FAST-ION-DRIVEN VERTICAL DISPLACEMENT OSCILLATORY MODES

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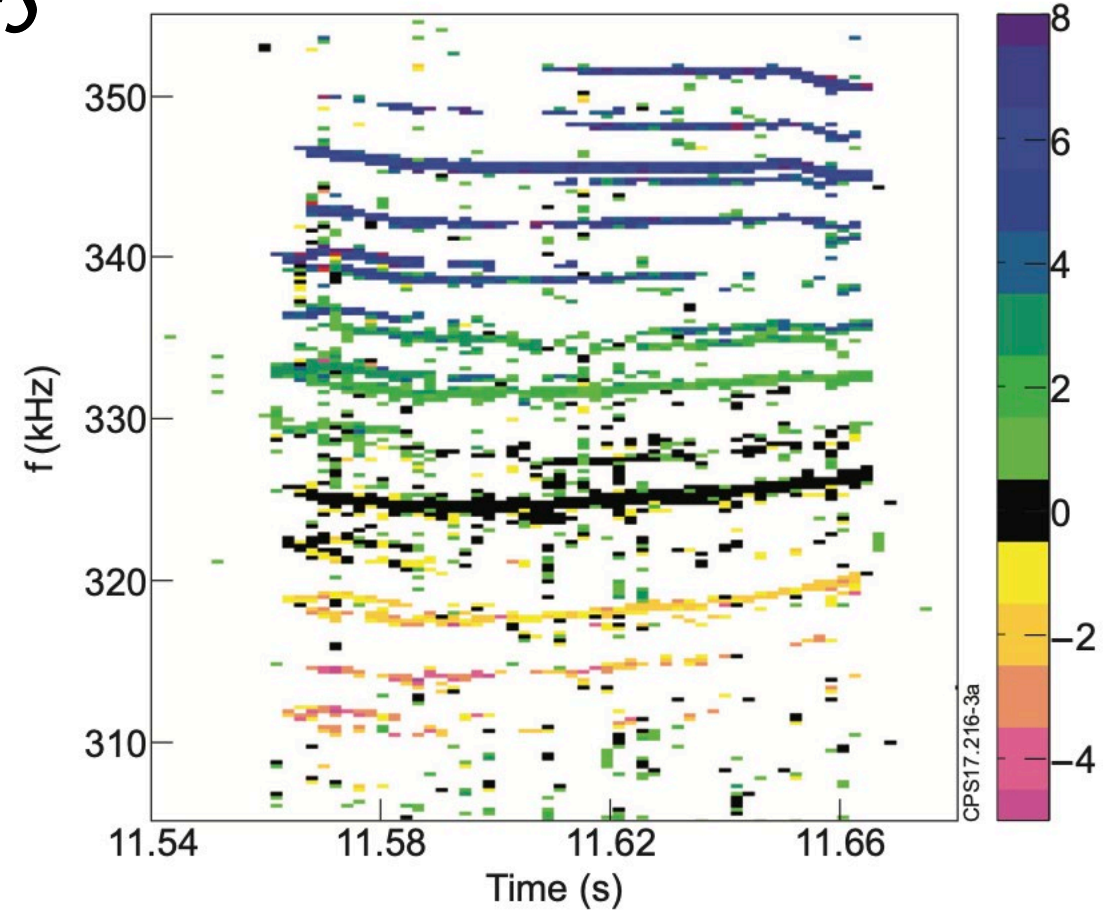
VERTICAL MODES $and\ n = 0$ *MODES IN THE ALFVÉN FREQUENCY RANGE IN JET DISCHARGES*

Axisymmetric mode (n=0, m=1) in shaped toroidal plasmas is a global plasma mode, normally associated with VDEs.

Passive feedback is required in order to stabilize the Ideal vertical instability. G. Laval, R. Pellat, and J. L. Soule, Phys. Fluids 17, 835 (1974).

Feedback stabilized n=0 mode presents an Alfvénic oscillation frequency. As a result, a resonant interaction with energetic particles can lead to the destabilization of the mode.

Experimental observation of n=0 Alfvén oscillations in recent JET experiments (H. J. C. Oliver et al. 2017 PoP, V.G. Kiptily et al. Nucl. Fusion 61, 114006 (2021), RT22-09 experimental campaign March 2023).



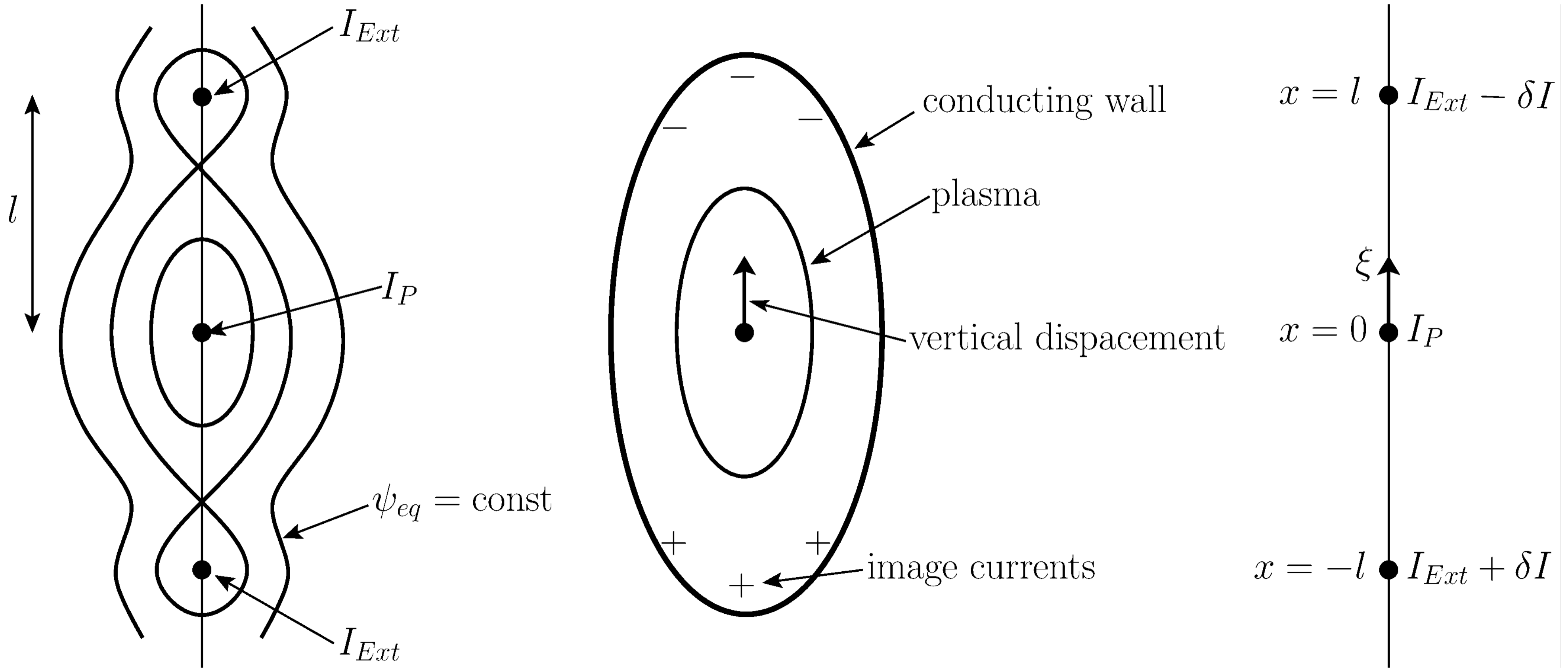
OUTLINE

A. Axysimmetric modes
   – Analytic Ideal MHD – Hybrid kinetic MHD

B. NIMROD JET simulations
   – Stable oscillations with realistic geometry and profiles

C. Conclusions

AN HEURISTIC MODEL CAN DESCRIBE THE VERTICAL STABILITY AND THE MODE MECHANISM



A

REDUCED MHD MODEL WITH PCS FOR ENERGETIC IONS AND SIMPLIFIED EQUILIBRIUM

$$\overline{\boldsymbol{P}}_h = p_{\perp h}\overline{\boldsymbol{I}} + (p_{\|} - p_{\perp})_h \boldsymbol{e}_{\|}\boldsymbol{e}_{\|}$$

Energetic ions pressure tensor

$$\boldsymbol{B} = \boldsymbol{e}_\phi \times \nabla\psi + B_\phi \boldsymbol{e}_\phi \quad , \quad \boldsymbol{v} = \boldsymbol{e}_\phi \times \nabla\varphi + v_\phi \boldsymbol{e}_\phi$$

$$\frac{\partial\psi}{\partial t} + [\varphi, \psi] = 0$$

$$\partial_t \nabla\cdot(\varrho\nabla\varphi) + \left[\varrho, (\nabla\varphi)^2\right]/2 + U[\varphi, \varrho] + [\varphi, U] = [\psi, J] - \boldsymbol{e}_\phi \cdot \nabla\times\nabla\cdot\overline{\boldsymbol{P}}_h$$

where $U = \nabla^2\varphi$, $J = \nabla^2\psi$ and $[\eta, \chi] = \boldsymbol{e}_\phi \cdot \nabla\eta \times \nabla\chi$

Straight Tokamak approximation
No equilibrium plasma flows
Uniform current and mass density up to plasma elliptical boundary at $\mu = \mu_b$
(μ, θ) are the standard elliptical coordinates (x= Asinh(μ)cos (ϑ); y = Acosh(μ) sin(ϑ)).

$$J_{eq} = J_0 H(\mu_b - \mu)$$

$$\rho_{eq} = \rho_0 H(\mu_b - \mu)$$

A

WE CAN STUDY THE DISPERSION RELATION OBTAINED WITH THE QUADRATIC FORMS FORMALISM

Details on equilibrium solution in R. Gajewski, Phys. Fluids 15 70 (1972) and F. Porcelli and A. Yolbarsop, Phys. Plasmas 26, 054501 (2019)

Consider perturbation:

$\psi(\mu, \vartheta, t) = \psi_{eq}(\mu, \vartheta) + \tilde{\psi}(\mu, \vartheta)\, e^{\gamma t}$ and $\varphi(\mu, \vartheta, t) = \tilde{\varphi}(\mu, \vartheta)\, e^{\gamma t}$

From the momentum equation:

$$\gamma \nabla \cdot (\rho_{eq} \nabla \tilde{\varphi}) = [\tilde{\psi}, J_{eq}] + [\psi_{eq}, \tilde{J}] - \boldsymbol{e}_\phi \cdot \nabla \times \nabla \cdot \tilde{\bar{\boldsymbol{P}}}_h$$

It is possible to derive standard quadratic forms:

$$-\gamma^2 \int d^3x \rho \boldsymbol{\xi} \cdot \boldsymbol{\xi}^* = -\frac{1}{2} \int d^3x \boldsymbol{\xi}^* \cdot F(\boldsymbol{\xi}) + \frac{1}{2} \int d^3x \boldsymbol{\xi}^* \cdot \nabla \cdot \tilde{\bar{\boldsymbol{P}}}_h \longrightarrow \boxed{-\gamma^2 \delta I = \delta W_{MHD} + \delta W_{fast}}$$

A

WITH REDUCED IDEAL MHD WE OBTAIN THE DISPERSION RELATION FOR A SIMPLIFIED EQUILIBRIUM: RIGID SHIFT MODE STRUCTURE

Passive feedback → confocal resistive wall ⟶ $\delta W_{MHD} \rightarrow \delta W_{MHD}(\gamma)$

Cubic dispersion relation:

$$\gamma^3 + \gamma^2 \frac{1}{\tau_\eta}\frac{1}{1 - \hat{e}_0 D} + \gamma\omega_0^2 + \omega_0^2 \frac{1}{\tau_\eta}\frac{1}{1-D} = 0$$

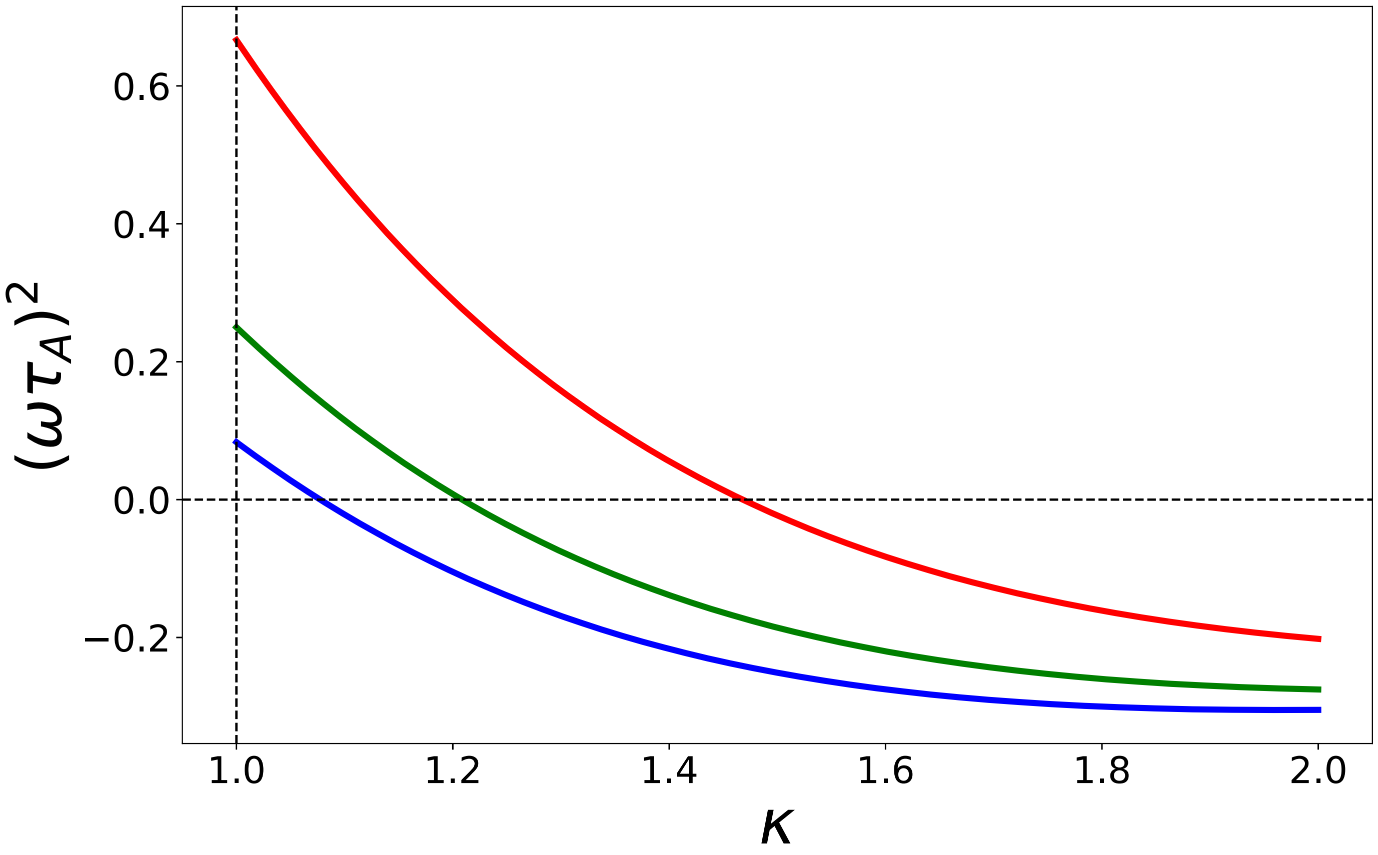
For plasma density and current density uniform inside elliptical plasma boundary

With τ_η resistive wall time, $\hat{e}_0 = e_0 b/(a+b)$, $e_0 = (b^2 - a^2)/(b^2+a^2)$, and geometrical parameter D(b/b_w, $\kappa = b/a$)

$$(\omega_0\tau_A)^2 = \left(\frac{D-1}{1 - \hat{e}_0 D}\right)(1 - e_0)\left(1 + e_0 - \sqrt{1-e_0^2}\right)$$

A

FOR AN IDEAL WALL, THE DISPERSION RELATION REDUCES TO $\omega^2 = \omega_0^2$



$$(\omega_0\tau_A)^2 = \left(\frac{D-1}{1-\hat{e}_0 D}\right)(1-e_0)\left(1+e_0-\sqrt{1-e_0^2}\right)$$

$D = 1$ Marginal stability

$D < 1$ Ideal vertical instability

$D > 1$ Oscillatory modes

$\rightarrow b/b_w = 1/5$

$\rightarrow b/b_w = 1/3$

$\rightarrow b/b_w = 1/2$

A

THE WALL POSITION DETERMINES THE STABILITY AND OSCILLATION FREQUENCY

$$D = \frac{b^2 + a^2}{(b-a)^2}\frac{b_w - a_w}{b_w} = D(\kappa, \frac{b}{b_w})$$

