Magnetorotational Turbulence and Dynamos in Accretion Disks

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In collaboration with
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Theory Seminar
Department of Astrophysical Sciences & PPPL
Princeton University
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Image credit: NRAO/AUI by J. Kagaya

Outlines

Aims:

- Disentangle diverse physical processes involved in sustaining MRI turbulence and dynamo activity.
- Identify the dominant mechanisms responsible for generating large-scale magnetic fields.

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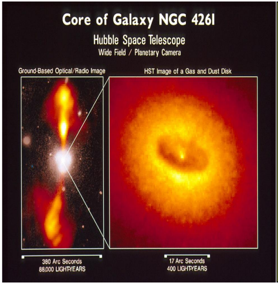
Direct Statistical Simulation (DSS)

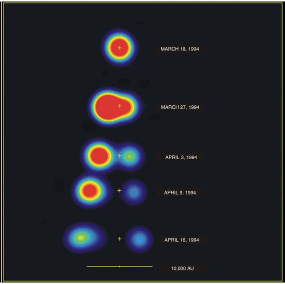
Key Results:

- We uncover various large-scale dynamo mechanisms.
- A path towards the construction of nonlinear mean-field dynamo theory.

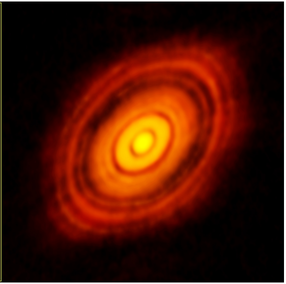
Accretion Power Sources (the incomplete list)



Active Galactic Nucleus



X-ray Binary



Protoplanetary Disk

Image credit: (Left) Optical/radio View of Elliptical Galaxy NGC 4261 (HST and NRAO,Caltech)
(Middle) Microquasars GRS 1915+105 (Very Large Array, NRAO/AUI/NSF)
(Right) Atacama Large Millimeter Array (ALMA) image of the protoplanetary disk around HL Tauri

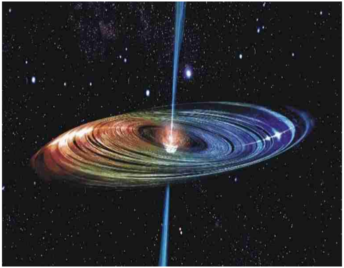


Image credit: NRAO/AUI by J. Kagaya

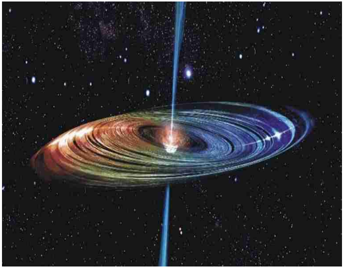
- Gas must lose angular momentum to spiral in

$$\boxed{l_K = \sqrt{GMr}}$$

- Gas must get rid of energy to remain Keplerian

$$\boxed{e = GM/2r}$$

- Gravitational energy is released as heat, radiation, outflows and jets.



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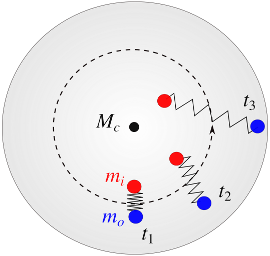
Angular momentum transport:

- Microscopic 'viscosity' in the gas is too weak to transport angular momentum
- Major Breakthrough: Magnetorotational Instability (MRI: Balbus & Hawley 1991)
- Weak magnetic field in a differentially rotating disk causes MRI
- Nonlinear development of MRI gives turbulence, which transports angular momentum outwards.



MRI mechanism with spring-mass analogue

Magnetic Fields in Accretion Flow

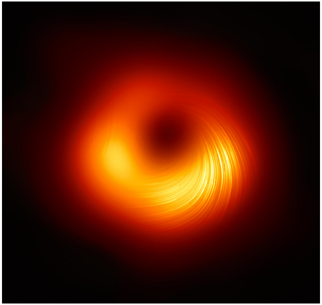


Image credit: EHT collaboration

Polarised image of M87 indicates strong poloidal flux at the black hole's edge.

