# Linear Stability of a weakly magnetized rotating plasma column

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Collaborators: Yann Camenen, Alexandre Poye, Alexandre Escarguel

# About me!

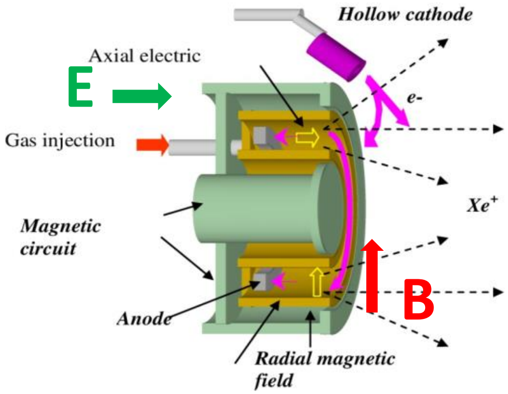
2017 | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 | 2024

- 2018: Summer internship at IPR, India
- 2019: Masters of Physics, GNDU, India
- 2020: Started Ph.D. in Energy, radiation and plasma physics at AMU
- 2023: Completed Ph.D. in Energy, radiation and plasma physics at AMU
- 2024: Postdoctoral researcher at PIIM, AMU until Jan. 2024

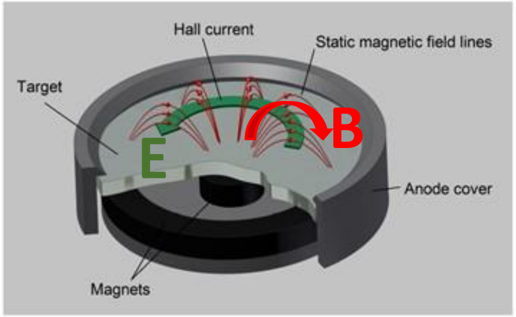
# Outline

- Background and Motivation
- Experimental results
- Theory
  I. Two-fluid formalism
  II. Linear Stability Analysis
- Results and Discussion
- Summary and Conclusions
- Outlook and Future Perspectives

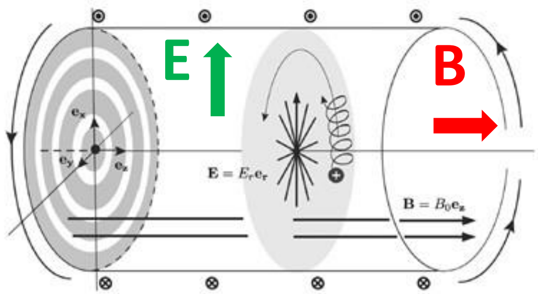
# Plasma devices in ExB configuration




(a). Hall-effect thruster



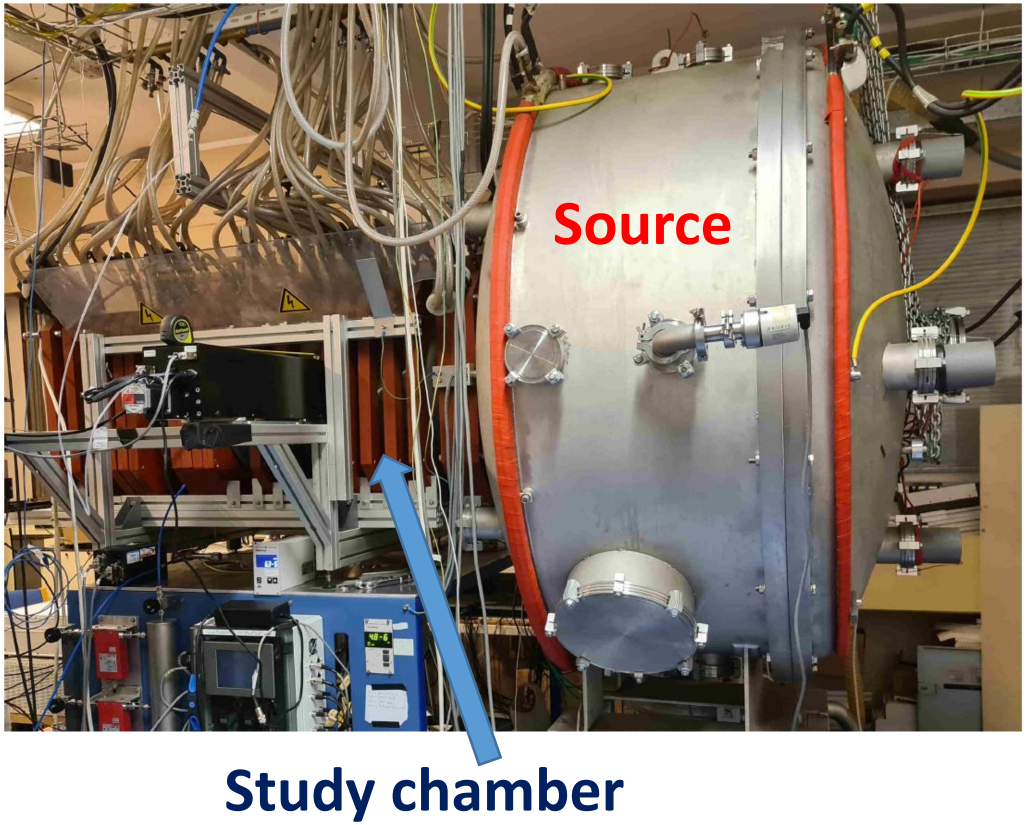

(b). Cross section of planar circular magnetron




(c). Configuration for rotating plasma separation in a uniform magnetic field

- Issue : Instabilities → appearance of coherent rotating structures → turbulent transport.
- Investigation of instabilities, coherent rotating structures in laboratory experiments.

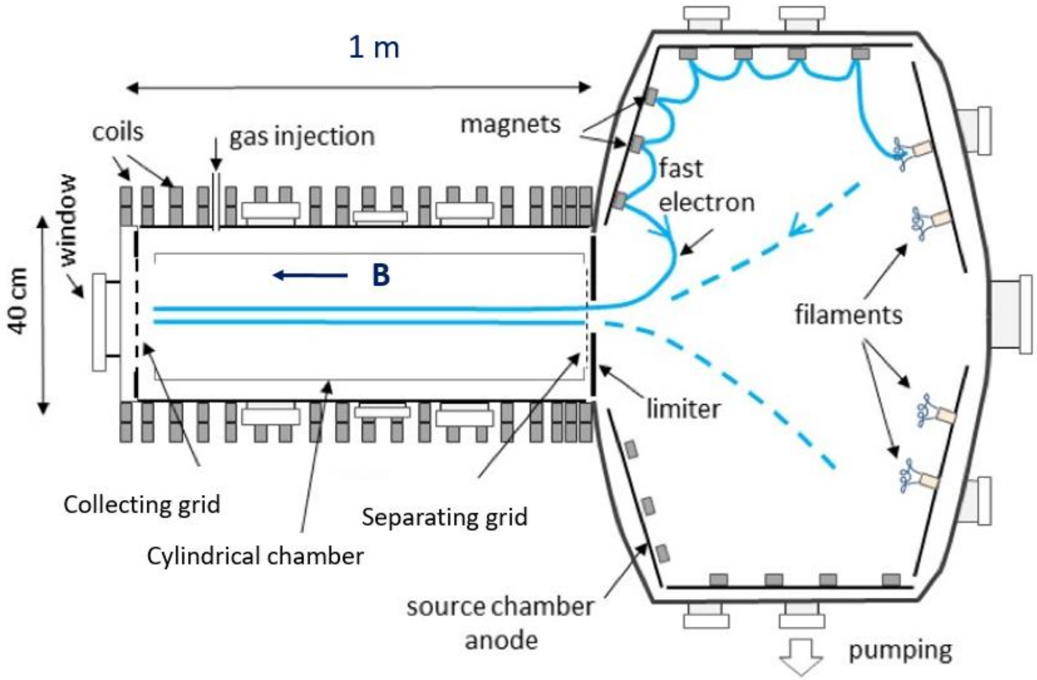
# MISTRAL experiment




## Typical MISTRAL parameters

| | |
|---|---|
| Magnetic field (B) | 100 – 360 Gauss |
| Column Length | 1 m |
| Column radius | 10 cm |
| Source type | Hot electron beam |
| $E_{primary\ electrons}$ | $\approx$ 40 eV |
| Pressure | $10^{-5} – 10^{-3}$ mbar |
| Electron density ($n_e$) | $10^{14} – 10^{16}\ m^{-3}$ |
| Electron temperature ($T_e$) | 2 – 6 eV |
| Ion temperature ($T_i$) | 0.2 eV |
| Neutral temperature ($T_n$) | Approx. 300 K |

# MISTRAL experiment

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- Plasma is created by the injection of primary electrons from source towards the study chamber.

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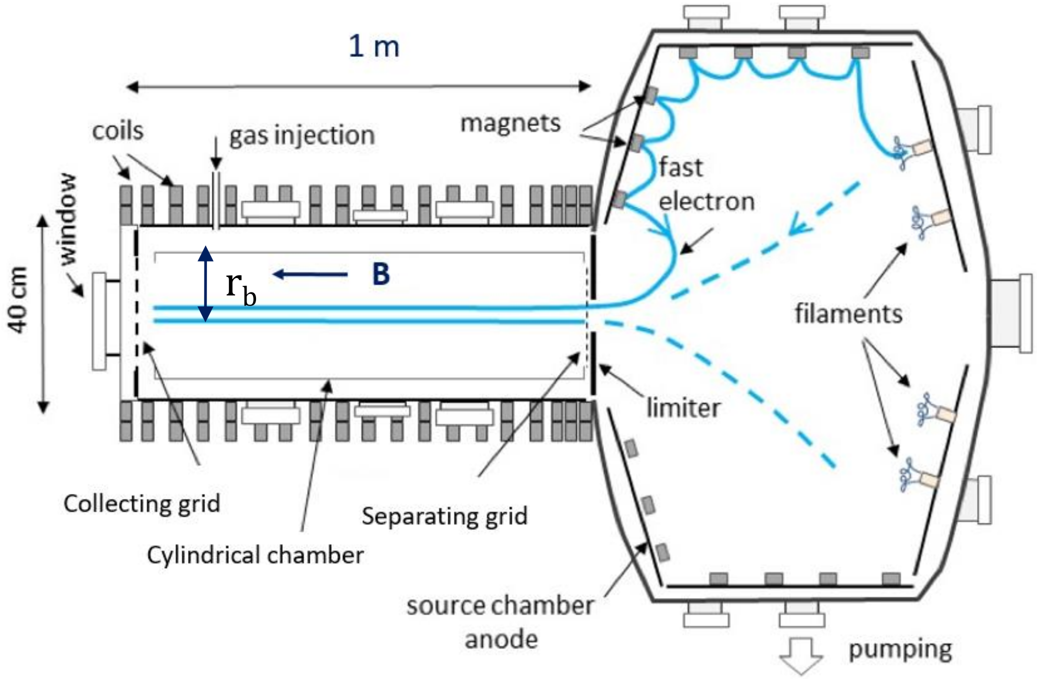


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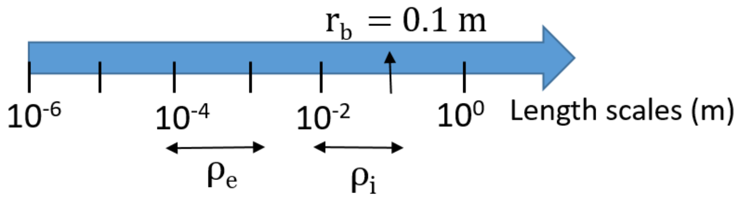
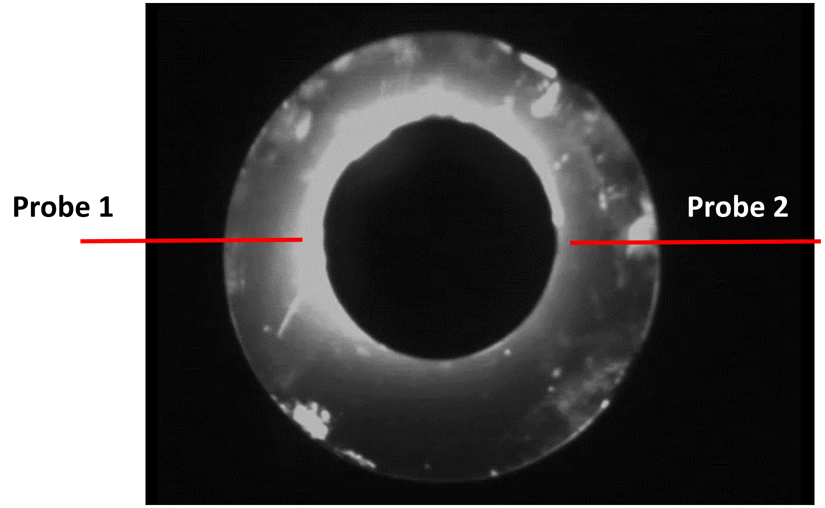
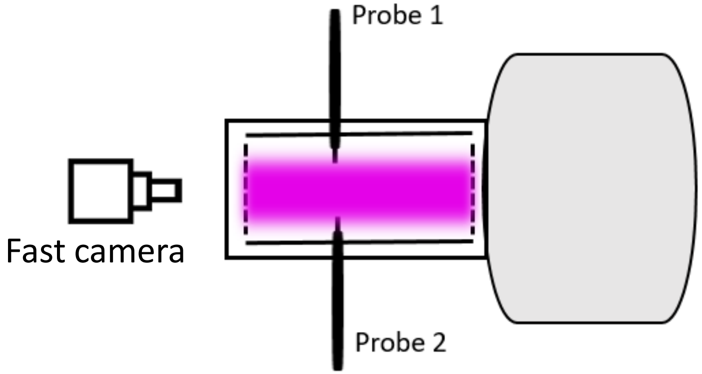
- Plasma is created by the injection of primary electrons from source towards the study chamber.
- Existence of coherent rotating spoke in MISTRAL plasma.

