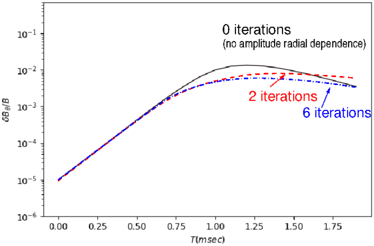
Outline

I. Resonant island EP dynamics is accounted for using Hamiltonian technique

EP islands for "Gaussian" mode

RBQ needs ~2 iterations to converge well. Lowers saturation ampl.





BB approach; RBQ "new" island accounts for RSAE radial amplitude variation;

- Low amplitude $\Delta P_\varphi \sim \Delta\Omega = 4\omega_b$ at $\delta B_\theta/B \lesssim (1 \div 5)\times 10^{-4}$ *(via ORBIT modeling, G.Meng, NF'18).* Supports resonant frequency approach for nonlinear wave particle interaction.
- Radial amplitude structure limits NL resonance frequency *(R.White et al., PoP'18).*

II. Analytic solution for amplitude evolution near threshold

Near marginal stability, the amplitude governed by

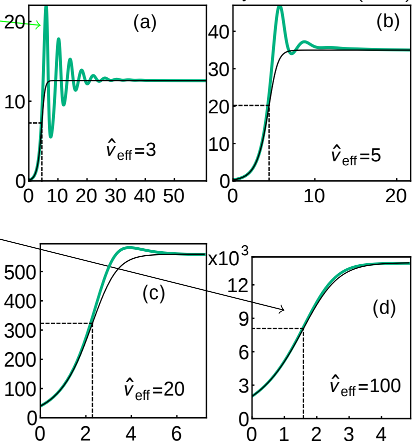
$$\frac{dA(t)}{dt} = A(t) - \frac{1}{2}\int d\Gamma \mathscr{H}\left\{\int_0^{t/2} dz z^2 A(t-z)\times \right.$$
$$\left. \times \int_0^{t-2z} dy e^{-\hat{\nu}_{eff}^3 z^2(2z/3+y)} A(t-z-y)A^*(t-2z-y)\right\}.$$

(Berk et al., PRL'96)
At large ν_{eff} (> net growth rate) only recent time history dictates the WPI dynamics, i.e. when $y,z \to 0$:

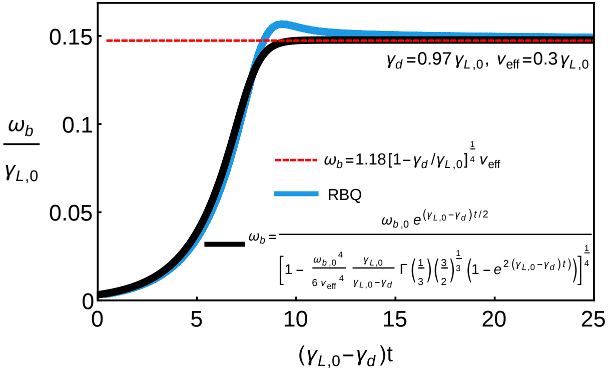
$$A(t) = \frac{A(0)e^t}{\sqrt{1-bA^2(0)(1-e^{2t})}}$$

where $A(0)$ is the initial amplitude and
$b \equiv \left[\int d\Gamma \mathscr{H} \frac{\Gamma(1/3)}{6\hat{\nu}_{eff}^4}\left(\frac{3}{2}\right)^{1/3}\right]$
(V.Duarte et al., submitted to NF).