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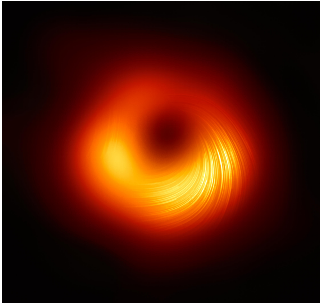

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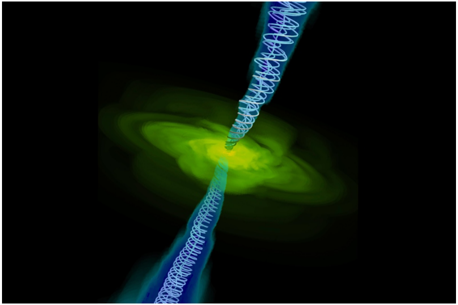

Image credit: Alexander Tchekhovskoy

Numerical simulations state the necessity of strong poloidal flux

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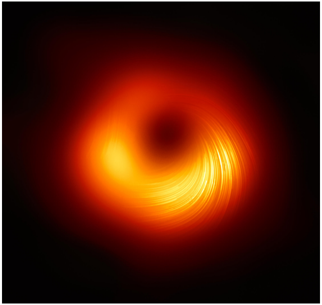

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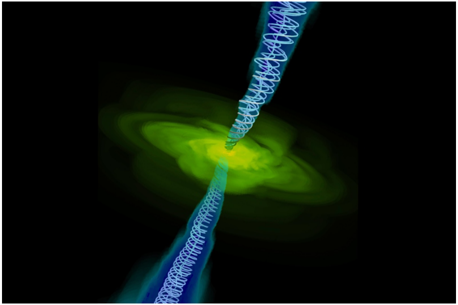

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Numerical simulations state the necessity of strong poloidal flux

What is the origin of such large-scale magnetic fields?

Direct numerical simulations show that the MRI generates its own large-scale magnetic fields through dynamo mechanisms.

Mean-field Theory

Mathematical model for the **Mean-field dynamo** &
Turbulent angular momentum transport

Mean-field Theory: Reynolds Averaging

The standard MHD equations

$$\frac{\mathscr{D}\rho}{\mathscr{D}t} = -\nabla\cdot(\rho\mathbf{u})\ , \tag{1}$$

$$\frac{\mathscr{D}\mathbf{u}}{\mathscr{D}t} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \frac{1}{\rho}\nabla P + f(\mathbf{u}) + \frac{1}{\rho}\mathbf{J}\times\mathbf{B} + \nu\nabla^2\mathbf{u}\ , \tag{2}$$

$$\frac{\mathscr{D}\mathbf{A}}{\mathscr{D}t} = q\Omega_0 A_y\hat{x} + \mathbf{u}\times\mathbf{B} - \eta\mu_0\mathbf{J}\ , \tag{3}$$

Here, $\mathscr{D}/\mathscr{D}t \equiv \partial/\partial t - q\Omega_0 x\partial/\partial y$, and $\overline{\mathbf{U}}^0 = -q\Omega_0 x\hat{y}$ is the background shear flow.

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Reynolds decomposition: $u = \overline{U} + u'$ and $B = \overline{B} + b'$, with $\overline{\overline{U}} = \overline{U}$ and $\overline{u}' = 0$ ...