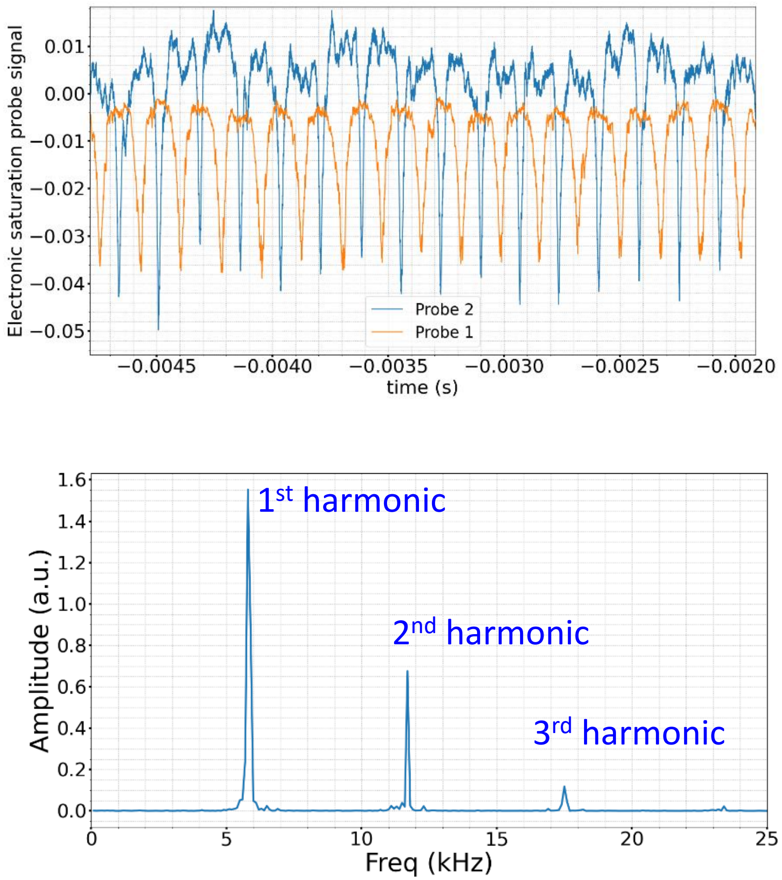


# Rotating spoke in MISTRAL plasma



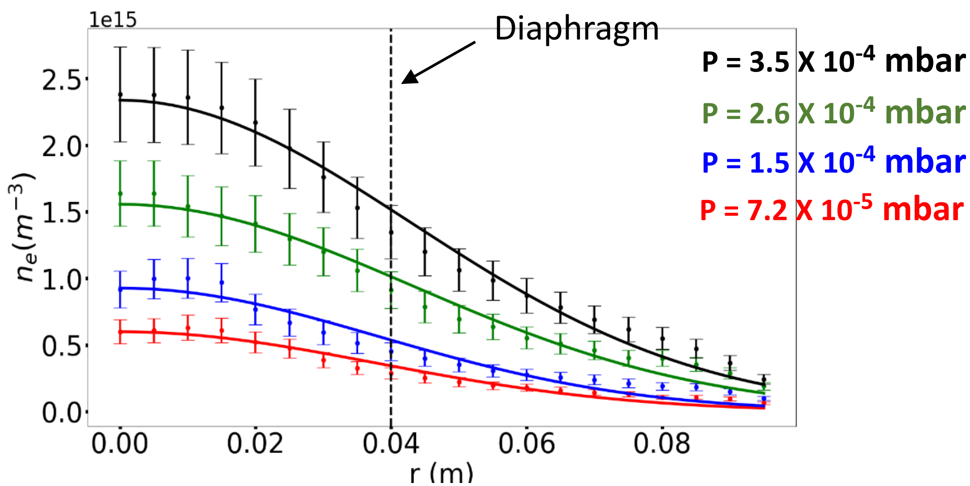

**m = 1**

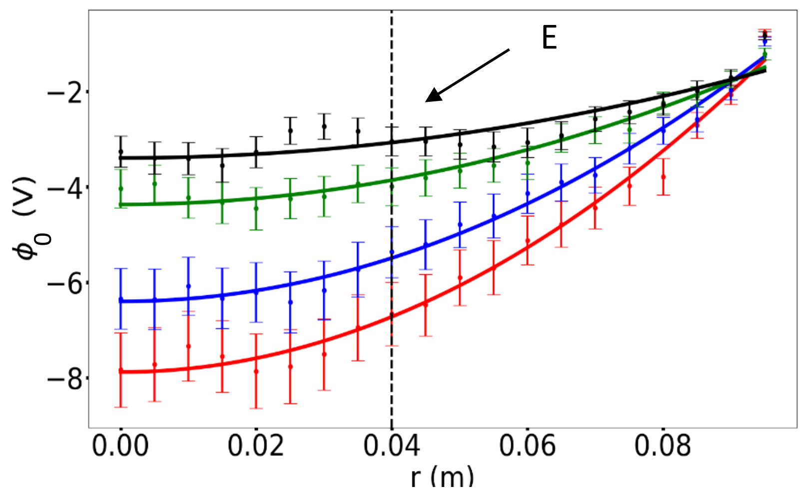



B = 160 G, P = $10^{-4}$ mbar

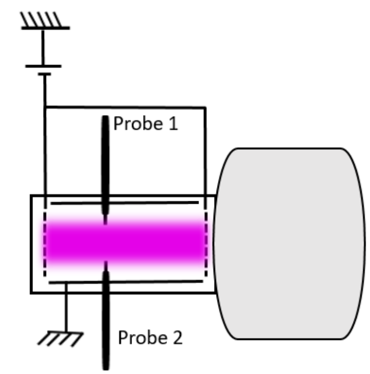
# Time averaged plasma profiles at B = 160 Gauss (Argon plasma)

- Time averaged over a few periods of rotating spoke (≈ 5-6).



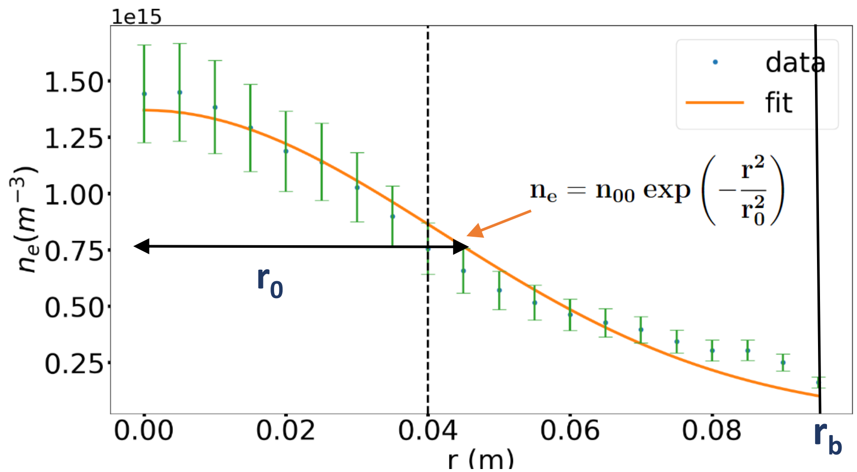



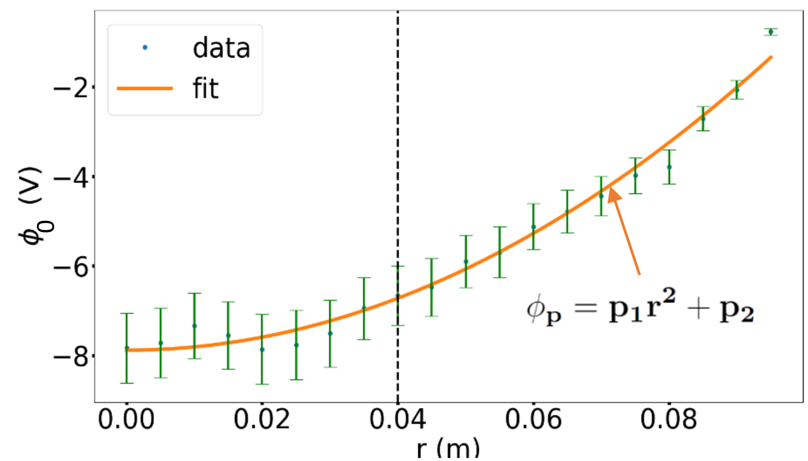

- $n_e$ → Gaussian profile
- $\phi_p$ → parabolic profile
- $\phi_p$ → Deviation from parabolic profile at high pressure




Experimental configuration

# Parameterization of density and potential profiles




- ExB flow frequency

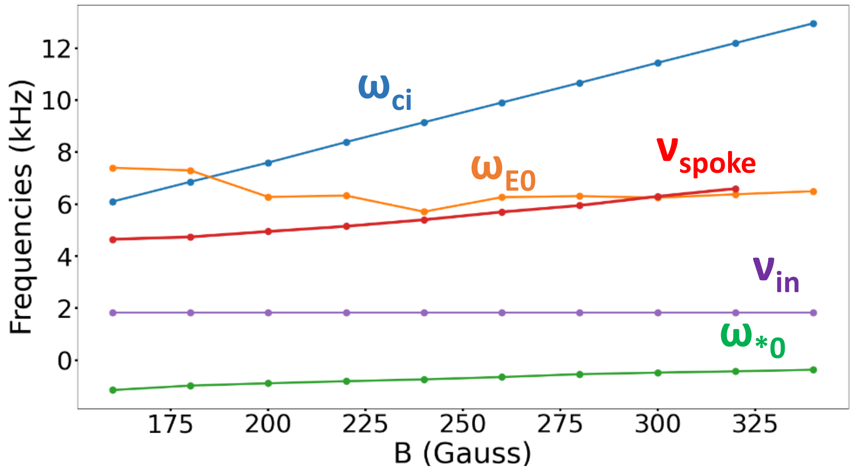
$$\omega_{E0} = \frac{B \times \nabla\phi_0}{rB^2} \cdot \hat{e}_\theta = \frac{\phi_0'}{rB} = \frac{2p_1}{B}$$

- Ion diamagnetic flow frequency

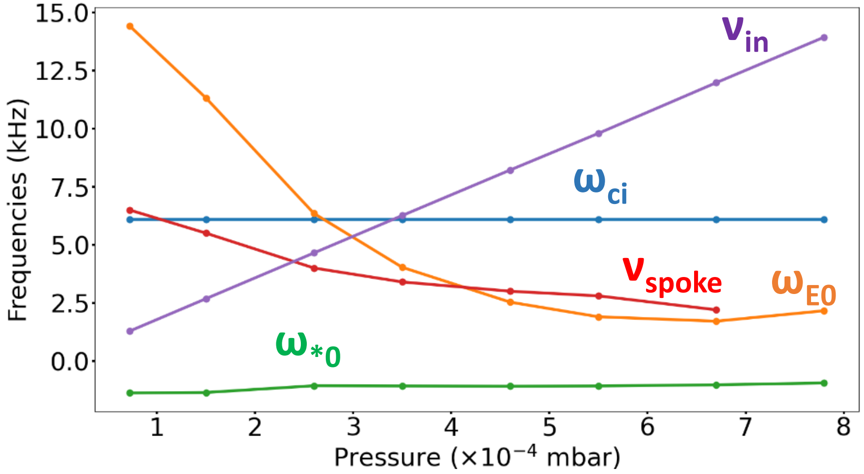
$$\omega_{*0} = \frac{T_i}{en_eB}\frac{B \times \nabla n_e}{rB} \cdot \hat{e}_\theta = \frac{T_i}{erB}\frac{n_e'}{n_e} = -\frac{T_i}{eB}\frac{2}{r_0^2}$$

# Characteristic frequencies in MISTRAL

At P = 1.04 X $10^{-4}$ mbar




At B = 160 Gauss




- $\omega_{ci} \uparrow$ as B $\uparrow$
- $\nu_{spoke} \uparrow$ as B $\uparrow$

- $\nu_{in} \uparrow$ as P $\uparrow$
- $\nu_{spoke} \downarrow$ as P $\uparrow$

- Ion – neutral collisions ($\nu_{in}$) are important → should be taken into account.
- $\omega_{E0}$ , $\nu_{spoke}$ , $\nu_{in} \approx \omega_{ci}$

# What’s the goal ?

- Experimental evidence on existence of rotating structures*/instabilities.

