Turbulent Dynamo action in a 3-dimensional magnetohydrodynamic plasma

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In Collaboration with

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Dr. Robert A. Ellis Fellowship Talk, PPPL, NJ-08540, USA

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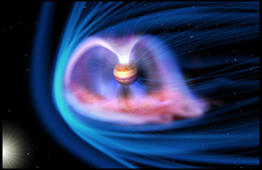
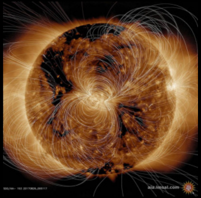
09.00 AM (ET) / 07.30 PM (IST)

Introduction & Motivation

- The presence of small, mean or large scale magnetic fields are very much common in astrophysical objects, planets, stars, interstellar medium, galaxy, accretion disks and also in the Sun.

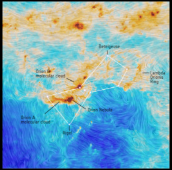


Planet [Credit: JAXA, Source: G]



Sun [Credit: NASA, Source: G]

Galaxy [Credit: Astrobin, Source: G]



Interstellar Medium [Credit: ESA & Planck, Source: G]

Accretion disks [Credit: M. GARLICK, Source: G]

Galaxy clusters [Credit: NARO & NASA, Source: G]

- What are the sources of these multi-scale magnetic energy?

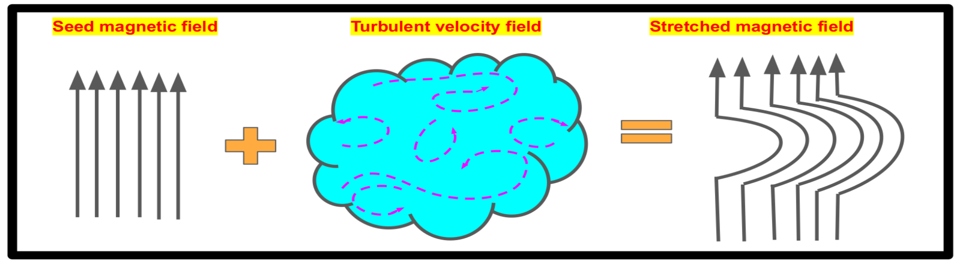
- According to Parker[1] [Parker E., 1979, Cosmical Magnetic Fields, Clarendon], these multi-scale magnetic fields are generated by the motion of conducting fluids through a transfer of kinetic to magnetic energy, that is, via Dynamo action.

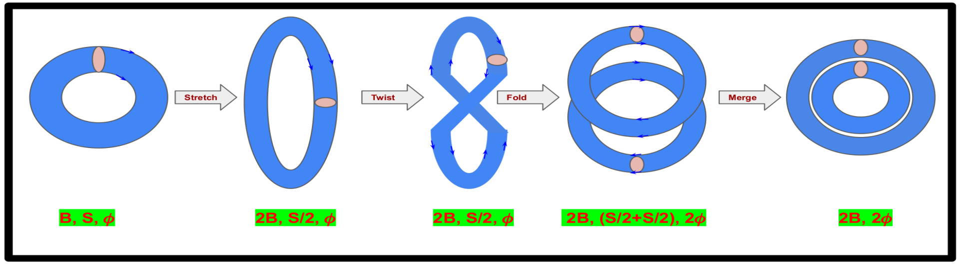
Classification of Dynamo models at a glance:

1. Large Scale Dynamo (LSD) or mean field [lack of reflectional symmetry or, nonzero fluid helicity is required].
2. Small Scale Dynamo (SSD) [no restrictions on fluid helicity].
3. Fast dynamos [growth rate remains finite in the limit $R_m \to \infty$] $\rightsquigarrow$ **STF** STF
4. Slow dynamos [magnetic diffusion plays a significant role].

- Linear or Kinematic [magnetic field dynamics does not "back react" with the velocity field]
- Non-linear or Self-consistent [the flow and the B-field "back react" on each other leading to a nonlinear saturation]. [F. Rincon, Journal of Plasma Physics 85, 205850401 (2019)]

Basic Dynamo Mechanism





Stretch-Twist-Fold (STF) Dynamo Mechanism

- Energy will obey: $2^n E_0$

Talk outline

1. Governing Equations.
2. Numerical Solver and its Development & Benchmarkings.
3. Role of kinetic (fluid) helicity in Dynamo instability.
4. Self-Consistent Dynamo with Helical & Non-Helical Drive.
5. Effect of flow shear on the onset of dynamos.
6. Conclusions.
7. Future Directions.

Dynamic equations

The following equations govern the dynamics of single fluid magnetohydrodynamic plasma (Conservative form),

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \rightarrowtail \textbf{Continuity Eqn.} \tag{1}$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \vec{\nabla} \cdot \left[\rho \vec{u} \otimes \vec{u} + \left(P + \frac{B^2}{2}\right) \mathbf{I} - \vec{B} \otimes \vec{B}\right] = \frac{1}{R_e} \nabla^2 \vec{u} + \vec{f} \rightarrowtail \textbf{Momentum Eqn.} \tag{2}$$

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \cdot \left(\vec{u} \otimes \vec{B} - \vec{B} \otimes \vec{u}\right) = \frac{1}{R_m} \nabla^2 \vec{B} \rightarrowtail \textbf{Induction Eqn.} \tag{3}$$

$$P = C^2 \rho \rightarrowtail \textbf{Eqn of state.} \tag{4}$$

$\vec{u}$, $\vec{B}$, P represent the velocity, magnetic and pressure fields, μ and η the kinematic viscosity and magnetic diffusivity. The dimensionless numbers are defined as: $Re = \frac{u_0 L}{\mu}$, $Rm = \frac{u_0 L}{\eta}$, $Pm = \frac{R_m}{R_e}$. The maximum fluid speed is u_0 and the Alfven speed is $V_A = \frac{u_0}{M_A}$, where M_A is the Alfven Mach number. Similarly the Sound speed is $C_s = \frac{u_0}{M_s}$, where M_s is the Sonic Mach number. Time is normalized as $t = t_0 * t'$ $t_0 = \frac{L}{V_A}$. The symbol "$\otimes$" represents the dyadic between two vector quantities (in Eq. 2 and 3).