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Mean equations

$$\mathscr{D}_t\bar{U}_i = -\bar{U}_j\partial_j\bar{U}_i - \frac{1}{\rho}\partial_i\left(\bar{P} + \frac{\bar{M}}{2}\right) + f_i(\bar{U}) + \frac{1}{\rho}\epsilon_{ijk}\bar{J}_j\bar{B}_k - \boxed{\frac{1}{\rho}\partial_j(\bar{R}_{ij} - \bar{M}_{ij})} + \nu\partial_{jj}\bar{U}_i \ , \quad (4)$$

$$\mathscr{D}_t\bar{A}_i = q\Omega_0\bar{A}_y\hat{x} + \epsilon_{ijk}\bar{U}_j\bar{B}_k + \boxed{\bar{\mathscr{E}}_i} - \eta\mu_0\bar{J}_i \ , \quad (5)$$

where $\bar{\mathscr{E}}_i = \overline{(u'\times b')}_i = \epsilon_{ijk}\bar{F}_{jk}$ is the mean electromotive force (EMF).
$\bar{M}_{ij} = \overline{b'_i b'_j}/\mu_0$, $\bar{R}_{ij} = \overline{\rho u'_i u'_j}$, and $\bar{F}_{ij} = \overline{u'_i b'_j}$ are the Maxwell, Reynolds and Faraday tensors.

Two traditionally decoupled mean-field theories

Angular Momentum Transport

$$
\begin{aligned}
\mathscr{D}_t \bar{M}_{ij} = -\bar{U}_k \partial_k \bar{M}_{ij} + \bar{M}_{ik}\partial_k \bar{U}_j + \bar{M}_{jk}\partial_k \bar{U}_i \\
+\frac{1}{\mu_0}\Big[\bar{B}_k \langle b'_i \partial_k u'_j + b'_j \partial_k u'_i \rangle \\
-(\bar{F}_{ki}\partial_k \bar{B}_j + \bar{F}_{kj}\partial_k \bar{B}_i)\Big] + \boxed{\mathscr{N}^M_{3P}},
\end{aligned}
$$

$$
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\mathscr{D}_t \bar{R}_{ij} = -\bar{U}_k \partial_k \bar{R}_{ij} - \bar{R}_{ik}\partial_k \bar{U}_j - \bar{R}_{jk}\partial_k \bar{U}_i \\
-2\epsilon_{jkl}\Omega_k \bar{R}_{il} - 2\epsilon_{ikl}\Omega_k \bar{R}_{jl} \\
+\frac{1}{\mu_0}\Big[\bar{B}_k \langle u'_i \partial_k b'_j + u'_j \partial_k b'_i \rangle \\
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Large-scale dynamo is switched off!
$\bar{F}_{ij} = 0$ and threaded by constant $\bar{B}_z$.

Kato+ 1995; Ogilvie 2003; Pessah+ 2006

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Mean-field Dynamo Theory

$$\partial_t \bar{B} = \nabla \times (\bar{U} \times \bar{B}) + \boxed{\nabla \times \bar{\mathscr{E}}} + \eta \nabla^2 \bar{B},$$

The closure on mean EMF

$$\begin{aligned} \bar{\mathscr{E}}_i &= \langle u' \times b' \rangle_i = \epsilon_{ijk}\bar{F}_{jk} \\ &\simeq \alpha_{ij}\bar{B}_j - \eta_{ij}\epsilon_{jkl}\partial_k \bar{B}_l. \end{aligned}$$

The mean-field dynamo theory has ignored the transport dynamics.

Brandenburg & Subramanian 2005

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Objective: To construct **a unified mean-field model** that combines large-scale dynamo and transport phenomena self-consistently.

Mean-field Dynamo

Induction equation

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Mean field evolution:

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The $\alpha - \Omega$ dynamo:

$$\text{Poloidal } (\overline{\mathbf{B}}_P) \xrightarrow{\Omega - \text{effect}} \text{Toroidal } (\overline{\mathbf{B}}_T)$$

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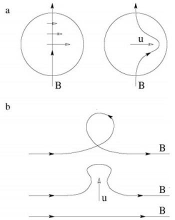

Schematic of the $\alpha - \Omega$ dynamo cycle

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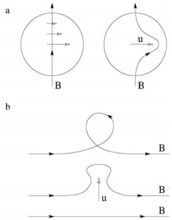
What happens when the α-effect is absent?