- Existing models** to study instabilities are based on:
  - Low frequency approximation (LFA) → characteristic frequencies << $\omega_{ci}$
  - No collisionality, If considered then based on LFA
  - Local analysis, if global analysis is present then it’s based on LFA

- These assumptions are not compatible with parameters relevant to MISTRAL.

→ Objective

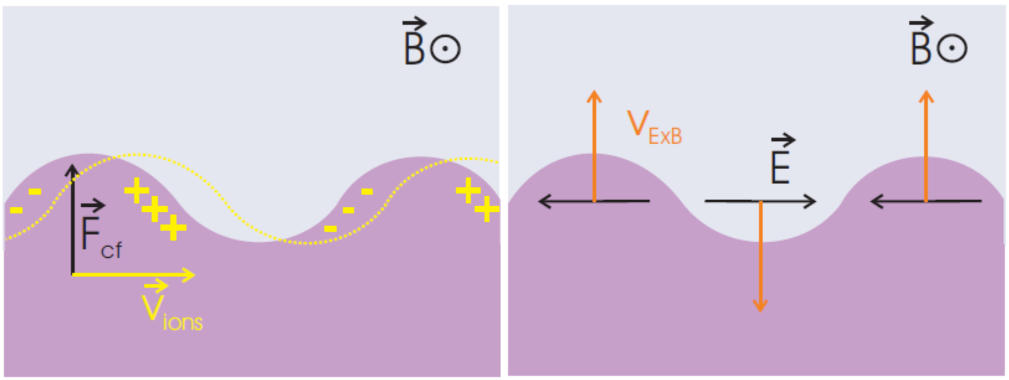
Model development to investigate instabilities in weakly magnetized plasma devices like MISTRAL:

- Valid at arbitrary frequency values
- Accounts for ion neutral collisionality
- Radially global model

*[Escarguel EPJD 2010]
**[Rosenbluth et al., Nucl.Fusion Suppl. (1962)]
**[Chen, PoF (1966)]

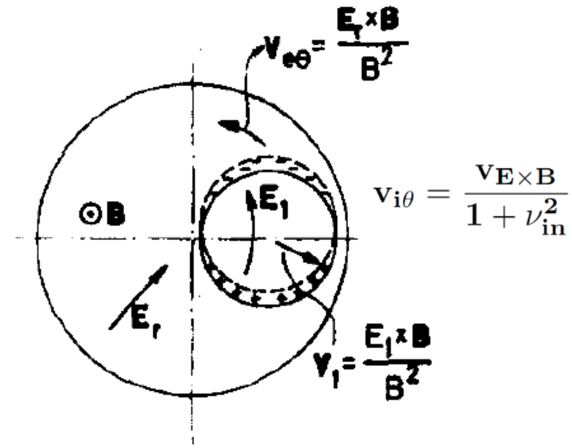
# Possible instabilities in MISTRAL

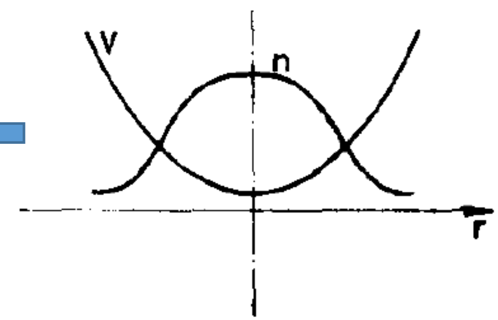
## Centrifugal instability




*S. Oldenburger thesis, 2010.*

## Neutral drag instability




*F. C. Hoh, Phys. Fluids* , 1963

- Instability arises due to difference in flow of ions and electrons.
- This difference is either brought about by inertia or by collisions or by combination of both.

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# Two-fluid model

*Continuity Equation :* $$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v}_j) = S_{ion} \; ; \quad j = i, e \quad , \; S_{ion} = n_{ep}\nu_{ion}$$

*Ion momentum :* $$\left(\frac{\partial \mathbf{v_i}}{\partial t} + \mathbf{v_i} \cdot \nabla \mathbf{v_i}\right) = \frac{e}{m_i}\left(-\nabla\phi + \mathbf{v_i} \times \mathbf{B}\right) - \frac{1}{n_i m_i}\nabla p_i - \frac{1}{n_i m_i}\nabla \cdot \pi_i - \nu_{in}\mathbf{v}_i - \frac{S_{ion}\mathbf{v_i}}{n_i}$$

Inertia | Electromagnetic force | Pressure gradient | Gyro-viscosity | Collisional friction | Source term

*Electron momentum :* $$0 = -en_e(-\nabla\phi + \mathbf{v}_e \times \mathbf{B}) - \nabla p_e$$

*Closure :* $$\frac{\partial T_i}{\partial r} = 0 \quad , \qquad \nabla p_j = \nabla n_j T_j$$

*Quasi – neutrality :* $$n_i = n_e$$

# Assumptions

- Electrostatic approximation $\Longrightarrow$ $\dfrac{\partial \boldsymbol{B}}{\partial t} = 0$
- No axial variations ($k_{||}$= 0) and uniform magnetic field i.e. $\boldsymbol{B} = B\hat{e}_z$
- No neutral flow ($v_n$ = 0)
- Rigid body rotation → Gaussian density + parabolic potential profiles (deviation at high pressure)

Gyro-viscosity is neglected
i.e. $\nabla \cdot \pi_i = 0$

- $$\left(\frac{\partial \mathbf{v_i}}{\partial t} + \mathbf{v_i} \cdot \nabla \mathbf{v_i}\right) = \frac{e}{m_i}\left(-\nabla\phi + \mathbf{v_i} \times \mathbf{B}\right) - \frac{1}{n_i m_i}\nabla p_i - \cancel{\frac{1}{n_i m_i}\nabla} \cdot \pi_i - \nu_{in}\mathbf{v}_i - \frac{S_{ion}\mathbf{v_i}}{n_i}$$

# Normal mode analysis

System of equations: ion and electron momentum and continuity equations

Linearization

$$n(r) = n_o(r) + n_1(r)\, exp[i(m\theta - \omega t)]$$
$$\mathbf{v}(r) = \mathbf{v_o}(r) + \mathbf{v_1}(r)\, exp[i(m\theta - \omega t)]$$
$$\phi(r) = \phi_o(r) + \phi_1(r)\, exp[i(m\theta - \omega t)]$$

Equilibrium (zero order part)

$$n(r) = \boxed{n_o(r)} + n_1(r)\, exp[i(m\theta - \omega t)]$$
$$\mathbf{v}(r) = \boxed{\mathbf{v_o}(r)} + \mathbf{v_1}(r)\, exp[i(m\theta - \omega t)]$$
$$\phi(r) = \boxed{\phi_o(r)} + \phi_1(r)\, exp[i(m\theta - \omega t)]$$

$\mathbf{n_0},\ \phi_\mathbf{0},\ \mathbf{v_0}$

Perturbed part (first order part)

$$n(r) = n_o(r) + \boxed{n_1(r)\, exp[i(m\theta - \omega t)]}$$
$$\mathbf{v}(r) = \mathbf{v_o}(r) + \boxed{\mathbf{v_1}(r)\, exp[i(m\theta - \omega t)]}$$
$$\phi(r) = \phi_o(r) + \boxed{\phi_1(r)\, exp[i(m\theta - \omega t)]}$$

$\mathbf{n_1},\ \phi_\mathbf{1},\ \mathbf{v_1}$

Dispersion relation in $\omega$

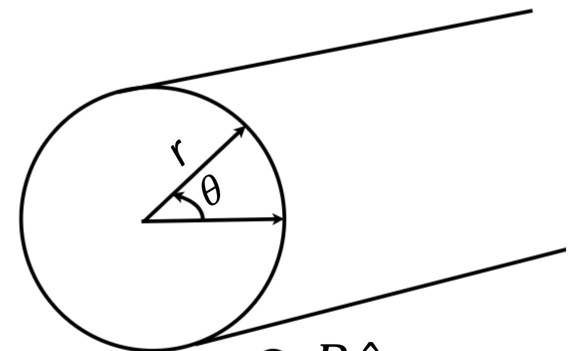
1. mode frequency $\omega_r$ and growth rate $\gamma$
2. Amplitude of density and potential fluctuations
3. Phase difference between density and potential fluctuations ........

# Equilibrium description

$$\left(\frac{\partial \mathbf{v_{i0}}}{\partial t} + \mathbf{v_{i0}} \cdot \nabla \mathbf{v_{i0}}\right) = \frac{e}{m_i}\left(-\nabla\phi + \mathbf{v_{i0}} \times \mathbf{B}\right) - \frac{1}{n_{i0} m_i}\nabla p_i - \nu_{in}\mathbf{v}_{i0} - \frac{S_{ion}\mathbf{v_{i0}}}{n_{i0}}$$

$$\mathbf{v_{i0}}(r) = \underbrace{v_{ir_0}\hat{e}_r}_{\text{radial flow}} + \underbrace{v_{i\theta_0}\hat{e}_\theta}_{\text{azimuthal flow}}$$

$$v_{i\theta_0} = r\omega_0$$




$\odot\, B\hat{z}$

$$\bar{\nu}_{eff} = \bar{\nu}_{in} + (n_{ep}/n_{i0})\bar{\nu}_{ion}$$

$$\bar{v}_{ir_0} = \frac{-\bar{\nu}_{eff}\bar{r}\bar{\omega}_0}{1 + 2\bar{\omega}_0} = \boxed{-\bar{r}\epsilon}$$

In absence of collisions, no radial flow

$$\bar{\omega}_0 = \frac{1}{2}\sqrt{\frac{1}{2}\left[\{1 + 4(\bar{\omega}_{*0} + \bar{\omega}_{E0}) - \bar{\nu}_{in}^2\} + \sqrt{\{1 + 4(\bar{\omega}_{*0} + \bar{\omega}_{E0}) - \bar{\nu}_{in}^2\}^2 + 4\bar{\nu}_{in}^2}\right]} - \frac{1}{2}$$