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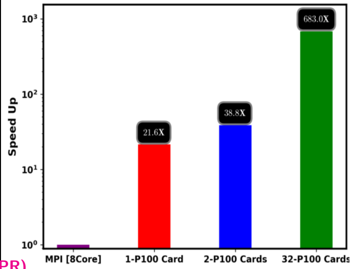
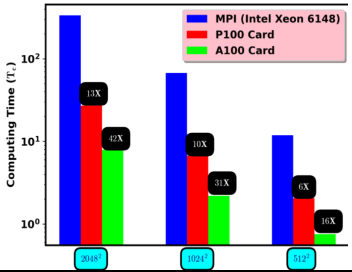
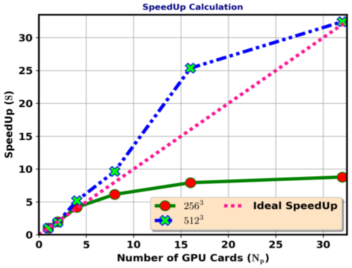
About the code GMHD3D and it's development

- We have upgraded existing hydrodynamic and magnetohydrodynamic solver to multi-node multi-card GPU architecture for simulating larger problems in large grid sizes and for better performances using OpenACC and OpenMPI.
- The solver uses pseudo-spectral technique. CUDA based FFT library [cuFFT library] & AccFFT libraries are used to perform Fourier transforms and Adams-Bashforth time solver for time updation.
- As the grid resolution increases, Solver approaches towards ideal scaling. We have also achieved linear speedup across 32 GPU cards on ANTYA cluster at IPR.
- It speeded up to 3.2x on A100 cards w.r.t P100 cards [*A100 scalings are done on **PARAM Siddhi-AI cluster** at CDAC, Pune, India during Hackathon 2021].

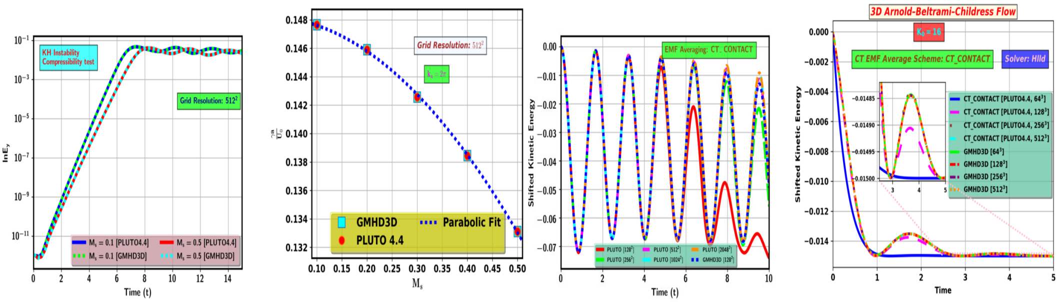






Benchmarking: GMHD3D vs PLUTO4.4

- For gaining confidence and further exploration, we have performed a detail comparative study between PLUTO 4.4 (an open source MHD solver) and GMHD3D for a subclass of physics problems.
- Good agreement between GMHD3D and PLUTO 4.4 in weakly compressible limit as well.
- With all available latest modules in PLUTO 4.4, it is observed that the spectral solver (GMHD3D) is much superior than finite difference solver (PLUTO 4.4) in various aspects.



S. Biswas & R. Ganesh, [Computers and Fluids 272 (2024) 106207]

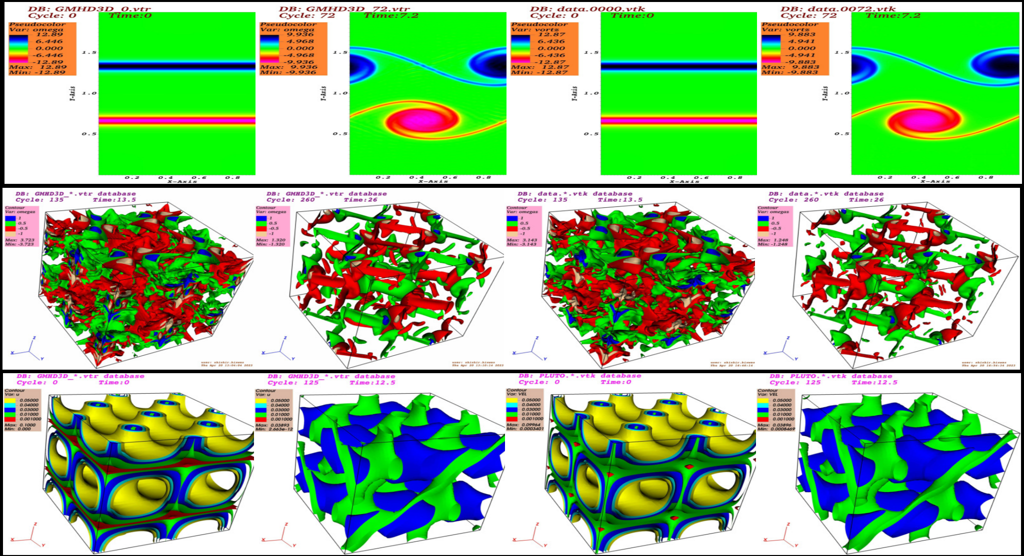
Benchmarking: GMHD3D vs PLUTO4.4

←— — — — — — GMHD3D — — — — — —→ ←— — — — — — PLUTO4.4 — — — — — —→

KH Instability

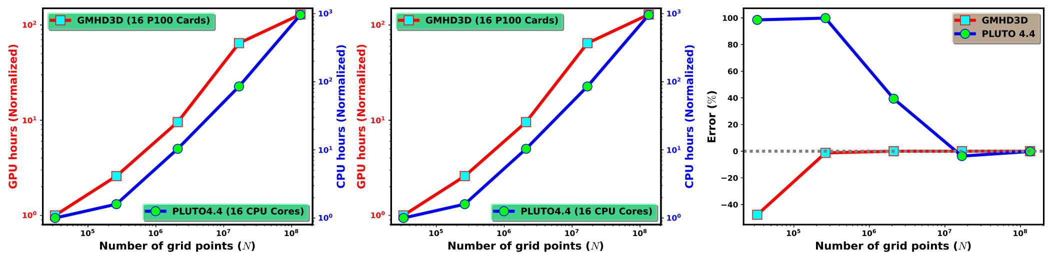
3D Turbulence

Recurrence



Efficiency of the GPU solver: Cost vs Accuracy

- Normalized computational costs for the GPUs and CPUs in relation to the grid resolutions has been plotted. The computational expenses increases linearly for both CPUs and GPUs as grid points increases. [1st & 2nd Fig from left]
- The normalized computational time of 16 GPUs is nearly similar to the computational time of 400 CPUs in cases when the computational workload is significant. [2nd Fig]



- Percentage of error for the GPU solver and CPU solver in relation to the grid resolutions is shown. The GPU solver converged rapidly to errors of the order of less than 1%, whereas the CPU solver requires a more precise grid (higher grid resolution) in order to converge. [3rd Fig]
- GMHD3D code fulfills the accuracy criteria at the lowest cost, whereas the PLUTO 4.4 code is the most expensive. This indicates the cost-effectiveness and accuracy of the GPU solver compared to the CPU solver for the subclass of problems considered.