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  - To mitigate correlated noise, options include setting **some coefficients to zero** and addressing **degeneracies among different coefficients**.
- A general form of EMF: The assumption of expansion solely with respect to $\bar{B}$ may not be sufficient; disregarding the influence of $\bar{U}$.

Model

A Unified Mean-Field Model

Direct Numerical Simulations

$$
\begin{aligned}
\frac{\mathscr{D}\rho}{\mathscr{D}t} &= -\nabla\cdot(\rho\mathbf{u})\ , \\
\frac{\mathscr{D}\mathbf{u}}{\mathscr{D}t} &= -(\mathbf{u}\cdot\nabla)\mathbf{u} - \frac{1}{\rho}\nabla P + f(\mathbf{u}) \\
&\quad + \frac{1}{\rho}\mathbf{J}\times\mathbf{B} + \nu\nabla^2\mathbf{u}\ , \\
\frac{\mathscr{D}\mathbf{A}}{\mathscr{D}t} &= q\Omega A_y\hat{x} + \mathbf{u}\times\mathbf{B} - \eta\mu_0\mathbf{J}\ .
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The model consists of a set of 7 coupled partial differential equations.

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Direct Statistical Simulations

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$$\frac{\mathscr{D}\bar{A}}{\mathscr{D}t} = q\Omega\bar{A}_y\hat{x} + \bar{U}\times\bar{B} + \bar{\mathscr{E}}(\bar{F}_{ij}) - \eta\mu_0\bar{J} ,$$

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$$\frac{\mathscr{D}\bar{R}_{ij}}{\mathscr{D}t} = \cdots + \mathscr{N}_{3\mathrm{rd}}^{R}; \quad \frac{\mathscr{D}\bar{F}_{ij}}{\mathscr{D}t} = \cdots + \mathscr{N}_{3\mathrm{rd}}^{F}.$$

It consists of a set of 28 coupled PDEs.

Direct Statistical Simulation (DSS) with PENCIL Code

I developed a new **special module** in the Pencil code to study the MRI turbulence and dynamo in the unstratified shearing box with DSS method.

The closure model

In our closure model for the third-order correlators, we employ an approach inspired by the **CE2.5 approximation**, a generalization of the eddy-damped quasi-normal Markovian (**EDQNM**) approximation for inhomogeneous flows..

Li, Marston & Tobias 2021

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$$
\begin{aligned}
\mathcal{N}_{ij}^{M} &= \frac{2}{L}\left[\sqrt{M}(c_1\bar{R}_{ij} - c_2\bar{M}_{ij}) - c_3\sqrt{R}\bar{M}_{ij} - \frac{c_4}{2}\sqrt{F}(\bar{F}_{ij} + \bar{F}_{ji})\right] \\
\mathcal{N}_{ij}^{R} &= \frac{2}{L}\left[\sqrt{M}(c_2\bar{M}_{ij} - c_1\bar{R}_{ij}) - c_5\sqrt{R}\bar{R}_{ij} - \frac{c_4}{2}\sqrt{F}(\bar{F}_{ij} + \bar{F}_{ji}) - \frac{c_6}{2}\sqrt{R}(\bar{R}_{ij} - \frac{1}{3}\bar{R}\delta_{ij})\right]
\end{aligned}
$$

interaction, dissipation, isotropization

Li, Marston & Tobias 2021

Direct Statistical Simulation (DSS) with PENCIL Code

I developed a new **special module** in the Pencil code to study the MRI turbulence and dynamo in the unstratified shearing box with DSS method.