Ion diamagnetic flow frequency

ExB flow frequency

Ion-neutral collision frequency

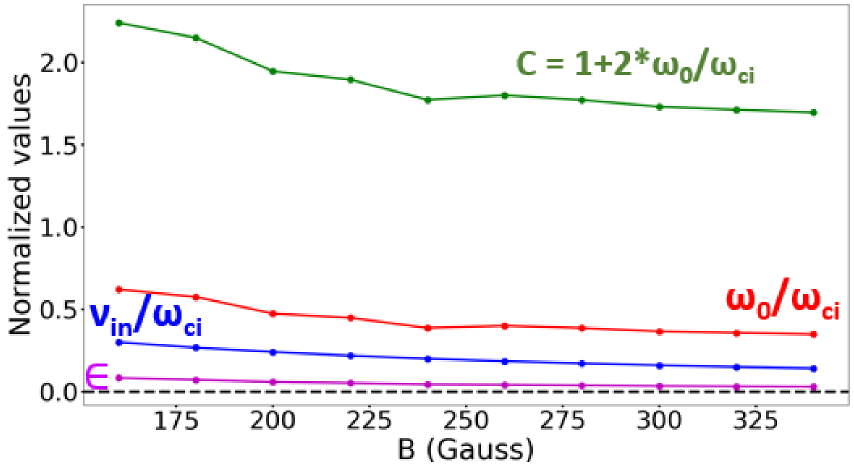
Normalizations used →

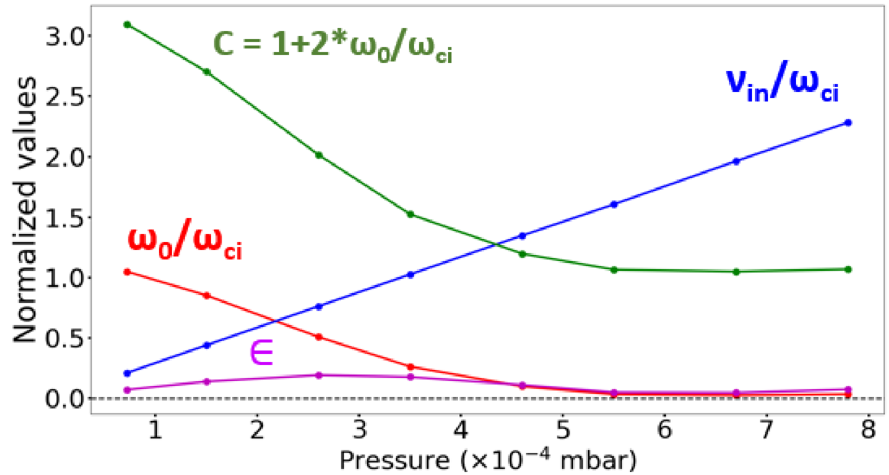
$$\bar{\omega} \rightarrow \omega/\omega_{ci}$$
$$\bar{r}, 1/\bar{L}_n \rightarrow r/\rho_i, \rho_i/L_n$$
$$\bar{\phi}_1 \rightarrow e\phi_1/T_{e0_{ref}}$$
$$\bar{v}_{i,e} \rightarrow v_{i,e}/v_{thi}$$

# Experimental observation: $\epsilon \ll 1$

Radial ion equilibrium flow

$\bar{v}_{ir_0} = -\bar{r}\,\epsilon$




- Verification of the assumption $\epsilon \ll 1$ for MISTRAL plasma motivates this choice.

$$\bar{\mathbf{v}}_{i1} = \bar{\mathbf{v}}_{i1}^{(0)} + \epsilon \bar{\mathbf{v}}_{i1}^{(1)}$$

# Dispersion relation

$$\boxed{\left(\frac{\partial \mathbf{v_i}}{\partial t} + \mathbf{v_i}\cdot\nabla\mathbf{v_i}\right) = \frac{e}{m_i}\left(-\nabla\phi + \mathbf{v_i}\times\mathbf{B}\right) - \frac{1}{n_i m_i}\nabla p_i} - \nu_{in}\mathbf{v}_i - \frac{S_{ion}\mathbf{v_i}}{n_i}$$

**Inertia**

$$\Phi_1'' + \left(\frac{1}{\bar r} - \frac{1}{\bar L_n}\right)\Phi_1' - \frac{m^2}{\bar r^2}\Phi_1 + \frac{1}{\bar r \bar L_n}N\Phi_1 = 0$$

**Inertia + collisional friction**

$$\epsilon\Phi_1''' + \left[\frac{C_{NC}}{\bar r} + \epsilon\left(\frac{3}{\bar r} - \frac{1}{\bar L_n}\right)\right]\Phi_1'' + \left[\frac{C_{NC}}{\bar r}\left(\frac{1}{\bar r} - \frac{1}{\bar L_n}\right) + \epsilon\left(-\frac{m^2}{\bar r^2} + \frac{1}{\bar r}\left(\frac{1}{\bar r} - \frac{1}{\bar L_n}\right) + \mathcal{A} - \mathcal{B}\right)\right]\Phi_1' + \left[\frac{C_{NC}}{\bar r}\left(-\frac{m^2}{\bar r^2} + \frac{N}{\bar r \bar L_n}\right) - \epsilon\mathcal{B}\left(\frac{2}{\bar r} - \frac{1}{\bar L_n}\right)\right]\Phi_1 = 0 \quad (42)$$

**Inertia + collisional friction + ionization source**

$$\epsilon\Phi_1''' + \left[\frac{C_{NC}}{\bar r} + \epsilon\left(\frac{3}{\bar r} - \frac{1}{\bar L_n}\right) - \lambda\epsilon\xi\bar r\right]\Phi_1'' + \left[\frac{C_{NC}}{\bar r}\left(\frac{1}{\bar r} - \frac{1}{\bar L_n}\right) + \epsilon\left(-\frac{m^2}{\bar r^2} + \frac{1}{\bar r}\left(\frac{1}{\bar r} - \frac{1}{\bar L_n}\right) + \mathcal{A} - \mathcal{B}\right) + \lambda D - \lambda\epsilon\left(im\bar\omega_0 + \xi\bar r\left(\frac{5}{\bar r} - \frac{1}{\bar L_n}\right) - C_{NC}\right)\right]\Phi_1' + \left[\frac{C_{NC}}{\bar r}\left(-\frac{m^2}{\bar r^2} + \frac{N}{\bar r \bar L_n}\right) - \epsilon\mathcal{B}\left(\frac{2}{\bar r} - \frac{1}{\bar L_n}\right) + \lambda\mathcal{D}\left(\frac{m\bar\omega_c}{\bar r C} + \frac{2}{\bar r} - \frac{1}{\bar L_n}\right) - \lambda\epsilon\left(\frac{2im\bar\omega_0}{\bar r} + 2\xi\left(\frac{2}{\bar r} - \frac{1}{\bar L_n}\right) - \left(\frac{\mathcal{F}}{\bar r} + C_{NC}\left(\frac{2}{\bar r} - \frac{1}{\bar L_n}\right)\right)\right)\right]\Phi_1 = 0$$

# Outline

# Centrifugal instability

$$\Phi_1'' + \left(\frac{1}{\bar{r}} - \frac{1}{\bar{L}_n}\right)\Phi_1' - \frac{m^2}{\bar{r}^2}\Phi_1 + \frac{1}{\bar{r}\bar{L}_n}N\Phi_1 = 0$$

$$N = m\left[\frac{C}{\bar{\omega}_{ph}} - \frac{C^2 - \bar{\omega}_c^2}{\bar{\omega}_{ph} + m\delta\bar{\omega}_0}\right]$$

Non-singular solution ➡

$$\Phi_1(r) = \left(\frac{\bar{r}}{\bar{r}_0}\right)^m \underbrace{F\left(\frac{m-N}{2}, 1+2m, \left(\frac{\bar{r}}{\bar{r}_0}\right)^2\right)}_{\text{Confluent Hypergeometric function}}$$

- For a given *m* number → find *N* for which $\Phi_1(r_b) = 0$
- Once *N* is known, $\overline{\omega}_{ph}$ can be computed.


*S. Aggarwal et al., J. Plasma Phys.* (2023), *vol.* 89, 905890310

*F. F. Chen, Phys. Fluids* 9, 965–981 (1966)

# Centrifugal instability

eigenvalue obtained using Whittaker equation

$\delta\bar{\omega}_0 = \bar{\omega}_0 - \bar{\omega}_{E0} - \bar{\omega}_{*0}$

$$\bar{\omega}_{ph}^3 - \frac{N}{m}\bar{\omega}_{ph}^2 - \left(C - C^2 + N\delta\bar{\omega}_0\right)\bar{\omega}_{ph} + mC\delta\bar{\omega}_0 = 0$$

$C = 1 + 2\bar{\omega}_0$

Azimuthal mode number, m=1,2,3,….

Due to inertial effects only

Contribution due to magnetic force and inertial effects in the ion-momentum equation

- Instability mechanism.
- Impact of low-frequency approximation.
- Characteristics of Centrifugal instability.

*S. Aggarwal et al., J. Plasma Phys.* (2023), *vol.* 89, 905890310

# Instability mechanism

$$\bar{\omega}_{ph}^3 - \frac{N}{m}\bar{\omega}_{ph}^2 - \left(C - C^2 + N\delta\bar{\omega}_0\right)\bar{\omega}_{ph} + mC\delta\bar{\omega}_0 = 0$$

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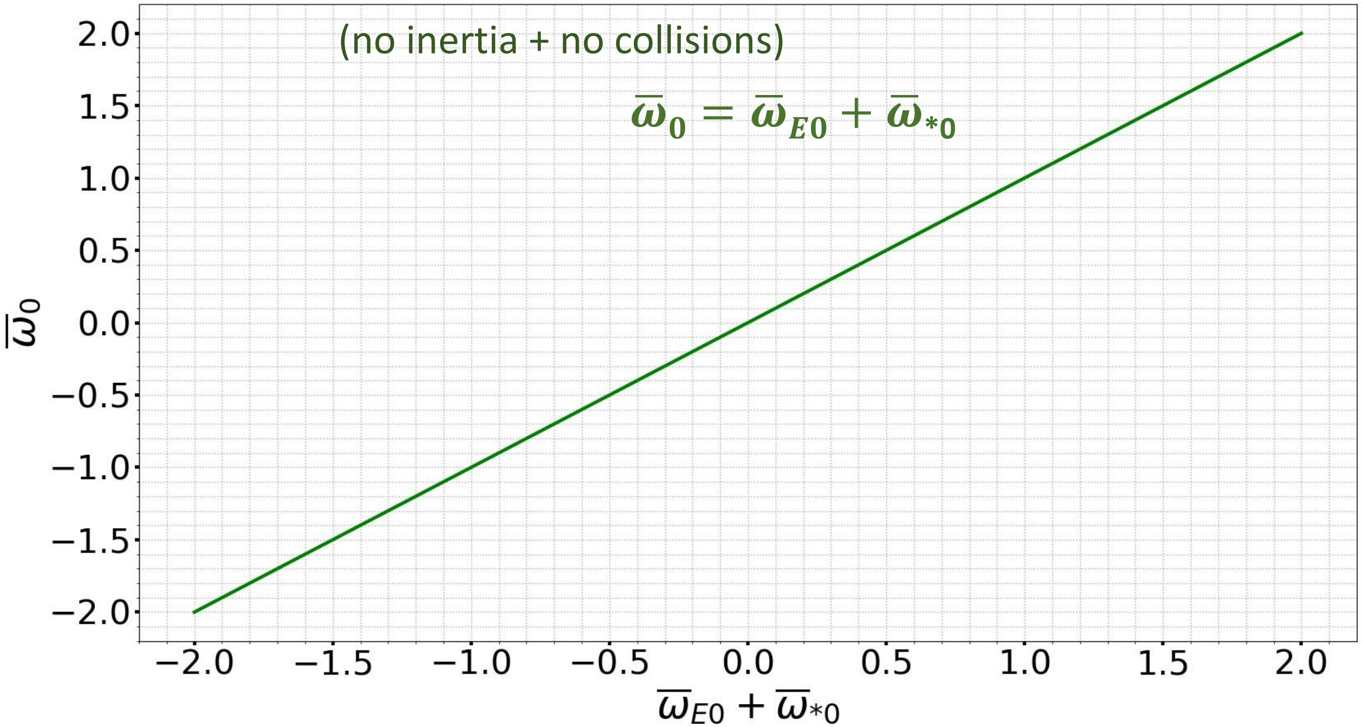
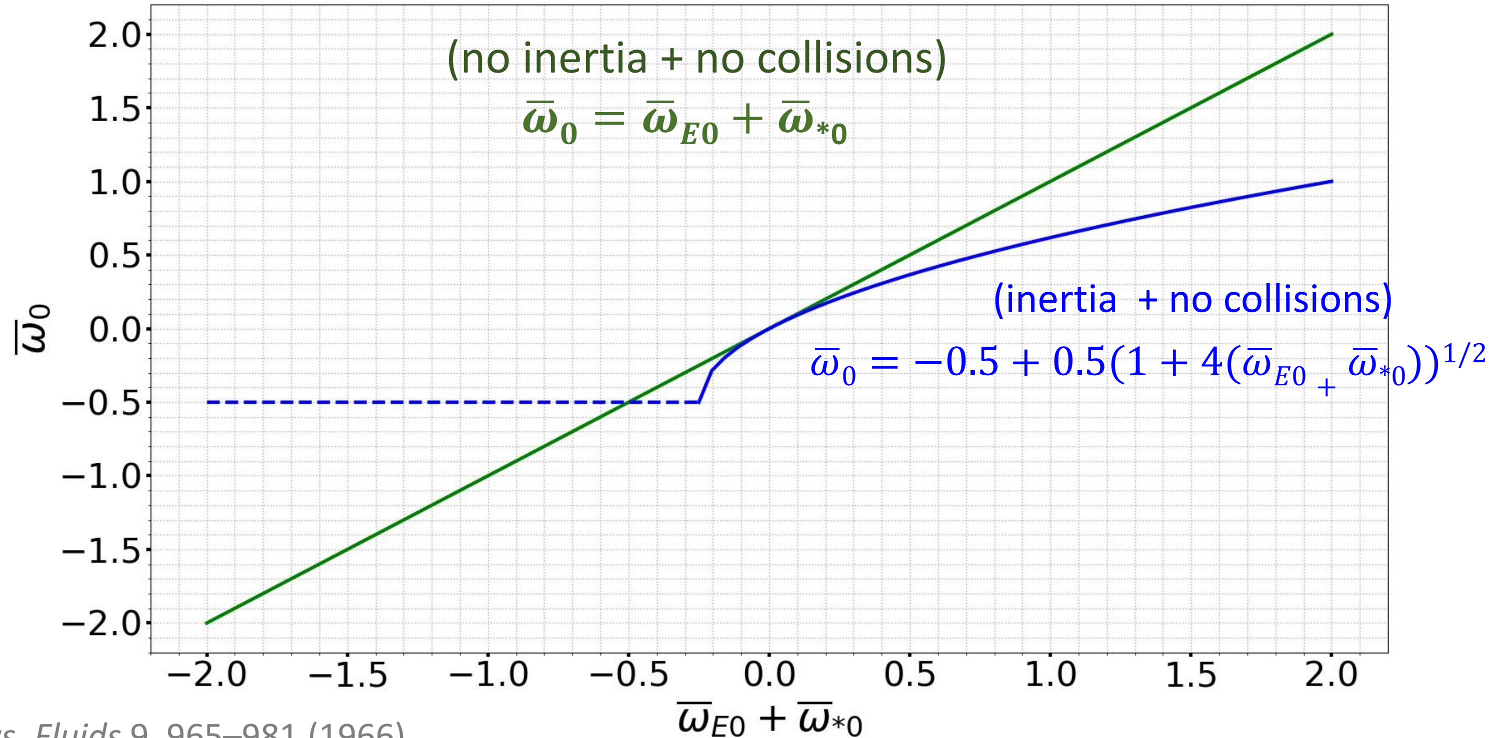