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Role of kinetic (fluid) helicity in Dynamo instability

Recently Yoshida et al. [Yoshida et al., Phys. Rev. Lett., 119, 244501 (2017)] proposed a new intermediate class of flow, that establishes a topological bridge between quasi-2dimensional and 3-dimensional flow classes. The flow is formulated as follows:

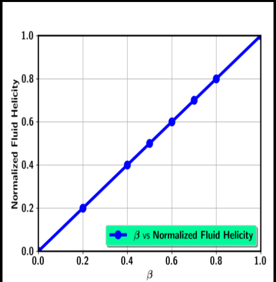
$$
\begin{aligned}
u_x &= [\sin(k_0 y) - \cos(k_0 z)] \\
u_y &= \beta[\sin(k_0 z) - \cos(k_0 x)] \\
u_z &= [\sin(k_0 x) - \beta\cos(k_0 y)]
\end{aligned}
$$

where β, is arbitrary real constants. We dub this flow as Yoshida-Morrison flow or YM flow.

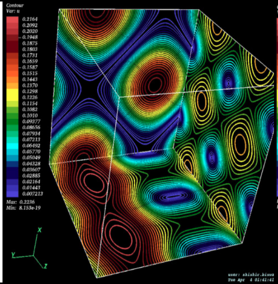
- The variation of β value in YM flow leads to new classes of flows.
- For $\beta = 0$, Yoshida et al. classifies it as EPI-2dimensional flow or in short EPI-2D flow.
- And for $\beta = 1$ the above equation becomes well known ABC like flow.

Helicity as a initial parameter

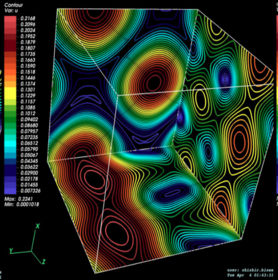
- By systematically varying β value, it is possible to inject finite fluid helicity $[\int_V \vec{u}.(\vec{\nabla} \times \vec{u})dV]$ in the system.
- $H_f = \int_V \vec{u}.(\vec{\nabla} \times \vec{u})dV = (8\pi)^3 \beta u_0^2. \Rightarrow$ $\boxed{\text{Normalized fluid helicity } H_f = \beta.}$



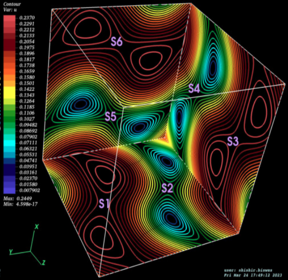
(a) **Helicity injection**



(b) $\beta = 0.0$



(c) $\beta = 0.2$

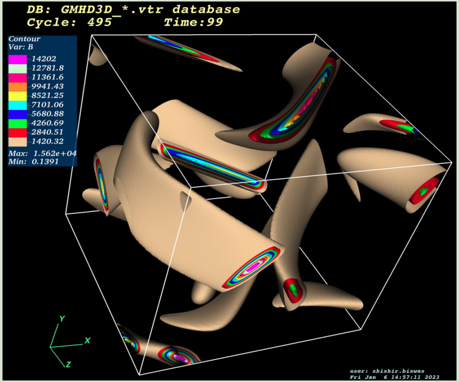


(d) $\beta = 1.0$

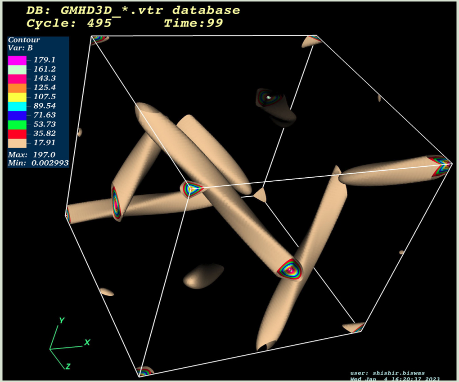
- For increasing β values, the velocity contours become increasingly chaotic and visible separatrix form emerges, which is a crucial ingredient of chaotic flows.

Iso-B Dynamics [Grid: 256^3]

$\beta = 0.2$ [Ribbon Like]



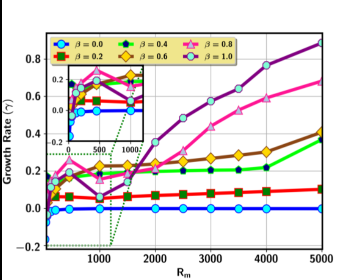
$\beta = 1.0$ [Cigar Like]



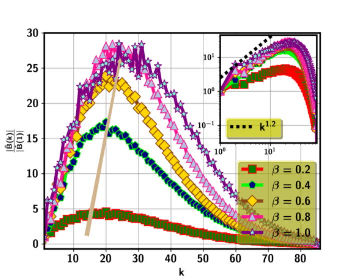
- We demonstrate as to, how a 'twisted ribbon' like iso-surface gets converted into classical 'cigar' like iso-surface with the injection of fluid helicty.

Non-Dynamo to Dynamo transition

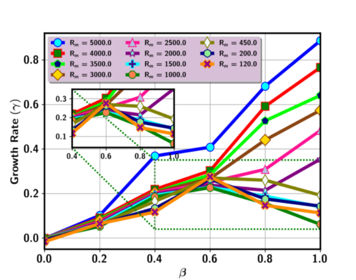
- Evidence of Fast Dynamo activity is observed for a range of YM flow.
- The spectrum clearly indicates that the dynamo is a small-scale dynamo (SSD).
- Using a simple kinematic dynamo model, for the considered flow, we demonstrate unambiguously a strong dependency of short scale dynamo on fluid helicity. In contrast to conventional understanding, it is shown that fluid helicity does strongly influence the physics of short scale dynamo for the flow profiles considered.
- This work brings out, for the first time, the role of fluid helicity in the transition from 'non-dynamo' to 'dynamo' regime systematically.



