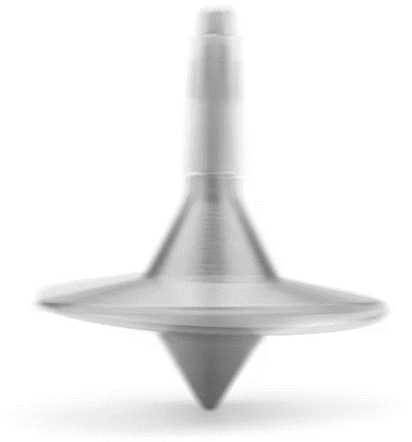
Parity-Broken Fluids: Theory and Applications



Dylan Reynolds

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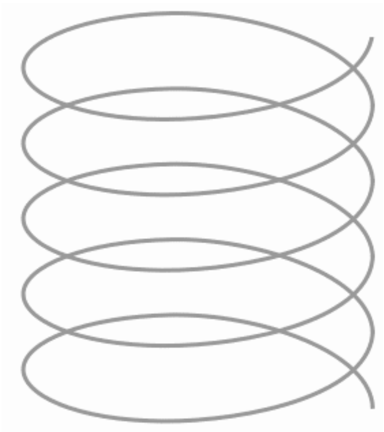
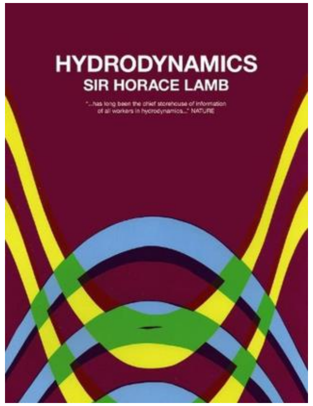
Oct. 5th, 2023

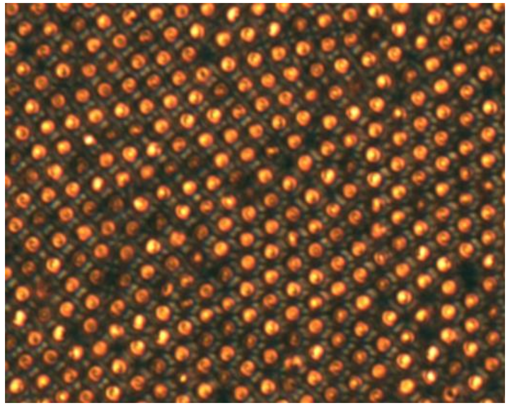
Princeton Plasma Physics Lab

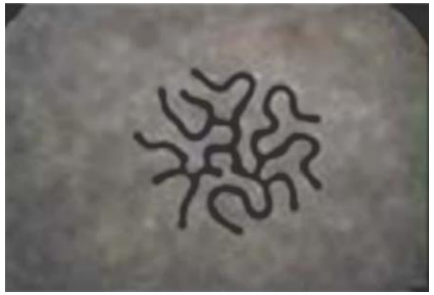


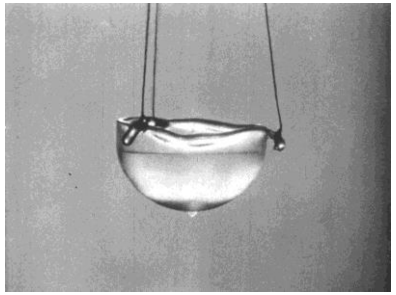
Outline

- Motivation
- Hydrodynamics
- Odd (gyro) viscosity
- Ferrofluids
- Hele-Shaw Cell
- Microscopic Origins
- Superfluids









Goal: *Parity-breaking (odd viscosity) is common to many systems and leads to a general type of observable phenomena*

Our Research Group

Sriram Ganeshan (CCNY, CUNY)

Gustavo Monteiro (CSI, CUNY)

Also: Sudheesh Srivastava, Chunyin Chan, Zeshui Song

Research Interests:

- Hydrodynamics of parity broken flows
- Active matter and odd viscosity
- Boundary dynamics of fractional quantum Hall fluids
- Kuramoto oscillators and machine learning

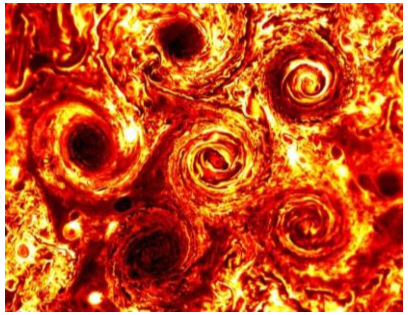
Nonlinear shallow water dynamics with odd viscosity
Gustavo M. Monteiro and Sriram Ganeshan
Phys. Rev. Fluids **6**, L092401 – Published 7 September 2021

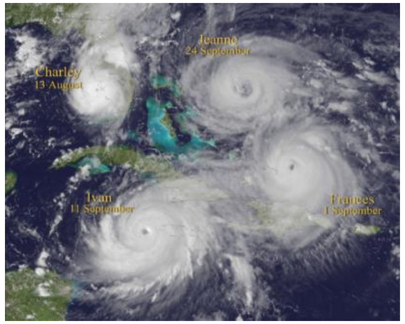
Hele-Shaw flow for parity odd three-dimensional fluids
Dylan Reynolds, Gustavo M. Monteiro, and Sriram Ganeshan
Phys. Rev. Fluids **7**, 114201 – Published 16 November 2022

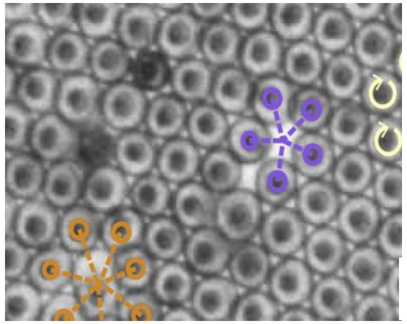
Kardar-Parisi-Zhang Universality at the Edge of Laughlin States.
Monteiro, G. M., Reynolds, D., Glorioso, P., & Ganeshan, S.
preprint arXiv:2305.16511 May 2023

Parity Breaking "spinny particles"

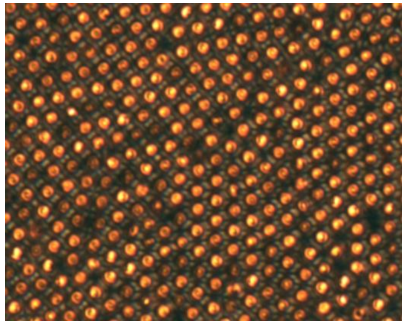
Ubiquitous across all scales…







[1]



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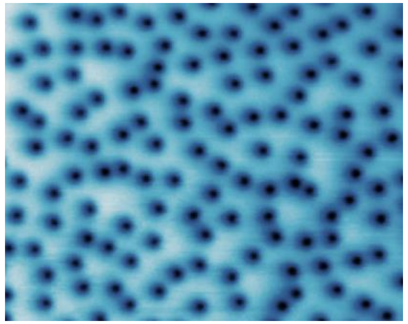
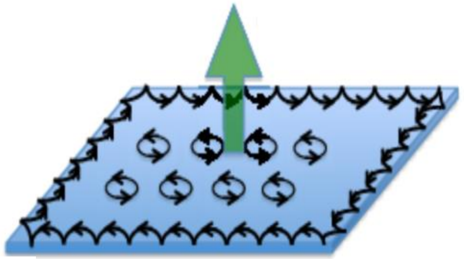


[3]



[4]

Some possible methods of study:

Odd ideal gasses – collisions of chiral particles, kinetic theory [5]

Odd elasticity – stress/strains in chiral crystals [6]

Odd fluids - odd viscosity [7]

[1] Tan, Tzer Han & Mietke, Alexander & Higinbotham, Hugh & Li, Junang & Chen, Yuchao & Foster, Peter & Gokhale, Shreyas & Dunkel, Jörn & Fakhri, Nikta. (2021). Odd dynamics of living chiral crystals. 10.48550/arXiv.2105.07507.
[2] Soni, V., Bililign, E.S., Magkiriadou, S. *et al.* The odd free surface flows of a colloidal chiral fluid. *Nat. Phys.* 15**,** 1188–194 (2019).
[3] Wells, F., Pan, A., Wang, X. *et al.* Analysis of low-field isotropic vortex glass containing vortex groups in $YBa_2Cu_3O_{7-x}$ thin films visualized by scanning SQUID microscopy. *Sci Rep* **5**, 8677 (2015).
[4] Weng, H., Dai, X., & Fang, Z. (2015). From Anomalous Hall Effect to the Quantum Anomalous Hall Effect. *arXiv: Materials Science*.
[5] Fruchart, M., Han, M., Scheibner, C., & Vitelli, V. (2022). The odd ideal gas: Hall viscosity and thermal conductivity from non-Hermitian kinetic theory. *arXiv preprint arXiv:2202.02037*.
[6] Scheibner, C., Souslov, A., Banerjee, D. *et al.* Odd elasticity. *Nat. Phys.* 16, 475–480 (2020).
[7] Khain, T., Scheibner, C., Fruchart, M., & Vitelli, V. (2022). Stokes flows in three-dimensional fluids with odd and parity-violating viscosities. *Journal of Fluid Mechanics, 934*, A23

Parity Breaking in Plasmas

- If Larmor radius is smaller than plasma length scale → Gyro Viscosity
- Split into two terms, perpendicular to B-field and parallel to B-field [8]

$$\eta_{\alpha\perp} = \frac{v_\alpha^2}{2\omega_\alpha} \qquad \eta_{\alpha\parallel} = \frac{v_\alpha^2}{\omega_\alpha} \qquad \omega_a = \frac{e_\alpha B}{m_\alpha}$$

- Contributes to gyro viscous stress in Braginskii Eq. (B-field along z direction)

$$\frac{1}{n_\alpha m_\alpha}(\nabla \cdot \boldsymbol{\pi})_\perp = -\left(\eta_{\alpha\perp}\nabla_\perp^2 + \eta_{\alpha\parallel}\frac{\partial^2}{\partial z^2}\right)\boldsymbol{v}_\alpha \times \boldsymbol{z} - \eta_{\alpha\parallel}(\nabla \times \boldsymbol{z})\frac{\partial}{\partial z}v_{\alpha z}$$

$$\frac{1}{n_\alpha m_\alpha}(\nabla \cdot \boldsymbol{\pi})_z = -\eta_{\alpha\parallel}\frac{\partial}{\partial z}\mathbf{z} \cdot (\nabla \times \mathbf{v}_\alpha),$$

- Dissipationless, forces always perpendicular to flow and appears in real part of dispersion

Note: Units of $\eta_{\alpha\perp}$ are L^2/T

Gyro viscosity is like specific angular momentum

$$evB = \frac{mv^2}{R}$$

$$Rv = \frac{mv^2}{eB}$$

[8] Kono, M., and J. Vranjes. "Gyro-viscosity and linear dispersion relations in pair-ion magnetized plasmas." *Physics of Plasmas* 22.11 (2015).