# Instability mechanism

$$\bar{\omega}_{ph}^3 - \frac{N}{m}\bar{\omega}_{ph}^2 - \left(C - C^2 + N\delta\bar{\omega}_0\right)\bar{\omega}_{ph} + mC\delta\bar{\omega}_0 = 0$$

$$\boxed{C \;=\; 1 + 2\bar{\omega}_0}$$

$$\boxed{\delta\bar{\omega}_0 = \bar{\omega}_0 - \bar{\omega}_{E0} - \bar{\omega}_{*0}}$$



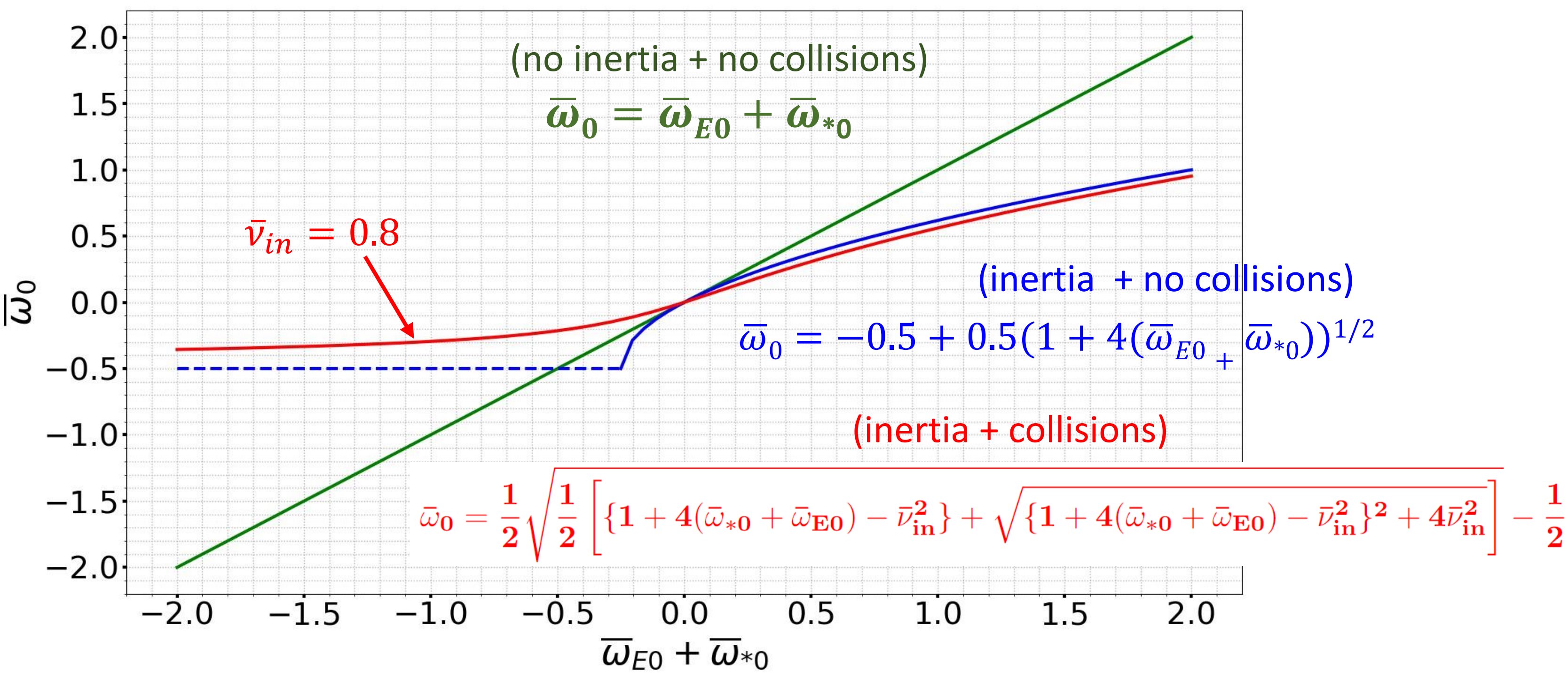

*F. F. Chen, Phys. Fluids 9, 965–981 (1966)*

# Instability mechanism

$$\bar{\omega}_{ph}^3 - \frac{N}{m}\bar{\omega}_{ph}^2 - \left(C - C^2 + N\delta\bar{\omega}_0\right)\bar{\omega}_{ph} + mC\delta\bar{\omega}_0 = 0$$

$$C = 1 + 2\bar{\omega}_0$$

$$\delta\bar{\omega}_0 = \bar{\omega}_0 - \bar{\omega}_{E0} - \bar{\omega}_{*0}$$

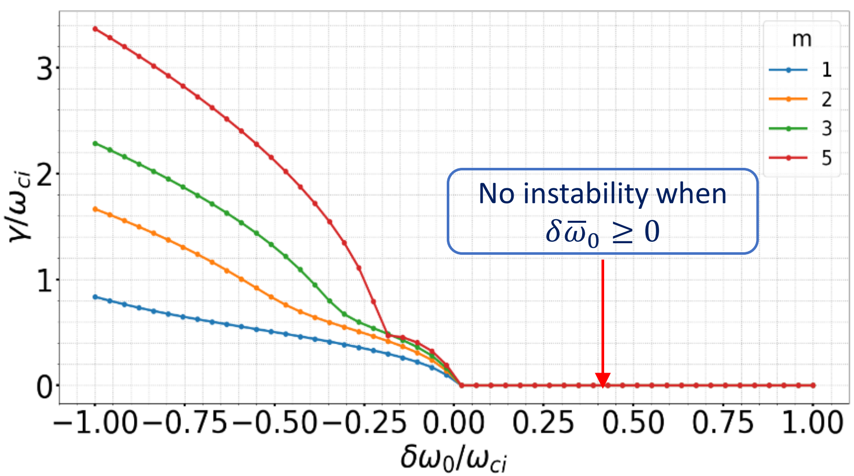


# Instability mechanism

$$\bar{\omega}_{ph}^3 - \frac{N}{m}\bar{\omega}_{ph}^2 - \left(\cancel{C} - \cancel{C^2} + N\delta\bar{\omega}_0\right)\bar{\omega}_{ph} + mC\delta\bar{\omega}_0 = 0$$

$$\boxed{\delta\bar{\omega}_0 = \bar{\omega}_0 - \bar{\omega}_{E0} - \bar{\omega}_{*0}}$$

Contribution from electron momentum equation



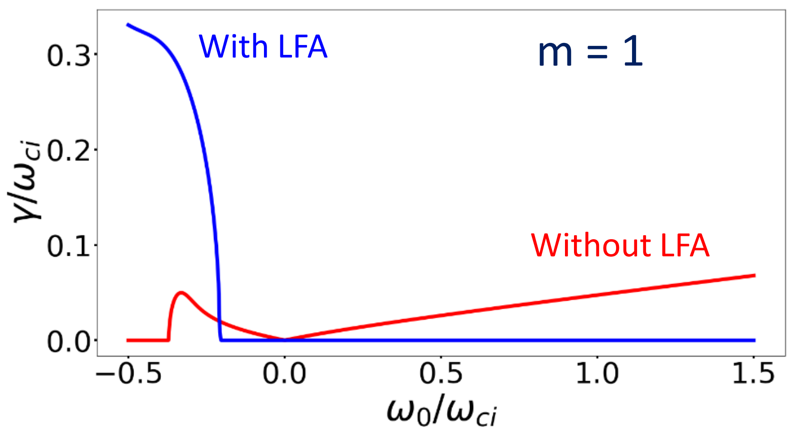

- Instability arises mainly from the difference between the ion and electron azimuthal flow.
- In absence of inertia or collisions, $\delta\bar{\omega}_0$ = 0 → no instability

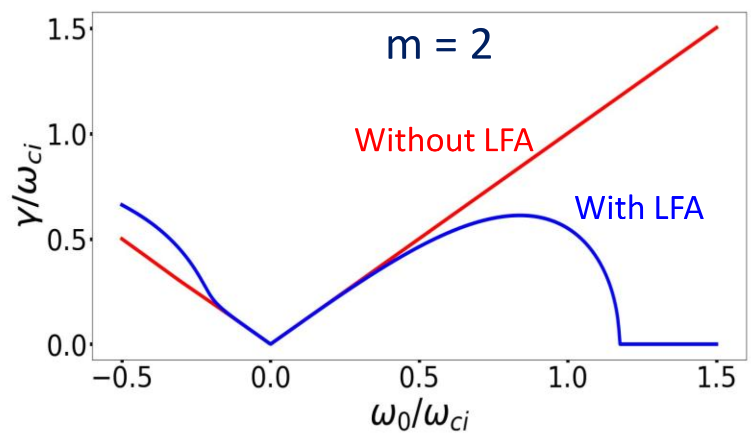
# Invalidity of LFA

Cubic term due to removal of low-frequency assumption

Dispersion relation without LFA → $$\bar{\omega}_{ph}^3 - \frac{N}{m}\bar{\omega}_{ph}^2 - \left(C - C^2 + N\delta\bar{\omega}_0\right)\bar{\omega}_{ph} + mC\delta\bar{\omega}_0 = 0$$

Dispersion relation with LFA → $$\frac{N}{m}\bar{\omega}_{ph}^2 + \left(C - C^2 + N\delta\bar{\omega}_0\right)\bar{\omega}_{ph} - mC\delta\bar{\omega}_0 = 0$$