$D = 1$ Marginal stability

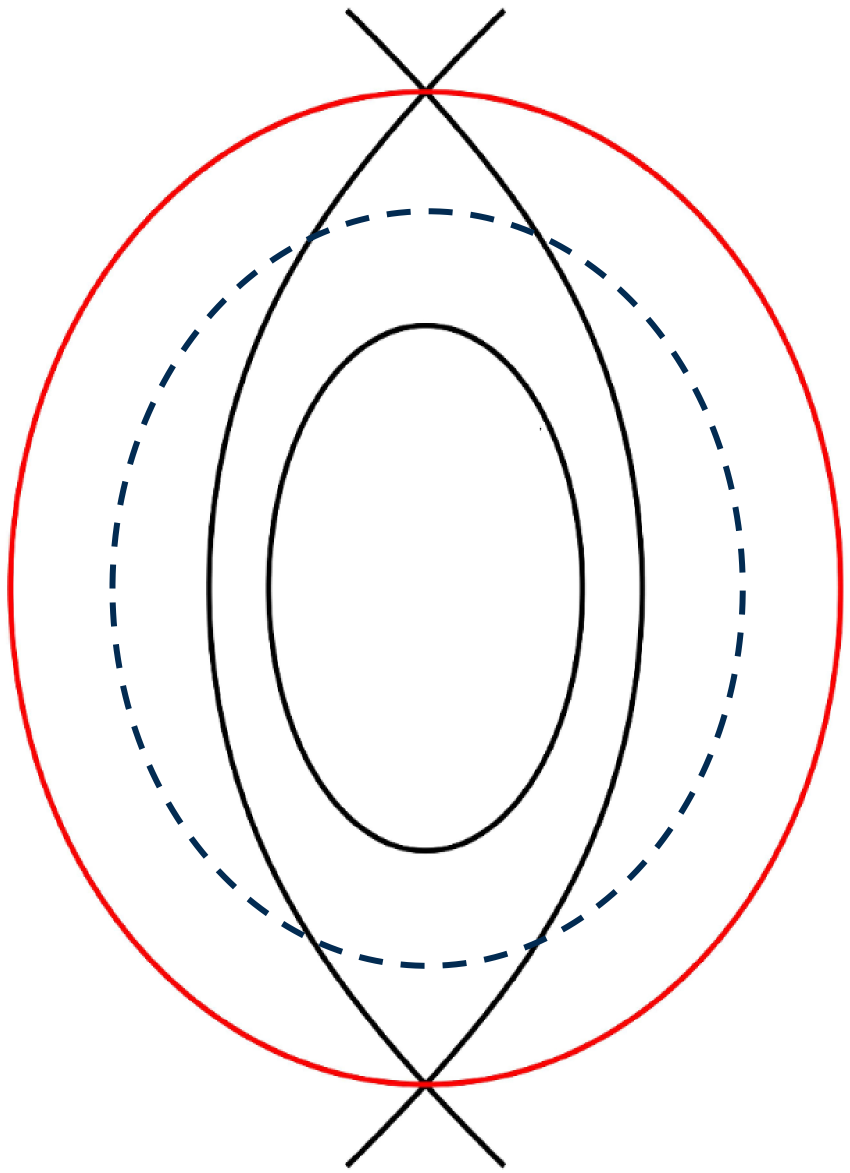
$D < 1$ Ideal vertical instability

$D > 1$ Oscillatory modes

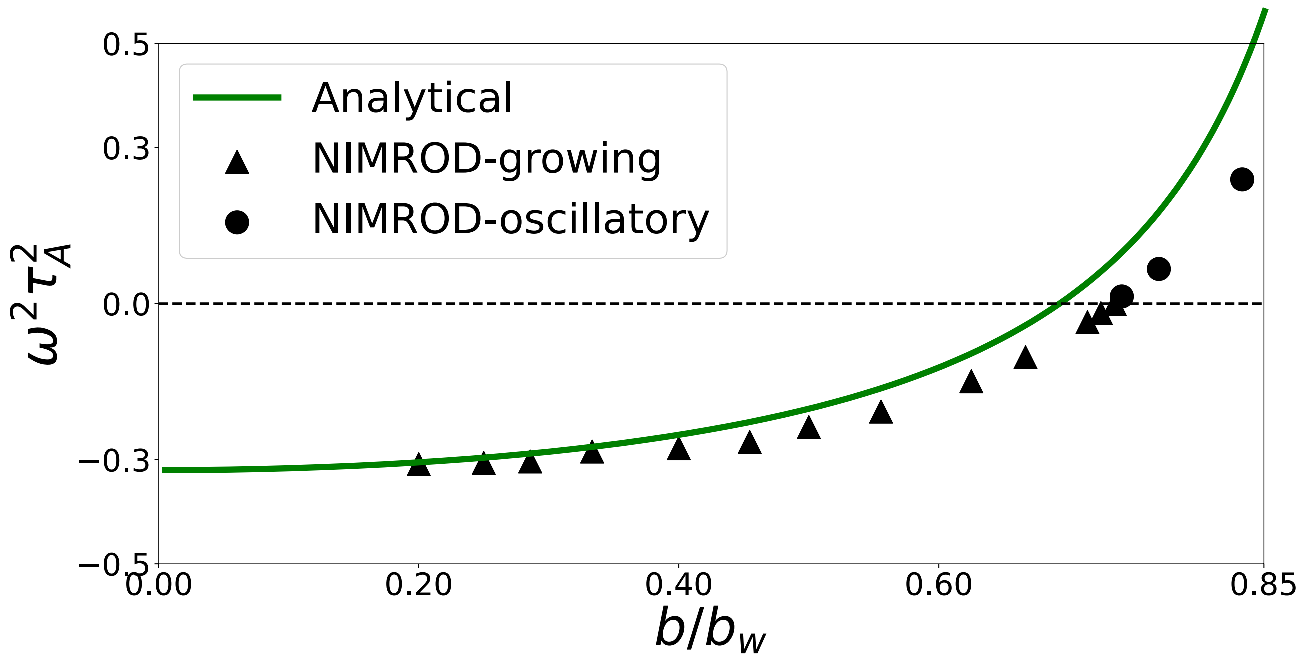
In relevant conditions ($D > 1$), the solutions of the cubic dispersion relation (resistive wall) are:

$$\omega = \pm\omega_0 - i\underbrace{\boxed{\frac{1}{2\tau_\eta}\frac{D(1-\hat{e}_0)}{(D-1)(1-\hat{e}_0 D)}}}_{\gamma_\eta}$$

$$\longrightarrow \quad \gamma = \frac{1}{(D-1)\tau_\eta}$$

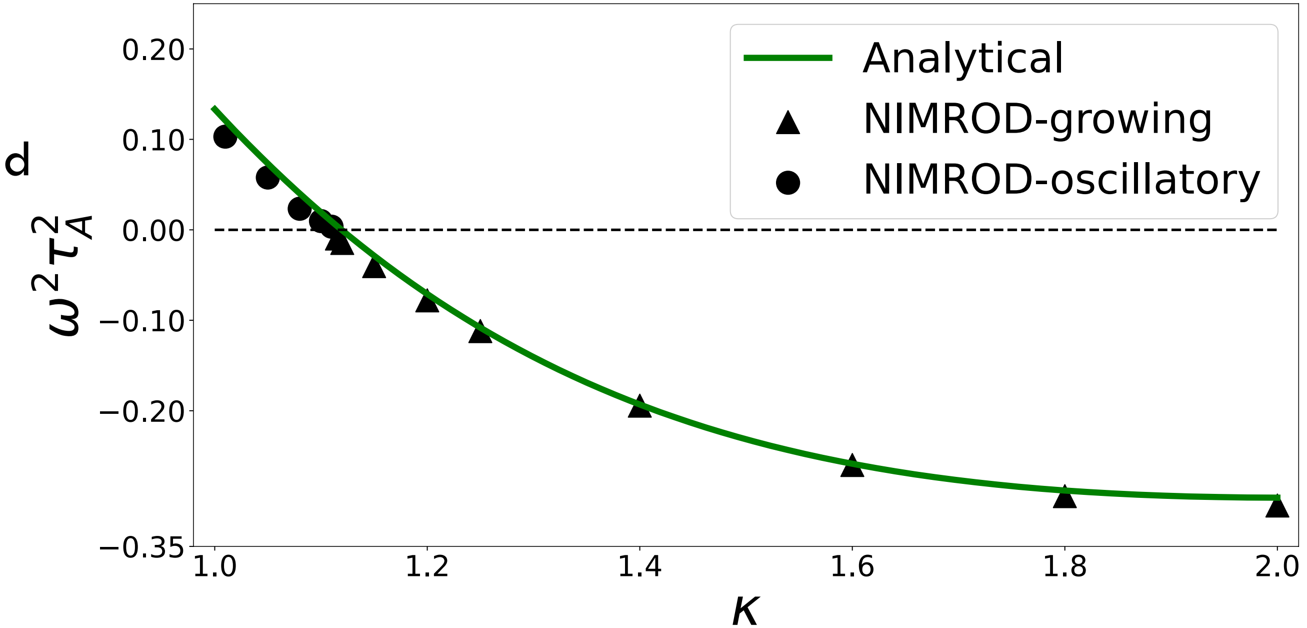


A STRAIGHT TOKAMAK NIMROD SIMULATION RESULTS SHOW VERY GOOD AGREEMENT WITH ANALYTIC THEORY



Main differences between simulation and analytic theory:

- Wall shape rectangular instead of confocal ellipse
  wall further away with same b/b_w
- Low density/High resistivity «halo» plasma instead of vacuum



A

CONSIDER THE FAST IONS EFFECTS PERTURBATIVELY IN THE EXPANSION PARAMETER $\epsilon_h = n_h/n_p \ll 1$

$$-\gamma^2 \delta I = \delta W_{MHD} + \delta W_{fast}$$

- Zeroth order in $\epsilon_h \rightarrow$ Ideal MHD solution
- First order in $\epsilon_h \rightarrow$ neglect $O(\epsilon_h)$ correction to the oscillation frequency and keep only contributions to the imaginary part of δW_{fast}

Dispersion relation: $\omega = \pm\omega_0 - i2\omega_0\gamma_\eta + i\delta\widehat{W}_h$

Assume $\delta\widehat{W}_h \sim Im\left(\delta\widehat{W}_{fast}\right) = i\omega_0^2\lambda_h + O(\gamma^2/\omega_0^2) \rightarrow \omega = \omega_0 + i\gamma_{tot}$ with $\gamma_{tot} = -\gamma_\eta + \frac{1}{2}\omega_0\lambda_h$

We consider the competition between the damping rate due to wall resistivity and the fast ion drive

$$\lambda_h > \frac{\gamma_\eta}{2\omega_0}$$

A | FAST IONS WITH MeV RANGE ENERGY MAY DESTABILIZE THE OSCILLATORY MODE

$$\delta W_{fast} = -\frac{1}{2}\int d^3x d^3v\ \tilde{L}^*\ \tilde{f}_h$$ Where $\tilde{L}$ is the perturbed Lagrangian

$$\delta W_{fast} = \cancel{\delta W_{h,ad}} + \delta W_{h,nad}(\omega)$$ The "adiabatic" part of δW_{fast} is real → neglect

$$\delta W_{fast} = \zeta_1 \sum_{\sigma} \int dP_\varphi dE d\mu\, \tau_{b,t} \left(\omega \cancel{\frac{\partial F}{\partial E} - n} \frac{\partial F}{\partial P_\varphi}\right) \sum_{p=-\infty}^{+\infty} \frac{|\Upsilon_p|^2}{\omega + p\omega_{b,t} \cancel{+ n\langle\varphi\rangle}}$$

$$\lambda_h = \zeta_2 \sum_{p=1}^{+\infty} \int dr d\Lambda r \frac{(v_p^*)^3}{h_\Omega} \frac{\partial F}{\partial \mathcal{E}}\Bigg|_{v=v_p^*} |\Upsilon_p|^2 \Bigg|_{v=v_p^*}$$

$$\left.\frac{\partial F}{\partial E}\right|_\mu = \left.\frac{\partial F}{\partial E}\right|_\Lambda - \left.\frac{\Lambda}{\mathrm{E}}\frac{\partial F}{\partial \Lambda}\right|_{\mathrm{E}} > 0$$

required for mode destabilization ($\lambda_h > 0$)

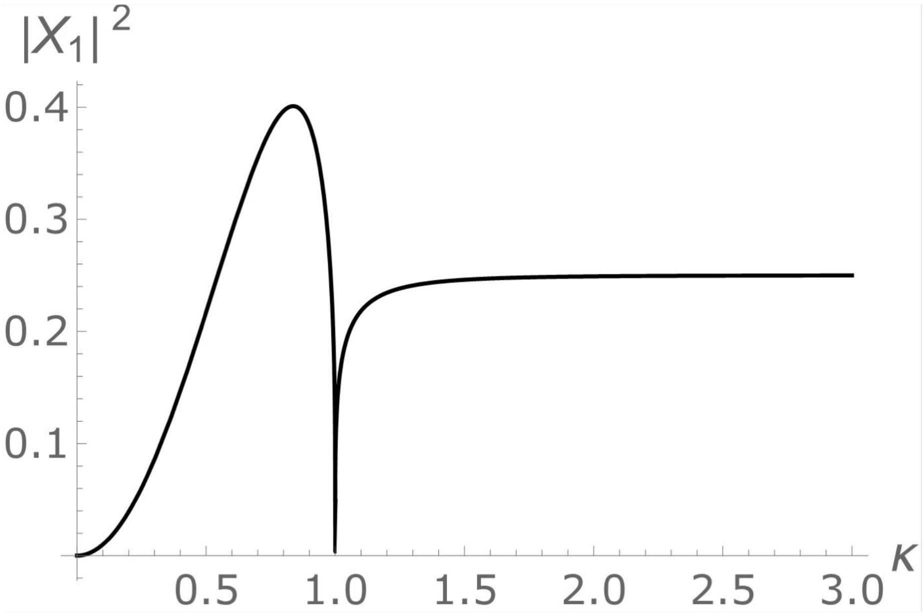
See Porcelli, Stankiewicz, Kerner and Berk 1994 PoP. *and T. Barberis, et al. 2022, Nucl. Fusion* ***62*** *064002*

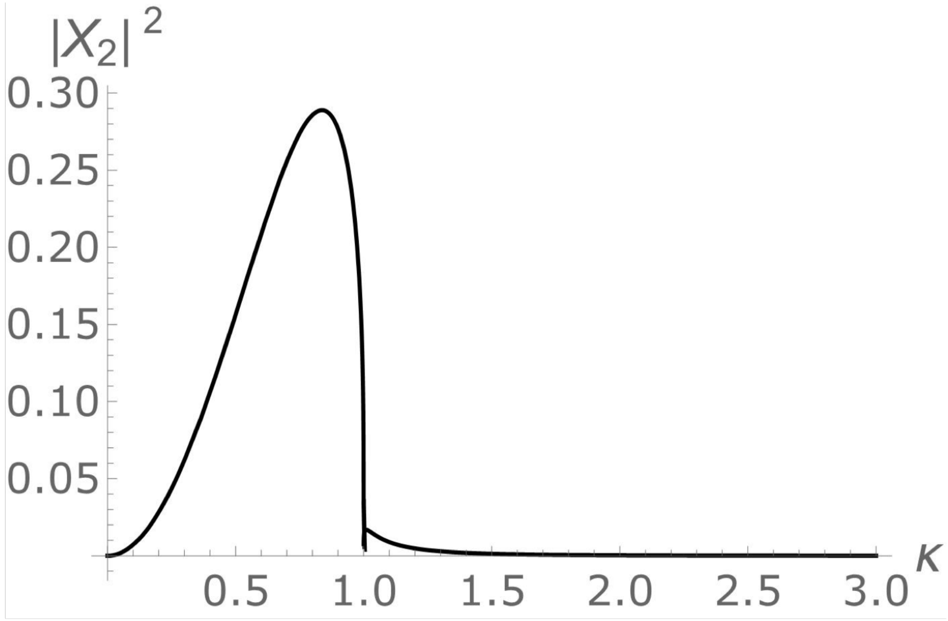
C. FOURIER COEFFICIENTS

TRAPPED AND PASSING PARTICLES CONTRIBUTIONS

$$\Upsilon_p(\mathcal{E}, \mu, P_\varphi) = \oint \frac{d\tau}{\tau_{b/t}} \tilde{L} exp(ip\omega_\Omega \tau), \quad \tilde{L} \approx \epsilon^2 \mathcal{E}(2-\Lambda)\xi \sin(\theta)/r$$

Assuming a small radial excursion, we are interested in the quantity: $X_p = \langle \sin(\theta) \exp(ip\omega_\Omega \tau) \rangle$

