PPPL Seminar
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RADIATIVE DAMPING OF TAE IN LOW-SHEAR PLASMAS

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Outline

- **What physics we study**

    Non-ideal factors (FLR and finite electron conductivity) couple TAE to KAW and cause TAE damping via radiation of KAWs

- **What is new**

    Real-space analytic calculation of radiative damping

- **What are the benefits**

    More simple and transparent formalism than the pre-existing Fourier-space approach

    Explicit use of mode symmetry

    More accurate numerical factor in the result

Earlier Work

- J.W. Connor et al., *Non-ideal Effects on TAE Stability*, in: Proceed. of the 21st EPS Conference on Control. Fusion and Plasma Phys., Montpellier, 27 June 1994, edited by E. Joffrin, P. Platz, P.E. Stott, Vol.18B, p. 616.
- R. Nyqvist and S.E. Sharapov, *Asymmetric radiative damping of low shear toroidal Alfvén Eigenmodes*, Physics of Plasmas **19**, 082517 (2012).
- H. L. Berk, R. R. Mett, D. M. Lindberg, *Arbitrary mode number boundary-layer theory for nonideal toroidal Alfvén* modes Phys. Fluids **B 5**, 3969, (1993)
- G.Y. Fu, C.Z. Cheg, R. Budny et al., *Analysis of alpha particle-driven toroidal Alfven eigemodes in Tokamak Fusion Test Reactor deuterium-tritium experiments*, Phys. Plasmas **3,** 4036 (1996)

Basic Equations

- Shear Alfvén Wave in a cylinder:

$$L_m\varphi_m \equiv \frac{d}{dr}\left(\frac{\omega^2}{V_A^2}-k_{\|m}^2\right)\frac{d\varphi_m}{dr}-\frac{m^2}{r^2}\left(\frac{\omega^2}{V_A^2}-k_{\|m}^2\right)\varphi_m=0$$

- Coupled equations for two poloidal harmonics of the wave field in a torus:

$$\rho^2\frac{d^4}{dr^4}\varphi_m+L_m\varphi_m+\frac{\hat{\varepsilon}}{4q^2R^2}\frac{d^2}{dr^2}\varphi_{m-1}=0$$

$$\rho^2\frac{d^4}{dr^4}\varphi_{m-1}+L_{m-1}\varphi_{m-1}+\frac{\hat{\varepsilon}}{4q^2R^2}\frac{d^2}{dr^2}\varphi_m=0$$

Coupling terms
Nonideal terms

Ideal TAE

- Local dispersion relation for Shear Alfvén Wave describes Alfvén Continuum:

$$\omega = \left| k_{\parallel m} V_A \right| = \frac{V_A}{R} \left| n - \frac{m}{q(r)} \right|$$

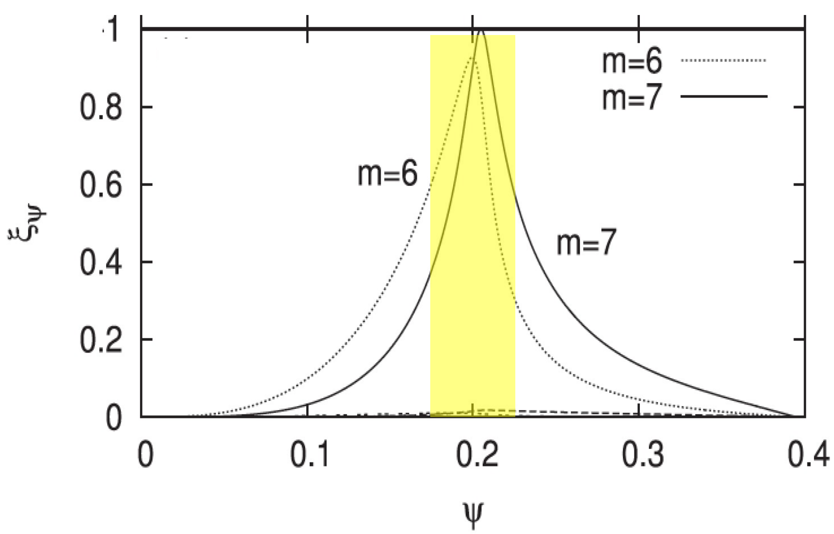
- Toroidicity couples two poloidal harmonics (m and $m-1$) and produces a gap in the Alfvén Continuum at