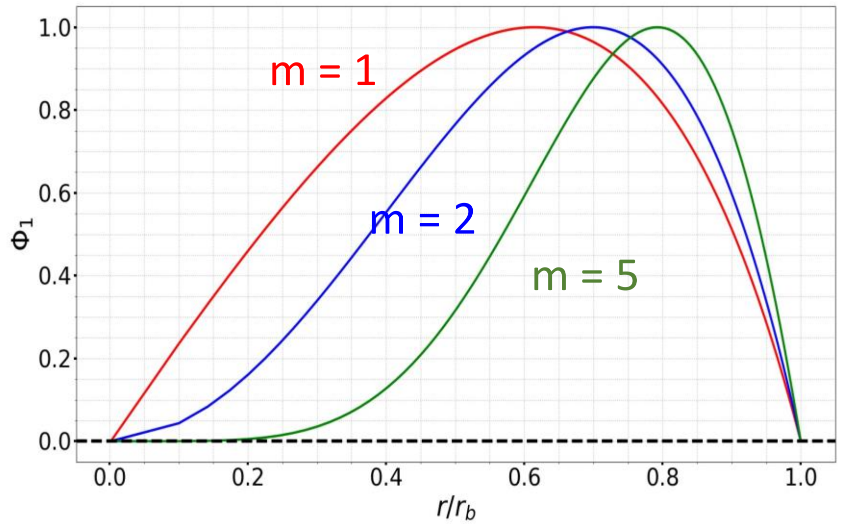




*S. Aggarwal et al., J. Plasma Phys.* (2023), *vol.* 89, 905890310

*F. F. Chen, Phys. Fluids* 9, 965–981 (1966)

# Radially global Eigen-modes

Eigen-function corresponding to n=0 for different azimuthal mode numbers m




$r_b$ = 10 cm → radial boundary or radius of cylinder

$$\Phi_1 = \bar{n}_1 + \tau\bar{\phi}_1$$

Normalized potential perturbation

Normalized density perturbation

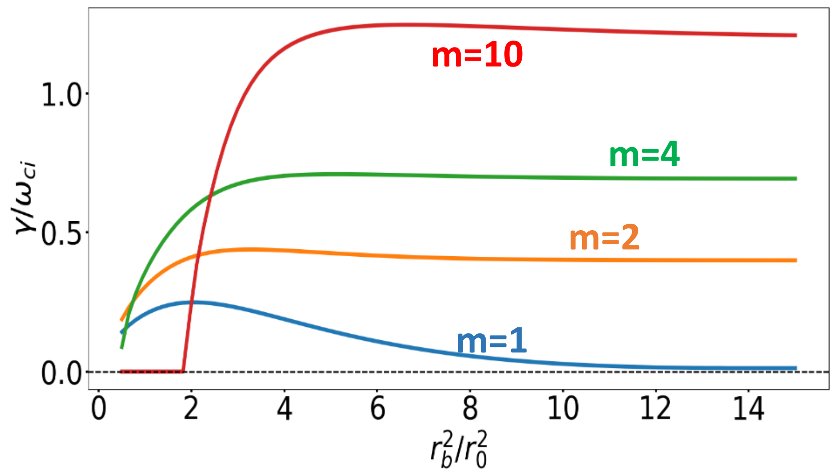
$$\tau = T_{e0}/T_{i0}$$

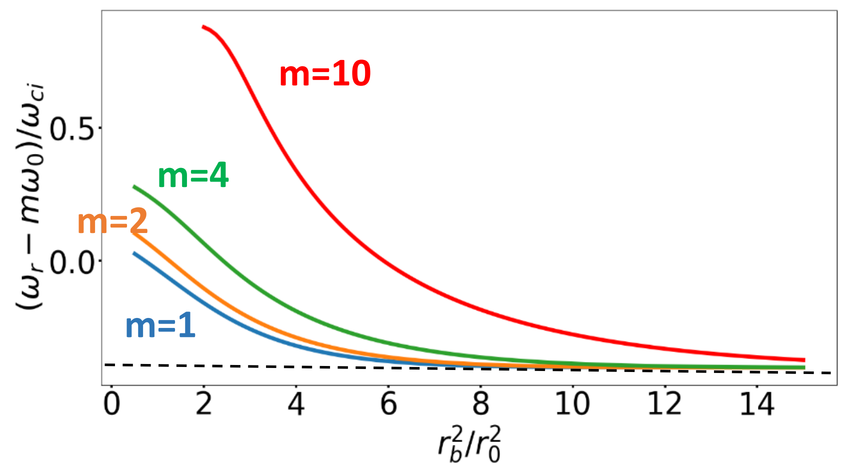
$$\boxed{\bar{n}_1 = \frac{m}{\bar{r}\bar{L}_n}\frac{1}{\bar{\omega} - m\bar{\omega}_{E0}}\tau\bar{\phi}_1}$$

➡ Mode localization towards outer radial boundary.

# Effect of radial boundary on growth rate and frequency

For $\omega_0/\omega_{ci} = 0.4$




- Analytical solution at large $r_b^2/r_0^2$ and n=0 → $\bar{\omega}_{ph} = -\bar{\omega}_0 + i\left(\sqrt{m-1}\right)|\bar{\omega}_0|$

- Deviation from the analytical solution at small $r_b^2/r_0^2$ → m=1 gets destabilized

*S. Aggarwal et al., J. Plasma Phys.* (2023), *vol.* 89, 905890310

# Local solution not applicable for MISTRAL plasma

*Dispersion relation :*

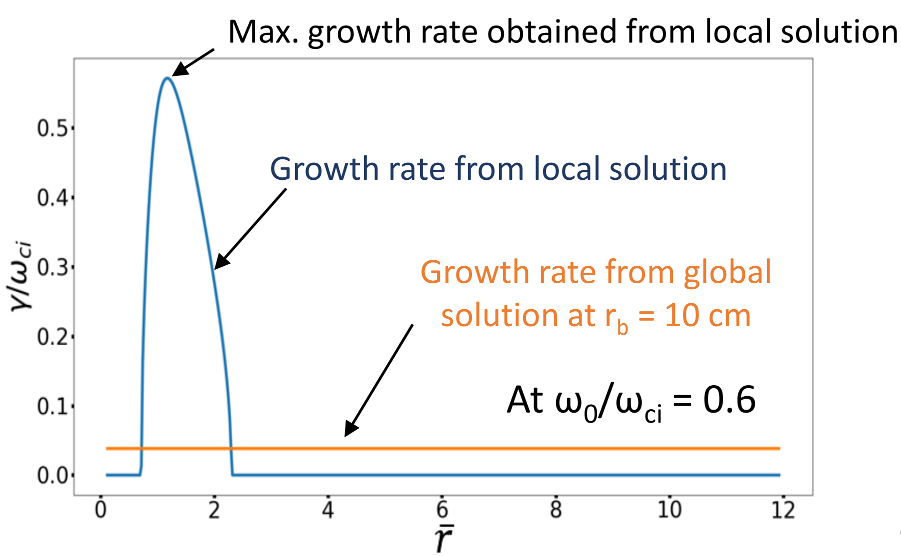
$$\cancel{\Phi_1''} + \left[\frac{1}{\bar{r}} - \frac{1}{\bar{L}_n}\right]\cancel{\Phi_1'} - \frac{m^2}{\bar{r}^2}\Phi_1 + \frac{1}{\bar{r}\bar{L}_n}N\Phi_1 = 0$$

$$N = m\left[\frac{C}{\bar{\omega}_{ph}} - \frac{C^2 - \bar{\omega}_{ph}^2}{\bar{\omega}_{ph} - m\bar{\omega}_0^2}\right]$$

*Background flow :*

$$\bar{\omega}_0^2 + \bar{\omega}_0 - (\bar{\omega}_{E0} + \bar{\omega}_{*0}) = 0$$

$$\begin{aligned} C &= 1 + 2\bar{\omega}_0 \\ \Phi_1 &= \bar{n}_1 + \tau\bar{\phi}_1 \\ \tau &= \frac{T_i}{T_e} \\ \bar{\omega}_{ph} &= \bar{\omega} - m\bar{\omega}_0 \\ \frac{1}{\bar{L}_n} &= -\frac{2\bar{r}}{\bar{r}_0^2} \end{aligned}$$




*Local* → *R. Gueroult et al., Phys. Plasmas* 24, 082102 (2017)

*Global* → *S. Aggarwal et al., J. Plasma Phys.* 89, 905890310 (2023)

# Dispersion relation with collisionality and inertia in the limit ∈ =0

$$\cancel{\epsilon\Phi_1'''} + \left[\frac{C_{NC}}{\bar{r}} + \epsilon\left(\frac{3}{\bar{r}} - \frac{1}{\bar{L}_n}\right)\right]\Phi_1'' + \left[\frac{C_{NC}}{\bar{r}}\left(\frac{1}{\bar{r}} - \frac{1}{\bar{L}_n}\right) + \cancel{\epsilon}\left(-\frac{m^2}{\bar{r}^2} + \frac{1}{\bar{r}}\left(\frac{1}{\bar{r}} - \frac{1}{\bar{L}_n}\right) + \mathcal{A} - \mathcal{B}\right)\right]\Phi_1'$$

$$+ \left[\frac{C_{NC}}{\bar{r}}\left(-\frac{m^2}{\bar{r}^2} + \frac{N}{\bar{r}\bar{L}_n}\right) - \cancel{\epsilon\mathcal{B}}\left(\frac{2}{\bar{r}} - \frac{1}{\bar{L}_n}\right)\right]\Phi_1 = 0 \quad (42)$$

$$C_{NC} = i\bar{\omega}_c\left(\frac{C^2 - \bar{\omega}_c^2}{C^2 + \bar{\omega}_c^2}\right) \qquad \bar{\omega}_c = \bar{\omega}_{ph} + i\bar{\nu}_{in}$$

eigenvalue obtained using Whittaker equation

$$\delta\bar{\omega}_0 = \bar{\omega}_0 - \bar{\omega}_{E0} - \bar{\omega}_{*0}$$

$$\bar{\omega}_{ph}^3 - \left(\frac{N}{m} - 2i\bar{\nu}_{in}\right)\bar{\omega}_{ph}^2 - \left(C - C^2 + \frac{N}{m}(m\delta\bar{\omega}_0 + i\bar{\nu}_{in}) + \bar{\nu}_{in}^2\right)\bar{\omega}_{ph} + (mC - iN\bar{\nu}_{in})\,\delta\bar{\omega}_0 = 0$$

Azimuthal mode number, m=1,2,3,….

$$C = 1 + 2\bar{\omega}_0$$

Contribution due to magnetic force and inertial effects in the ion-momentum equation

Due to inertial effects only

Reminder ! $$\bar{\omega}_0 = \frac{1}{2}\sqrt{\frac{1}{2}\left[\{1 + 4(\bar{\omega}_{*0} + \bar{\omega}_{E0}) - \bar{\nu}_{in}^2\} + \sqrt{\{1 + 4(\bar{\omega}_{*0} + \bar{\omega}_{E0}) - \bar{\nu}_{in}^2\}^2 + 4\bar{\nu}_{in}^2}\right]} - \frac{1}{2}$$

# Dual effect of collisions (m=1)

C=1

$$\bar{\omega}_{ph}^3 - \left(\frac{N}{m} - 2i\bar{\nu}_{in}\right)\bar{\omega}_{ph}^2 - \left(C - C^2 + \frac{N}{m}(m\delta\bar{\omega}_0 + i\bar{\nu}_{in}) + \bar{\nu}_{in}^2\right)\bar{\omega}_{ph} + (mC - iN\bar{\nu}_{in})\,\delta\bar{\omega}_0 = 0$$

C=1

$$\boxed{\delta\bar{\omega}_0 = \bar{\omega}_0 - \bar{\omega}_{E0} - \bar{\omega}_{*0}}$$

$$\bar{\omega}_0 = \frac{1}{2}\sqrt{\frac{1}{2}\left[\{1 + 4(\bar{\omega}_{*0} + \bar{\omega}_{E0}) - \bar{\nu}_{in_0}^2\} + \sqrt{\{1 + 4(\bar{\omega}_{*0} + \bar{\omega}_{E0}) - \bar{\nu}_{in_0}^2\}^2 + 4\bar{\nu}_{in_0}^2}\right]} - \frac{1}{2}$$