(a) YM flow dynamo



(b) Spectra (Evidence of SSD)



(c) Dynamo Transition

S. Biswas & R. Ganesh, [Phys. Scr. 98 075607 (2023)]

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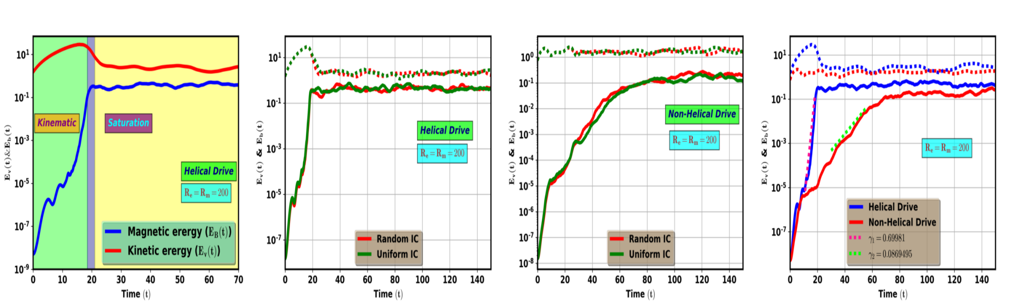
Self Consistent Dynamo with Helical & Non-Helical Drive

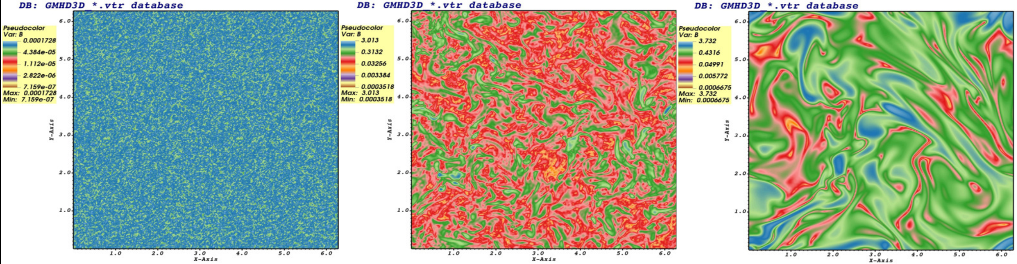
- What do we mean by helical & non-helical drive?
- YM flow drive (with & without helicity) has been utilized.
- Strong magnetic field is generated through "Dynamo Process". This starts affecting the topology of velocity field and leads to non-linear saturation [1st Fig from left].



- Independent of initial magnetic field [2nd & 3rd].
- Helical dynamo saturates faster [4th].

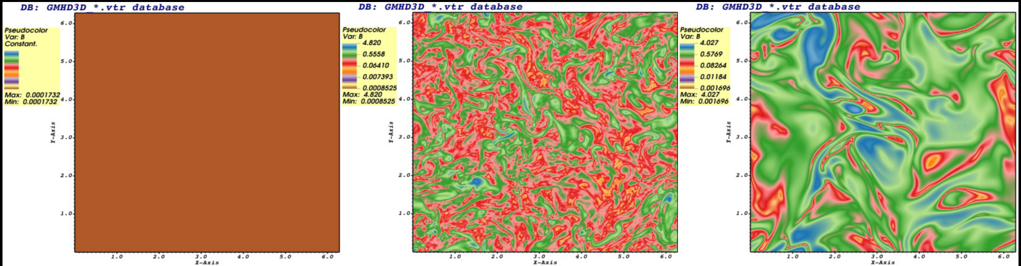
Magnetic field Dynamics [Grid: 256^3]

B_0 = Seed



Initial Kinematic Self-Consistent

B_0 = Uniform



What is P_m and why it is important?

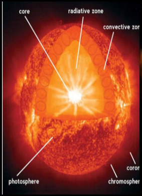
- $P_m = \frac{\nu}{\eta} = \frac{R_m}{R_e} \rightsquigarrow$ dimensionless parameter $\rightsquigarrow$ characterizes the relative importance of magnetic diffusivity (η) to kinematic viscosity (ν) in a plasma.
- P_m is defined as $10^{-5}\frac{T^4}{n}$ for a fully ionized medium [where $T \rightsquigarrow$ measured in kelvin and n $\rightsquigarrow$ particle concentration in cm^{-3}].
- High P_m limit is suitable for low-density, warmed and diffuse plasmas and low P_m limit implies dense environments.

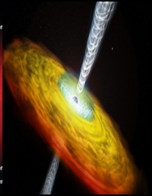
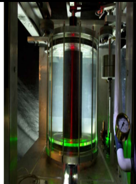
High P_m Limits

- Intracluster medium of galaxy clusters.
- Protogalaxies.
- High-temperature phases of the interstellar medium.
- The early universe, and so forth.

Low P_m Limits

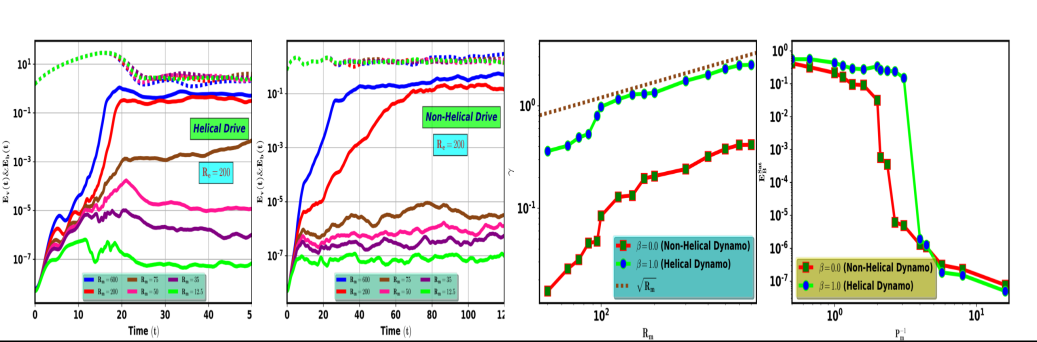
- Liquid-metal cores of planets, star convective zones.
- Protostellar disks.
- Liquid-metal (mercury, sodium, gallium) investigations (e.g., Dynamo & MRI experiments).
- Tokamak plasmas (e.g., Fusion experiments).