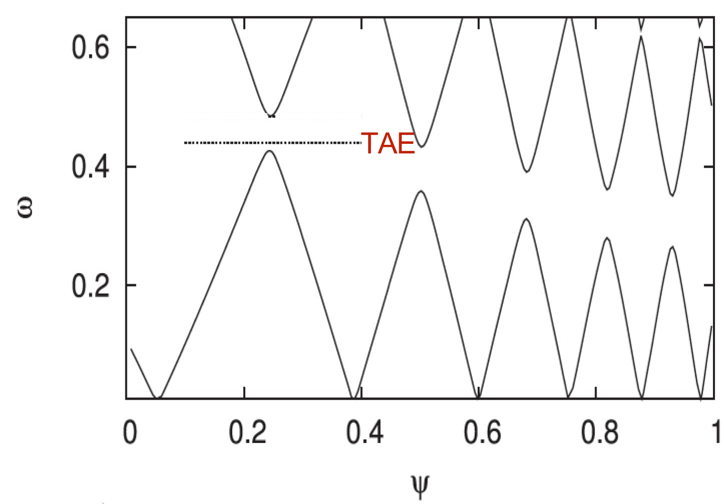
$$q(r_m) = \frac{2m-1}{2n}$$

- At low shear, the TAE frequency is near the lower tip of the Alfvén Continuum gap

Characteristic Scales of Ideal TAE at Low Shear

- Inner structure of TAE: $\Delta^{in} = \frac{\varepsilon r_m}{m}$
- Outer structure of TAE: $\Delta^{out} = \frac{r_m}{m} \gg \Delta^{in}$
- Distance between neighboring gaps : $\left| r_m - r_{m+1} \right| = \frac{r_m}{mS} \gg \Delta^{out}$





Ideal Inner Layer

- Dimensionless variables:

$$z = 4n\left[q(r) - q_m\right]/\hat{\varepsilon}$$

$$U \equiv \frac{\partial \varphi_m}{\partial z};\ V \equiv \frac{\partial \varphi_{m-1}}{\partial z}$$

$$g \equiv \left(\omega^2 - \omega_0^2\right)/\hat{\varepsilon}\omega_0^2;\ \omega_0\left(r_m\right) = \frac{V_A}{2qR};\ \hat{\varepsilon} = \frac{5}{2}\frac{r_m}{R}$$

- Inner layer equations:

$$\left(g+z\right)U + V = C_m$$

$$\left(g-z\right)V + U = -C_{m-1}$$

C_m and C_{m-1} are integration constants

- Matching condition to the outer solution:

$$\int\left(U+V\right)dz = \left(C_m - C_{m-1}\right)\pi^2 S/4$$

$$\int\left(U-V\right)dz = \left(C_m + C_{m-1}\right)\pi^2 S/4$$

Symmetry of the Ideal TAE at the Gap Bottom

- $C_m = C_{m-1}$

- Mode structure:

$$U_0 + V_0 = \frac{2C_m z}{z^2 + 1 - g^2};\ U_0 - V_0 = -\frac{2C_m\left(g+1\right)}{z^2 + 1 - g^2}$$

$$g = -1 + \pi^2 S^2 / 8$$

- $U_0 + V_0$ is an odd function of z

- $U_0 - V_0$ is an even function of z

- $\left|U_0 + V_0\right| \gg \left|U_0 - V_0\right|$

Nonideal Inner layer

- Nonideal inner layer preserves the ideal layer symmetry

- Inner layer equations:

$$\lambda^2 (U+V)'' + (g+1)(U+V)V + z(U-V) = 0$$

$$\lambda^2 (U-V)'' + (g-1)(U-V) + z(U+V) = 2C_m$$

$$\lambda^2 \equiv (4/\hat{\varepsilon})^3 \frac{m^2 S^2}{4 r_m^2} \frac{T_i}{m_i \omega_{Bi}^2} \left[\frac{3}{4} + \frac{T_e}{T_i} \right] \ll 1$$