Amplitude A vs time t for the full cubic equation (green) and the analytical solution (black)



RBQ verification against analytical solution for amplitude evolution near threshold

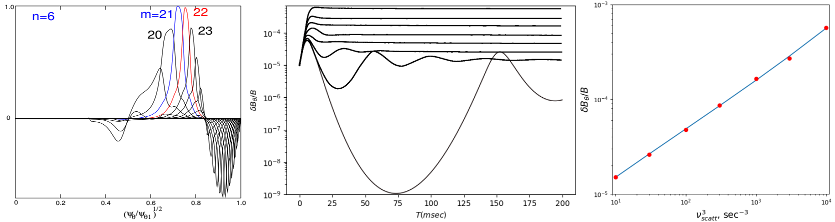


Both RBQ diffusion solver and the analytical solution recover:

- time evolution
- expected saturation level near marginal stability [Berk, Breizman '90]!

III. RBQ verification via Coulomb collisions

Global $n = 6$ TAE saturates over $\sim msec$ RBQ shows $\delta B_\theta/B \propto \nu_{eff}^{1.65}$



- TAE amplitude scales with fast ion Coulomb scattering frequency, $\delta B_\theta/B \sim \nu_{eff}^{2} \sim \nu_\perp^{2/3}$, where $\nu_{eff}^{3} = \nu_\perp \left|\frac{\partial\Omega}{\partial\chi}\right|^2$ *(Berk et al.,Phys. Fluids B'90)*.
- Dirichlet boundary conditions, $f_h(\bar{\psi}_\theta \to 0) = const$ and $f_h(\bar{\psi}_\theta \to 1) = 0$, are required to account for Coulomb scattering.

- Intermittency (fluctuations in losses) is expected in predictive RBQ simulations!!

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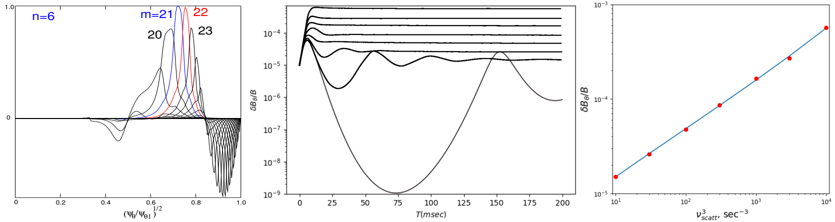

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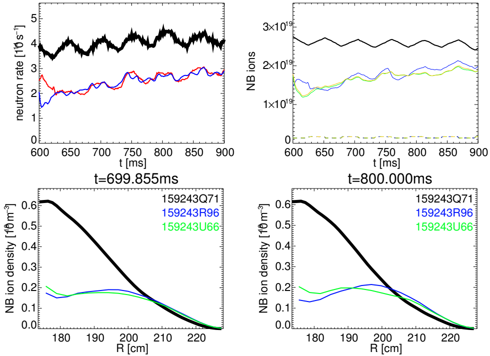
11 observed modes are employed similar to kick model interpretive version

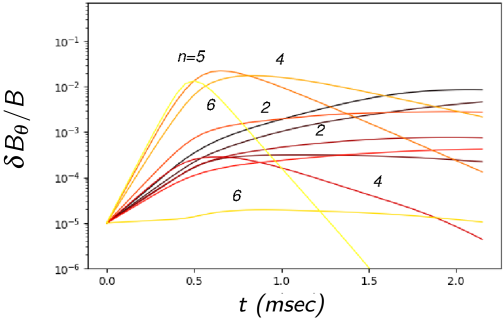
Color-coded (on the right) evolution/profiles



- Classical NUBEAM calculations no AE effects
- Kick model + with neutron flux constraint
- RBQ with neutron flux constraint
- AE amplitudes are computed in RBQ1D at $t = 805msec$ time of measurements.

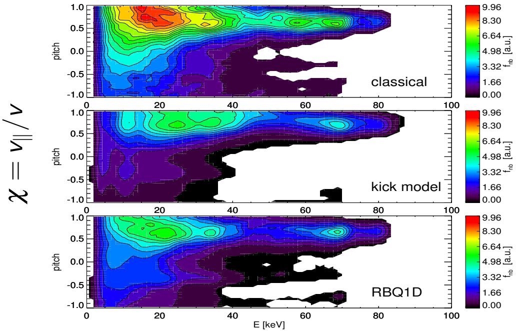
RBQ applications in the interpretive mode are consistent with measurements. PDFs need to be reworked to a better representation of EP diffusion in the COM phase space.