Active matter

- Collection of agents that convert environment energy into directed mechanical motion

  bacteria, cells, liquid crystals, Janus particles, active colloids, spinny particles…

- Fluid description → properties of active matter captured by viscosity coefficients

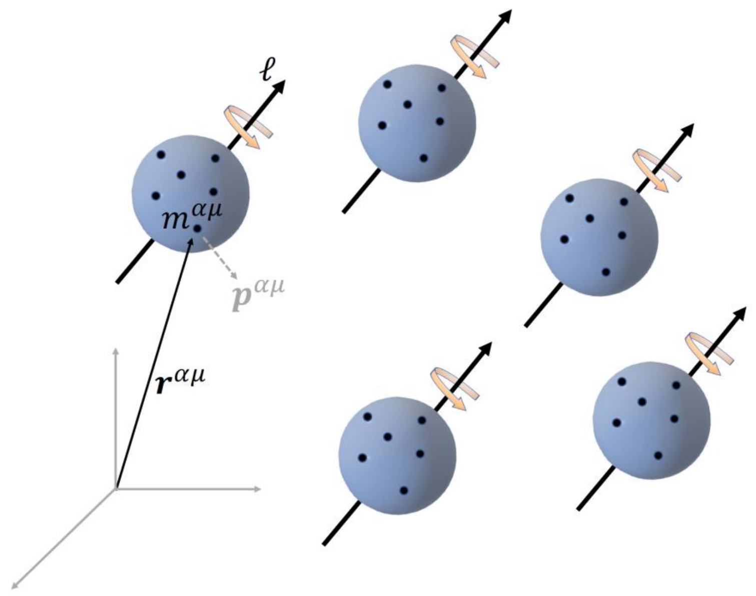
- Studied extensively in 2D, less so in 3D

- Markovich & Lubensky 2021 [9]:

  - Coarse-grain collection of complex molecules

  - Introduce intrinsic angular momentum density $\vec{\ell}$

  - Gives parity-broken viscosity in both 2D and 3D



[9] Markovich, T., & Lubensky, T. C. (2021). Odd viscosity in active matter: microscopic origin and 3d effects. *Physical Review Letters*, *127*(4), 048001.

Possible microscopic descriptions

3D odd viscosity can arise from relaxation of $\vec{\ell}$:

$$\hat{g}_i(\mathbf{r}) = \sum_{\alpha\mu} p_i^{\alpha\mu} \delta\left(\mathbf{r} - \mathbf{r}^{\alpha\mu}\right)$$

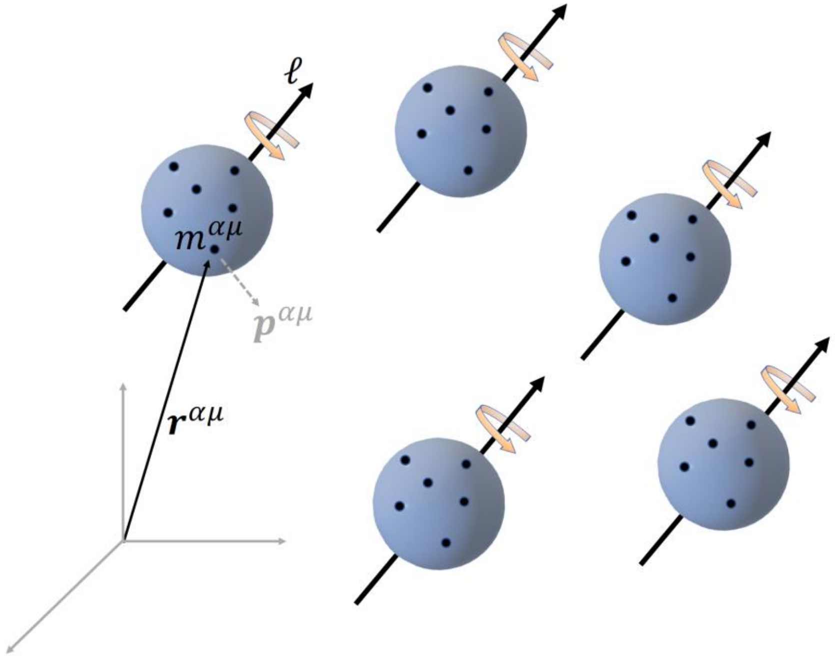
which gives “coarse grained” momentum

$$\mathbf{g}(\mathbf{r}) = \mathbf{g}^c + \frac{1}{2}\boldsymbol{\nabla} \times \boldsymbol{\ell}$$

If $\vec{\ell}$ is relaxed, then momentum conservation gives:

$$\dot{g}_i + \nabla_j\left(v_j g_i\right) = -\nabla_i \tilde{P} + \eta \nabla^2 v_i + \frac{1}{2}\boldsymbol{\ell} \cdot \nabla \omega_i$$

with $\quad \tilde{P} \equiv P + \boldsymbol{\ell} \cdot \boldsymbol{\omega}$



Note: - Same form as ferrofluid (later)
- 3D odd viscosity is stemming from angular momentum; common to most 3D cases, including gyro viscosity

[9] Markovich, T., & Lubensky, T. C. (2021). Odd viscosity in active matter: microscopic origin and 3d effects. *Physical Review Letters*, *127*(4), 048001.

Hydrodynamics

Study of global conserved quantities Q: $\frac{dQ}{dt} = 0$

Gives rise to local conservation laws for density q: $\partial_t q + \partial_i \Pi_i^q = 0$

Flux $\Pi_i^q(q, v_i, \ldots)$ given by constitutive relation

$Q = \int d\vec{r}\; q(\vec{r})$:
mass
momentum
energy
entropy
angular momentum
…

We focus on mass and momentum:

$$\partial_t \rho + \partial_i g_i = 0 \qquad g_i = \rho v_i$$

$$\partial_t g_i + \partial_j \Pi_{ij} = 0 \qquad \Pi_{ij} = \rho v_i v_j - T_{ij}$$

$\rho =$ mass density
$v_i =$ flow velocity
$g_i =$ momentum density
$\Pi_{ij} =$ momentum flux

Ignore any thermal effects, so energy conservation comes automatically

Hydrodynamics

Governing equations typically written: $\partial_t \rho + \partial_i(\rho v_i) = 0$

$$\rho D_t v_i = \partial_j T_{ij} + f_i$$

with material derivative $D_t = \partial_t + v_j \partial_j$

In first order hydro T_{ij} is first order in gradients of velocity: $T_{ij} = -\delta_{ij} P + \eta_{ijkl} \partial_k v_l + \cdots$

Viscosity tensor η_{ijkl} is in general a function of ρ and $\vec{r}$

Simplifications: - If compressible: $P = P_0 + c^2(\rho - \rho_0) + \cdots$

- If incompressible: $\partial_i v_i = 0$

Viscosity Tensor in 2D

- In 2D compressible fluid a general viscosity tensor η_{ijkl} has 16 independent components
- For isotropic 2D fluids symmetry leaves only 6 unique transport coefficients [10]

$$\eta_{ijkl} = \underbrace{\eta\left(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}\right)}_{\text{Shear}} + \underbrace{\zeta\delta_{ij}\delta_{kl}}_{\text{Bulk}} + \underbrace{\Gamma\epsilon_{ij}\epsilon_{kl}}_{\text{Rotational}} + \underbrace{\eta_H\left(\epsilon_{ik}\delta_{jl} + \epsilon_{jl}\delta_{ik}\right)}_{\text{Odd/Hall}} + \underbrace{\zeta_H\delta_{ij}\epsilon_{kl}}_{\text{Odd pressure}} + \underbrace{\Gamma_{\mathrm{H}}\epsilon_{ij}\delta_{kl}}_{\text{Odd torque}}$$

- In the incompressible limit, and no internal torque (T_{ij} symmetric under $i \leftrightarrow j$)

$$\eta_{ijkl} = \eta\left(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk}\right) + \eta_H\left(\epsilon_{ik}\delta_{jl} + \epsilon_{jl}\delta_{ik}\right) + \zeta_H\delta_{ij}\epsilon_{kl}$$

$$\boxed{\begin{array}{c} \partial_i v_i = 0, \qquad \delta_{kl} \to 0 \\ T_{ij} \sim \eta_{ijkl}\partial_k v_l \end{array}}$$

- To be written as a Hamiltonian system, only η_H remains (with constant ρ)

[10] Monteiro, G.M., Abanov, A.G., & Ganeshan, S. (2021). Hamiltonian structure of 2D fluid dynamics with broken parity. *SciPost Physics*.

Basic Results in 2D

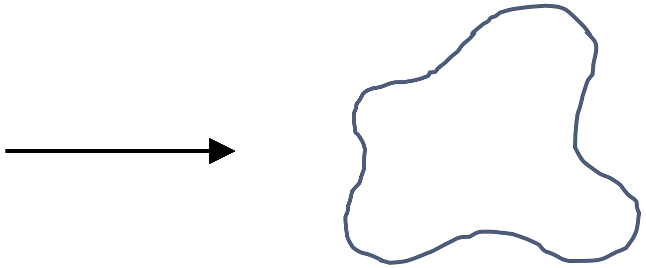
Odd viscosity has no effect on incompressible bulk:

$$D_t v_i = -\partial_i P + \partial_j \sigma_{ij} \,, \qquad \sigma_{ij} = \nu_e\left(\partial_i v_j + \partial_j v_i\right) + \nu_o\left(\partial_i v_j^* + \partial_i^* v_j\right)$$

$$D_t v_i = -\partial_i \tilde{P} + \nu_e \nabla^2 v_i \,, \qquad \tilde{P} = P - \nu_o \omega$$

Thus, if boundary conditions depend only on flow:

- Flow does not depend on ν_o
- Net force on closed contour is independent of ν_o
- Net torque depends on change in area



$$\tau_o = 2\nu_o \frac{dA}{dt}$$

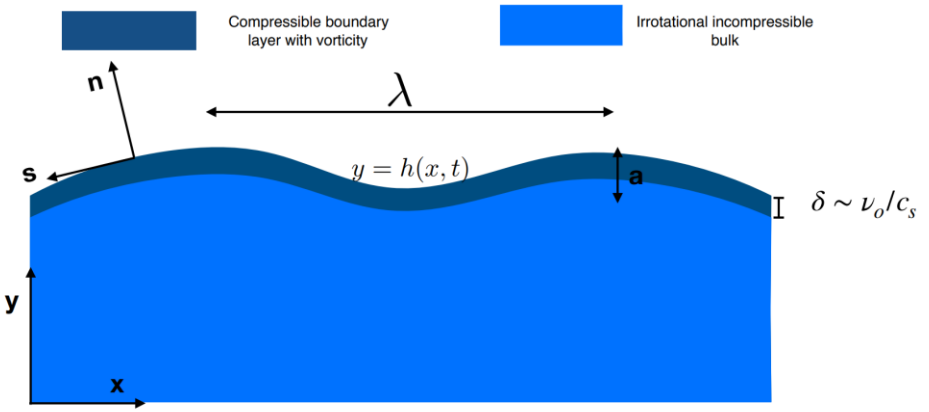
Odd viscosity does not produce any drag or lift forces in the incompressible case

[11] Ganeshan, Sriram, and Alexander G. Abanov. "Odd viscosity in two-dimensional incompressible fluids." *Physical review fluids* 2.9 (2017): 094101.