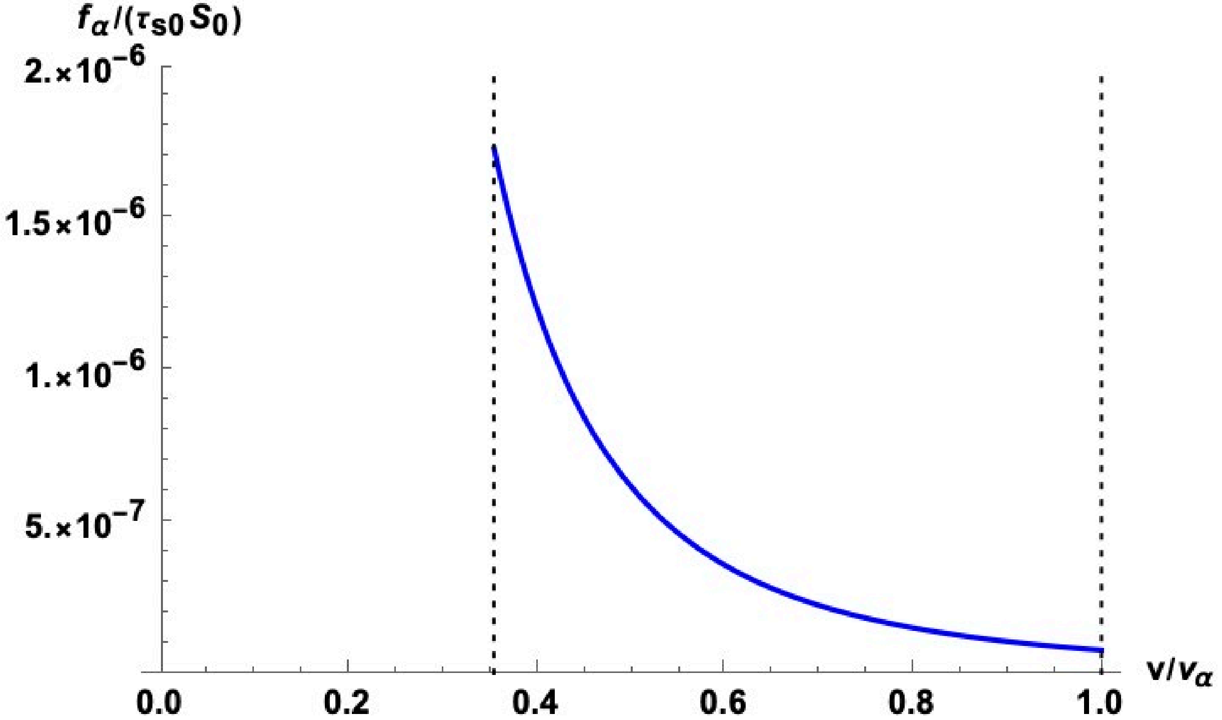


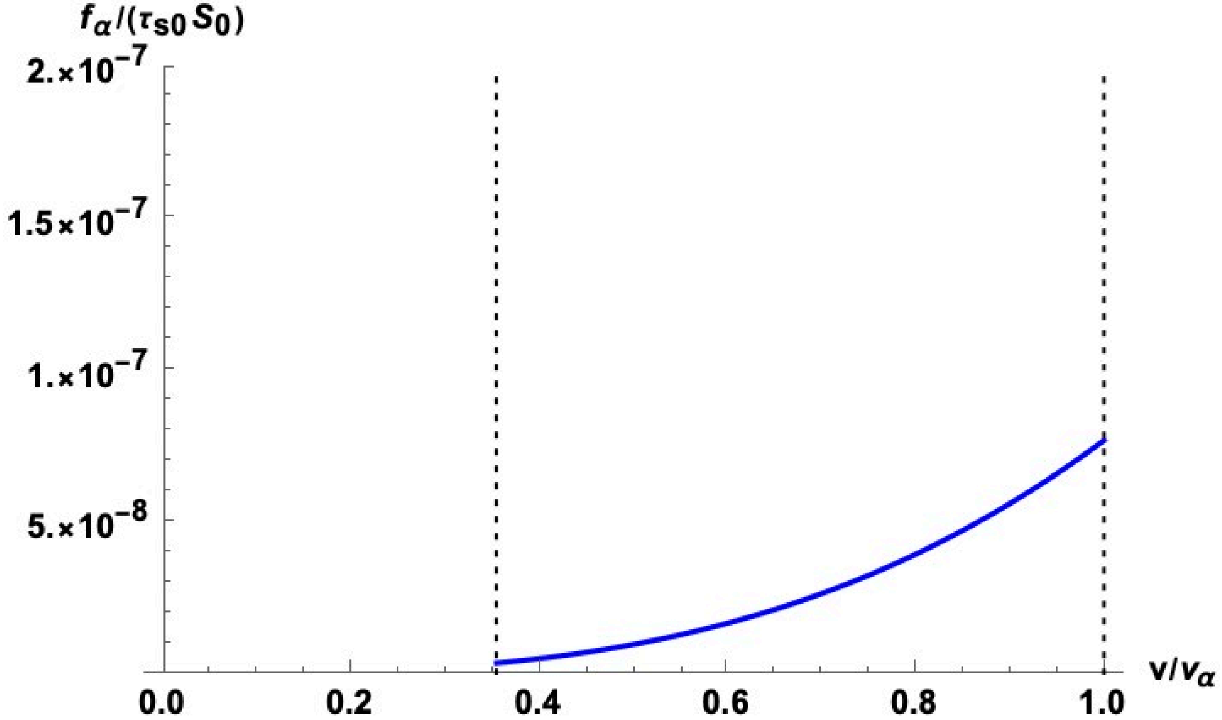


$\kappa^2 = 1/2 + (1-\Lambda)/(2\epsilon)$, (trapped particles for $0 < \kappa < 1$ and passing particles for $\kappa > 1$)

A | FOR AN ISOTROPIC DISTRIBUTION FUNCTION WITH $\partial \mathrm{F}/\partial \Lambda = 0$ A LOSS TERM CAN LEAD TO $\partial \mathrm{F}/\partial E > 0$

- $\frac{\partial v^3 F}{\partial t} - \frac{v}{\tau_s}\frac{\partial v^3 F}{\partial v} + \nu_\alpha v^3 F = v^3 S_\alpha$ For $v > v_c$

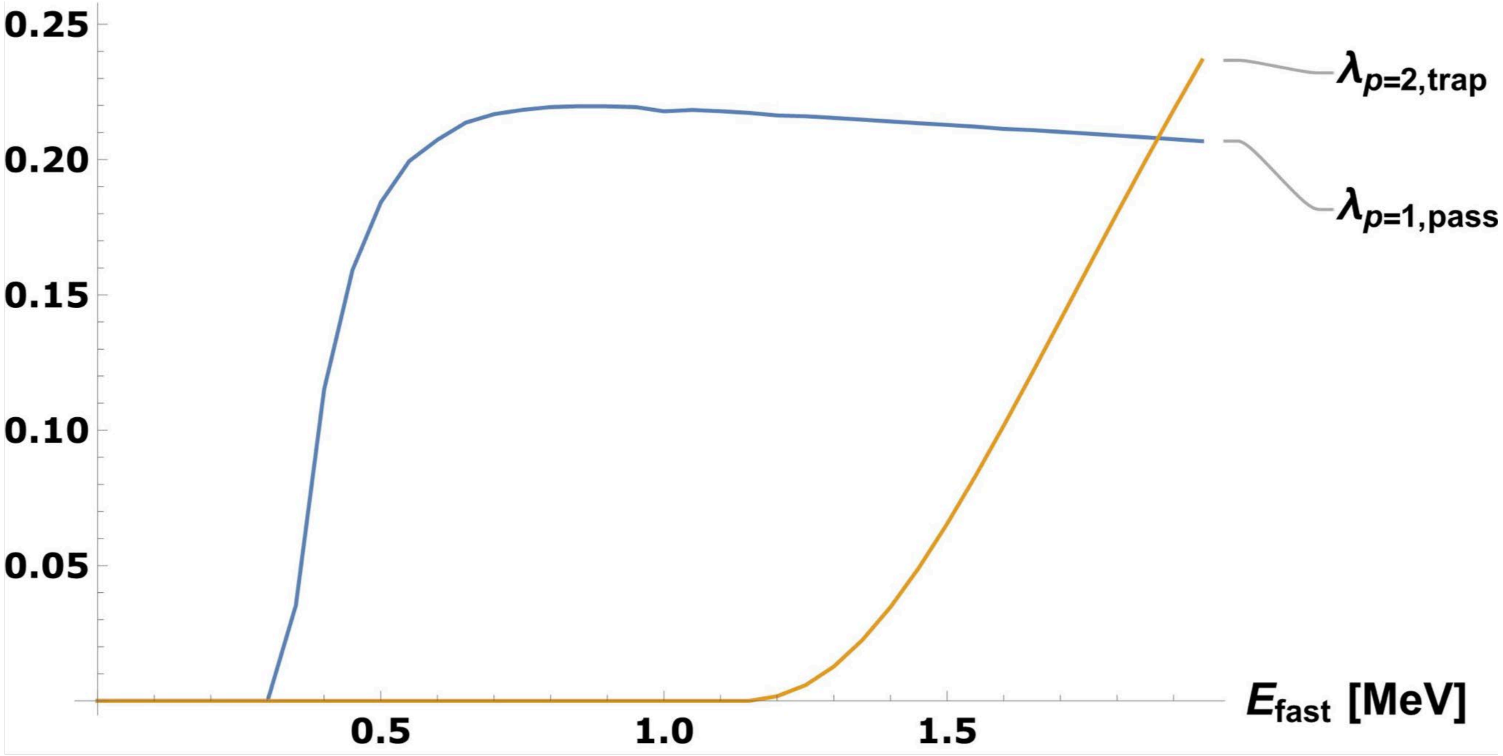




A FOR BIRTH ENERGY $\sim 1MeV$ THE MAIN CONTRIBUTION COMES FROM PASSING PARTICLES, TRAPPED PARTICLES DOMINATE AT $E_b \sim 2MeV$

For the simple slowing down distribution with losses, the coefficient λ_h can be written as:

$$\lambda_h = \left(\frac{n_h}{n_p}\right)\left(\frac{m_h}{m_c}\right)\left(q^2\frac{a^2}{R_0^2}\right)\sum_{p,\Omega}\lambda_{p,\Omega}$$



First harmonic resonance for trapped particles requires energies higher than $5MeV$

→ For $E_b = 1.5\ MeV$ threshold for destabilization $n_h/n_p \sim 10^{-2} - 10^{-3}$

Contribution from trapped and passing Deuterium considering: $a = 0.9m, R_0 = 3m, f_0 \sim 300kHz, \nu_\alpha = 6/\tau_s$

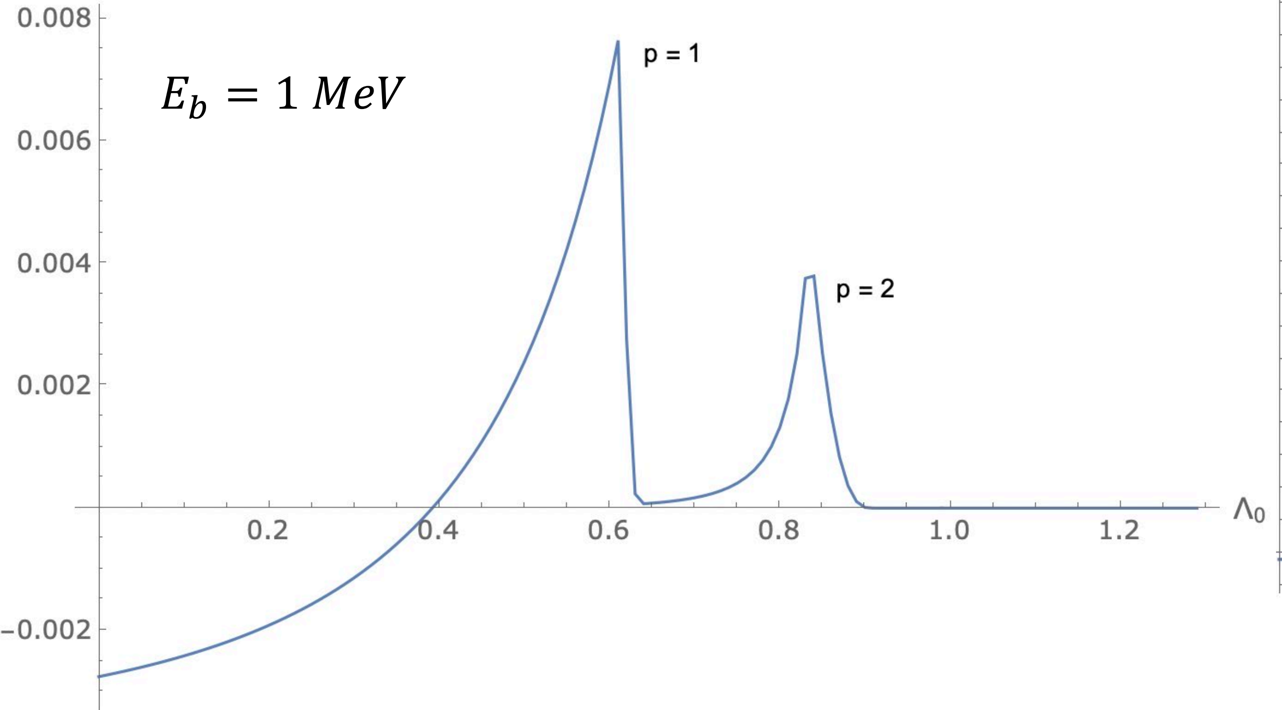
ALSO A SINGLE PITCH ANGLE SLOWING DOWN DISTRIBUTION FUNCTION CAN PROVIDE THE FREE ENERGY FOR MODE DESTABILIZATION

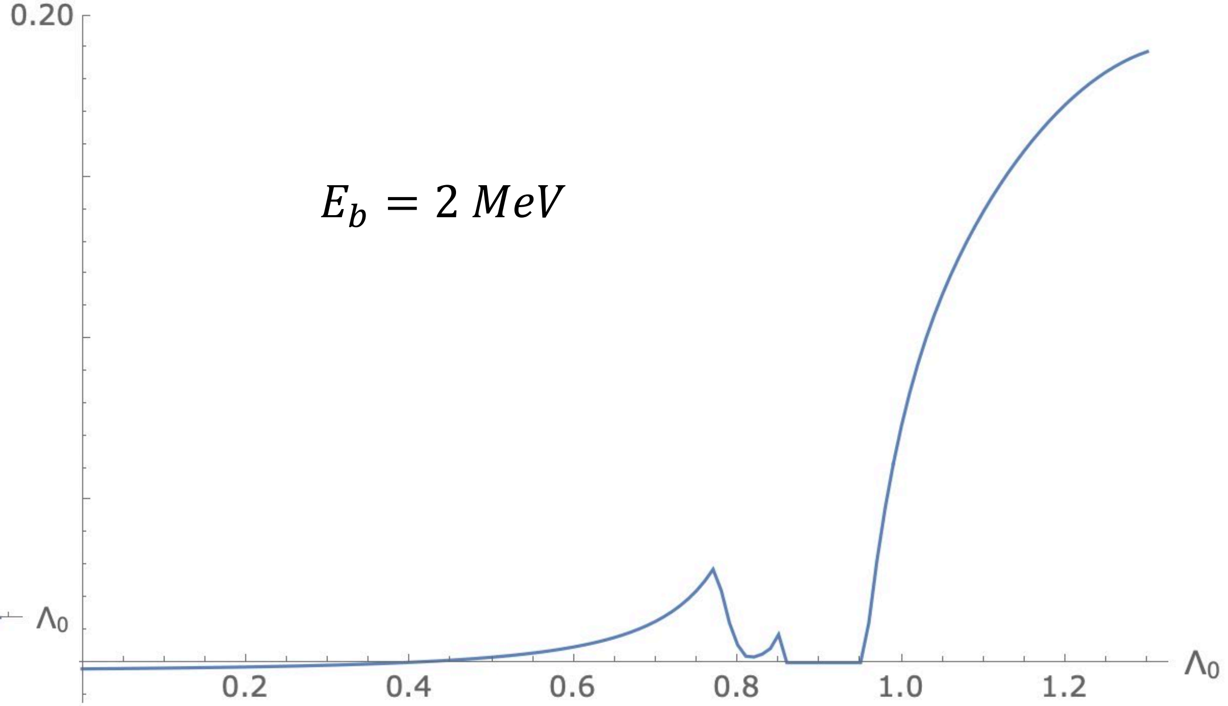
$$F(r,\Lambda,v) = C(r)\delta(\Lambda-\Lambda_0)\frac{H(v_b-v)}{v^3+v_c^3}$$

$$\left.\frac{\partial F}{\partial E}\right|_{\Lambda} - \frac{\Lambda}{\mathrm{E}}\left.\frac{\partial F}{\partial \Lambda}\right|_{\mathrm{E}} > 0$$

$\rightarrow$ Threshold at $\Lambda_0 = 2/5$, same threshold found for EGAMS

- *Qiu Z.,Zonca F. and Chen L. Plasma Sci. Technol. 13 257 (2011)*





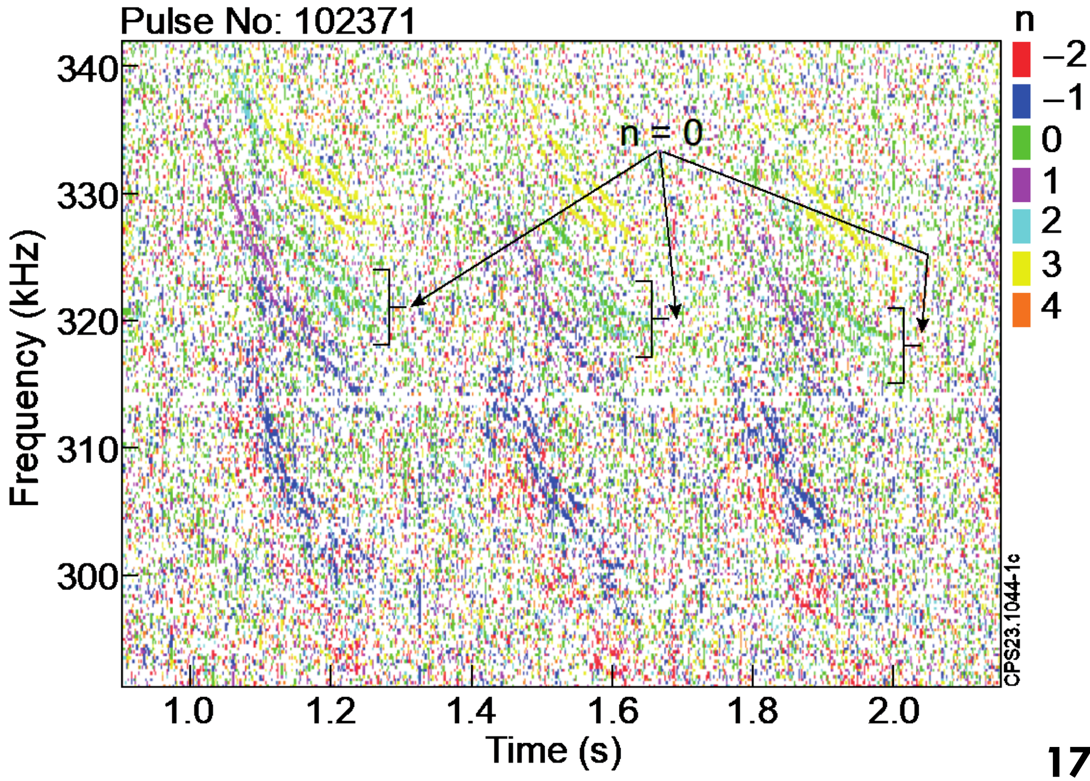
A DISTRIBUTION WITH $\partial F/\partial E > 0$ CAN ALSO BE OBTAINED TRANSIENTLY DUE TO NBI MODULATION OR SAWTOOTH OSCILLATIONS