Effect of magnetic Prandtl numbers: Helical & Non-Helical Drive

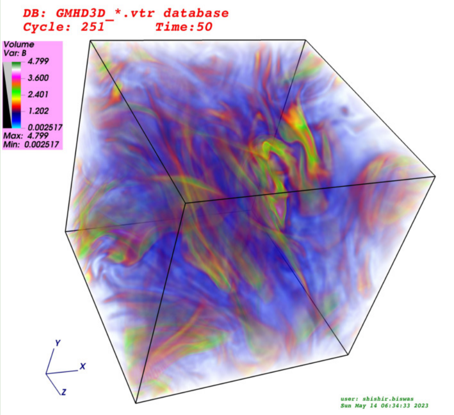
- What is the effect of P_m on Dynamos?
- At lower P_m limit Dynamo action is seen to be suppressed for both helical and non-helical cases [1st & 2nd Fig from left].



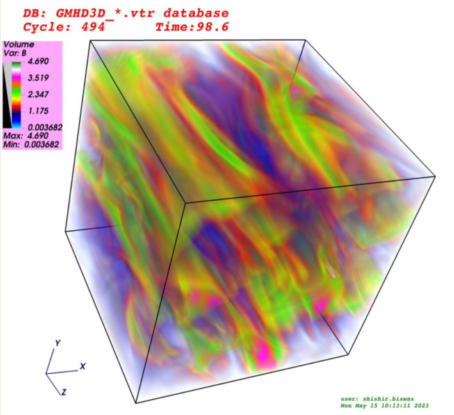
- The Dynamo growth rate, γ is $\propto \sqrt{R_m}$ with helical as well as non-helical drive (at high R_m) [3rd Fig].
- Saturation value of magnetic energy (E_B^{Sat}) is seen to be decreased with P_m for both helical and non-helical dynamos [4th Fig].

Helical & Non-Helical Dynamo: $P_m = 2.0$ [Grid: 256^3]

Helical Dynamo (Volume-Rendering)

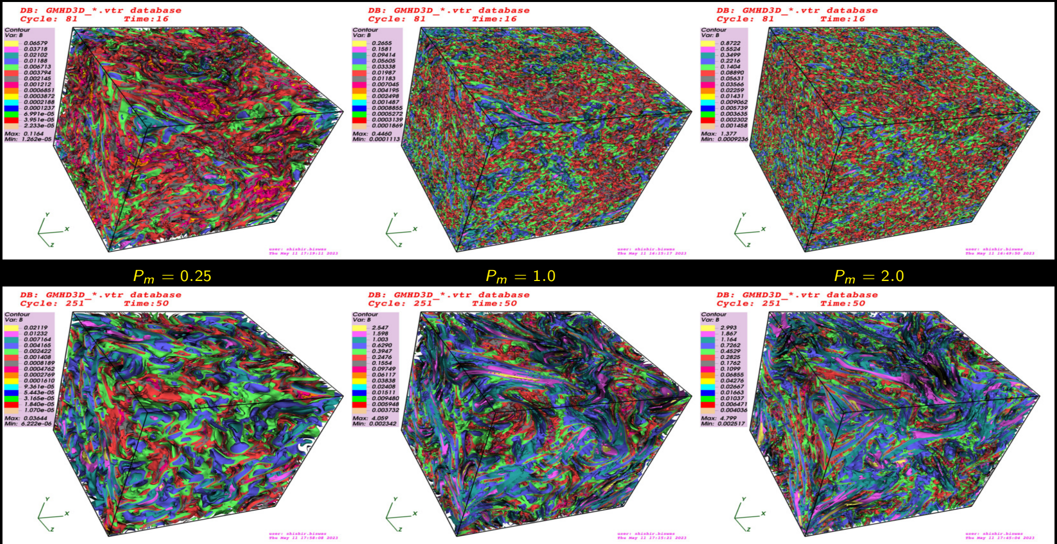


Non-Helical Dynamo (Volume-Rendering)



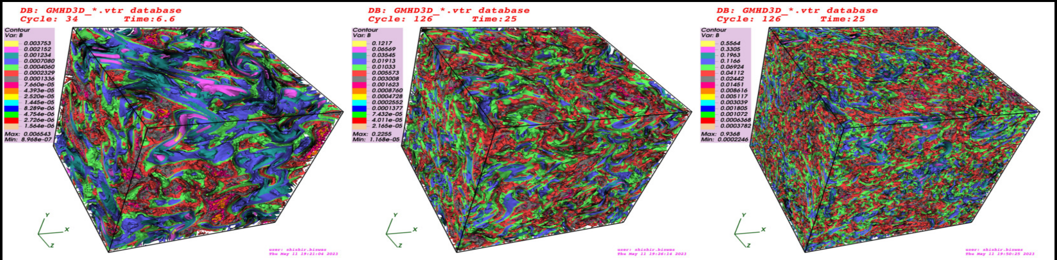
- The magnetic fields in the saturated stages exhibit structures larger than those in the kinematic stages.

Helical Dynamo at Different P_m: Magnetic field dynamics

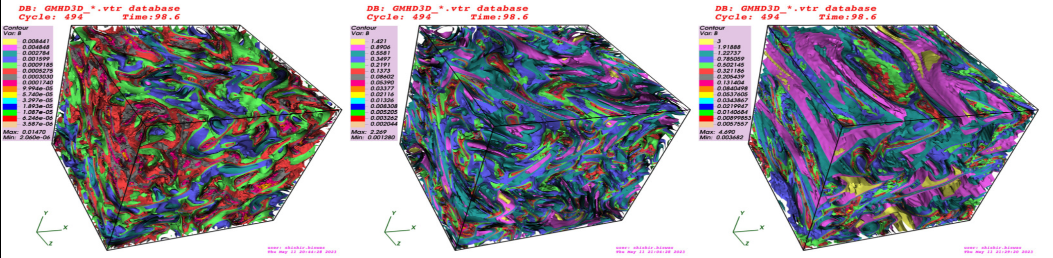


- The magnetic field structures increase in size when P_m $(= \frac{R_m}{R_e})$ decreases in both the kinematic and saturation stages.