Results in 2D

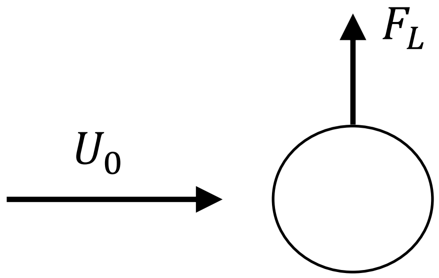
- For a compressible ocean (free surface), effects of odd viscosity or confined to compressible boundary layer [12]
- In the nonlinear regime, odd viscosity modifies the KdV equation and soliton solutions [13]

$$\eta_x \pm \frac{\eta_t}{\sqrt{gh}} + \frac{3}{2h}\eta\,\eta_x + h^2\left(\frac{1}{6} \pm \frac{\nu_o}{\sqrt{gh^3}}\right)\eta_{xxx} = 0$$



- If particle number conservation is violated, lift is possible [14]

$$\partial_t \rho + \partial_k(\rho v_k) = -\frac{1}{\kappa}(\rho - \rho_0)$$



- Current project: Odd compressible Oseen equation

$$\vec{v} = U_o\hat{x} + \epsilon\vec{u}$$

[12] Abanov, Alexander G., et al. "Hydrodynamics of two-dimensional compressible fluid with broken parity: variational principle and free surface dynamics in the absence of dissipation." *Physical Review Fluids* 5.10 (2020): 104802.
[13] Monteiro, Gustavo M., and Sriram Ganeshan. "Nonlinear shallow water dynamics with odd viscosity." *Physical Review Fluids* 6.9 (2021): L092401.
[14] Lier, Ruben, et al. "Lift force in odd compressible fluids." *Physical Review E* 108.2 (2023): L023101.

Viscosity Tensor in 3D

- In 3D, a general viscosity tensor η_{ijkl} has 81 independent components (compressible case)
- For rotational invariance in a single plane (xy), 19 independent coefficients [15]

$$
\begin{bmatrix}
\Gamma_1 & \eta^R & 0 & 0 & 0 & 0 & 0 & \eta^e_{Q,1}+\eta^o_{Q,1} & \eta^e_{Q,2}+\eta^o_{Q,2} \\
-\eta^R & \Gamma_1 & 0 & 0 & 0 & 0 & 0 & \eta^e_{Q,2}+\eta^o_{Q,2} & -\eta^e_{Q,1}-\eta^o_{Q,1} \\
0 & 0 & \Gamma_3 & \eta^e_A+\eta^o_A & 0 & 0 & \eta^e_{Q,3}+\eta^o_{Q,3} & 0 & 0 \\
0 & 0 & \eta^e_A-\eta^o_A & \frac{3}{2}\zeta & 0 & 0 & \eta^e_s+\eta^o_s & 0 & 0 \\
0 & 0 & 0 & 0 & \mu_1 & \eta^o_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\eta^o_1 & \mu_1 & 0 & 0 & 0 \\
0 & 0 & \eta^e_{Q,3}-\eta^o_{Q,3} & \eta^e_s-\eta^o_s & 0 & 0 & \mu_3 & 0 & 0 \\
\eta^e_{Q,1}-\eta^o_{Q,1} & \eta^e_{Q,2}-\eta^o_{Q,2} & 0 & 0 & 0 & 0 & 0 & \mu_2 & \eta^o_2 \\
\eta^e_{Q,2}-\eta^o_{Q,2} & -\eta^e_{Q,1}+\eta^o_{Q,1} & 0 & 0 & 0 & 0 & 0 & -\eta^o_2 & \mu_2
\end{bmatrix}
$$

η^o_1 =2D odd viscosity (transverse)

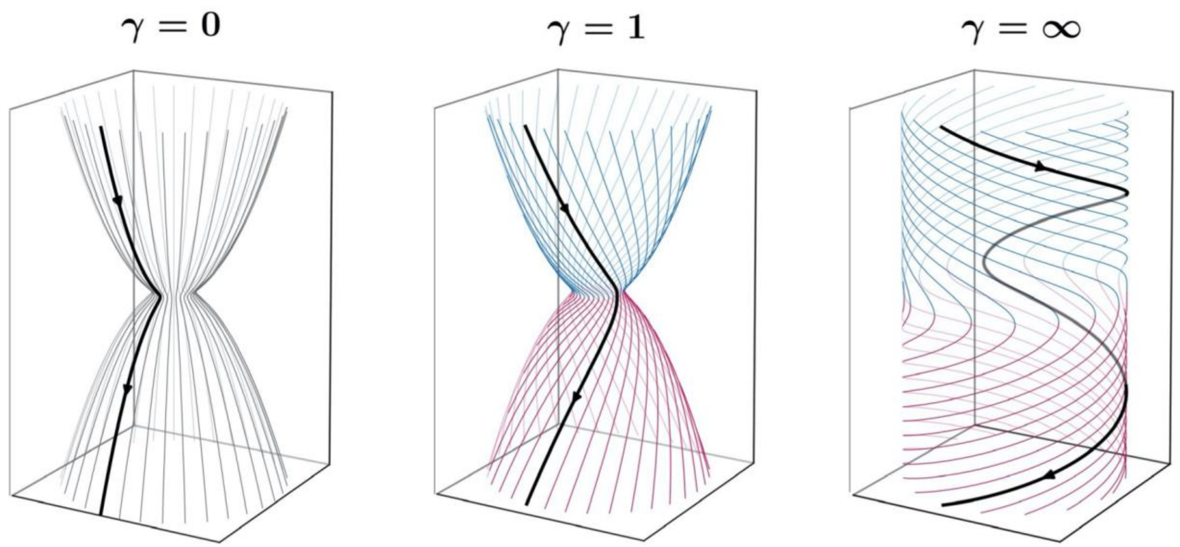
η^o_2 =3D odd viscosity (longitudinal)

- Importantly, there exist **2** ‘odd viscosity’ coefficients
- Underlying microscopic theory determines the precise relation between coefficients

[15] Khain, T., Scheibner, C., Fruchart, M., & Vitelli, V. (2022). Stokes flows in three-dimensional fluids with odd and parity-violating viscosities. *Journal of Fluid Mechanics*, *934*, A23.

Results in 3D

- Bulk flow is modified (Stokeslet) [15]



$$\vec{F} = -\hat{z}\, F_z\, \delta^3(\vec{x})$$

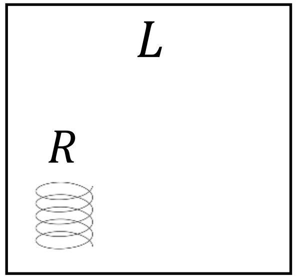
$$\gamma = \frac{\eta_2^o}{\mu} \ , \text{ with } \ \eta_1^0 = -2\eta_2^0$$

- Novel "odd" mechanical waves are stable [9]
- Still active area of research; flows without "-2" constraint, and for non- isotropic not understood

[9] Markovich, T., & Lubensky, T. C. (2021). Odd viscosity in active matter: microscopic origin and 3d effects. *Physical Review Letters*, *127*(4), 048001.
[15] Khain, T., Scheibner, C., Fruchart, M., & Vitelli, V. (2022). Stokes flows in three-dimensional fluids with odd and parity-violating viscosities. *Journal of Fluid Mechanics*, *934*, A23.

3D Odd Viscosity in Plasmas

Role of gyro viscosity depends on relevant scaling (ordering) of the problem



$$\delta = \frac{R}{L} = \frac{\text{ion Larmor Radius}}{\text{length scale}}$$

- Hall MHD Ordering (large accelerations): gyro viscosity enters with pressure at 2nd order
- MHD Ordering (subsonic flows): gyro viscosity enters alone at 2nd order
- Drift Ordering: gyro viscosity enters at 2nd order WITH acceleration and viscosity

$$-\nabla p + \mathbf{J}\times\mathbf{B} = \delta^2\left(n\frac{d\mathbf{V}_i}{dt} + \underline{\nabla\cdot\Pi_i^{gv}} + \mathbf{b}\cdot\nabla\cdot\Pi_i^{nc} - n\mu_A\nabla^2\mathbf{V}_i\right)$$

gyroviscous cancellation:

$$n\left(\frac{\partial\mathbf{V}_{*i}}{\partial t} + \mathbf{V}_i\cdot\nabla\mathbf{V}_{*i}\right) + \underline{\nabla\cdot\Pi_i^{gv}}(\mathbf{V}_i) \approx \nabla\chi - \mathbf{b}n\mathbf{V}_{*i}\cdot\nabla V_{\parallel i}$$

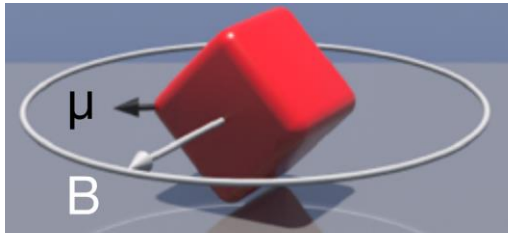
$$\boxed{\mathbf{V}_e = \mathbf{V}_{\parallel e} + \mathbf{V}_E + \mathbf{V}_{*e}}$$

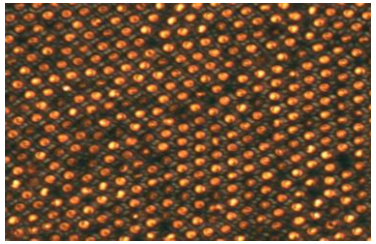
$$\chi = -p_i\mathbf{b}\cdot(\nabla\times\mathbf{V}_{\perp i})$$