- NBI modulation on timescales shorter than the slowing down time
  – Ya.I. Kolesnichenko and V.V. Lutsenko 2019 *Nucl. Fusion*, M.A. Van Zeeland et al *PoP* 2021
- Sawtooth oscillations with period shorter than slowing down time
  – V.G. Kiptily et al. 2021 *Nucl. Fusion*

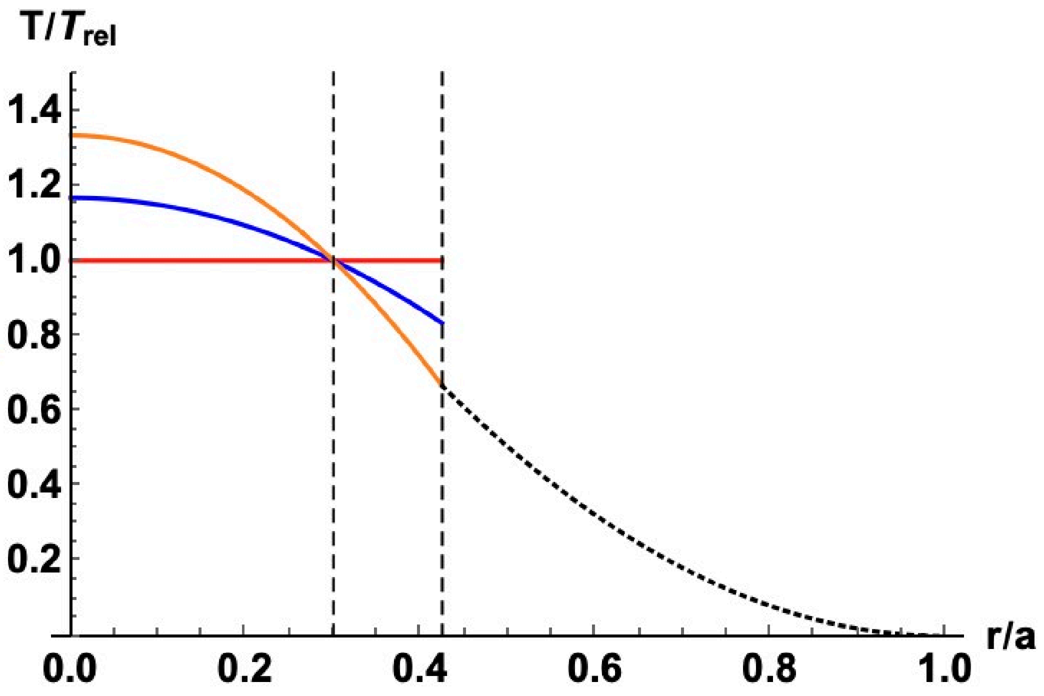
$$\frac{\partial v^3 F}{\partial t} - \frac{v}{\tau_s(t)}\frac{\partial v^3 F}{\partial v} = v^3 S_\alpha(t) \quad \rightarrow F(t) \quad \text{(For } v > v_c\text{)}$$

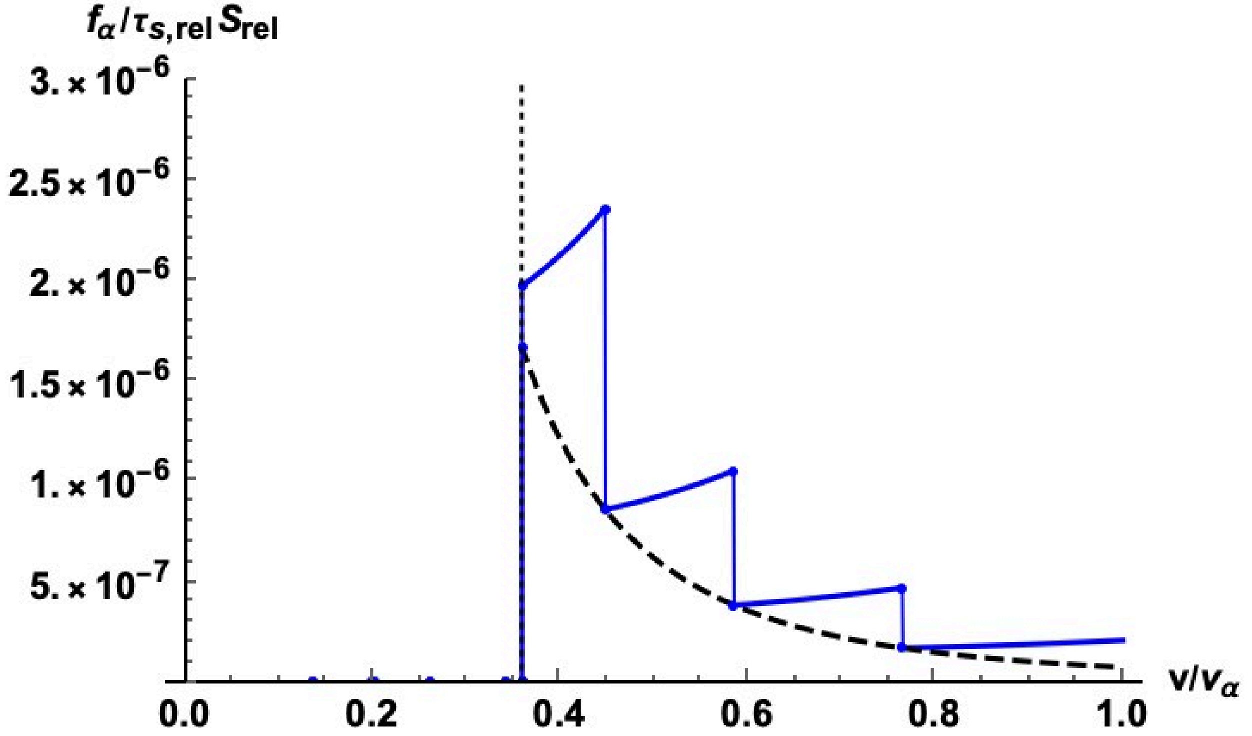
NBI modulation $\rightarrow$ time-dependent source:
if the modulation of the fast ion source is on a timescale faster than the slowing down time, $\partial F/\partial E > 0$

Sawtooth oscillation $\rightarrow$ time-dependent τ_s
For fusion α_s $\rightarrow$ source modulation
$\rightarrow$ $\tau_s/t_{saw} > 1$ can lead to $\partial F/\partial E > 0$



A | CONSIDERING KADOMTSEV RECONNECTION MODEL, A TIME DEPENDENT ANALYTIC DISTRIBUTION FUNCTION IS OBTAINED





$$f_{\alpha,n}(r,t,v) = S_{rel}\frac{v_\alpha^2}{v^3}H\left[v_\alpha - v\right]$$
$$\tau_{s,rel}\left\{1 + A(r)\left[\frac{\hat{t}_n(r,t,v)}{\tau_{saw}} - n\right]\right\}^{7/2} \qquad \rightarrow F(t) = \sum_{n=0}^{n_M} f_{\alpha,n}$$
$$H\left[v - \hat{v}_{n,min}(r,t)\right]H\left[\hat{v}_{n,max}(r,t) - v\right]$$

$\partial F/\partial E > 0$ requires $\tau_s/t_{saw} \approx 3$

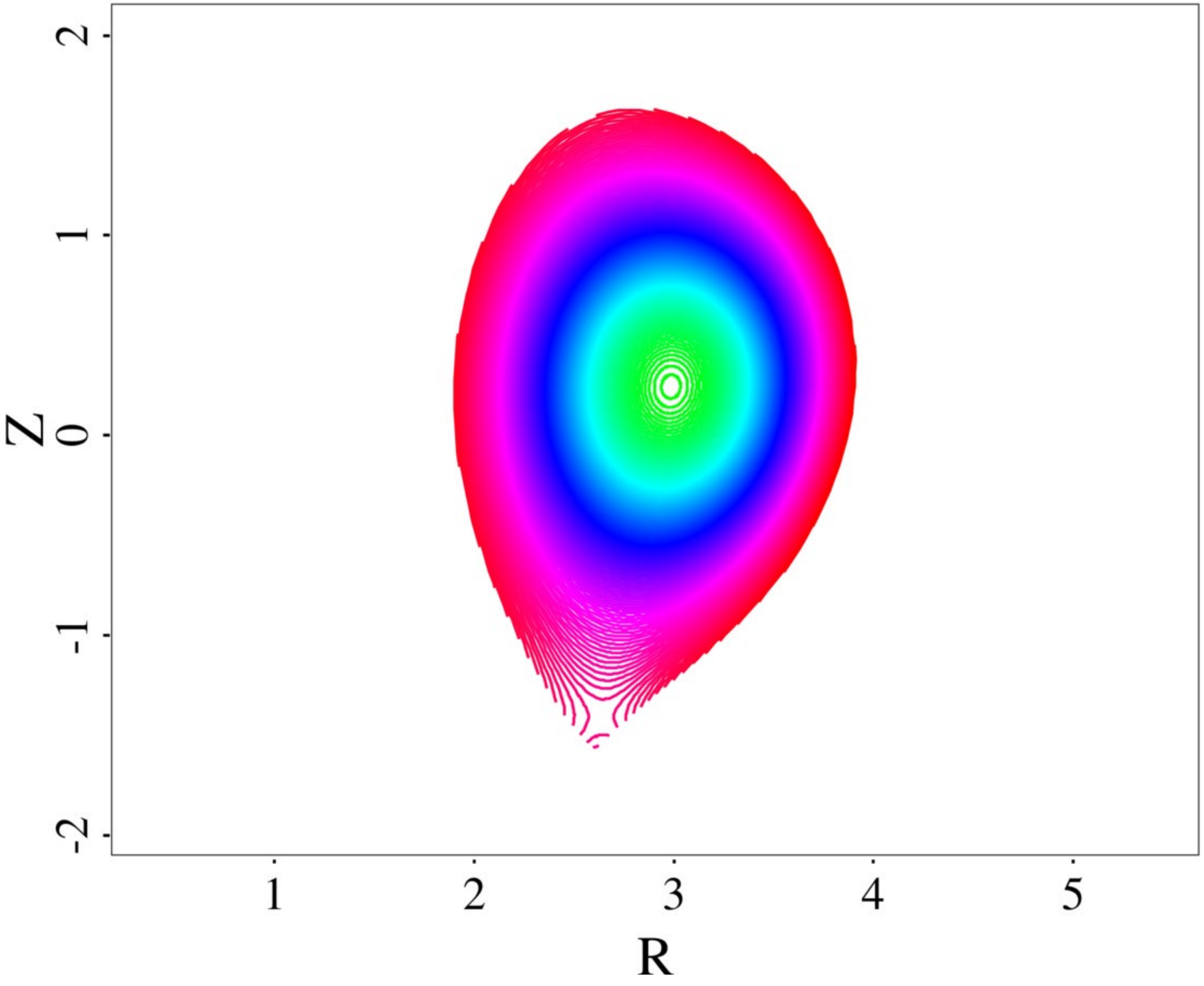
Details in T. Barberis, et al. 2023, 49th EPS conference and T. Barberis et al. 2024 submitted to PPCF

B NIMROD SIMULATIONS OF JET DISCHARGES

Pressure and current density profiles from EFIT equilibrium reconstruction of shot #102371 at t =51.000

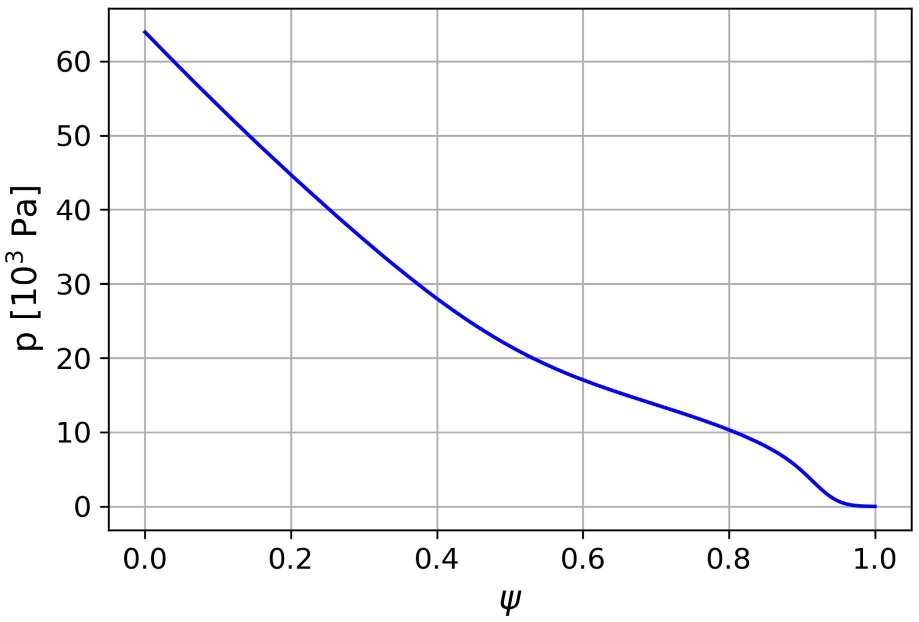
Look for stable oscillations
→ Minimize damping sources

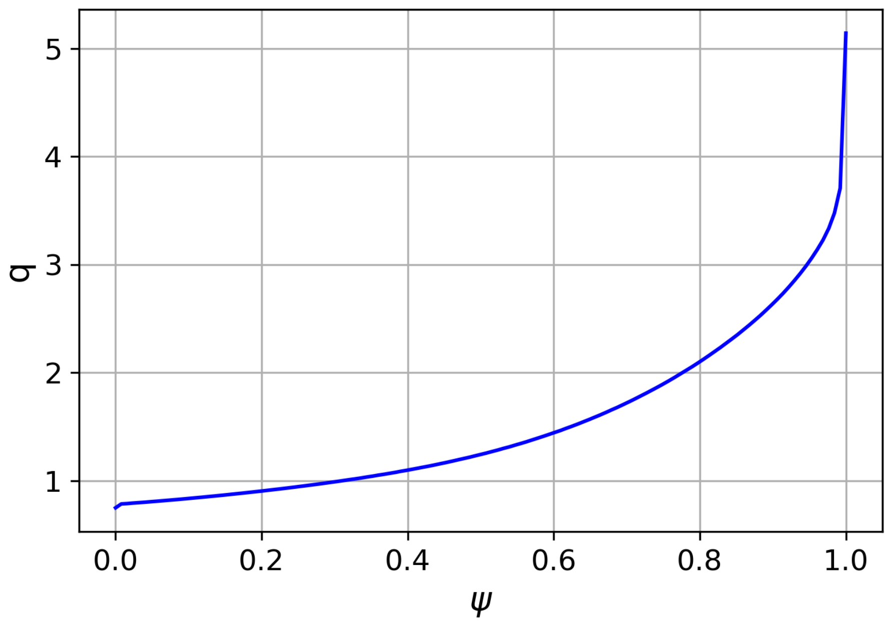
Single null configuration with non-zero triangularity
→ one X-point is in the domain (but in the halo region) while a second X-point still lies outside.



Details in T. Barberis, et al. 2024, submitted to NF

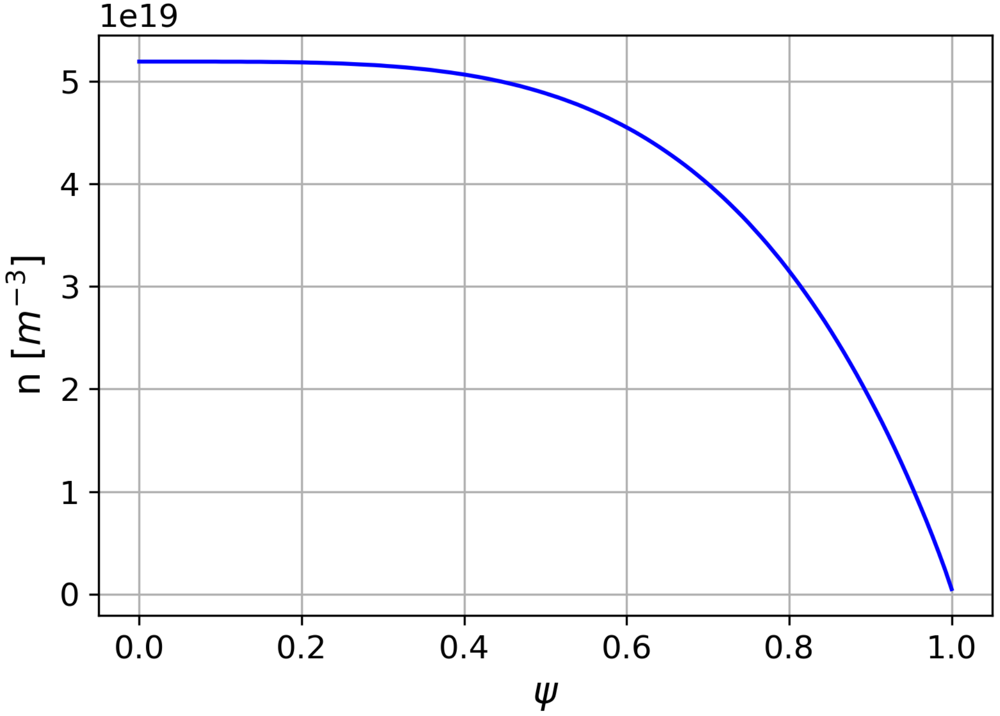
REALISTIC EQUILIBRIUM PROFILES → CURRENT AND PLASMA DENSITY PROFILES WILL IMPACT THE ANALYTIC MODEL MODE STRUCTURE