Non-Helical Dynamo at Different P_m: Magnetic field dynamics



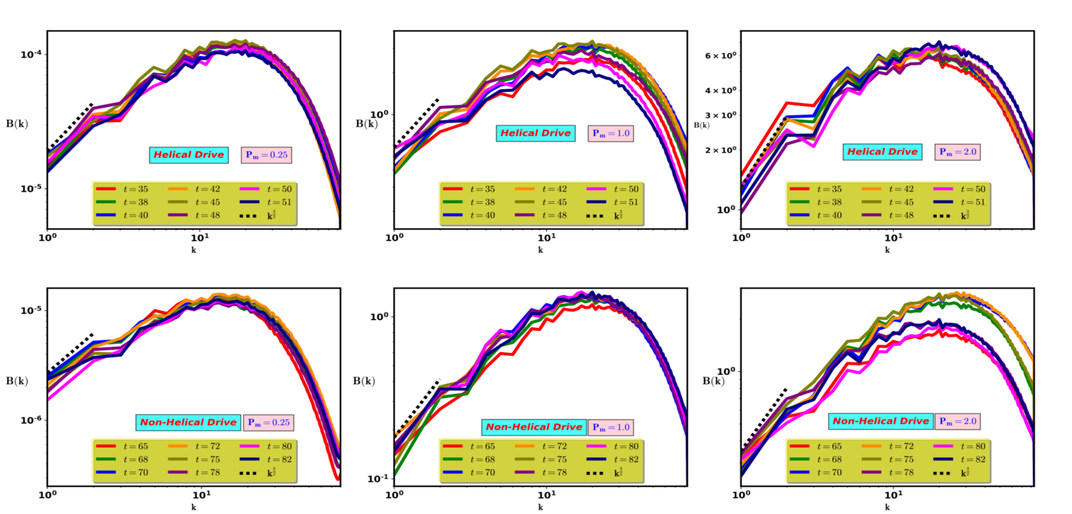
$P_m = 0.25$ $P_m = 1.0$ $P_m = 2.0$



- In the kinematic stage, the magnetic field structures increase in size as Pm $(= \frac{R_m}{R_e})$ decreases. However, in the saturated stage, the opposite occurs.

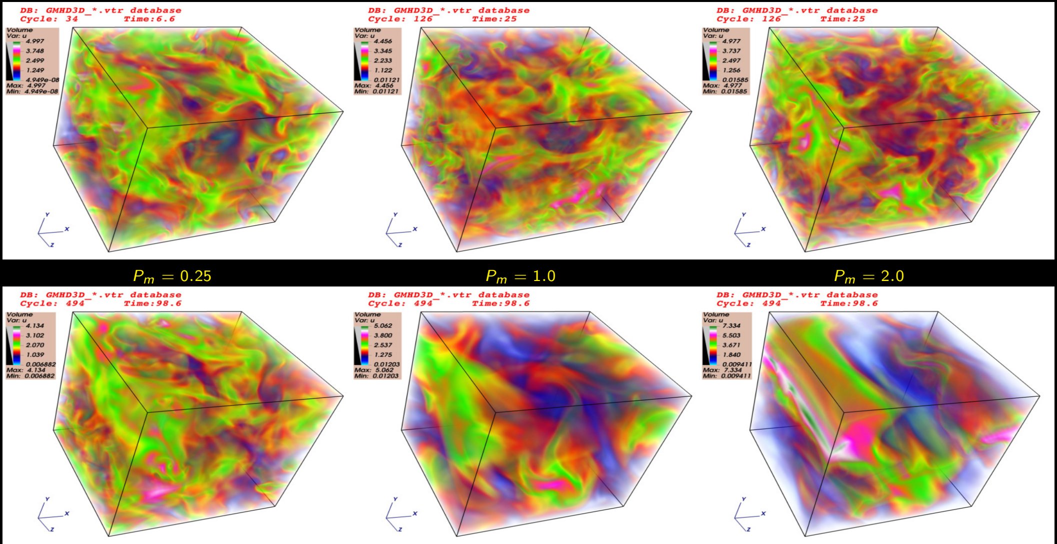
Spectral Calculation

- Small scale dynamos (SSD) are identified for $0.25 \leq P_m \leq 2.0$.



- Magnetic energy spectra follows well known Kazantsev's Scaling ($B(k) \propto k^{\frac{3}{2}}$).

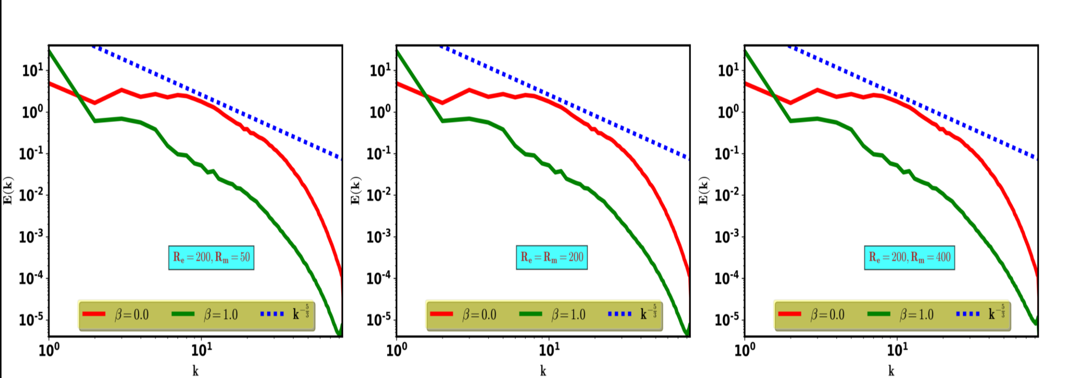
Non-Helical Dynamo at Different P_m: Velocity Field Dynamics



- Velocity field structures increase in size as P_m increases during the saturated stage. Therefore, the velocity field dynamics precisely mimic the magnetic field dynamics.

Spectral Calculation: Kinetic Energy Spectra

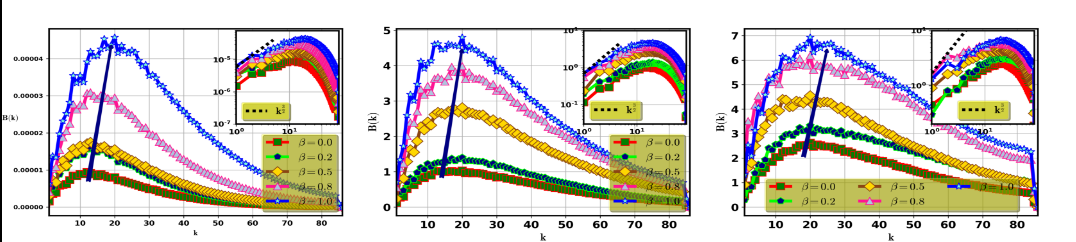
- Kinetic energy spectra has been calculated for both the helical and non-helical driven dynamos.