- Matching condition to the outer solution:

$$\int (U+V)\,dz = 0$$

$$\int (U-V)\,dz = C_m \pi^2 S / 2$$

$$|U+V| \gg |U-V|$$

Reduction of the Inner Layer Equations

- Short–wavelength KAW radiates from the inner layer
- Represent the total field as a sum of TAE and small perturbation (R):

$$U + V = U_0 + V_0 + R_+$$

$$U - V = U_0 - V_0 + R_- \qquad \boxed{|R_-| \ll |R_+|}$$

$$\lambda^2 R_+'' + (g+1)R_+ + zR_- + z\cancel{(U_0 - V_0)} + (g+1)\cancel{(U_0 + V_0)} = -\lambda^2 (U_0 + V_0)''$$

$$\lambda^2 R_-'' + (g-1)R_- + (g-1)\cancel{(U_0 - V_0)} + zR_+ + z\cancel{(U_0 + V_0)} = -\lambda^2 (U_0 - V_0)'' + \cancel{2C_m}$$

- Cancel the zeroth-order terms and express small R_- through R_+:

$$\boxed{\lambda^2 R_+'' + (g+1)R_+ + \frac{z^2}{1-g}R_+ = -\lambda^2 (U_0 + V_0)''}$$

WKB Formalism for KAW Radiation

- The TAE structure presents a weak source for the KAW:

$$\lambda^2 R_+'' + (g+1) R_+ + \frac{z^2}{1-g} R_+ = -\lambda^2 (U_0 + V_0)''$$

- The KAW field is tractable within the WKB-approximation:

$$R_+ = -\frac{1}{(\psi_+'\psi_- - \psi_-'\psi_+)} \left(\psi_+ \int_{-\infty}^{z} (U_0+V_0)'' \, \psi_- \, dz - \psi_- \int_{\infty}^{z} (U_0+V_0)'' \, \psi_+ \, dz \right)$$

$$\psi_\pm = \frac{1}{\sqrt{k}} exp(\pm i\Phi)$$

$$\Phi \equiv \int_0^z k \, dx$$

$$k \equiv \frac{1}{\lambda} \sqrt{g + 1 + z^2/(1-g)}$$

Damping Rate Evaluation

- Inner-outer matching condition:

$$\int_{-\infty}^{\infty}\left[2C_m - z\left(U_0+V_0\right) - zR_+\right]dz = \pi^2 S C_m$$

- Dispersion relation with radiation included:

$$g+1=\frac{\pi^2 S^2}{8}+\frac{S}{4C_m}\,\mathrm{Im}\int_{-\infty}^{\infty} zR_+\,dz$$

$$\left(\omega^2-\omega_0^2\right)/\hat{\varepsilon}\omega_0^2+1=\frac{\pi^2 S^2}{8}\left\{1-i\frac{a}{\pi}\left[\int_{-\infty}^{\infty}\frac{\sin\left(a\int_0^x\sqrt{1+y^2}\,dy\right)}{\left(1+x^2\right)^{1/4}}x\,dx\right]^2\right\}$$

$$a\equiv\frac{\pi^2 S^2}{4\sqrt{2}\lambda}$$

- Exponentially small damping rate at $a\equiv\dfrac{\pi^2 S^2}{4\sqrt{2}\lambda}>>1$:

$$\boxed{\frac{\gamma}{\omega_0}=-\frac{\pi^2 S^2\hat{\varepsilon}}{12}\exp\left[-\frac{\pi a}{2}\right]}$$

Previous Approach

- Transform the inner layer equations to Fourier space without splitting the fields of ideal TAE and KAW to obtain

$$-\lambda^2 k^2 V_k + gV_k - i\frac{dV_k}{dk} + U_k = -C_m\delta(k)$$

$$-\lambda^2 k^2 U_k + gU_k + i\frac{dU_k}{dk} + V_k = C_m\delta(k)$$

- Reduce this set to a second-order ODE for U_k

- Use WKB anzatz to solve the second-order ODE

- Satisfy the challenging boundary condition at $k = 0$, where the WKB approximation is problematic

Abstract

Instabilities of Alfvén eigenmodes (AEs) are of significant concern because they can enhance the cross-field transport of alpha particles beyond the neoclassical level in fusion plasmas. The threshold value of alpha-particle pressure for exciting AEs depends critically on the damping rate of AEs. The damping mechanisms include kinetic damping due to interactions with thermal particles, continuum damping due to Alfvén continuum crossing, and radiative damping due to emitting kinetic Alfvén waves (KAWs). The radiative damping is substantial and can even prevail in high-temperature burning plasmas. We revisit the radiative damping theory for TAEs in low-shear plasmas. In contrast to earlier papers, we provide the damping calculations in real space rather than Fourier space. This approach is straightforward technically and more enlightening from a physics standpoint.