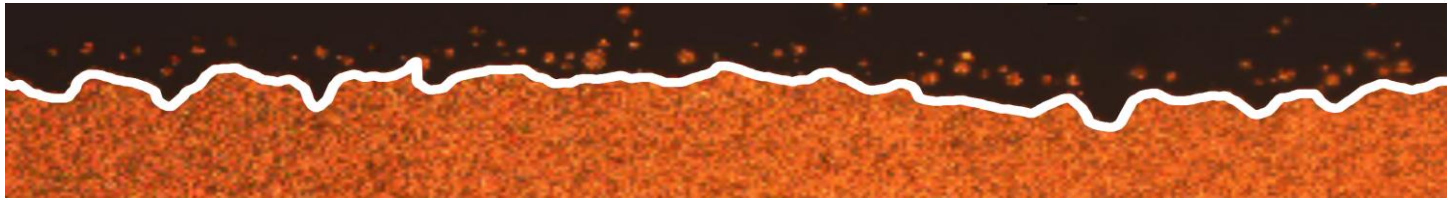
[16] Ordered Fluid Equations D. D. Schnack

Magnetic Colloids to Ferrofluids

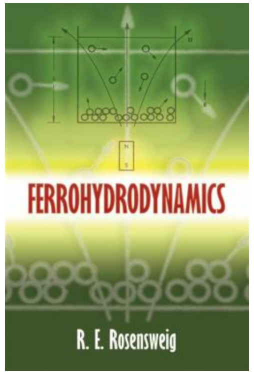
Magnetic colloids [17]:







- These relatively large ($\sim\mu m$) particles show odd viscosity experimentally
- For small particles like Ferrofluids ($\sim nm$) these effects 'washout'



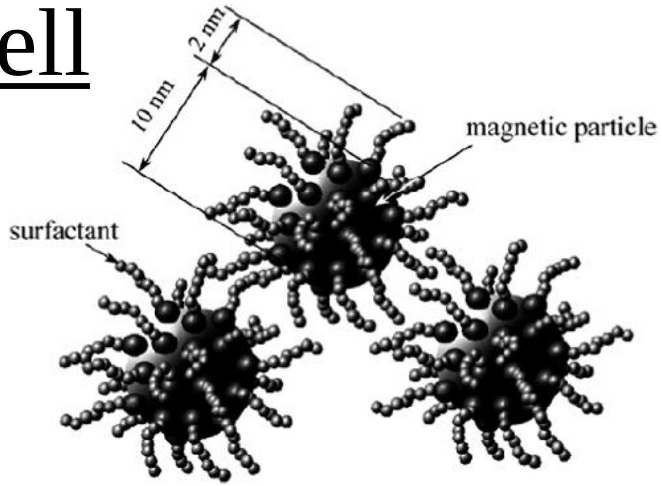
Seem to possess all essential ingredients for parity breaking

Is there a version of Ferrofluids that breaks parity?

[18]

[17] Soni, V., Bililign, E.S., Magkiriadou, S. *et al.* The odd free surface flows of a colloidal chiral fluid. *Nat. Phys.* 15, 1188–1194 (2019).
[18] Cowley, M. D. "Ferrohydrodynamics. By R. E. ROSENSWEIG. Cambridge University Press, 1985. 344 Pp. £45." *Journal of Fluid Mechanics* 200 (1989): 597-99. Print.

Ferrofluids in a nutshell



[19]

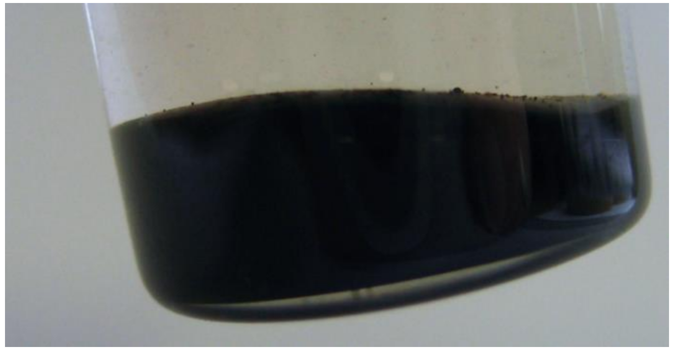
Colloidal suspension with 3 components

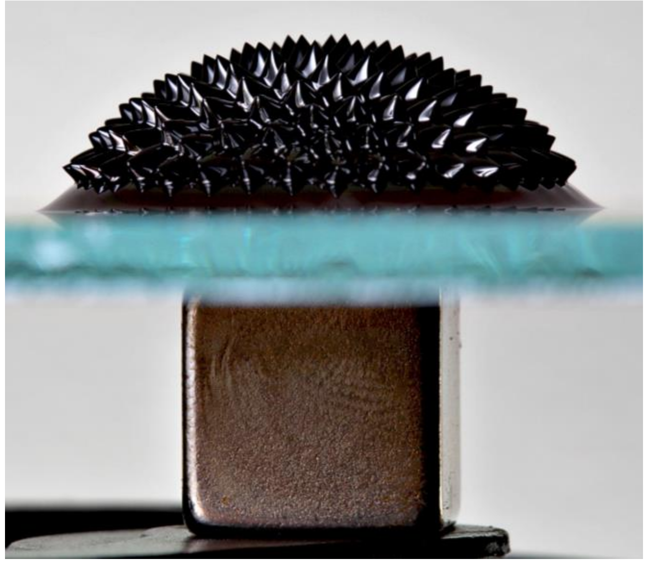
- 5% Magnetized particles (magnetite, hematite, ~10 nm)
- 10% Surfactant (prevents clumping via Van der Waals)
- 85% Carrier fluid

Regular fluid if no external B field: Brownian Motion randomizes magnetic moments

Large scale magnetization in the presence of external *B* field

$137





[19] Ghasemi, Jalal & Jafarmadar, Samad & Nazari, Meysam. (2015). Effect of magnetic nanoparticles on the lightning impulse breakdown voltage of transformer oil. Journal of Magnetism and Magnetic Materials. 389. 10.1016/j.jmmm.2015.04.045.

arXiv:2301.07096

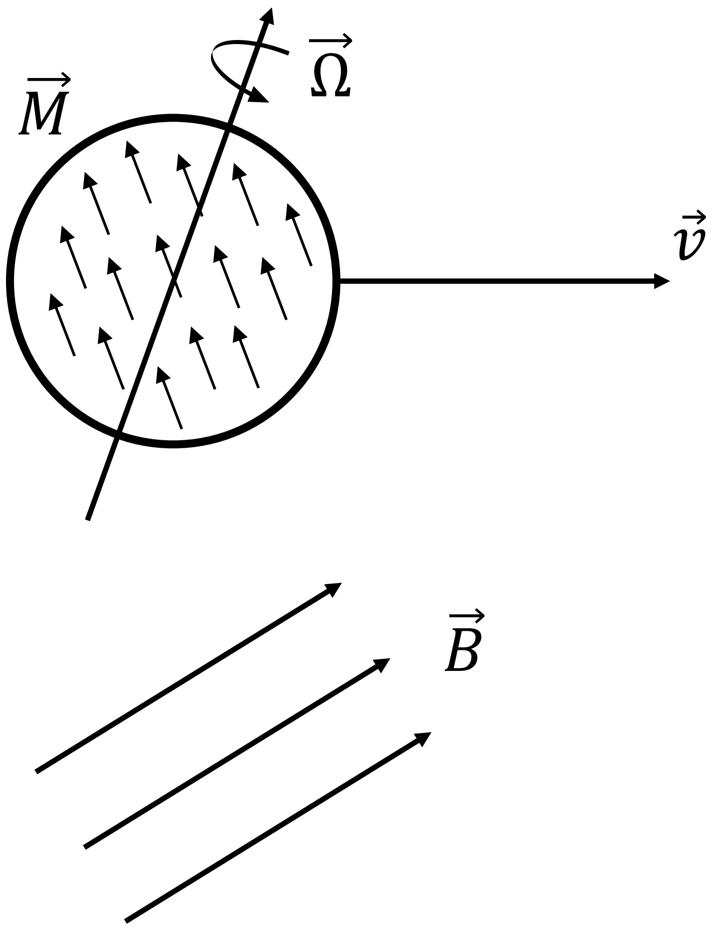
Ferrohydrodynamics

Degrees of Freedom

Fluid flow velocity:	$\vec{v} = (v_x, v_y, v_z)$
Particle rotation rate:	$\vec{\Omega} = (\Omega_x, \Omega_y, \Omega_z)$
Magnetization:	$\vec{M} = (M_x, M_y, M_z)$
Pressure:	$P = P(x, y, z)$

Fixed Quantities

Applied magnetic field:	$B = (B_x, B_y, B_z)$
Density:	$\rho = const.$
Moment of inertia per mass:	$I = const.$ (small!)



arXiv:2301.07096

Main Idea

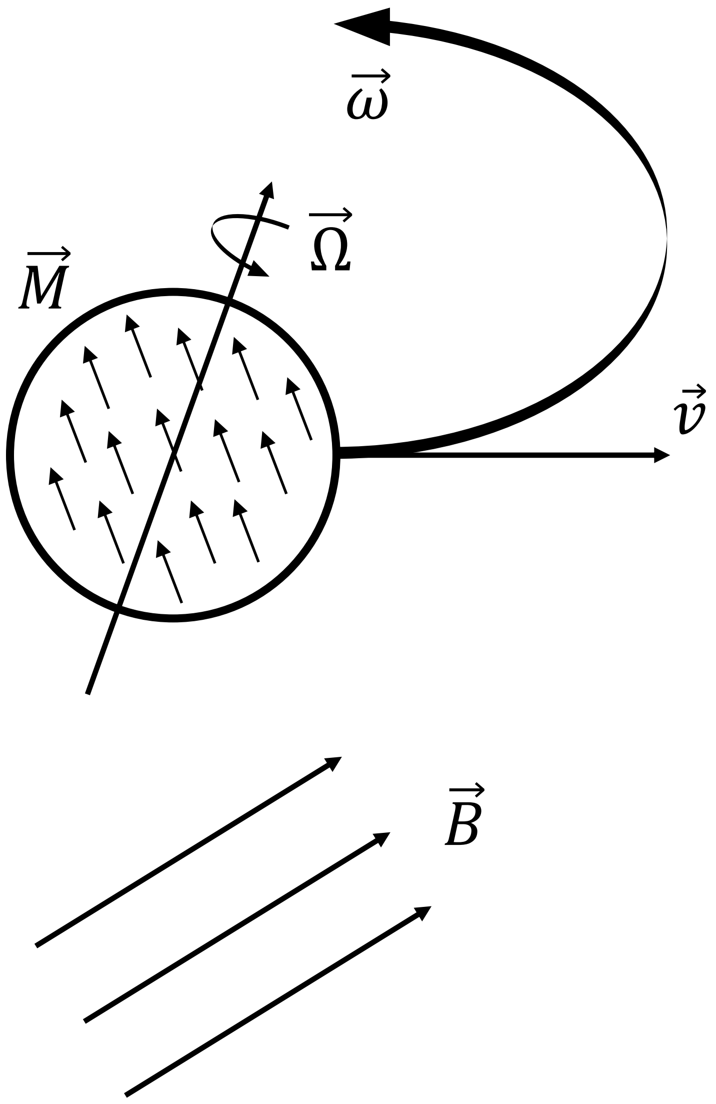
Recently:

- Markovich & Lubensky couple $\vec{\ell} = I\vec{\Omega}$ to $\vec{\omega} = \vec{\nabla} \times \vec{v}$

  → leads to 3D odd viscosity in active matter systems [9]

- Relevant for magnetic colloids & large complex molecules

- Ferrofluid particles are too small; these effects don't manifest on hydrodynamic scale [17]



Idea:

Couple magnetization $\vec{M}$ to $\vec{\omega}$

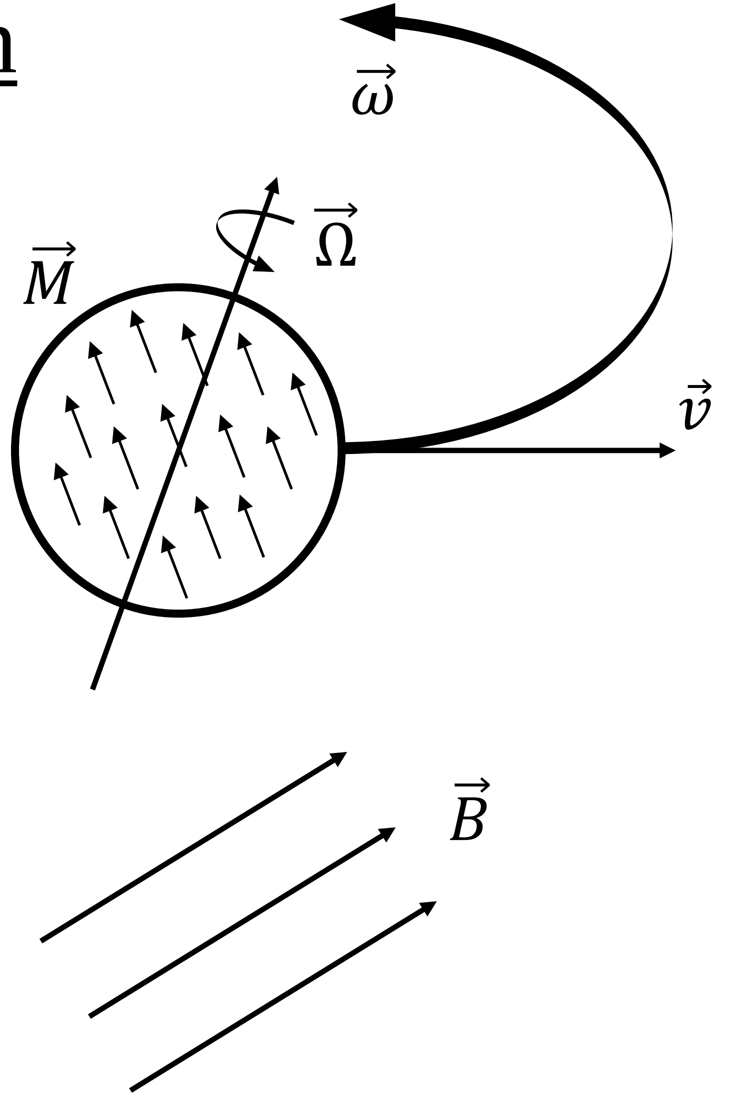
[9] T. Markovich and T. C. Lubensky. Odd Viscosity in Active Matter: Microscopic Origin and 3D Effects. Physical Review Letters, 127, 048001 (2021).

[17] V. Soni, E. S. Bililign, S. Magkiriadou, S. Sacanna, D. Bartolo, M. J. Shelley, and W. T. Irvine. The odd free surface flows of a colloidal chiral fluid. Nature Physics, 15, 1188–1194 (2019).

arXiv:2301.07096

Ferrofluid Hamiltonian

$$H = \int d^3r \left[\frac{1}{2}v_i v_i + \frac{1}{2I}\ell_i\ell_i + \underline{\frac{1}{2}\ell_i\omega_i} + \underline{\frac{\gamma}{2}\omega_i M_i} - M_i B_i\right]$$



$\rho = 1$ for simplicity

$\vec{\ell} = I\vec{\Omega}$ is angular momentum density (per particle)

γ = coupling constant

Markovich & Lubensky

New coupling

$$R = \int d^3r \left[\frac{1}{2}\nu\left(\partial_i v_j + \partial_j v_i\right)^2 + \frac{1}{2}\Gamma\left(2\Omega_i - \epsilon_{ijk}\partial_j v_k\right)^2 + \frac{1}{2\tau}\left(M_i - M_i^0\right)^2\right]$$

Use appropriate Poisson Brackets and dissipation function

→ derive ferrohydro equations

Note: Don't need Hamiltonian framework, but convenient for parity breaking (dissipationless)

arXiv:2301.07096

Governing Equations

Skip technical details…

Key Physical Steps:

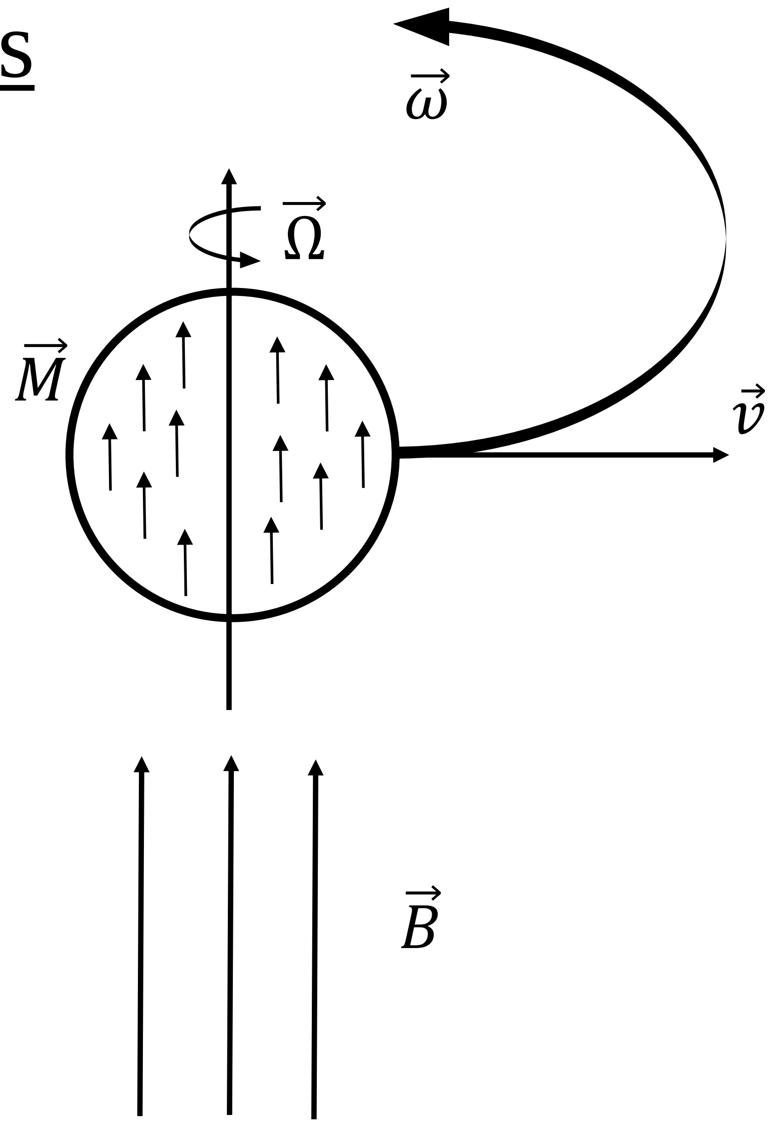
- Fix uniform $\vec{B}$ field in the z direction
- Small particle size (I goes to zero)

→ particles rotate before magnetization re-adjusts

(contrast to larger magnetic colloids)

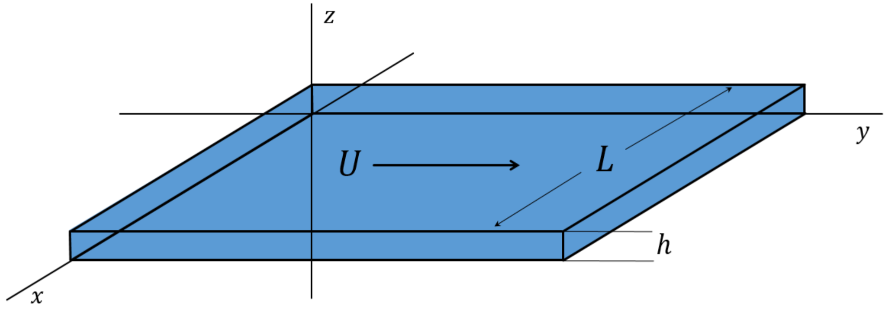
$$\boxed{D_t v_i + \partial_i P = \nu \nabla^2 v_i + \frac{\gamma}{4} M^0 \partial_z \omega_i}$$

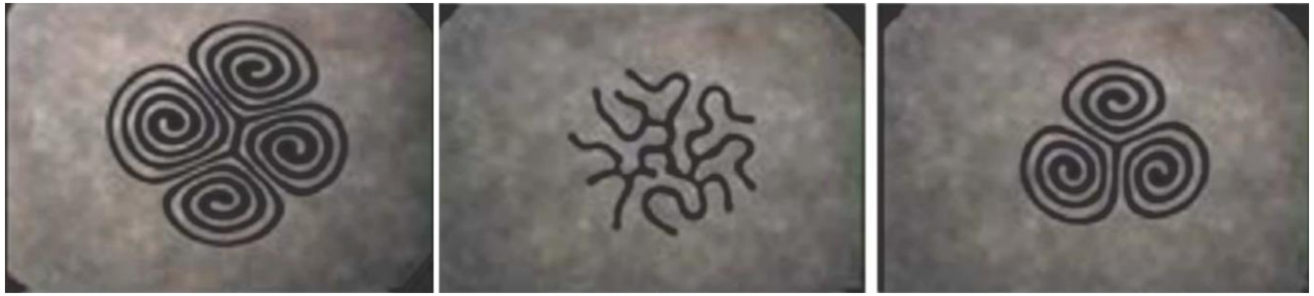
Unique to 3D flows! No term in standard ferrohydrodynamics



arXiv:2301.07096

Ferrofluids in Hele-Shaw Cells





[20]