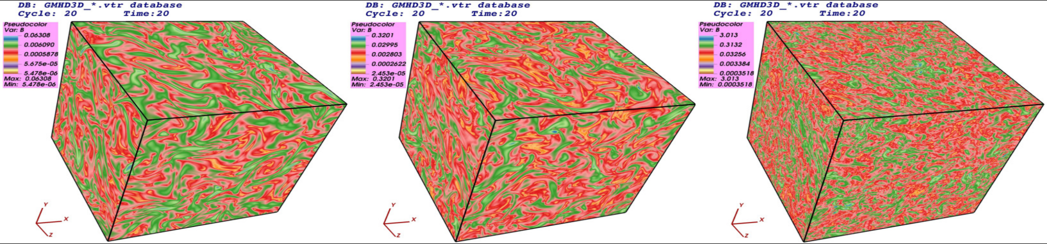
$P_m = 0.25$ $P_m = 1.0$ $P_m = 2.0$

- Kinetic energy spectra follows Kolmogrov's Scaling ($E(k) \propto k^{-\frac{5}{3}}$).
- This scaling is independent of nature of driving available in the system.

- Small scale dynamos (SSD) are seen to be affected by helicity injection.



$P_m = 0.25$ $P_m = 1.0$ $P_m = 2.0$



$\beta = 0.0$ $\beta = 0.5$ $\beta = 1.0$

S. Biswas & R. Ganesh, [To be Submitted (2024)]

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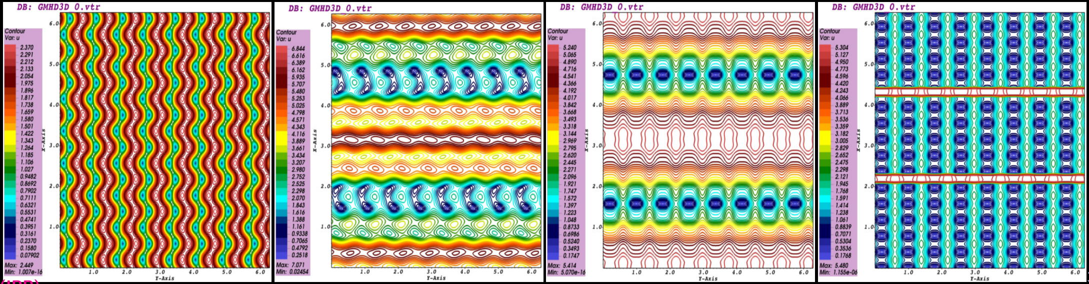
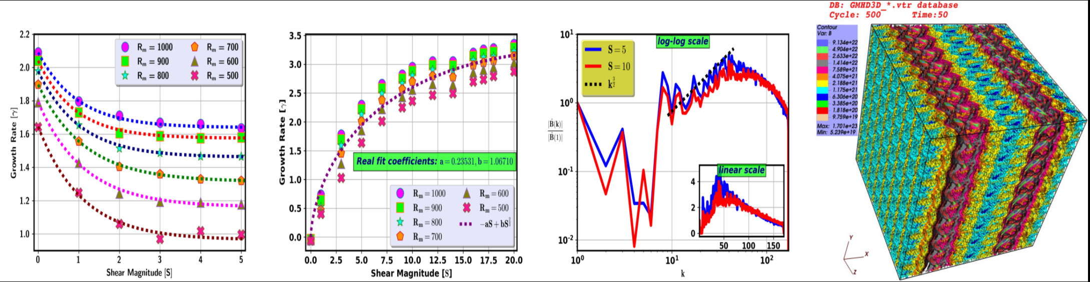
Effect of flow shear on the onset of dynamos

- Shear flows often coexist in astrophysical conditions and the role of flow shear on the onset of dynamo is only beginning to be investigated.
- To control magnetic fluctuations, Tobias & Cattaneo (2013) [S. M. Tobias and F. Cattaneo, Nature 497, 463 (2013)] proposed that, the **"suppression principle"** could be mediated, for instance, by shear and helical structure of the flows.
- So the impact of shear on helical and non-helical flows are interesting to explore in the context of dynamo action.
- To investigate the role of shear flows in the context of dynamo action, we introduce a periodic, large-scale shear flow of the form, $\vec{u_s} = (0, S\cos(k_s x), 0)$ with YM flow.



Helical & Non-Helical flow with shear

- It has been shown that the presence of a flow shear reduces the efficiency of small-scale dynamo action for Helical base flows.
- Unlike fully helical flows, it has been observed that, for EPI2D non-helical flows, the small-scale dynamo action (SSD) increases as the shear flow strength (S) increases. The spectral diagnostics also are found to be in agreement with the observation.
- A generalized algebraic scaling $[\gamma(S) = -aS + bS^{\frac{2}{3}}]$ for the magnetic energy growth rate (γ) as a function of the shear flow strength (S) has been obtained.



Suppression of SSD Enhancement of SSD Spectra (Evidence of SSD) Iso-B surface for the same

M. Proctor, [JFM, 697, 504-510(2012)] S. Biswas & R. Ganesh, [POP, 30, 112902 (2023)]

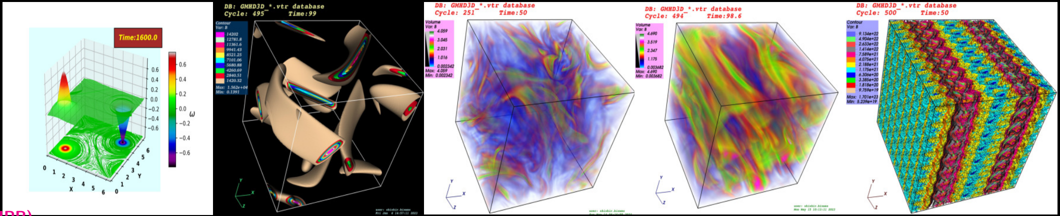
Talk outline

1. Governing Equations.
2. Numerical Solver and its Development & Benchmarkings.
3. Role of kinetic (fluid) helicity in Dynamo instability.
4. Self-Consistent Dynamo with Helical & Non-Helical Drive.
5. Effect of flow shear on the onset of dynamos.
6. **Conclusions.**
7. Future Directions.