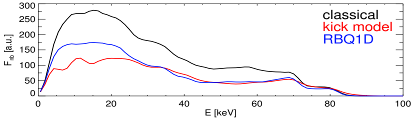
QL (RBQ1D) selfconsistent evolution of all 11 modes



Selfconsistent amplitudes of AEs are consistent with measured values $O(10^{-4} - 10^{-2})$.
Shown are AE evolutions with modified prescriptions for broadening *(White et al., PoP'18)*.

RBQ evolves distribution function in a similar manner as the kick model

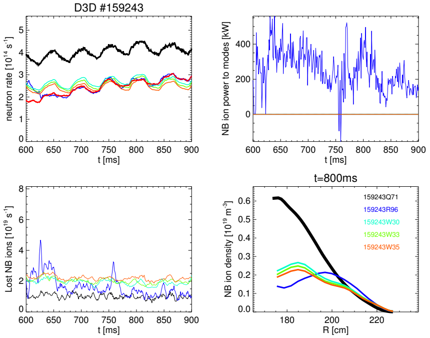




- Co-going passing ions are strongly redistributed.
- Amplitudes are kept constant throughout observed times.
- Neutron rate includes radial and energy dependence within TRANSP simulations.
- Fast ion diffusion is collected through the probability density function for NUBEAM.

Apply predictive RBQ1D analysis for multiple mode case

Color-coded (in figures) evolution/profiles with +0%, +10% and +20% higher amplitudes



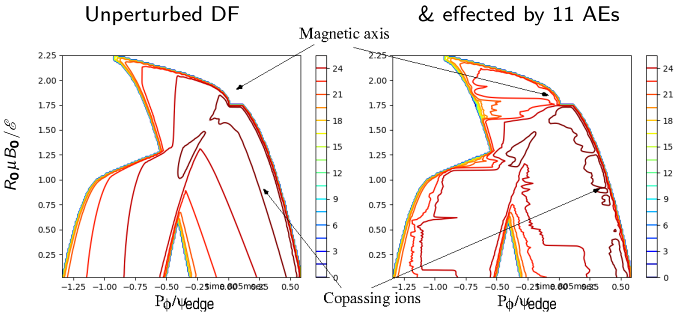
- Distributions are evolved by RBQ selfconsistently to saturation, $\gamma_L(t) \simeq \gamma_d$.
- Amplitudes are computed.
- Diffusion coefficients are transferred to NUBEAM.
- (Near) hollow EP density is due COM location sensitive diffusion.
- Rotation is ignored!!

  It can be significant, leading to EP energy shift $\sim E_0/2$ in DIII-D.

Status and plans for reduced models development

- RBQ model is built based on QL approach *(Berk, NF'95,PoP'96, Dupree, PhFl'66, Kaufman, PhFl'72)*
- Broadening technique is included for non-slanted 1D mesh in P_φ.
- RBQ employs full diffusion solver module prepared for single or multiple resonance problem.
- RBQ1D is applied to DIII-D cases for V&V (Gorelenkov, et al., IAEA TCM'17, NF'18). Low rotation is critical not to cause EP energies change significantly. Can we apply RBQ1D to NSTX-U with chirping frequency AEs?
- 2D extension will be developed within ISEP SciDac.
  - 1D problem requires direct knowledge of diffusion coefficients in WDM
  - conservative estimates for RBQ approach imply 2.5 *PFlops* computer power needed BP.

Within RBQ1D distribution function evolves due to all mode's effects



- Strong redistribution is seen for co-going passing ions.
- Shown are ions near injection energy $\mathscr{E} \simeq 55-70keV$.
- However the fast ion diffusion, as a result, is collected through the probability density function for NUBEAM using the kick model subroutines.