The closure model

In our closure model for the third-order correlators, we employ an approach inspired by the **CE2.5 approximation**, a generalization of the eddy-damped quasi-normal Markovian (**EDQNM**) approximation for inhomogeneous flows..

$$\mathcal{N}_{ij}^{M} = \frac{2}{L}\left[\sqrt{M}(c_1\bar{R}_{ij} - c_2\bar{M}_{ij}) - c_3\sqrt{R}\bar{M}_{ij} - \frac{c_4}{2}\sqrt{F}(\bar{F}_{ij} + \bar{F}_{ji})\right]$$

$$\mathcal{N}_{ij}^{R} = \frac{2}{L}\left[\sqrt{M}(c_2\bar{M}_{ij} - c_1\bar{R}_{ij}) - c_5\sqrt{R}\bar{R}_{ij} - \frac{c_4}{2}\sqrt{F}(\bar{F}_{ij} + \bar{F}_{ji}) - \frac{c_6}{2}\sqrt{R}(\bar{R}_{ij} - \frac{1}{3}\bar{R}\delta_{ij})\right]$$

interaction, dissipation, isotropization

$$\bar{\mathcal{N}}_{ij}^{F} = \frac{2}{L}\left[\sqrt{\bar{M}}(c_7\bar{F}_{ji} - c_8\bar{F}_{ij}) - c_9\sqrt{\bar{R}}\bar{F}_{ij} - \frac{c_{10}}{2}\sqrt{\bar{F}}\left(\bar{M}_{ij} + \bar{R}_{ij}\right) - \frac{c_{11}}{4}\sqrt{\bar{F}}\left(\bar{F}_{ij} + \bar{F}_{ji} - \frac{2}{3}\bar{F}\delta_{ij}\right)\right]$$

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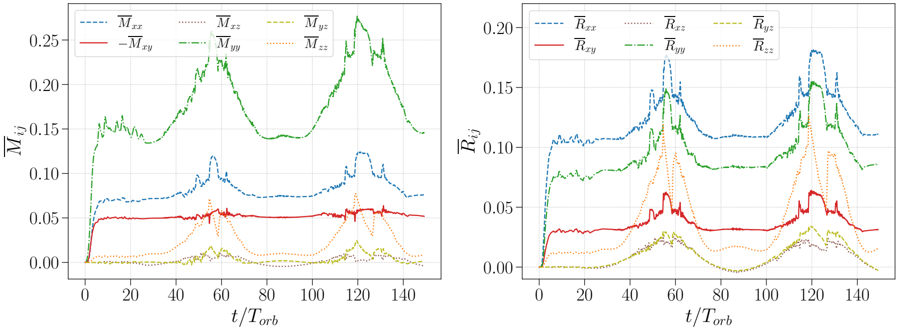
Results: Turbulent Angular Momentum Transport

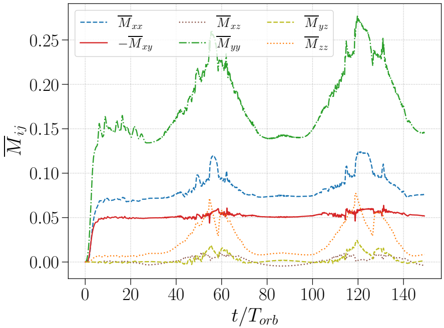
The responsible stress-tensors to transport angular momentum outward can be obtained from $i = y$

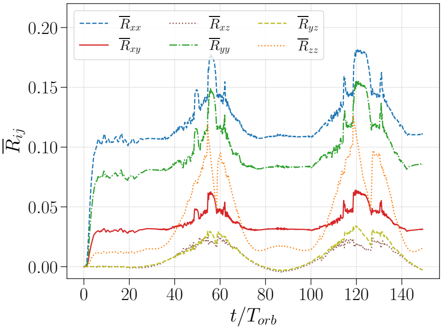
$$\mathscr{D}_t \bar{U}_i = -\bar{U}_j \partial_j \bar{U}_i - \frac{1}{\rho}\partial_i \left(\bar{P} + \frac{\bar{M}}{2} \right) + f_i(\bar{U}) + \frac{1}{\rho}\epsilon_{ijk} \bar{J}_j \bar{B}_k - \frac{1}{\rho}\partial_j (\bar{R}_{ij} - \bar{M}_{ij}) + \nu \partial_{jj} \bar{U}_i$$