← Left Safety factor and pressure profiles up to separatrix

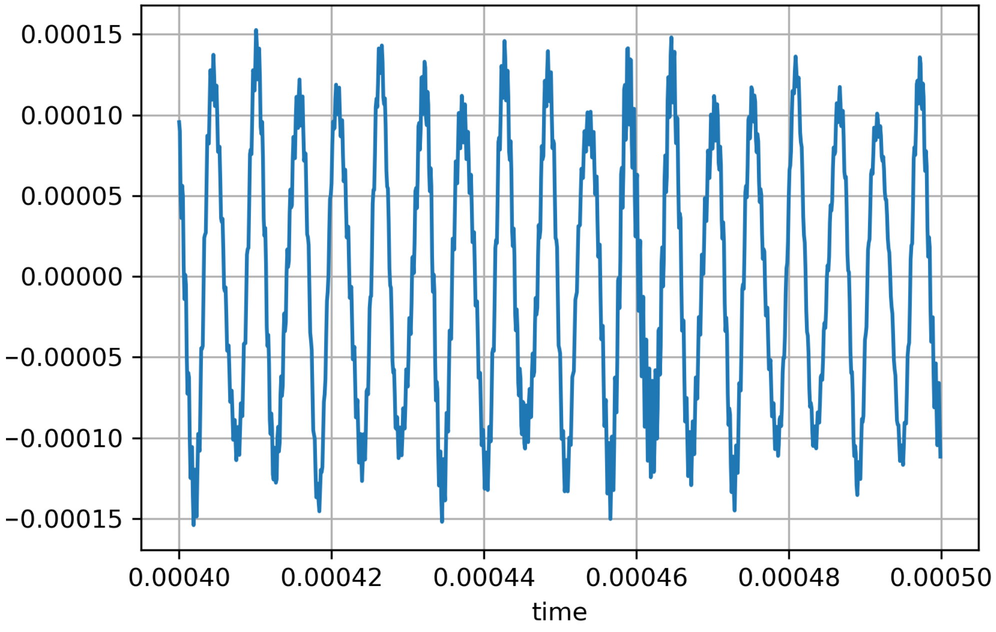
Right → Density profile up to the separatrix.
Halo region density uniform with $n_{halo} = 5 \times 10^{17}$



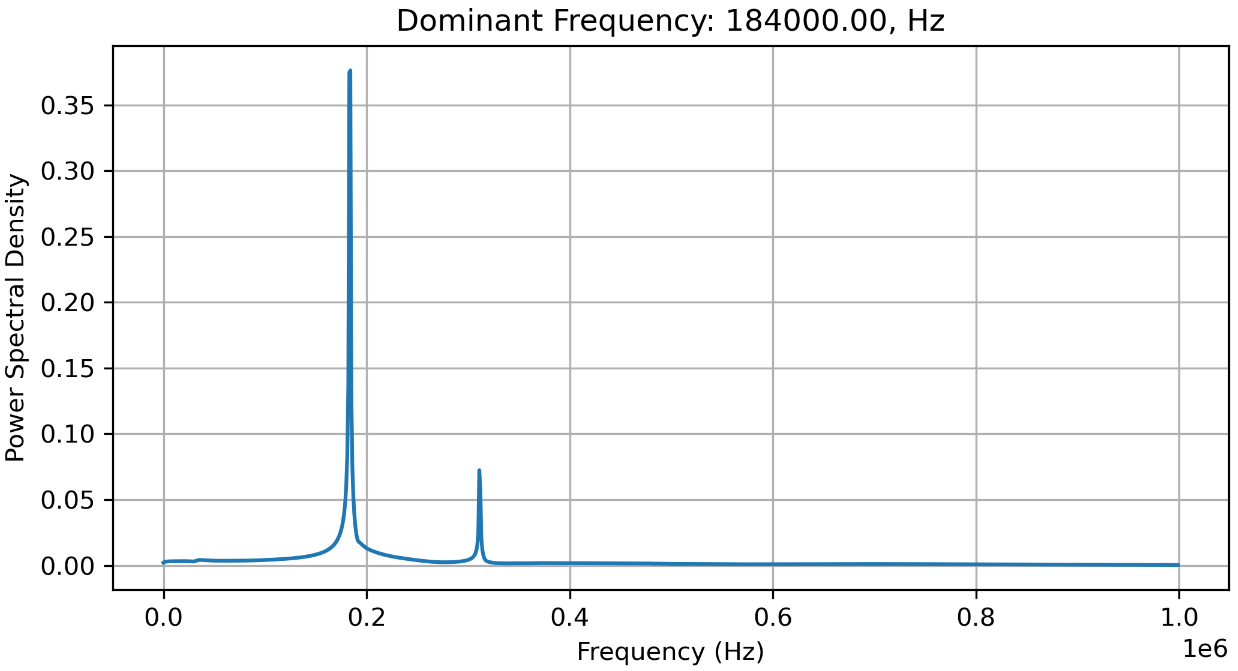
B

STABLE OSCILLATIONS FOUND WHEN PERTURBING THE EQUILIBRIUM WITH A «VERTICAL PUSH» OF THE PLASMA

Magnetic energy time trace

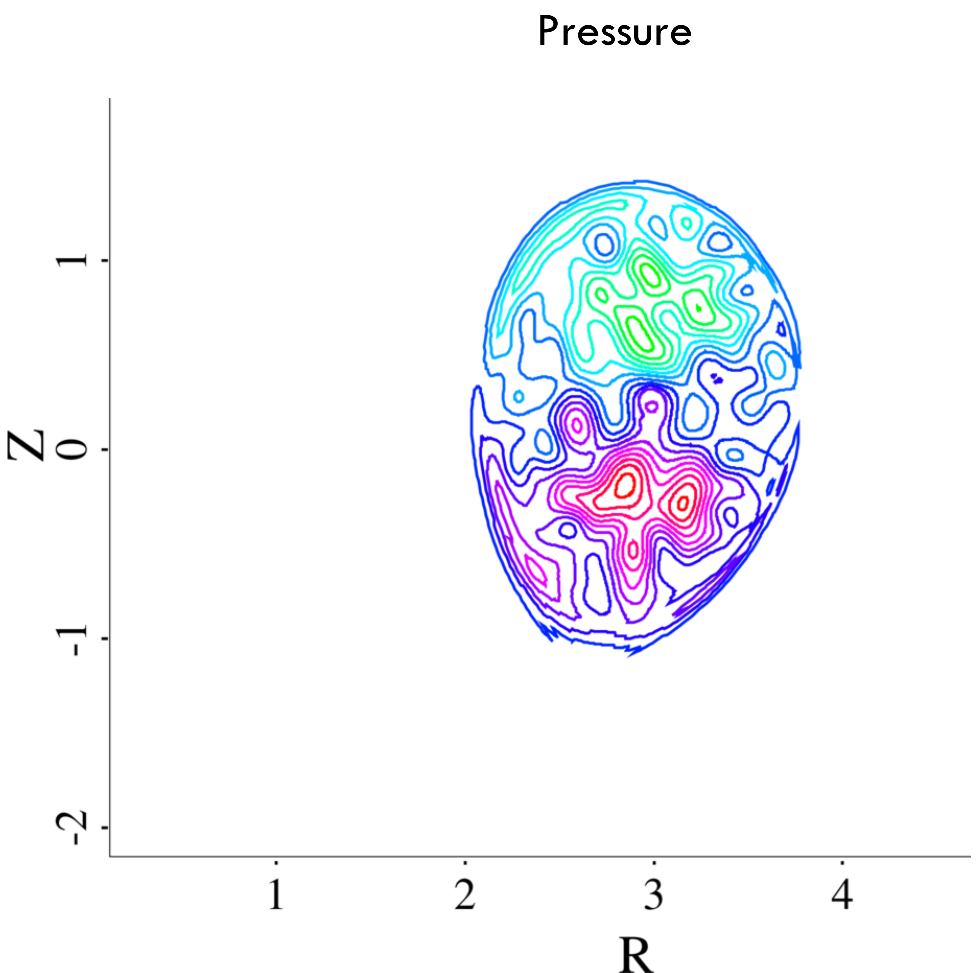


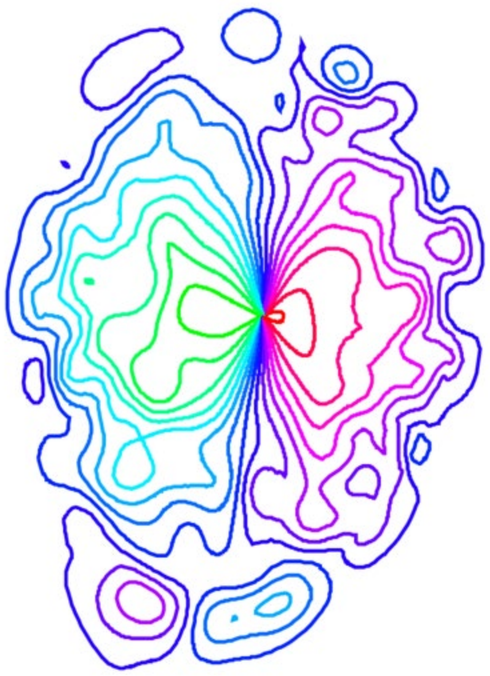
FFT signal shows two distinct peaks:
Dominant peak at $\sim 184\ kHz$ and secondary peak at $\sim 311\ kHz$



B

HINTS OF AN UP-DOWN SYMMETRIC PRESSURE PERTURBATION AND $m = 1$ MODE STRUCTURE



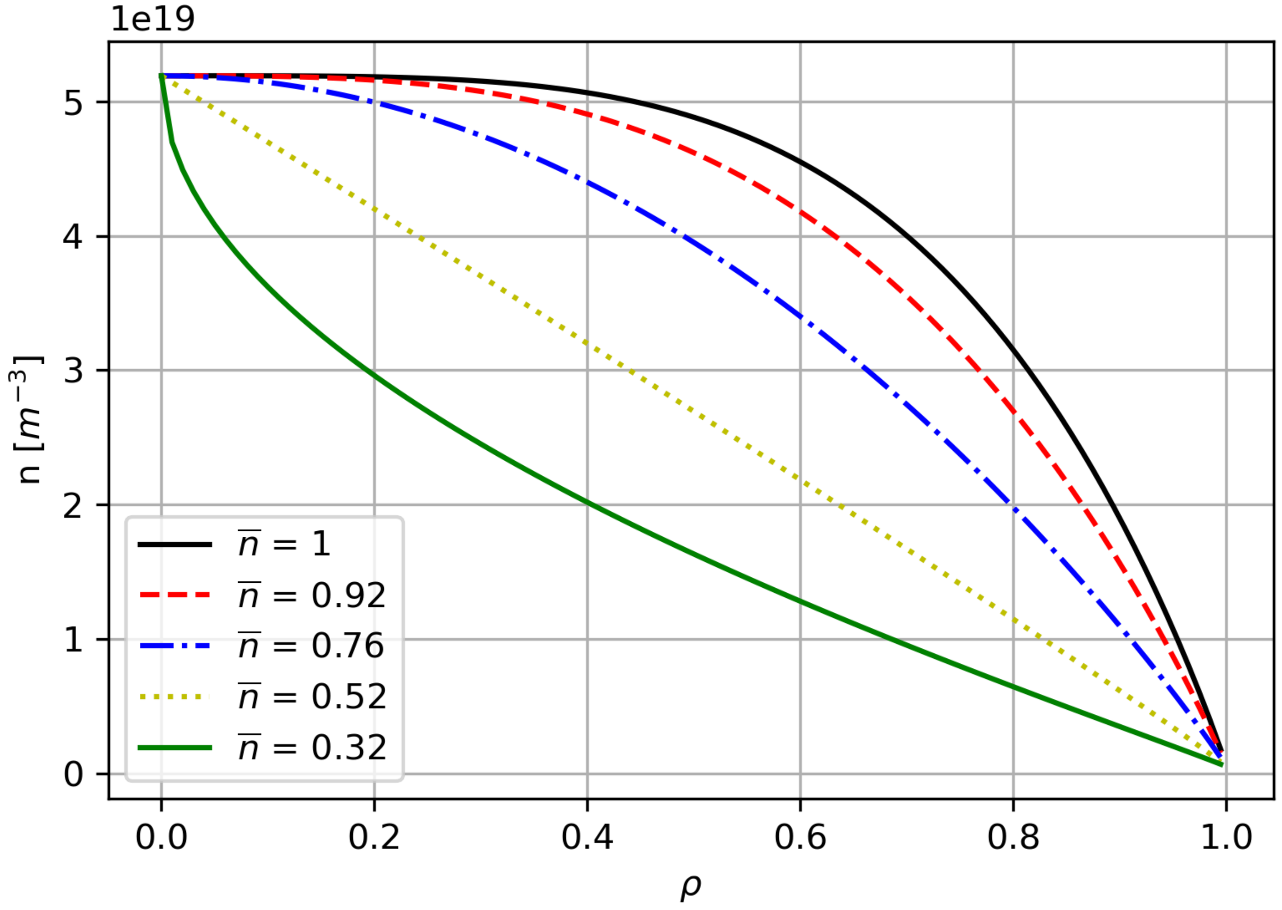


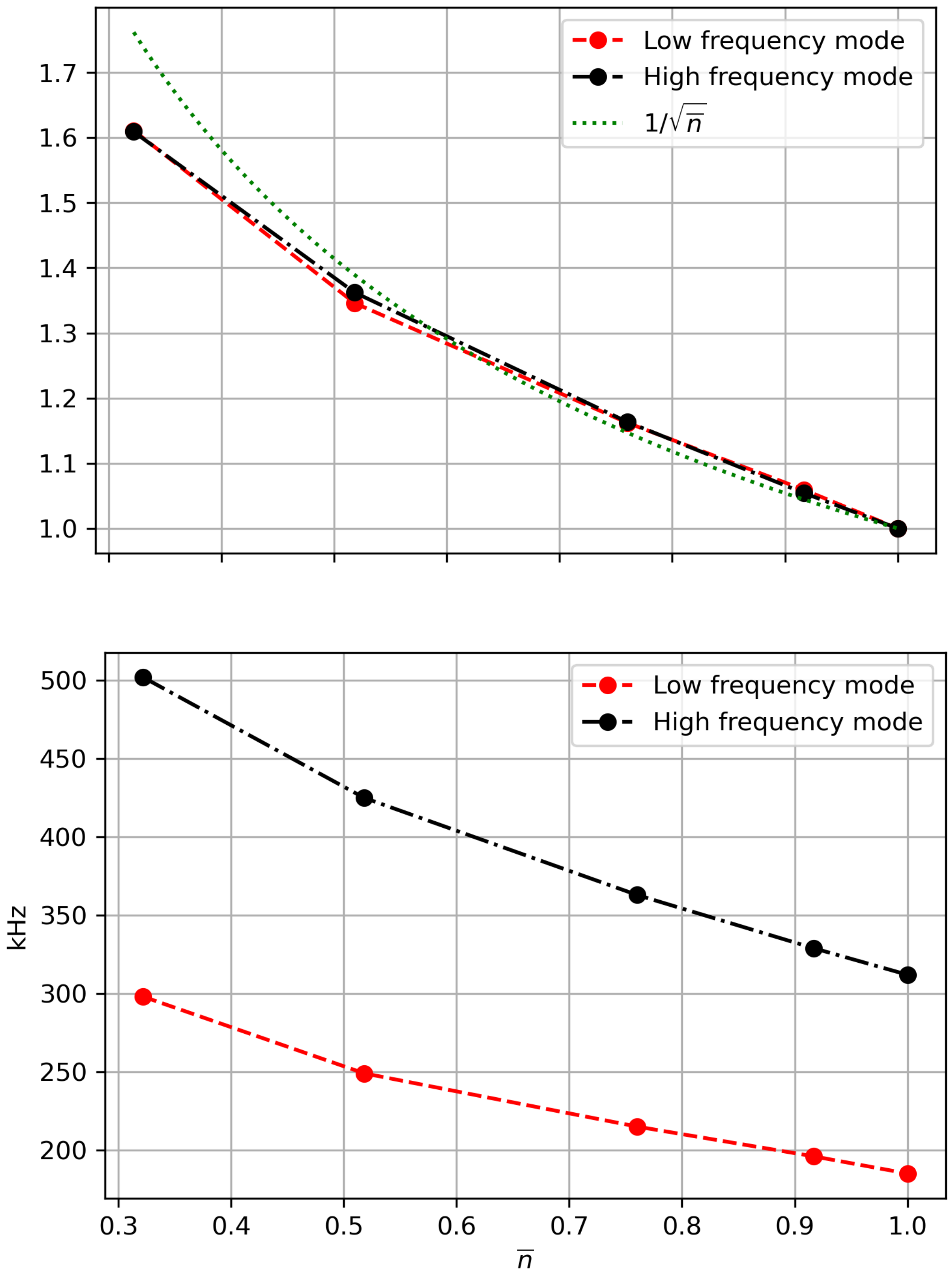
Perturbation in B norm.

Perturbation in B tang.