# Dual effect of collisions (m=1)

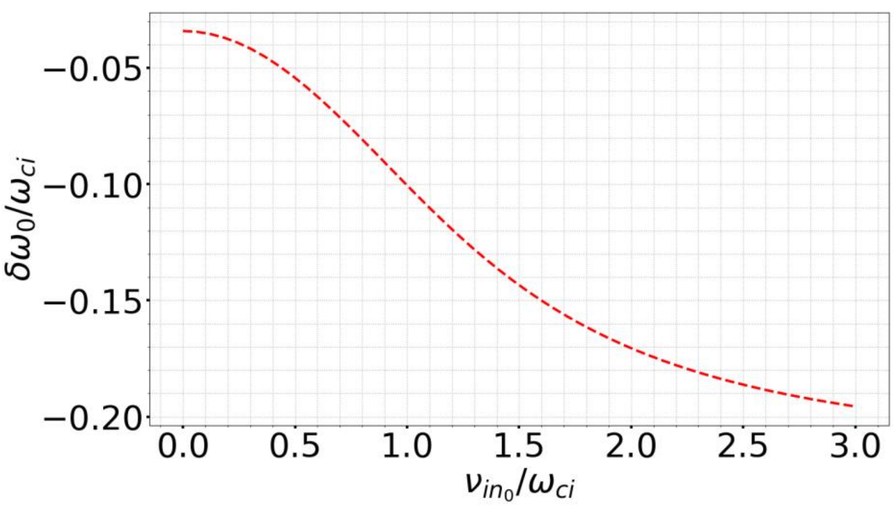
C=1

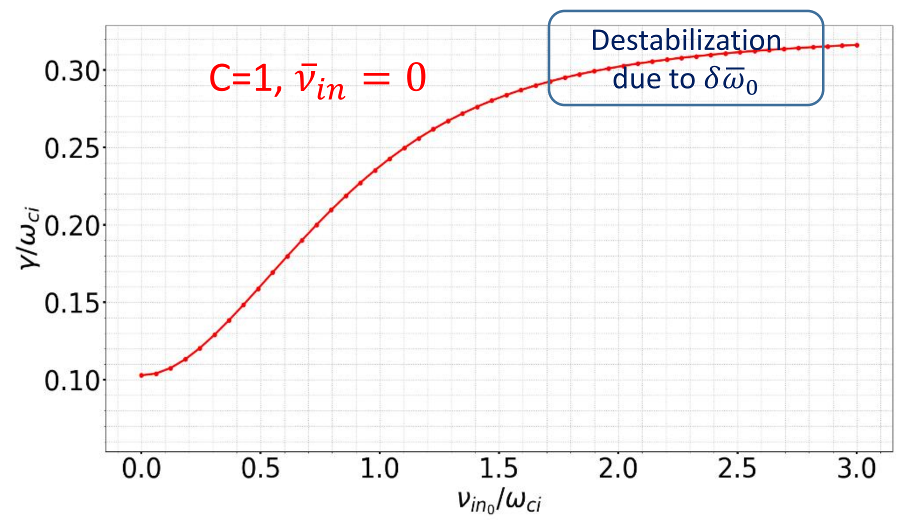
$$\bar{\omega}_{ph}^3 - \left(\frac{N}{m} - 2i\cancel{\bar{\nu}_{in}}\right)\bar{\omega}_{ph}^2 - \left(C - C^2 + \frac{N}{m}(m\delta\bar{\omega}_0 + i\cancel{\bar{\nu}_{in}}) + \cancel{\bar{\nu}_{in}^2}\right)\bar{\omega}_{ph} + (mC - iN\cancel{\bar{\nu}_{in}})\,\delta\bar{\omega}_0 = 0$$

C=1

$$\boxed{\delta\bar{\omega}_0 = \bar{\omega}_0 - \bar{\omega}_{E0} - \bar{\omega}_{*0}}$$

$$\bar{\omega}_0 = \frac{1}{2}\sqrt{\frac{1}{2}\left[\{1 + 4(\bar{\omega}_{*0} + \bar{\omega}_{E0}) - \bar{\nu}_{in_0}^2\} + \sqrt{\{1 + 4(\bar{\omega}_{*0} + \bar{\omega}_{E0}) - \bar{\nu}_{in_0}^2\}^2 + 4\bar{\nu}_{in_0}^2}\right]} - \frac{1}{2}$$




- Parameters used: $r_b = 10\rho_i$, $\bar{\omega}_{E0} = 0.4$, $\bar{\omega}_{*0} = -0.18$, $r_0 = 3\rho_i$

# Dual effect of collisions (m=1)

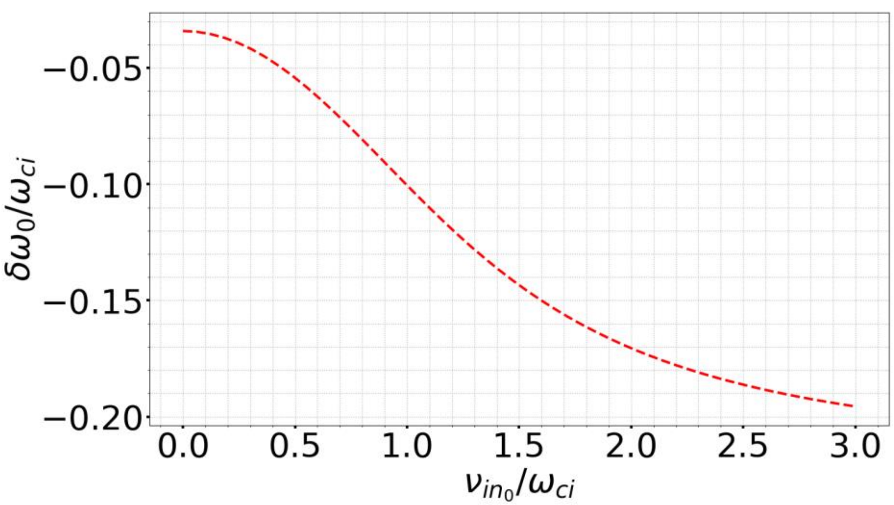
C=1

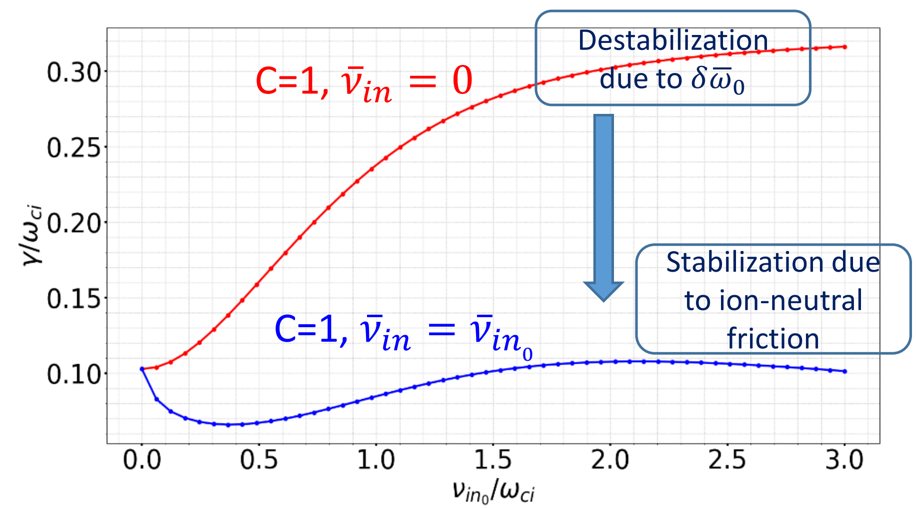
$$\bar{\omega}_{ph}^3 - \left(\frac{N}{m} - 2i\bar{\nu}_{in}\right)\bar{\omega}_{ph}^2 - \left(C - C^2 + \frac{N}{m}(m\delta\bar{\omega}_0 + i\bar{\nu}_{in}) + \bar{\nu}_{in}^2\right)\bar{\omega}_{ph} + (mC - iN\bar{\nu}_{in})\,\delta\bar{\omega}_0 = 0$$

C=1

$$\delta\bar{\omega}_0 = \bar{\omega}_0 - \bar{\omega}_{E0} - \bar{\omega}_{*0}$$

$$\bar{\omega}_0 = \frac{1}{2}\sqrt{\frac{1}{2}\left[\{1 + 4(\bar{\omega}_{*0} + \bar{\omega}_{E0}) - \bar{\nu}_{in_0}^2\} + \sqrt{\{1 + 4(\bar{\omega}_{*0} + \bar{\omega}_{E0}) - \bar{\nu}_{in_0}^2\}^2 + 4\bar{\nu}_{in_0}^2}\right]} - \frac{1}{2}$$

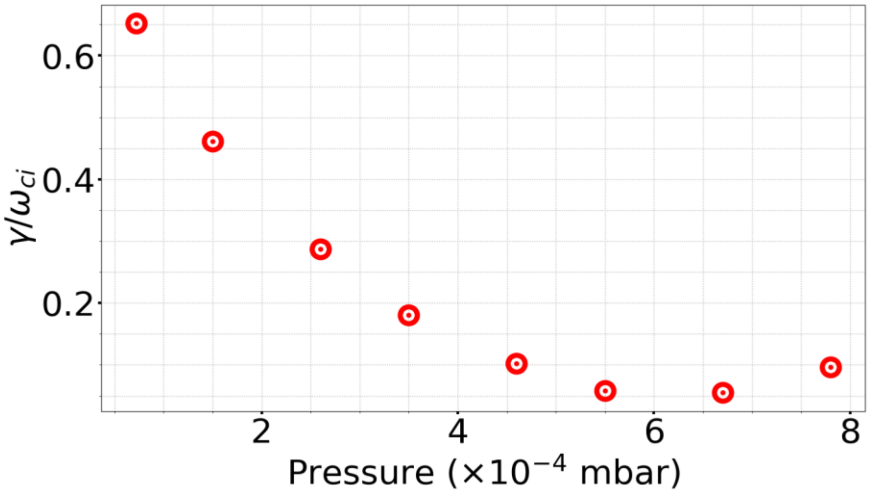


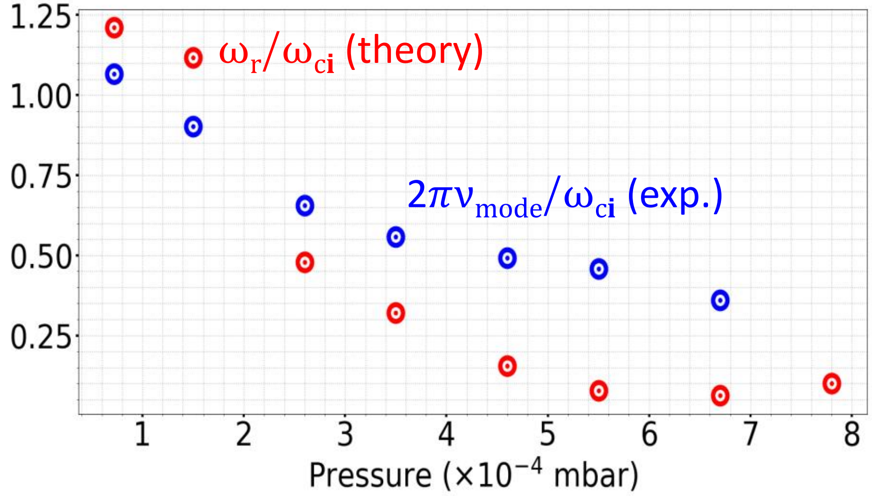

- Parameters used: $r_b = 10\rho_i$, $\bar{\omega}_{E0} = 0.4$, $\bar{\omega}_{*0} = -0.18$, $r_0 = 3\rho_i$

# Comparison of experimental data and theoretical predictions (m=1)

Normalized Growth rate




Normalized mode frequency




➡ Qualitative agreement found between the theoretical description and experimental data.

# Summary & conclusions

- ❑ Two – fluid model developed for MISTRAL plasma:
  - Without low-frequency approximation → Valid at arbitrary frequency values
  - Radially global model
  - Linear analysis in the limit $\in \rightarrow 0$

- ❑ Instability can be driven by:
  - Inertia
  - Ion – neutral collisions
  - Or combination of both

- ❑ For centrifugal instability, the mode's growth rate and frequency are significantly affected by the radial boundary position and background flow.