Results: Maxwell and Reynolds Tensors



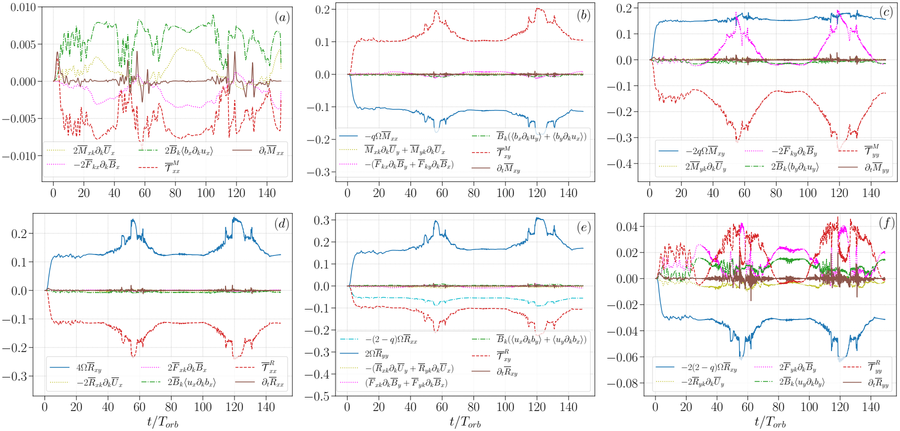
Results: Maxwell and Reynolds Tensors





- The dominant contribution in transport arises from the correlated magnetic fluctuations, rather than from their kinetic counterpart, i.e., $-\bar{M}_{xy} > \bar{R}_{xy}$.
- The turbulent magnetic energies: $\bar{M}_{yy} > \bar{M}_{xx} > \bar{M}_{zz}$.
- The turbulent kinetic energies: $\bar{R}_{xx} > \bar{R}_{yy} > \bar{R}_{zz}$.

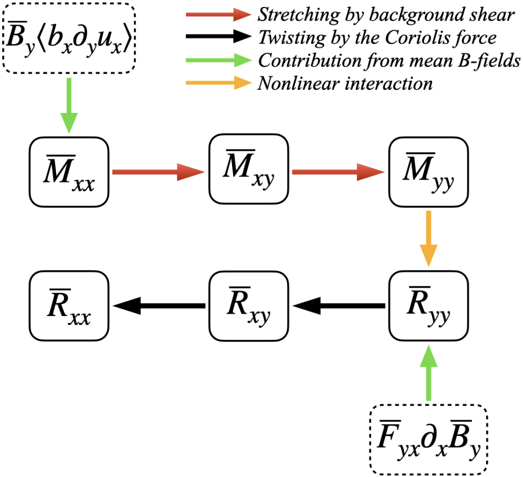
Turbulent Angular Momentum Transport



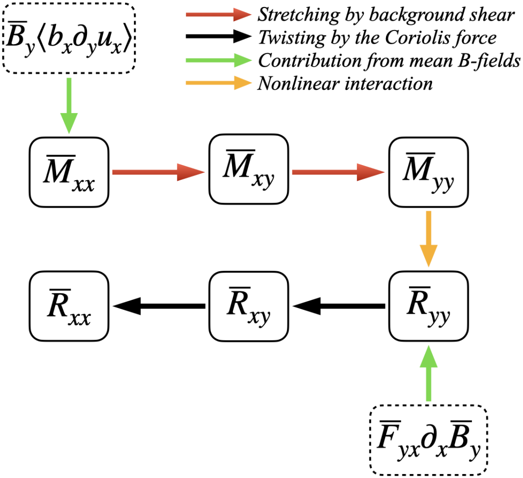
Time-evolution of individual terms appeared in the volume-averaged equations for Maxwell stress and Reynolds stress.