B

SCAN OF THE DENSITY PROFILE HIGHLIGHTS THE ALFVÉNIC NATURE OF BOTH OSCILLATIONS

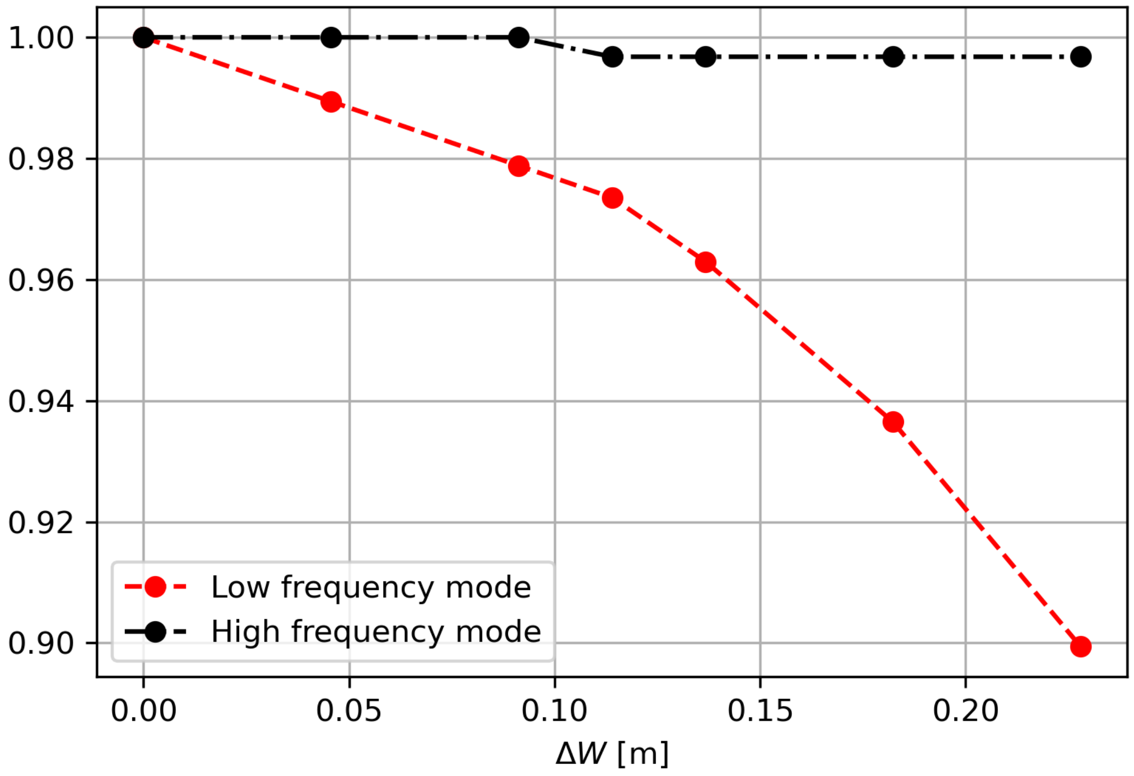


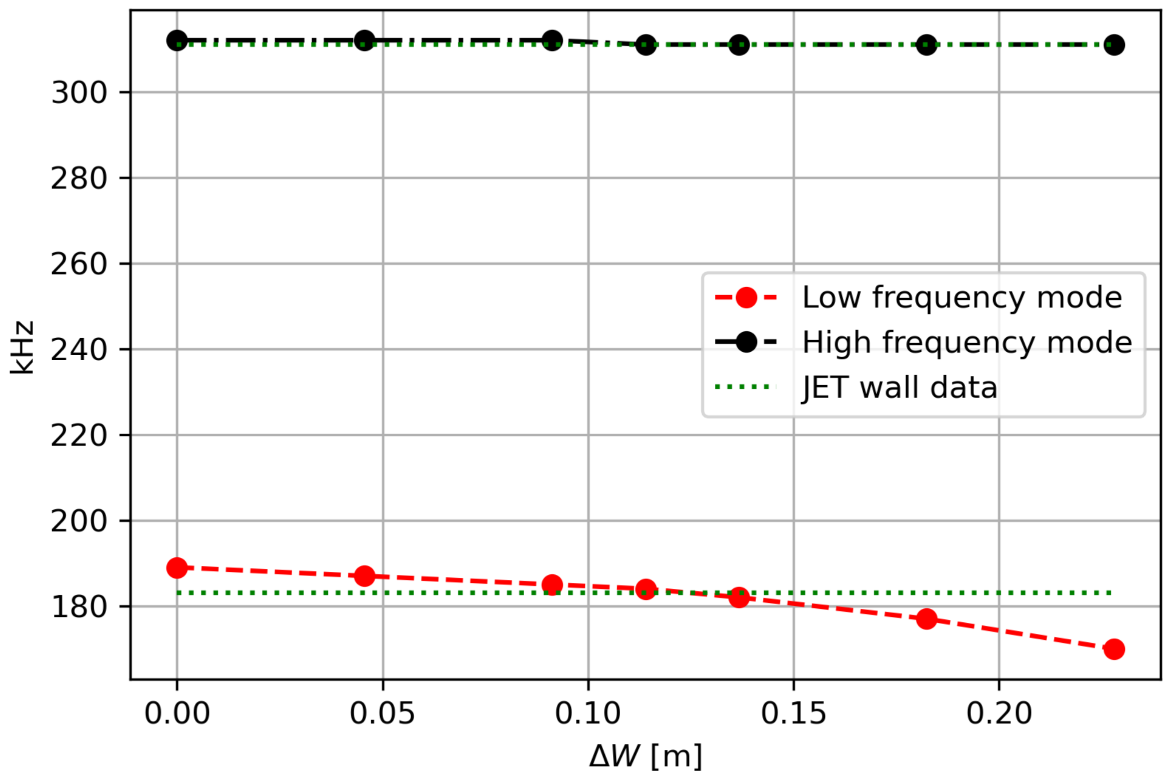


B

THE TWO FREQUENCIES SHOW DIFFERENT DEPENDENCE WITH RESPECT TO WALL POSITION

Frequency change of the two modes as a function of the wall distance in terms of the plasma boundary-wall distance ΔW.
High frequency mode (GAE) ~ insensitive (change of 1 kHz)
Low frequency mode (VDOM) shows a stronger dependence (189 kHz – 170 kHz).





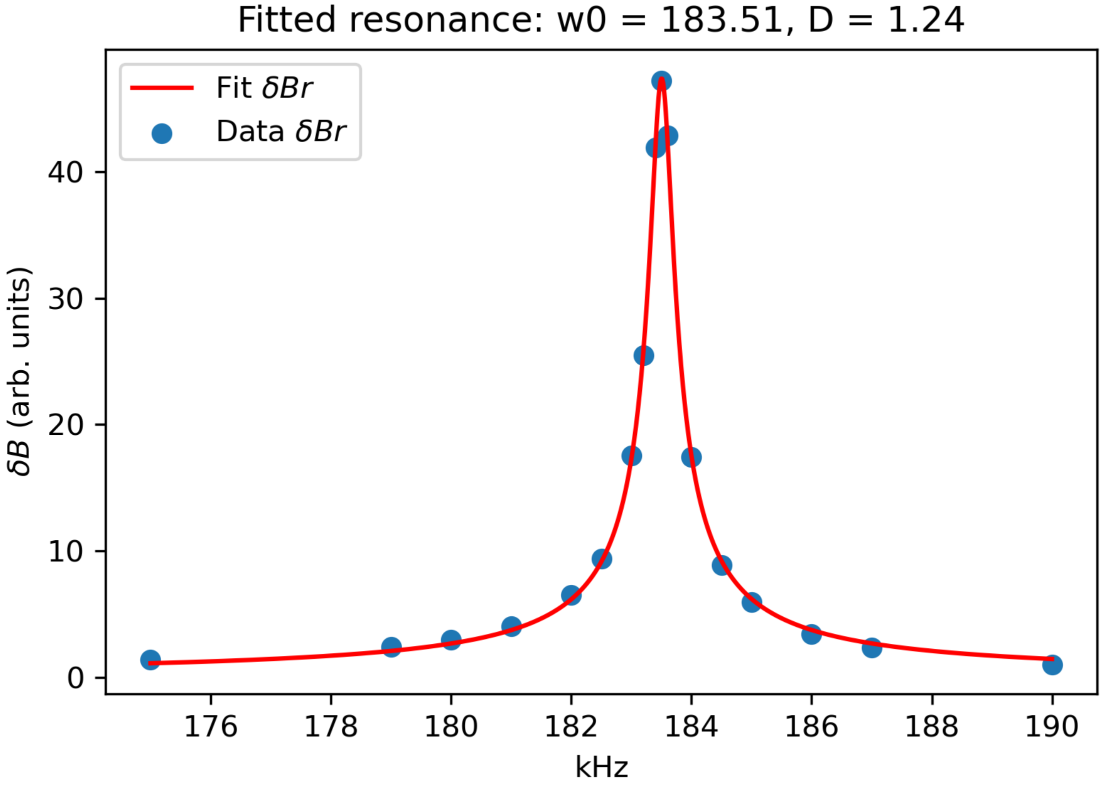
B

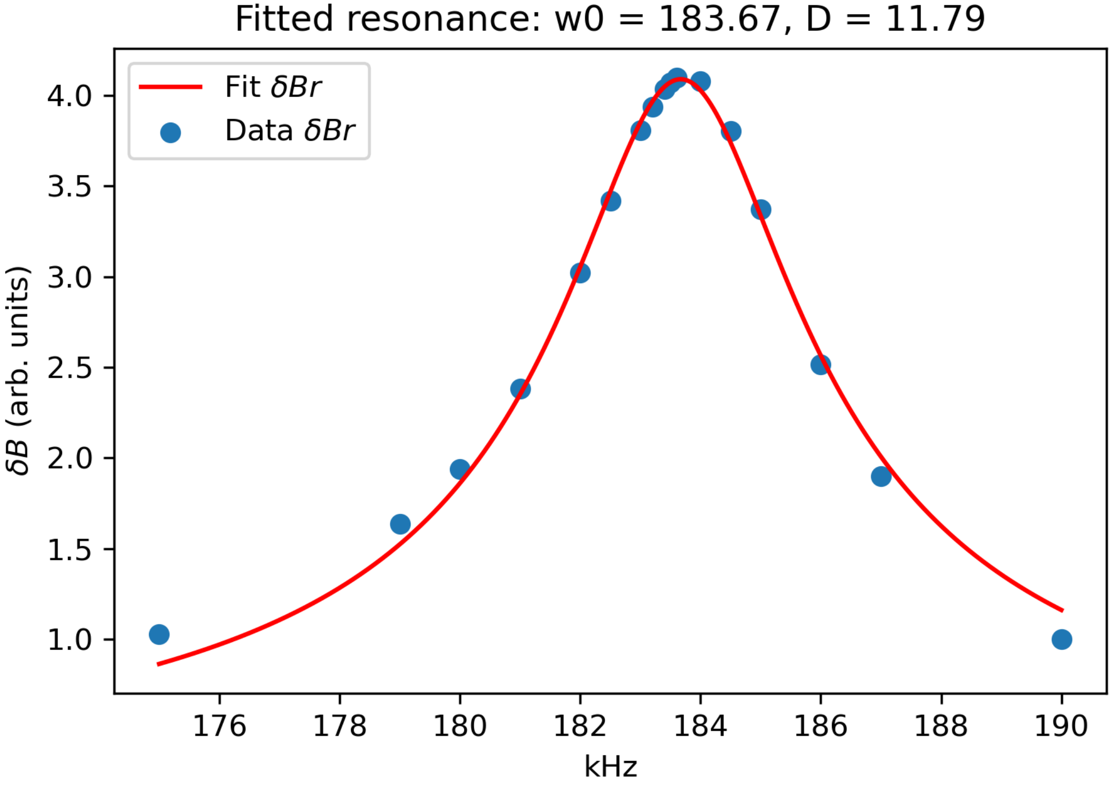
IN ORDER TO STUDY EACH MODE INDEPENDENTLY WE CONSIDER A DIFFERENT KIND OF INITIAL PERTURBATION: OSCILLATOR IN T

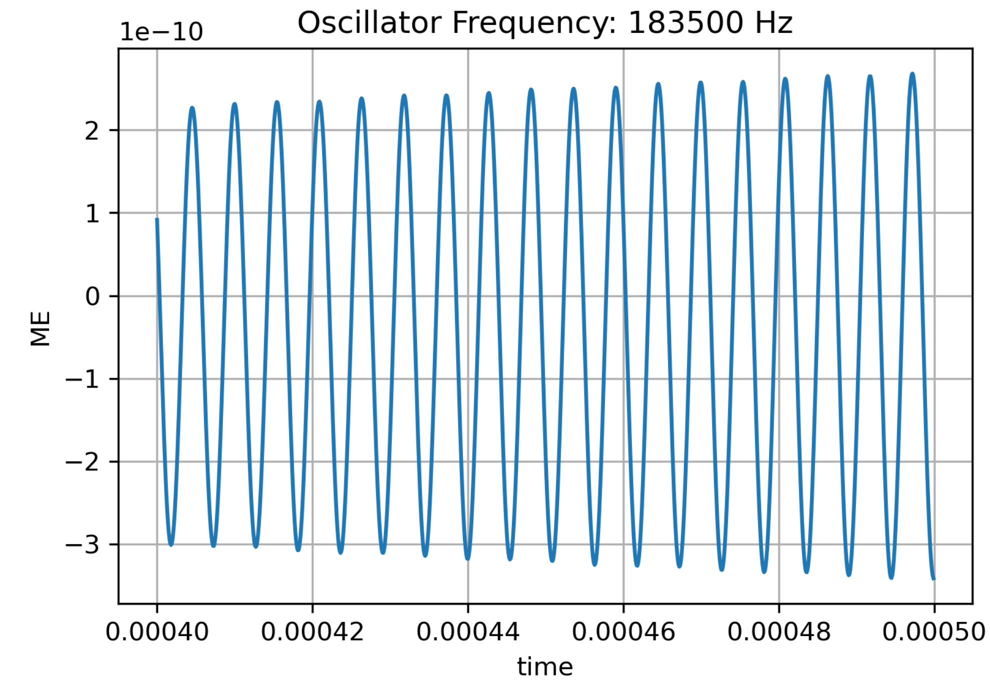
20x viscosity

Extra term in the temperature evolution $\propto \sin(\omega_0 t) * \exp(\frac{(r-r_0)^2+(z-z_0)^2}{\Delta^2})$

Data fitted with $|\mathrm{Y}| \propto \frac{C}{\sqrt{(\omega-\omega_0)^2+D^2}}$ assuming resonance condition $\mathrm{Y} \propto \frac{C}{(\omega-\omega_0)+iD}$

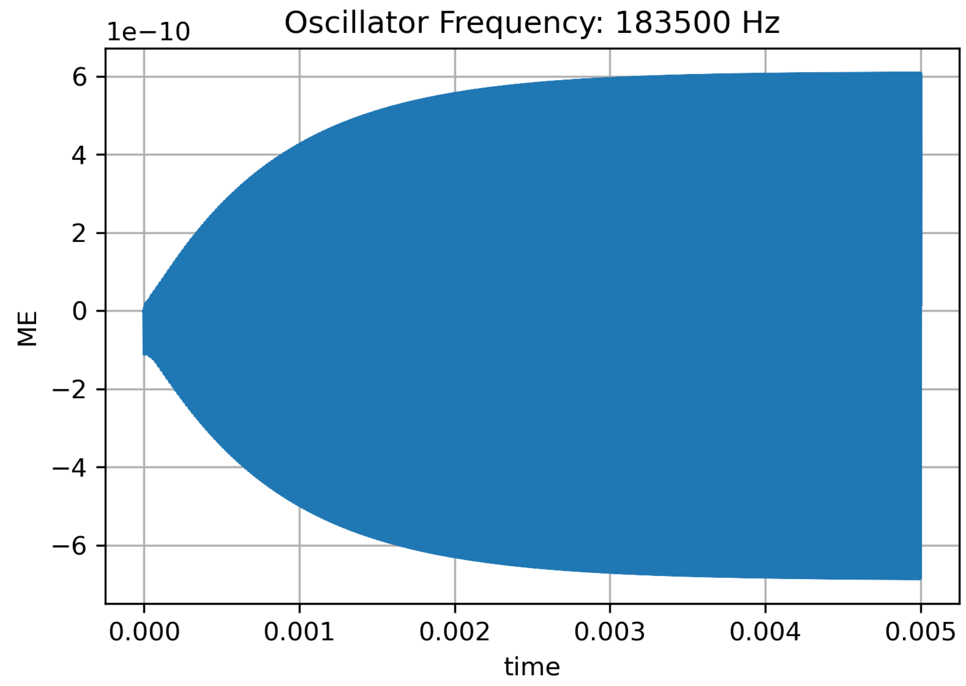


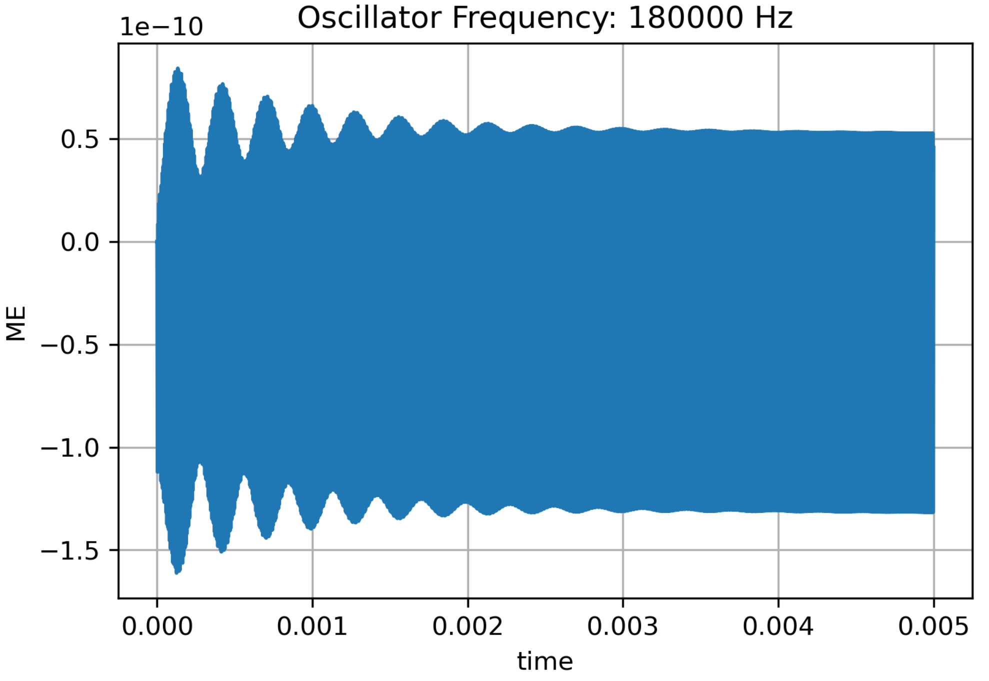




Resonant case:

Growing & saturation

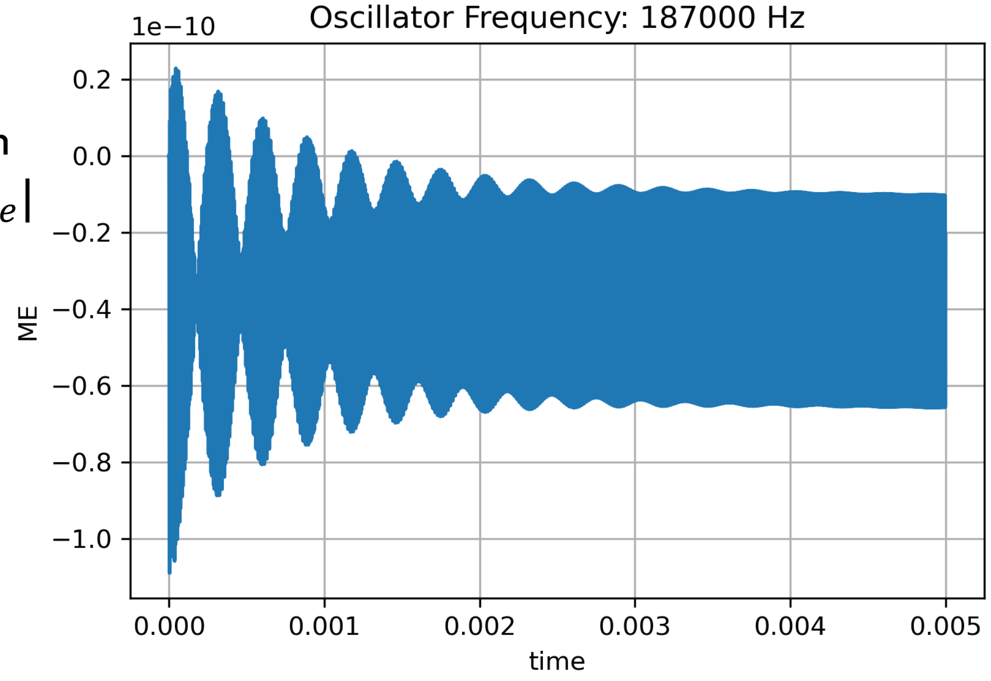




Off-resonant case:

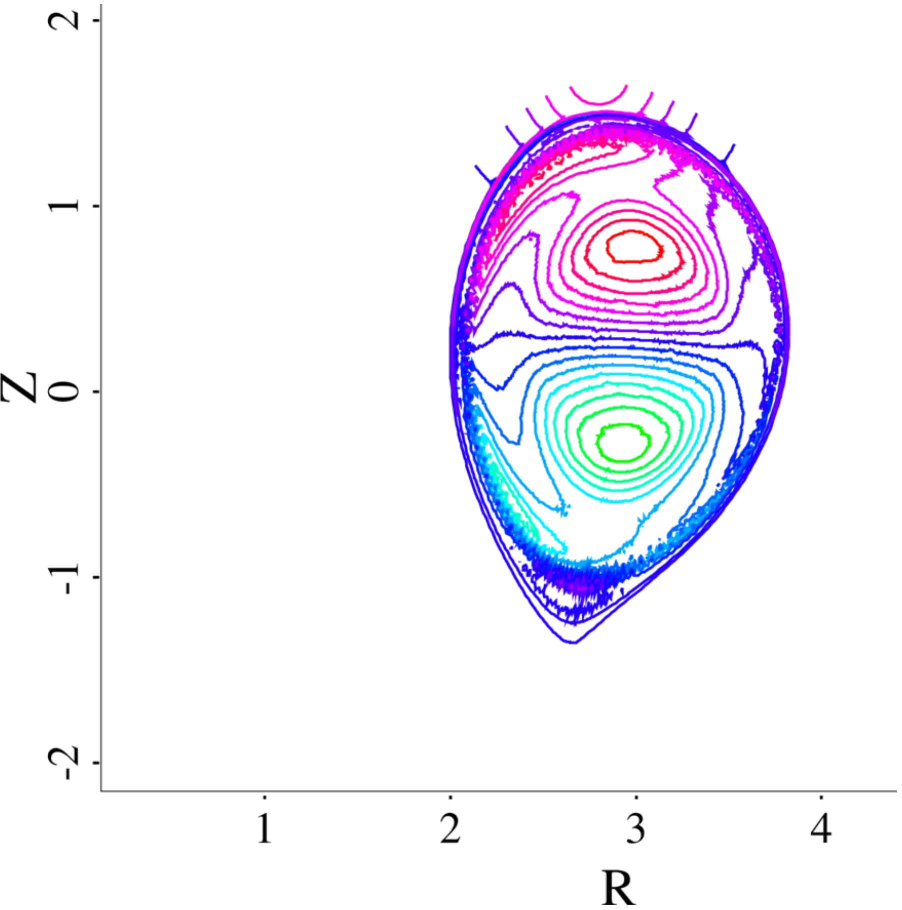
Beating & saturation

$$f_{beat} = |f_{osc} - f_{mode}|$$

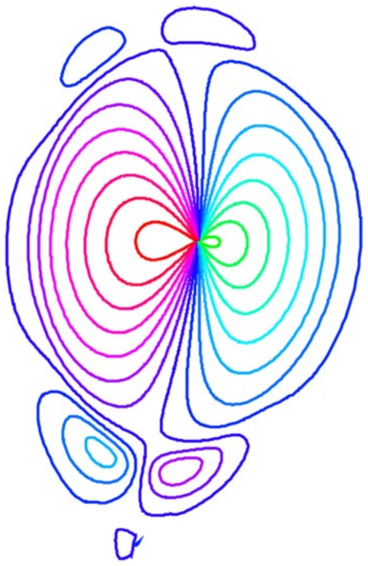


B

MODE STRUCTURE: UP-DOWN SYMMETRIC PRESSURE PERTURBATION AND $m = 1$ MODE STRUCTURE

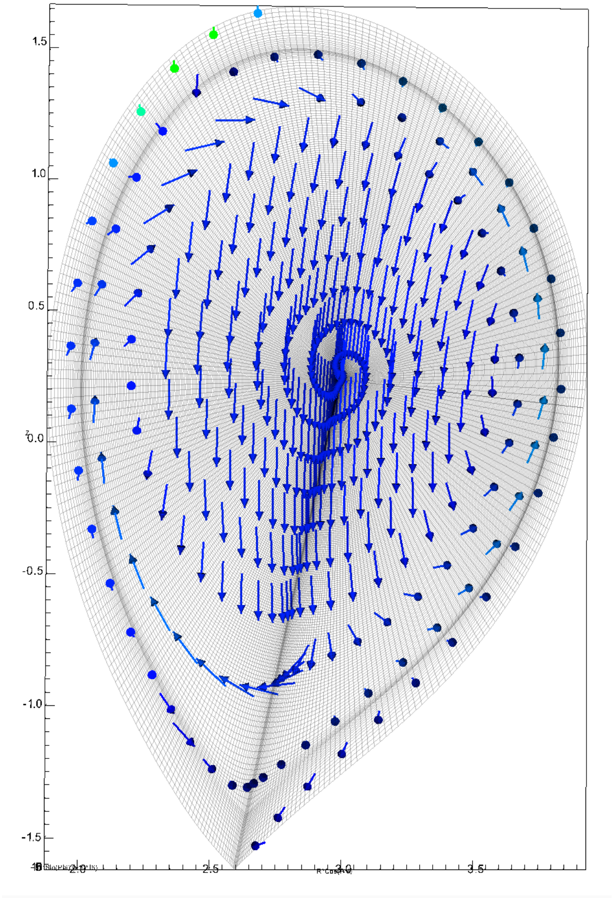


Perturbed pressure



Perturbation in B norm.

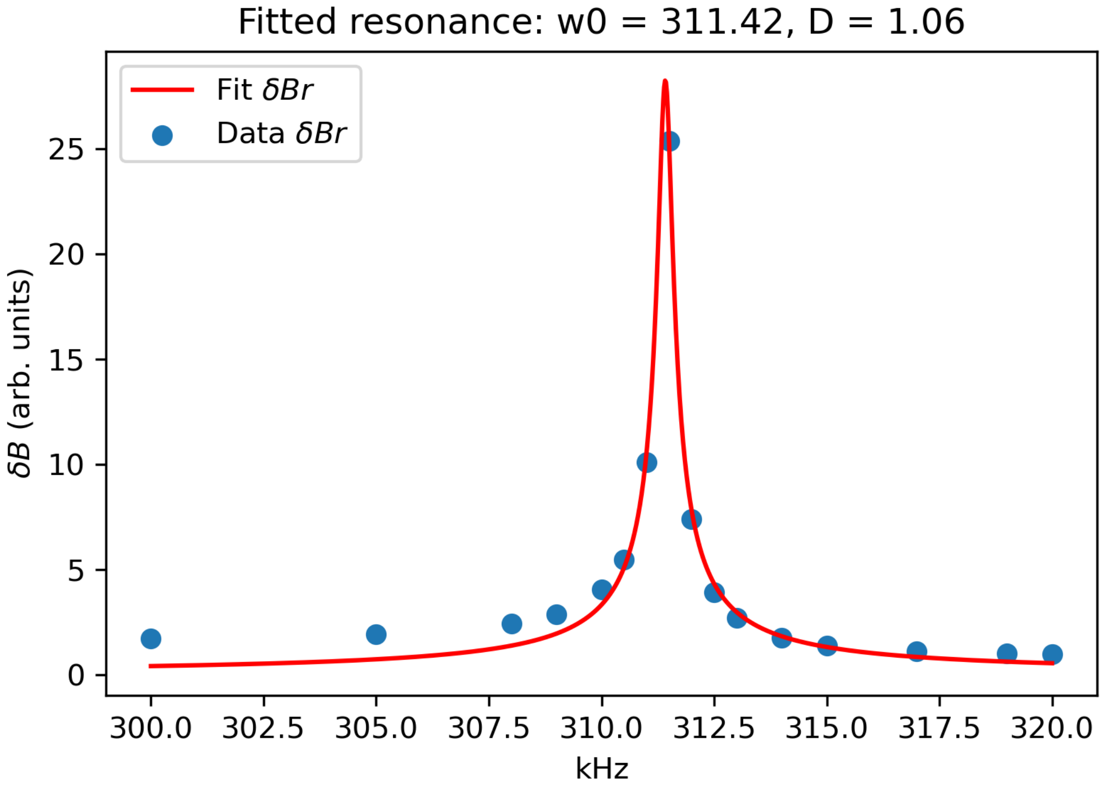
Perturbation in B tang.

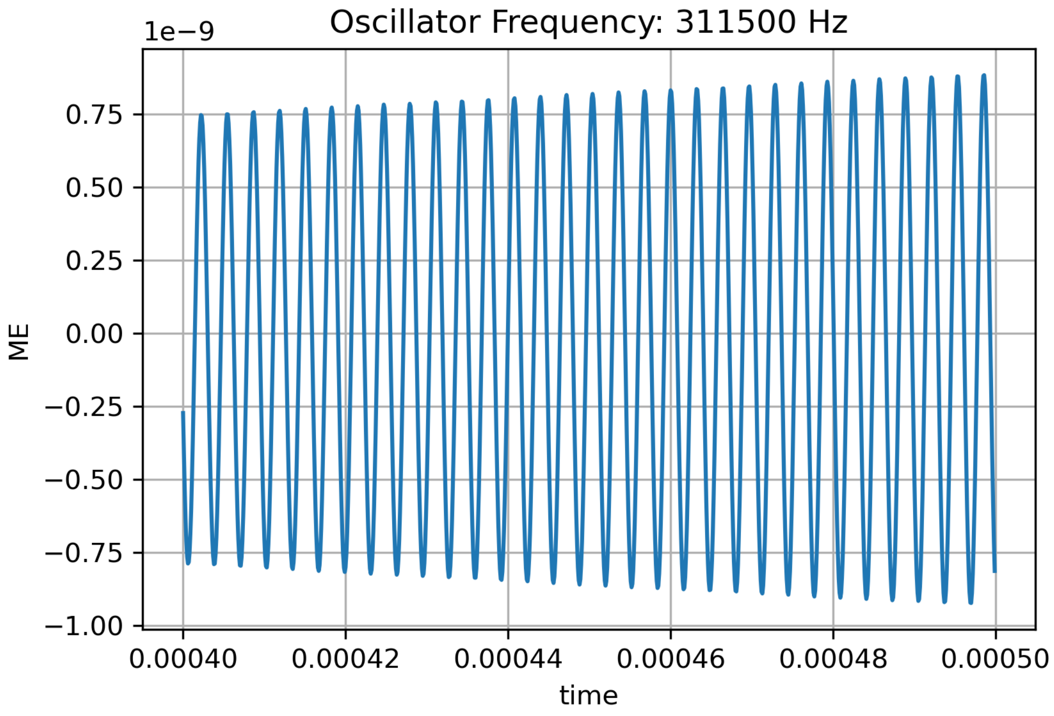


Velocity perturbation vector plot

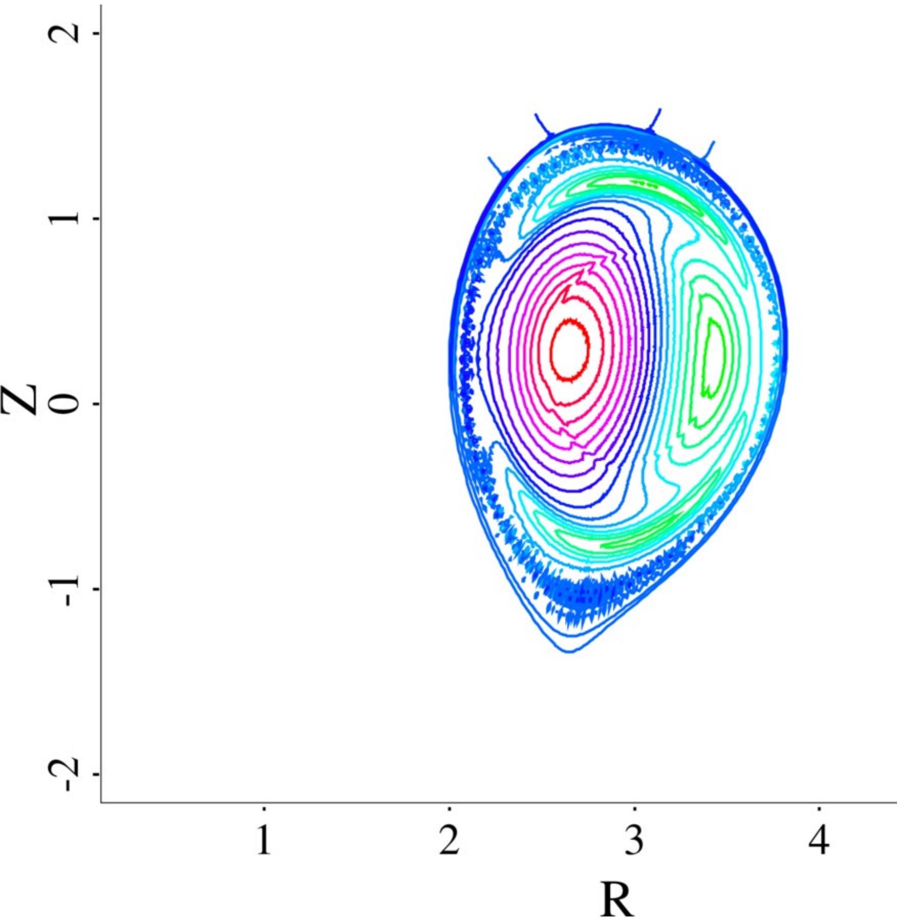
B

WITH THE OSCILLATOR WE CAN INVESTIGATE THE TWO MODES INDEPENDENTLY

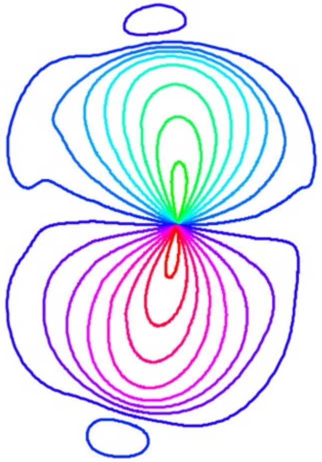




B | MODE STRUCTURE: «HORIZONTAL» PRESSURE PERTURBATION AND $m = 1$ MODE STRUCTURE

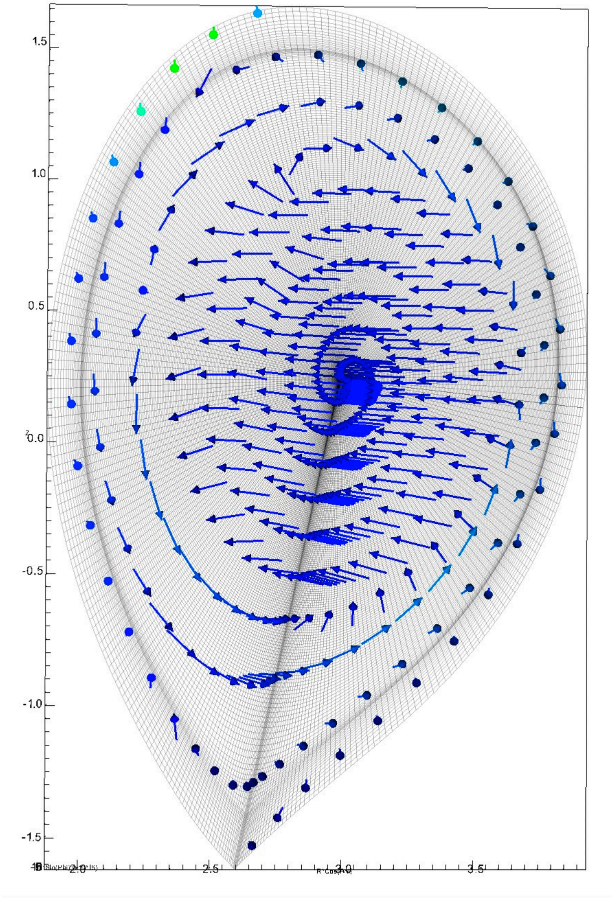


Perturbed pressure



Perturbation in B norm.