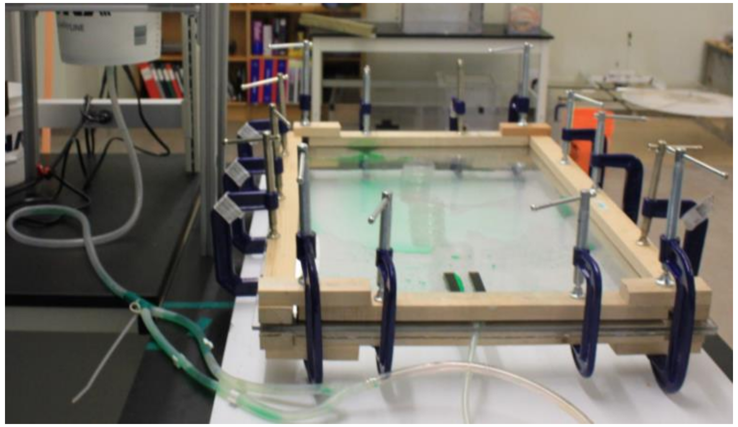
New coupling (Modified Darcy's Law):

$$0 = \partial_x V_x + \partial_y V_y$$

$$-\frac{h^2}{12}\partial_x P = \mu V_x - \frac{\gamma}{4} M^0 V_y$$

$$-\frac{h^2}{12}\partial_y P = \mu V_y + \frac{\gamma}{4} M^0 V_x$$

"Table-top" experiment to probe parity-odd fluid



[20] Elborai, S. & Kim, Do Kyung & He, Xiaowei & Lee, Se-Hee & Rhodes, S. & Zahn, Markus. (2005). Self-forming, quasi-two-dimensional, magnetic-fluid patterns with applied in-plane-rotating and dc-axial magnetic fields. Journal of Applied Physics. 97. 10Q303-10Q303. 10.1063/1.1851453.

arXiv:2301.07096

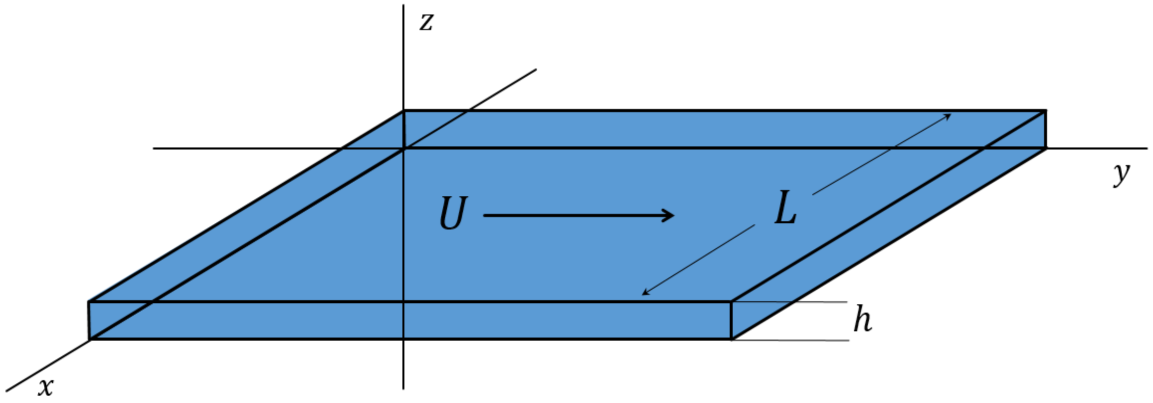
Darcy's Law

Incompressible 3D fluid with shear viscosity:

$$\partial_i v_i = 0$$

$$\rho\left(\partial_t v_i + v_j \partial_j v_i\right) = \partial_j T_{ij} + f_i$$

$$T_{ij} = -P\delta_{ij} + \eta\left(\partial_i v_j + \partial_j v_i\right),$$



Confine fluid between plates with separation h, assume xy variables vary over large scales compared to h:

$$V_i = -\frac{h^2}{12\eta}\partial_i P$$

$$\boxed{\begin{aligned} v_x &= 6z(1-z)V_x \\ v_y &= 6z(1-z)V_y \\ v_z &= 0 \\ P &= P(x,y) \end{aligned}}$$

Phys. Rev. Fluids 7, 114201 (2022)

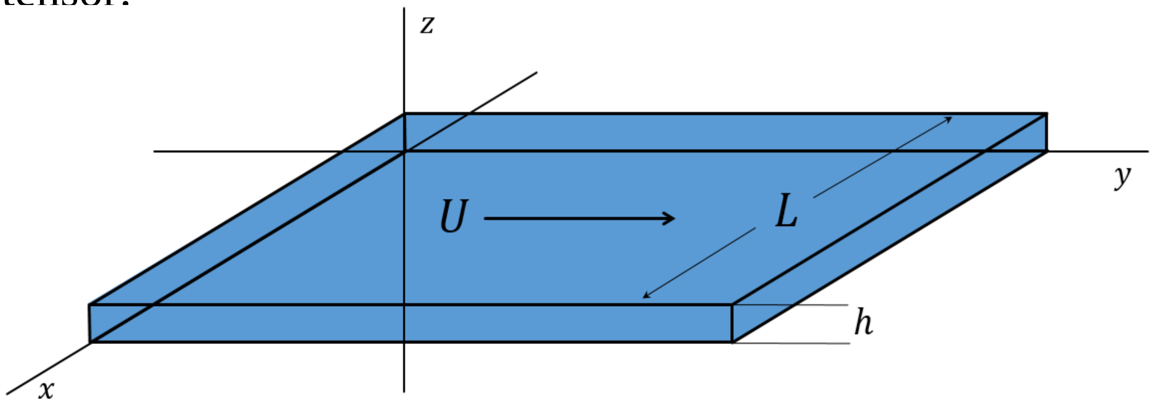
Modified Darcy's Law

Incompressible 3D fluid with general (uniform) viscosity tensor:

$$\partial_i v_i = 0$$

$$\rho\left(\partial_t v_i + v_j \partial_j v_i\right) = \partial_j T_{ij} + f_i$$

$$T_{ij} = -P\delta_{ij} + \eta_{ijkl}\partial_k v_l,$$



Same scaling analysis. Remarkably, only four coefficients remain (out of 64 in principle):

$$V_i = -\frac{h^2}{12}(\mathfrak{y}^{-1})_{ij}\partial_j P \qquad \mathfrak{y} = \begin{pmatrix} \eta_{xzzx} & \eta_{xzzy} \\ \eta_{yzzx} & \eta_{yzzy} \end{pmatrix}$$

Phys. Rev. Fluids 7, 114201 (2022)

Modified Darcy's Law

$$V_i = -\frac{h^2}{12}(\mathfrak{y}^{-1})_{ij}\partial_j P \qquad \mathfrak{y} = \begin{pmatrix} \eta_{xzzx} & \eta_{xzzy} \\ \eta_{yzzx} & \eta_{yzzy} \end{pmatrix}$$

Analogous to:

- Flow through anisotropic porous media (symmetric unless parity is broken)
- Ohm's law with general conductivity (Hall Effect, $\vec{E} \sim \vec{\nabla} P$)

We examine special case of cylindrical symmetry

$\eta = \eta_{xzzx} = \eta_{yzzy}$ (shear viscosity)

$\eta_* = \eta_{xzzy} = -\eta_{yzzx}$ (3d odd viscosity)

Coefficient η_* is one of the **2** parity odd viscosities associated with 3D cylindrical symmetry

Longitudinal… Not 2D Hall viscosity!

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Exp 1: Channel Flow

- Misalignment between flow and pressure gradient

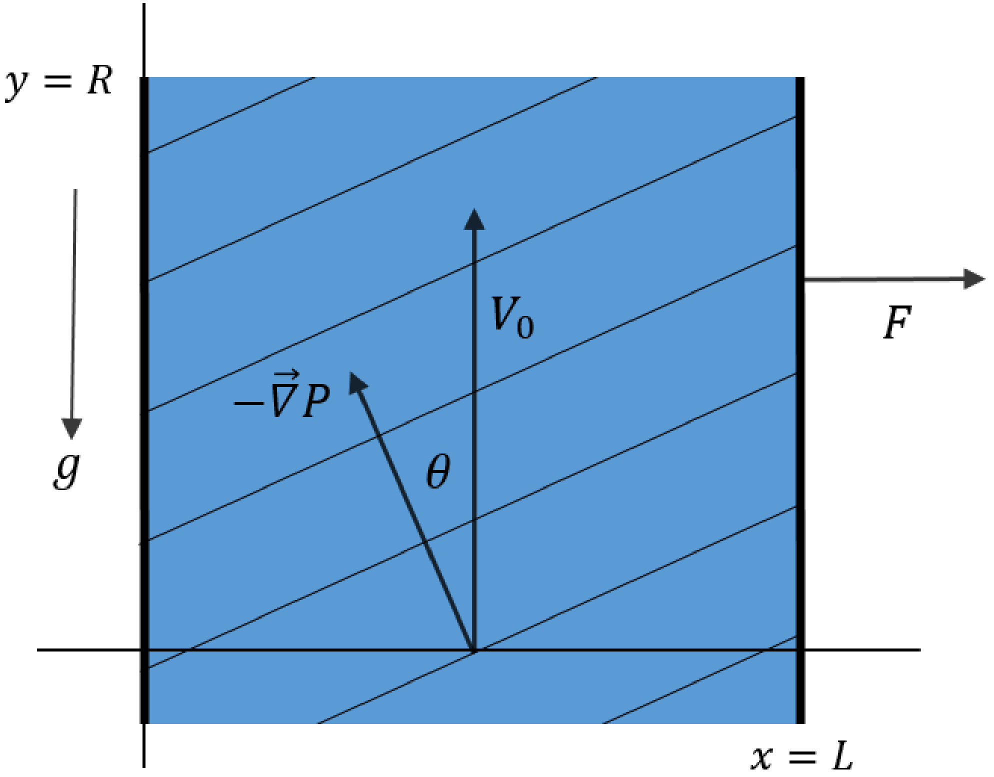
$$\theta = \arctan\left(\frac{\eta_*}{\eta}\right)$$

- Transverse force on channel walls

$$F = 2\eta_* V_0 (RLh)$$

- Hall Analogy:

  Transverse electric field E_x is required to maintain vertical current



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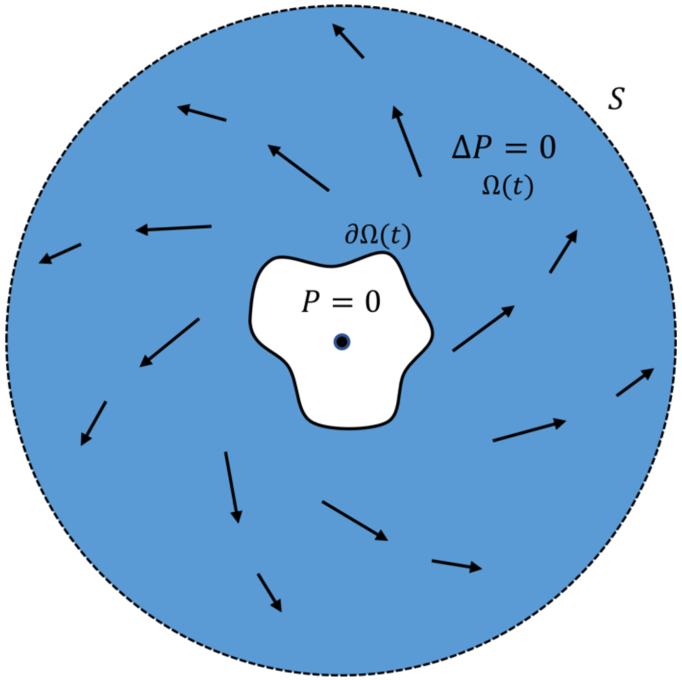
Exp 2: Expanding Bubble

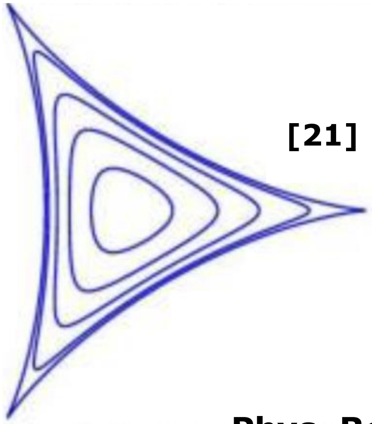
- Laplacian Growth, well studied
- Polubarinova-Galin equation:

$$\left(\frac{\partial P}{\partial t} - \frac{\eta}{\eta^2+\eta_*^2}|\vec{\nabla}P|^2\right)\Big|_{\partial\Omega(t)} = 0$$

- Modification to cusp formation time
- Circulation at infinity gives measure of parity odd-odd terms

$$\Gamma \sim \frac{\eta_*}{\eta^2+\eta_*^2}\frac{d\mathcal{A}}{dt}$$





[21]

[21] Singularities in nonlocal interface dynamics, B. Shraiman and D. Bensimon, Phys. Rev. A 30, 2840(R) – Published 1 November 1984

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Exp 3: Saffman/Taylor Instability

- Perform linear stability analysis on free surface of the form:

$$H(x,t) = \epsilon \,\mathrm{Re}\left(e^{ikx+\Omega t}\right)$$

- Interface boundary conditions

  - Kinematic: V_n continuous & matched with interface

  - Forces: P continuous

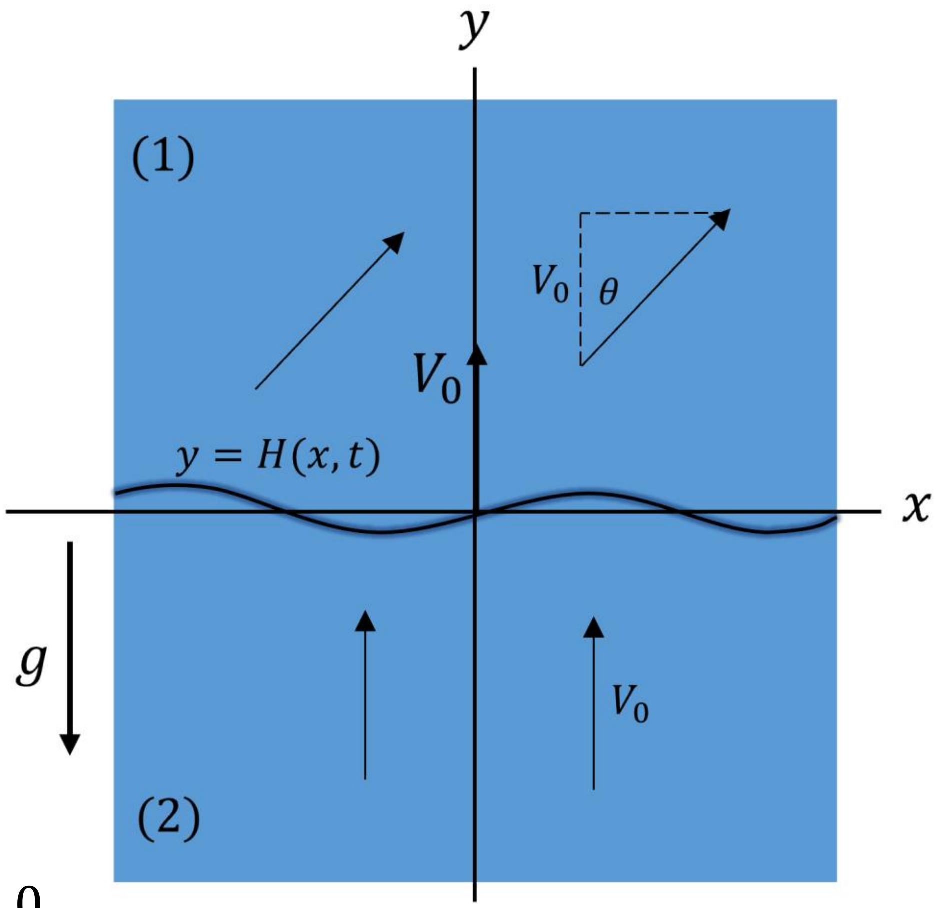
- Constraint on asymptotic flow angle

$$\theta = \arctan\left(\frac{\eta_*^{(2)} - \eta_*^{(1)}}{\eta^{(1)}}\right)$$

- Modified stability condition

$$\left(\rho^{(1)} - \rho^{(2)}\right)g + V_0\left(\eta^{(1)} - \eta^{(2)}\right) + \frac{V_0\left(\eta_*^{(1)} - \eta_*^{(2)}\right)^2}{\eta^{(1)} + \eta^{(2)}} < 0$$

- Parity-odd terms introduce more stable regions in the parameter space



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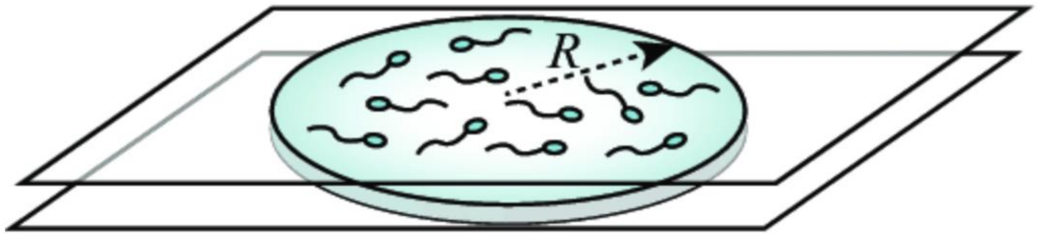
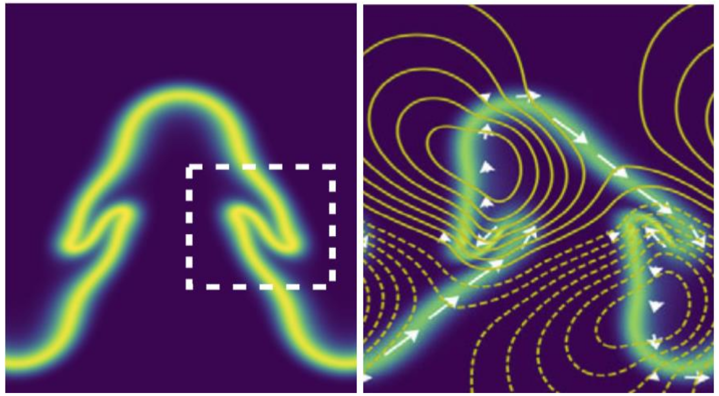
Applications

Facilitates the measurement of parity-odd viscosity coefficients in active matter systems, especially useful for complex biological systems



[22]

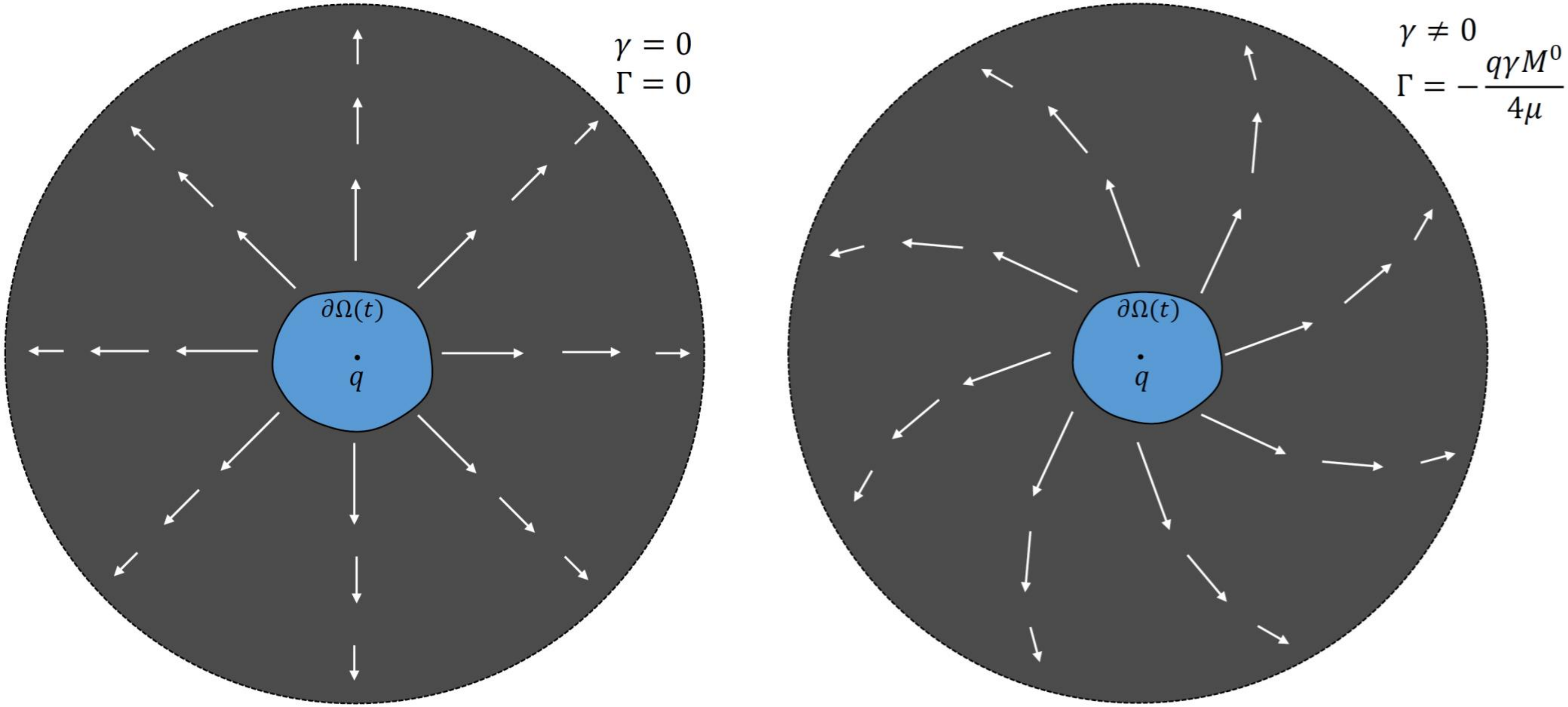


[23]