Conclusions

- We have recently upgraded an existing hydrodynamic and magnetohydrodynamic solver (for fluid and plasma turbulence) to multi-node, multi-card GPU architecture [GMHD3D] for better performances using OpenACC and OpenMPI and achieved linear speedup across 32 nos of P100 GPU cards.

  - Using this newly developed solver, we have predicted long-time relaxation of Navier-Stokes turbulence [1st Fig from left].
  - A numerical observation of a partially recurrent pattern has been observed in an initial EPI two-dimensional (EPI-2D) flow.
  - The effect of fluid helicity on dynamo activity has been studied in both linear (Kinematic) [2nd] and non-linear (Self-Consistent) [3rd & 4th] limits.
  - The impact of flow shear on the onset of kinematic dynamos is also investigated [5th].



Future Directions

1. Effect of scale separation between system length scale and energy injection scale! Does the helical drive generate LSD?
2. Effect of non-linearity in shear driven dynamos! Helical and Non-helical flow with shear as a drive.
3. Shear driven Dynamos in compressible limit! Interaction of Dynamo waves with Sound wave.
4. Effect of Hall MHD on Dynamo instability! Does the consideration of ion skin depth impacts the fate of dynamo instability?
5. Dynamo action for a weakly ionized medium (ISM plasma) using A two fluid approach. Damping effect on dynamo instability due to ion-neutral collision.
6. MRI Dynamo problem! Differential rotation $\rightsquigarrow$ Helical motion $\rightsquigarrow$ turbulence due to MRI $\rightsquigarrow$ Magnetic field amplification.
7. **Scaling efficiency of GMHD3D requires to be improved to address the above said problems.**

List of Publications

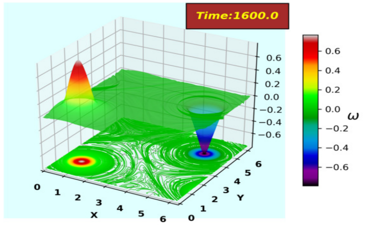
1. S. Biswas & R. Ganesh, **Physics of Fluids 34, 065101 (2022)** [**Selected as a Featured article**].
2. S. Biswas & R. Ganesh, **Phys. Scr. 98 075607 (2023)**.
3. S. Biswas & R. Ganesh, **Phys. Plasmas 30, 112902 (2023)**.
4. S. Biswas & R. Ganesh, **Computers and Fluids 272 106207 (2024)**.
5. S. Biswas, R. Mukherjee, N. V. Vydyanathan, & R. Ganesh, **Conference Proceedings of 28th IAEA Fusion Energy Conference, (2020)**.
6. S. Biswas & R. Ganesh, "Unveiling the Role of Helical and Non-Helical Drives on the evolution of Self-Consistent Dynamos"; **To be Submitted (2024)**.

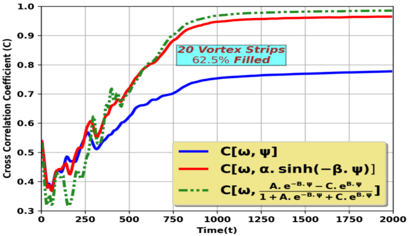
Acknowledgment

1. **ANTYA:** HPC facility at IPR, India.
2. **Prof. Abhijit Sen**, IPR, India.
3. **Dr. Rupak Mukherjee**, CUS, Gangtok, Sikkim, India.
4. **N. Vydyanathan**, Bengaluru India & **B. K. Sharma** at NVIDIA, Bengaluru, India.

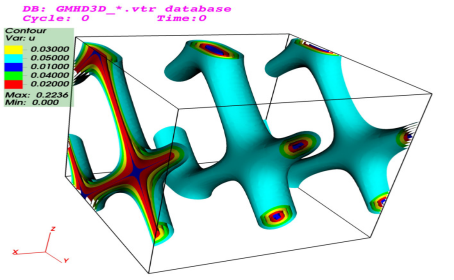
Thank You!

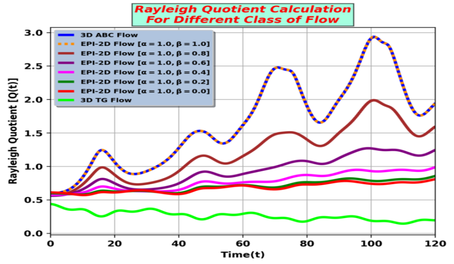
- Using newly developed GPU based hydrodynamic solver at high resolution and at very high Reynolds number, we investigate the systematic deviation of final relaxed state of 2-dimensional incompressible decaying NS turbulence from the sinh-Poisson model predicted by statistical mechanical theory of point vortices.
- We have also shown that the late time states agree quantitatively with the predictions of KMRS theory for increasing values of initial total circulation of either kind. [S. Biswas & R. Ganesh, Physics of Fluids 34, 065101 (2022) [Selected as a Featured article]].





- It has been discovered that the recurring TG flow has bounded Rayleigh Quotient [Q(t)], whereas the 3D ABC flow, which is non-recurring, has an unbounded Rayleigh Quotient [Q(t)] with time.
- Using newly developed multi-GPU MHD slover (GMHD3D), we have identified that the EPI-2D flow, which is partially recurring in nature is found to show semi bounded Rayleigh Quotient (i.e. stayed between ABC flow and TG flow) with time.
  [S. Biswas, R. Mukherjee, R. Ganesh, & A. Sen, [To be submitted soon]]





Parallel Scaling

The computational time of a code should ideally scale as, $T_c = \frac{N^3 logN}{N_p^{\gamma}}$. Here, N_p is the number of GPUs and N is the number of grid points in each direction and γ is fitting exponent.

$\rightsquigarrow$ Now Strong scaling implies, $\boxed{T_c = CN_p^{-\gamma}}$ $\Rightarrow$ the speedup for a fixed problem size with respect to the number of processors, and is governed by **Amdahl's law**.

$\rightsquigarrow$ The parallelization and performance of any code can be analyzed using two parameters,

1. SpeedUp (S): SpeedUp is defined as, $S = \frac{T_c^0}{T_c^N}$. Here, T_c^0 is the reference simulation time for minimum number of resources (N_0) and T_c^N is the reference simulation time for N number of resources.
2. Scaling efficiency (W): W is defind as, $\frac{T_c^0 N_0}{T_c^N N}$.