- ❑ Dual effect of ion-neutral collisions:
  - Destabilization as $|\delta\overline{\omega}_0| \uparrow$ → contribution due to $\overline{\omega}_0$
  - Stabilization due to ion-neutral friction → contribution due to perturbed ion velocity

# Work in progress & Future perspectives

- Linear analysis with finite ∈ and ionization source → work in progress

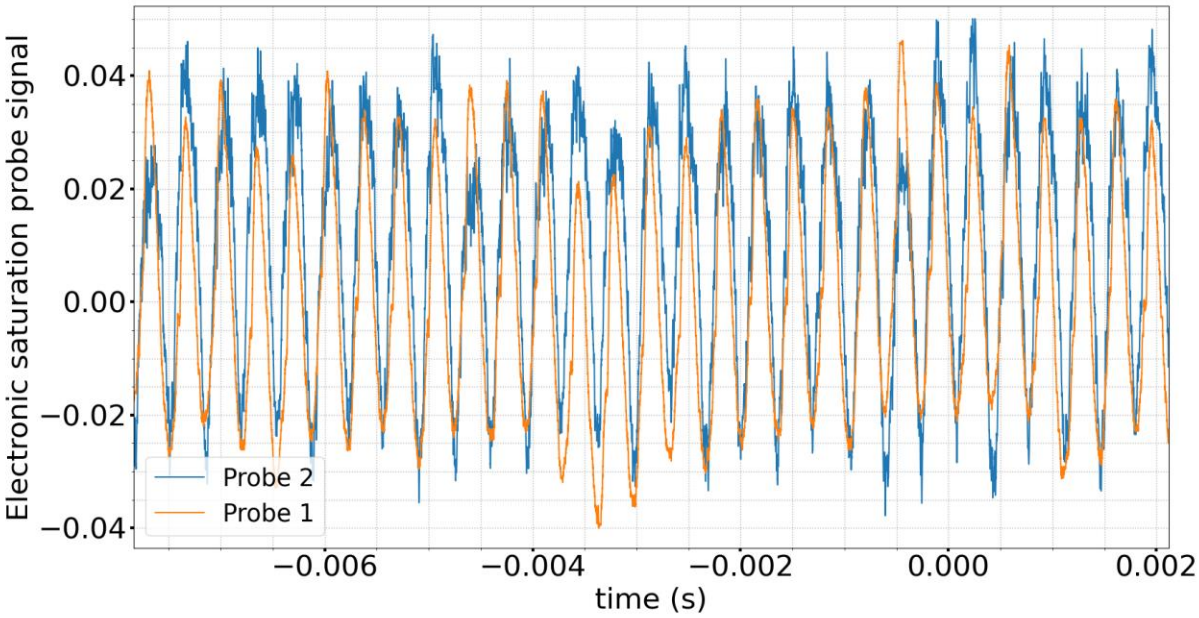
- Accounting for FLR effects is essential for achieving the stabilization of higher mode numbers.

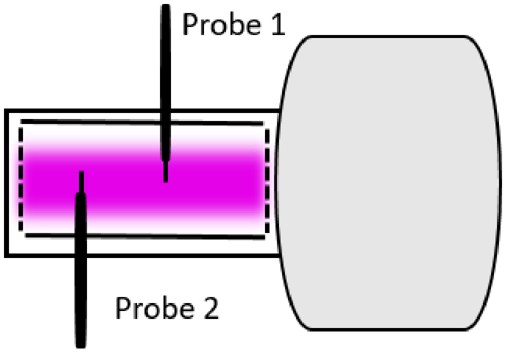
- Performing a comprehensive linear analysis with arbitrary radial equilibrium flow i.e. without $\epsilon \ll 1$.

- Non – linear simulations.

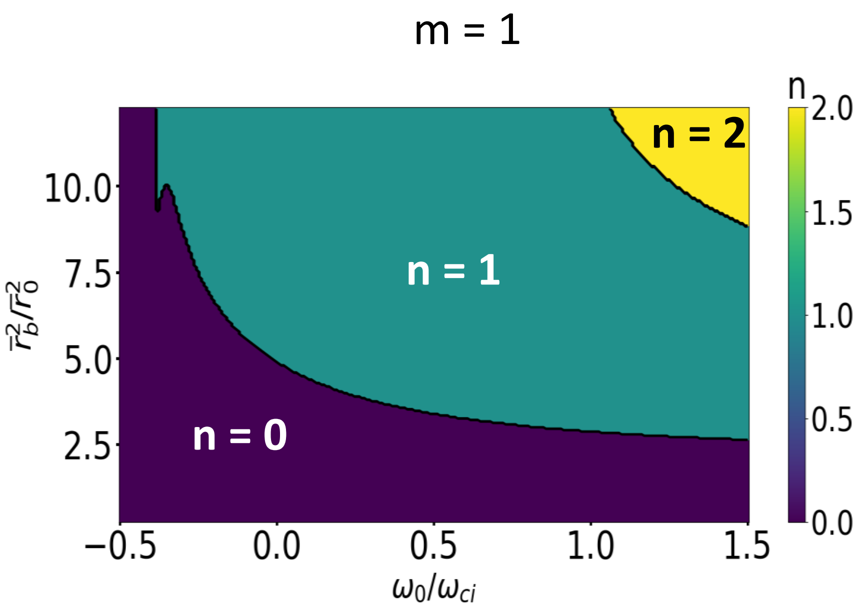
Thank you for your attention……

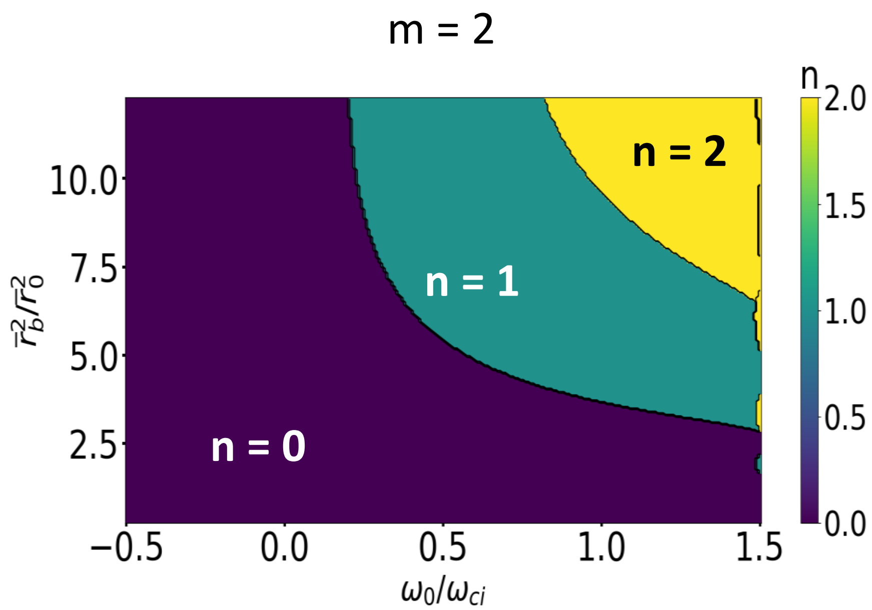
# $k_{||}$ measurement



# Maximum growth rate for radial mode number n

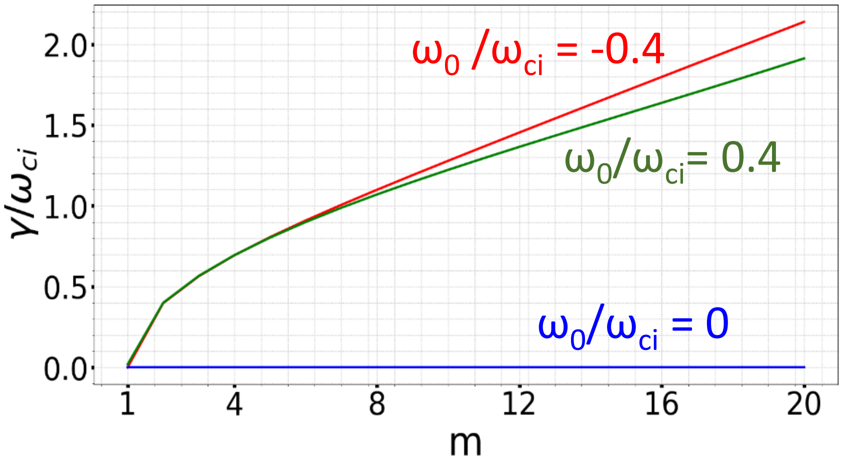


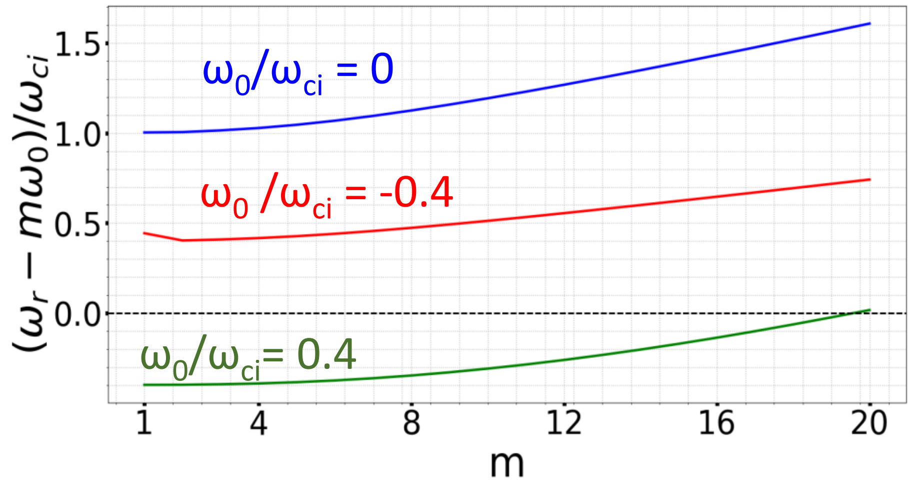




➡ n = 0 doesn't necessarily yield the largest growth rate.

# Azimuthal mode spectra for growth rate and frequency

At $r_b^2/r_0^2 \approx 11$ and n=0



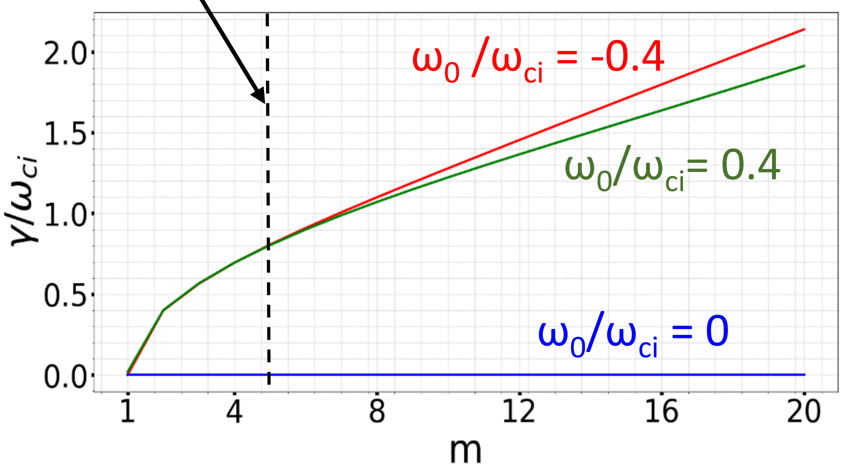

- Growth rate increases as $\left(\sqrt{m-1}\right)|\bar{\omega}_0|$
- Perturbed Doppler shifted frequency opposes the direction of equilibrium flow frequency → consistent with the analytical result.

*S. Aggarwal et al., J. Plasma Phys.* (2023), *vol.* 89, 905890310

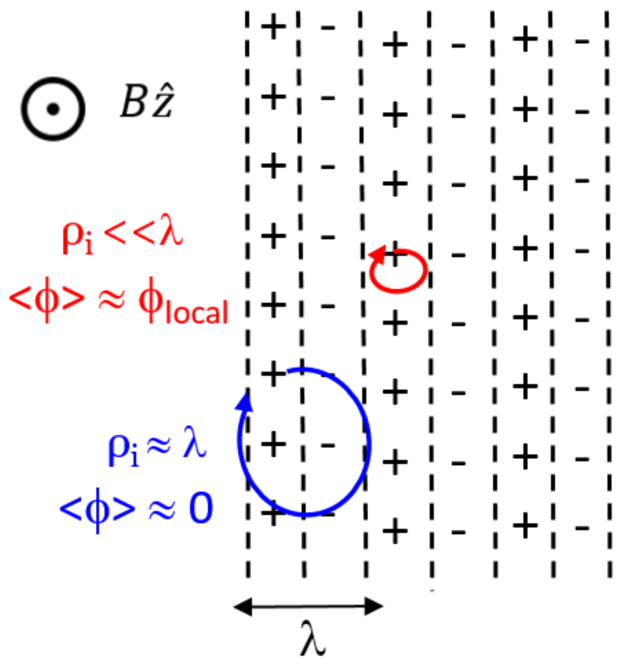
# No stabilization observed at large m numbers

$k_\theta \rho_i \approx 1 \rightarrow m \approx 5$

At $r_b^2/r_0^2 \approx 11$ and n=0




- No stabilization at high m numbers due to absence of Finite Larmor radius effects ($k_\theta \rho_i \approx 1$).

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