Perturbation in B tang.



Velocity perturbation vector plot

C

CONCLUSIONS

1. Ideal MHD analytic theory describing vertical displacement oscillatory modes (VDOM) successfully verified with NIMROD straight tokamak simulations.
2. Hybrid kinetic MHD analytic calculations show how MeV fast ions may lead to the destabilization of VDOM through resonant interaction with different distribution functions.
3. In JET discharge simulations two high-frequency modes have been observed investigating their resonance with an oscillator in temperature:

   - A mode with frequency 183.5 kHz that we identified as vertical displacement oscillatory mode with up-down symmetric density perturbation and nearly rigid shift mode structure.
   - Second mode with a frequency of 311.5 kHz almost independent from the wall position, with peaking perturbation close to the minimum of the Alfvén continuum and horizontal mode structure identified as $n = 0$ GAE (see H. J. C. Oliver et al. 2017 PoP.)

C

FUTURE WORK

1. Extension of the hybrid kinetic MHD analytic study with time-dependent distribution function induced by sawtooth oscillations
2. Implement a $\partial F/\partial E > 0$ distribution function in the NIMROD hybrid kinetic PiC module in order to provide the free energy for the mode destabilization
3. Modeling of JET D-T discharges where n=0 modes have been observed, aiming to provide the first identification of VDOM destabilized by fusion α particles in a tokamak experiment
4. Extension of the numerical analysis to the D-IIID tokamak device, determining the kind of energetic particles required for the VDOM destabilization in different devices.

THANKS FOR THE ATTENTION

A | WITH REDUCED IDEAL MHD WE OBTAIN THE DISPERSION RELATION FOR A SIMPLIFIED EQUILIBRIUM: RIGID SHIFT MODE STRUCTURE

Passive feedback → confocal resistive wall ⟶ $\delta W_{MHD} \rightarrow \delta W_{MHD}(\gamma)$

Cubic dispersion relation:

$$\gamma^3 + \gamma^2 \frac{1}{\tau_\eta} \frac{1}{1 - \hat{e}_0 D} + \gamma \omega_0^2 + \omega_0^2 \frac{1}{\tau_\eta} \frac{1}{1 - D} = 0$$

For plasma density and current density uniform inside elliptical plasma boundary

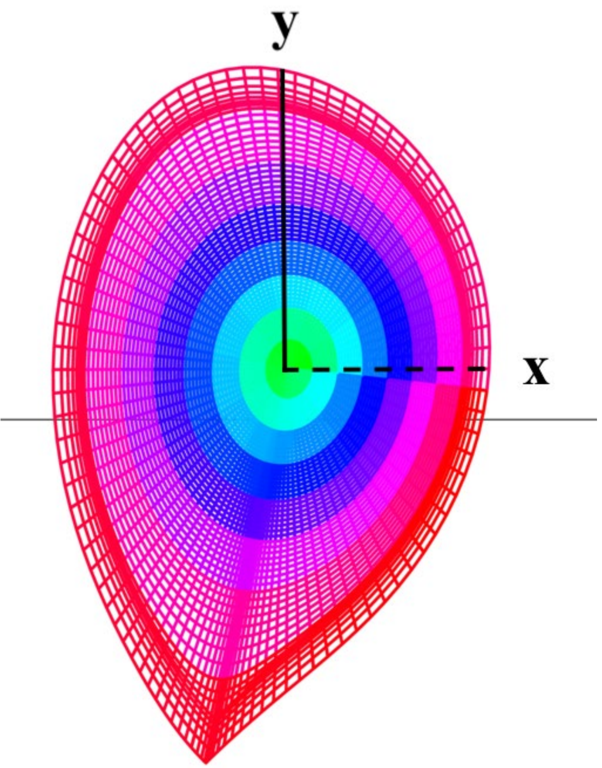
With τ_η resistive wall time, $\hat{e}_0 = e_0 b/(a+b)$, $e_0 = (b^2 - a^2)/(b^2 + a^2)$, and geometrical parameter D(b/b_w, $\kappa = b/a$)

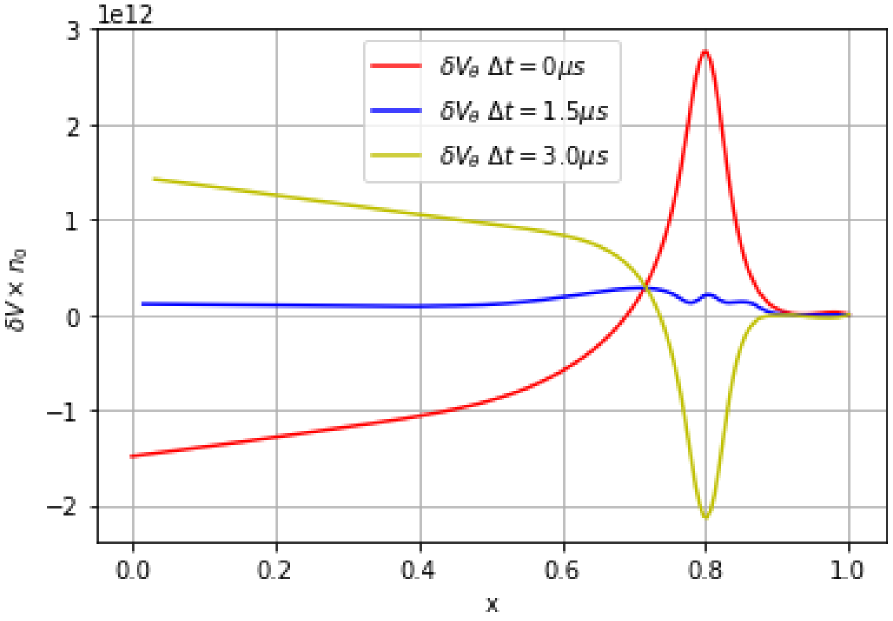
$$\omega_0 = \left[\frac{D-1}{1 - \hat{e}_0 D}\right]^{\frac{1}{2}} \left[(1 - e_0)\left(1 + e_0 - \sqrt{1 - e_0^2}\right)\right]^{\frac{1}{2}} \tau_A^{-1} \quad , \tau_A = \frac{V_A}{R_0 q}$$

Oscillation frequency in case of passive feedback stabilization

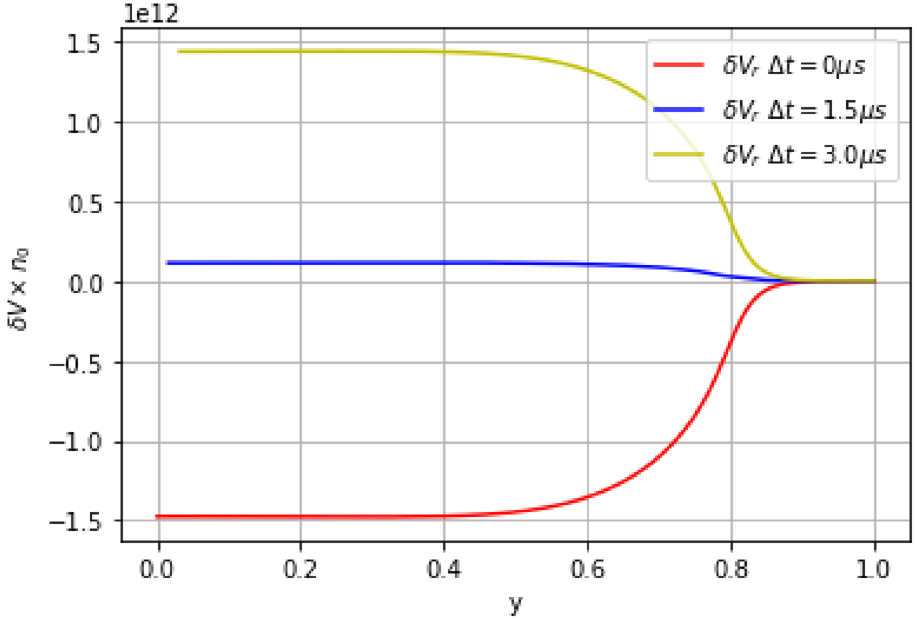
$$\tilde{L} \simeq -(m v_\parallel^2 + \mu_\perp B)\boldsymbol{\xi} \cdot \boldsymbol{\kappa}$$

B | MODE STRUCTURE: 1D PLOTS OF RELEVANT PERTURBED MOMENTUM → VERTICAL DIRECTION FOR VDOM





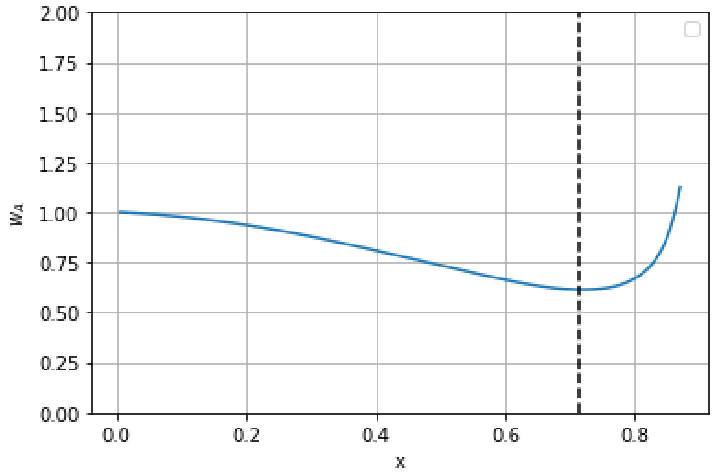
Perturbed momentum in the direction Tangential ($\sim\theta$) to flux surfaces along slice x



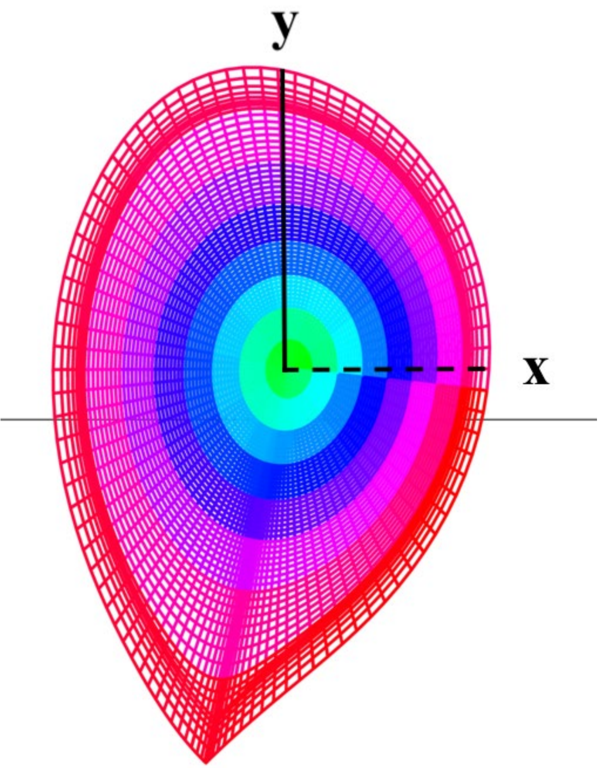
Perturbed momentum in the direction normal ($\sim r$) to flux surfaces along slice y

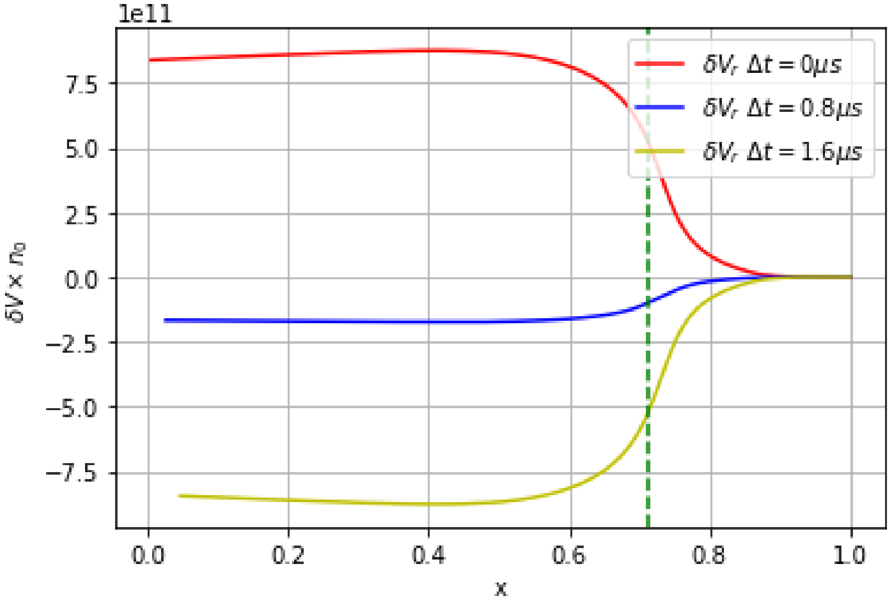
B

MODE STRUCTURE: 1D PLOTS OF RELEVANT PERTURBED MOMENTUM → HORIZONTAL DIRECTION FOR GAE

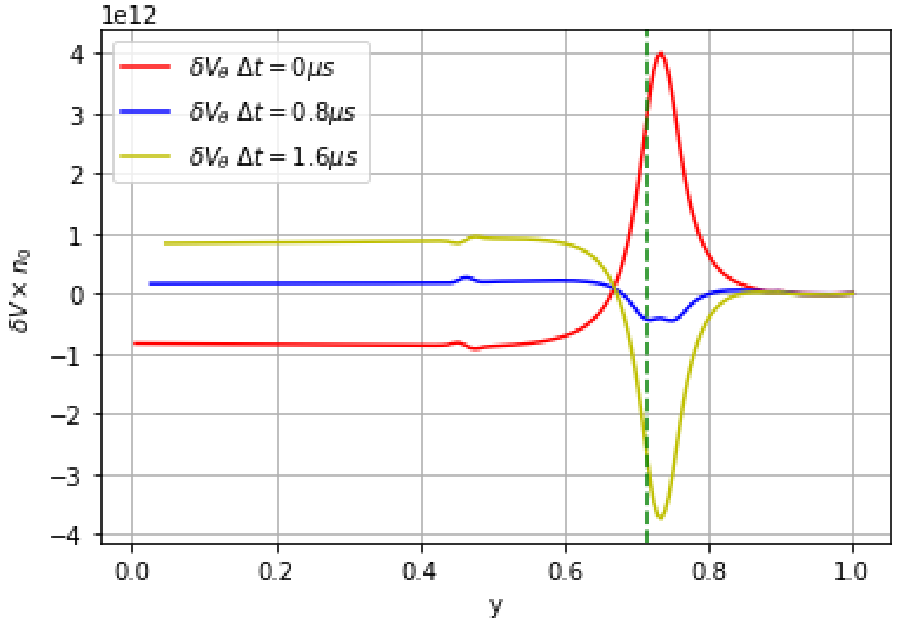


Analytic $n = 0$ Alfvén continuum





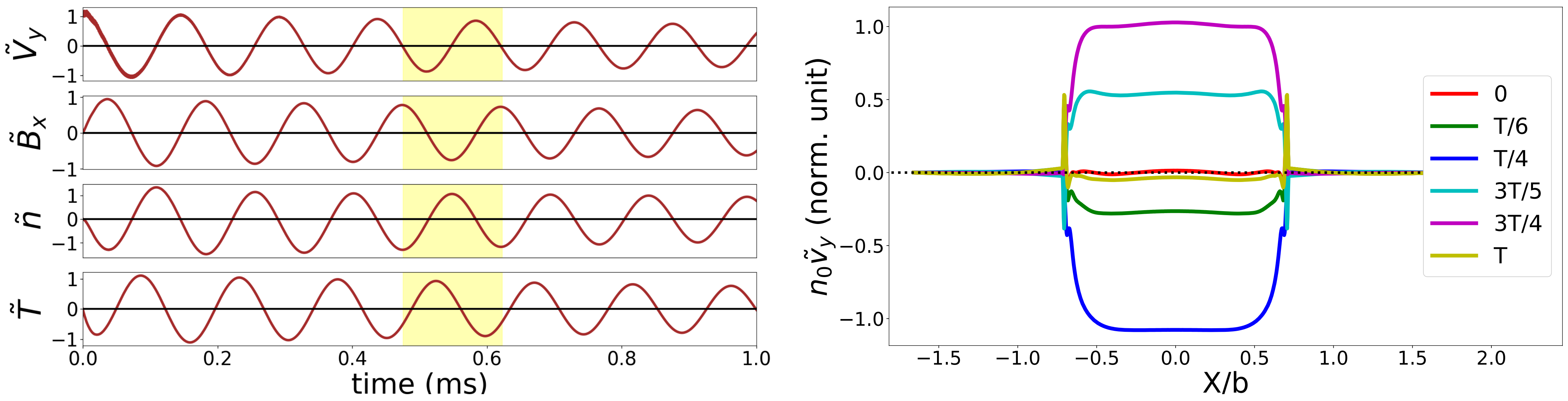
Perturbed momentum in the direction normal ($\sim r$) to flux surfaces along slice x



Perturbed momentum in the direction Tangential ($\sim\theta$) to flux surfaces along slice y

A

STABLE OSCILLATIONS CHARACTERIZED BY RIGID SHIFT $n = 0, m = 1$ MODE STRUCTURE



See *D. Banerjee, C.C. Kim, T. Barberis and F. Porcelli et al. 2024, PoP in press.*

TOROIDAL JET SIMULATIONS