[22] Circularly confined microswimmers exhibit multiple global patterns, Alan Cheng Hou Tsang and Eva Kanso, Phys. Rev. E 91, 043008 – Published 13 April 2015

[23] Active matter invasion of a viscous fluid: Unstable sheets and a no-flow theorem, Christopher J. Miles, Arthur A. Evans, Michael J. Shelley, and Saverio E. Spagnolie, Phys. Rev. Lett. 122, 098002 – Published 4 March 2019

Experimental Setup



(a) without parity breaking (b) with parity breaking

No coupling → no circulation (*yes or no result*)

arXiv:2301.07096

Quantum Superfluids

- Fluid behavior of many-body quantum system
- Map complex wavefunction ψ to hydrodynamic variables
- Constraint on vorticity $\vec{\omega} = \vec{\nabla} \times \vec{v}$
- Vortex solutions → density fluctuations important (compressible)

Typical Prescription: $\psi = \sqrt{\rho} e^{i\theta}$

$$\boxed{v_i = \frac{\hbar}{m} \partial_i \theta}$$

fluid-like equations: $\partial_t \rho + \partial_i(\rho v_i) = 0$

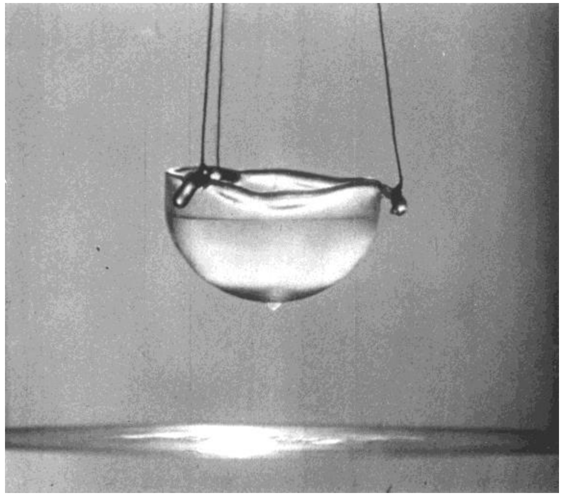
$$\rho D_t v_i = \partial_j T_{ij} + f_i$$

$$T_{ij} = -\delta_{ij} P + \cancelto{0}{\eta_{ijkl} \partial_k v_l} + \cdots$$

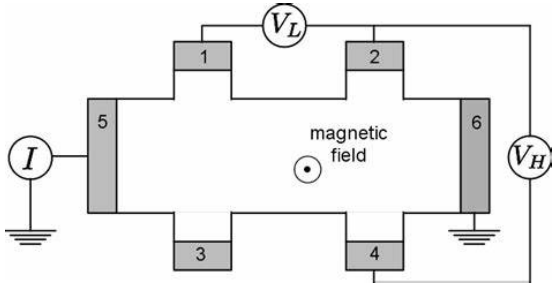
"Quantum Pressure"

$$P = \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}$$

- Helium-4
- Superconductivity
- Ultra cold atomic gases
- Bose-Einstein condensates
- Fractional quantum Hall
- Polaritons



Fractional Quantum Hall Effect



- Collective behavior of 2D electron system in strong magnetic field
- Laughlin states admit fluid description

Start with Chern-Simons-Ginzburg-Landau action

$$S_{\text{CSGL}} = \int d^3x \left[i\hbar\Phi^\dagger D_t \Phi - \frac{\hbar^2}{2m} |D_i\Phi|^2 - V(|\Phi|^2) - \frac{\hbar\nu}{4\pi}\epsilon^{\mu\lambda\kappa} a_\mu \partial_\lambda a_\kappa \right]$$

where Φ is bosonic matter field, A_μ is external electromagnetic field, a_μ is statistical Chern-Simons field

Define velocity:

$$v_i = \frac{\hbar}{m}\left(\partial_i\theta + a_i + \frac{e}{\hbar}A_i + \frac{\epsilon_{ij}}{2n}\partial^j n\right)$$

arXiv:2305.16511

Fluid Equations

Charge and Momentum conservation:

$$\partial_t n + \partial_i (n v_i) = 0$$

$$\partial_t v_i + v_j \partial_j v_i + \frac{eB}{m} \epsilon_{ij} v_j - \frac{1}{mn} \partial_j T_{ij} = 0$$

Hall Constraint:

$$n - \frac{\nu e B}{2\pi\hbar} + \frac{\nu}{4\pi} \partial_i \left(\frac{\partial_i n}{n} \right) + \frac{\nu m}{2\pi\hbar} \epsilon_{ij} \partial_i v_j = 0$$

2D Odd Viscosity!

$$\nu_o \sim \frac{\hbar}{2}$$

Stress Tensor:

$$T_{ij} = \left(V - nV' - \frac{\pi \hbar^2 n^2}{\nu m} \right) \delta_{ij} - \frac{\hbar n}{2} \left(\epsilon_{ik} \partial_k v_j + \epsilon_{jk} \partial_i v_k \right)$$

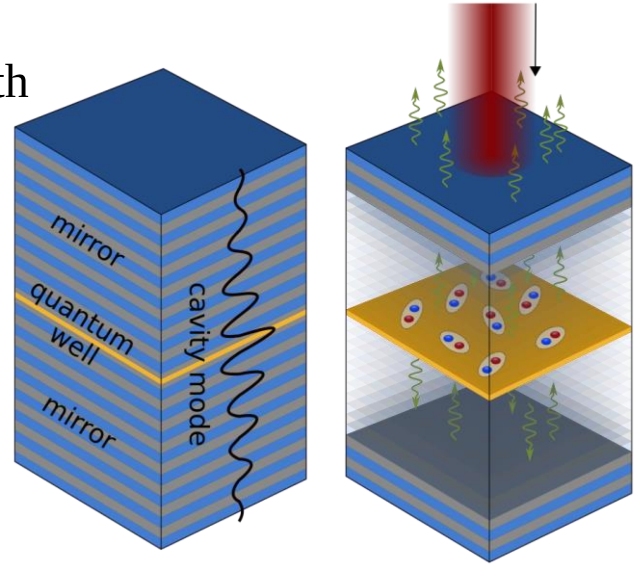
arXiv:2305.16511

Polaritons

- Exciton-Polaritons created when light modes (photons) hybridize with quantum well excitons. [24]
- Resulting quasiparticles are bosonic
- If a condensate is formed, described by wavefunction Ψ in 2D

  → admits a fluid description

- System is inherently out of equilibrium, so no action principle



[25]

Goals: *Find fluid description of 2D polariton system*

Drag and lift forces on impurities or obstacles?

[24] Sedov, E., Lukoshkin, V.A., Kalevich, V.K., Savvidis, P., & Kavokin, A.V. (2021). Circular polariton currents with integer and fractional orbital angular momenta.
[25] Michael D. Fraser

Wavefunction Description

Start with generalized Gross-Pitaevskii Equation (nonlinear Schrodinger) given by

$$i\hbar\partial_t\Psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r}) + \alpha|\Psi|^2 + \alpha_R n_R\right]\Psi + \frac{i\hbar}{2}\left(Rn_R - \gamma\right)\Psi,$$

where

$$\boxed{n_R(t,\vec{r}) \simeq \frac{P(\vec{r})}{\gamma_X}\left(1 - \frac{R}{\gamma_X}|\Psi|^2\right)}$$

$$\begin{aligned}
m &= \text{effective polariton mass}\\
V(\vec{r}) &= \text{obstacle potential}\\
\alpha &= \text{polariton-polariton interaction coupling}\\
\alpha_R &= \text{polariton-exciton interaction coupling}\\
R &= \text{stimulated scattering rate from reservoir to polariton mode}\\
\gamma &= \text{polariton decay rate}\\
P(\vec{r}) &= \text{pumping intensity}\\
\gamma_X &= \text{exciton decay rate.}
\end{aligned}$$

For this work, assume pumping intensity is uniform and constant

Fluid Description

$$\boxed{\Psi(t,\vec{r}) = \sqrt{\rho(t,\vec{r})}\, e^{i\theta(t,\vec{r})}}$$

$$\boxed{v_i = \frac{\hbar}{m}\left(\partial_i\theta + \frac{1}{2\rho}\epsilon_{ij}\partial_j\rho\right)}$$

gGPE splits into real and imaginary pieces which are fluid type equations:

$$\partial_t\rho + \partial_i\left(\rho v_i\right) = \frac{R}{\gamma_X}\rho P_0\left(1 - \frac{R}{\gamma_X}\rho\right) - \gamma\rho$$

$$\rho D_t v_i = \partial_j T_{ij} + \rho f_i$$

where the stress tensor and force are:

$$T_{ij} = -\frac{\alpha}{2m}\left(1 - \frac{\alpha_R}{\alpha}\frac{RP_0}{\gamma_X^2}\right)\delta_{ij}\rho^2 - \frac{\hbar R^2 P_0}{4m\gamma_X^2}\epsilon_{ij}\rho^2 - \frac{\hbar}{2m}\rho\left(\partial_i v_j^* + \partial_i^* v_j\right)$$

$$f_i = -\frac{1}{m}\partial_i V$$

Concluding Thoughts

- Parity breaking appears in many physical systems (active fluids, plasmas, superfluids)
- Odd viscosity, in 2D and 3D encapsulates these properties in a fluid description
- Observable effects and common between systems (edge dynamics in 2D, bulk spiral in 3D)
- 3D effects, in general, are not well understood

Thanks for your time…Questions?