Upper panels: (a) $\bar{M}_{xx}$, (b) $\bar{M}_{xy}$, (c) $\bar{M}_{yy}$; Lower panels: (d) $\bar{R}_{xx}$, (e) $\bar{R}_{xy}$, (f) $\bar{R}_{yy}$.

Turbulent Angular Momentum Transport



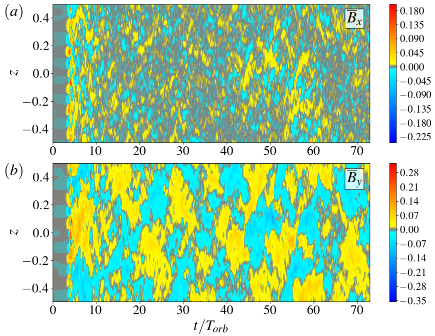
Turbulent Angular Momentum Transport



The turbulent transport is not possible without large-scale magnetic fields, i.e., the mean-field dynamo mechanism is necessary.

Mean-field Dynamo in Unstratified Shearing-box

Mean-field Dynamo in Unstratified Shearing-box



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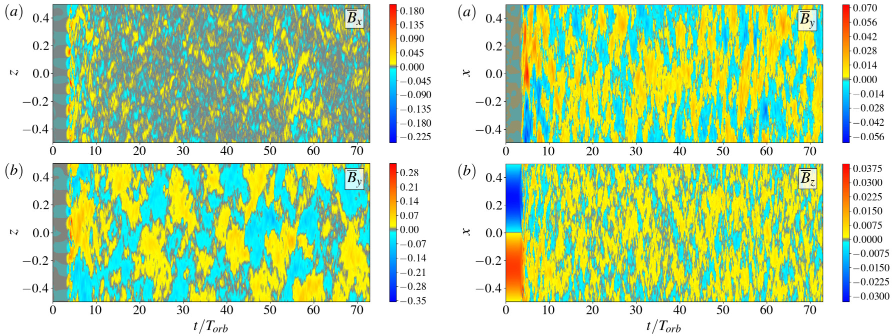
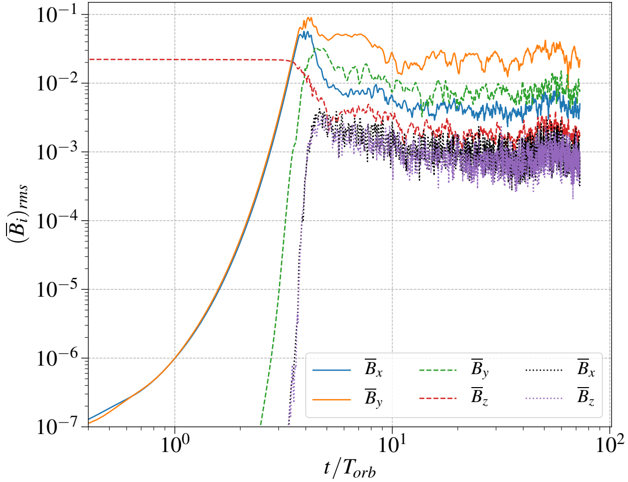


Mean-field Dynamo in Unstratified Shearing-box



— xy averaging $\bar{B}(z)$, - - - yz averaging $\bar{B}(x)$, $\cdots$ xz averaging $\bar{B}(y)$

Mean-field Dynamo

What are the roles of the different terms in generating mean magnetic fields from the planar-averaged induction equations?

Mean-field Dynamo

The mean field equations in x–y averaging are given by

$$\partial_t \bar{B}_x = -\partial_z \bar{\mathscr{E}}_y - \bar{U}_z \partial_z \bar{B}_x + \bar{B}_z \partial_z \bar{U}_x,$$
$$\partial_t \bar{B}_y = -q\Omega \bar{B}_x + \partial_z \bar{\mathscr{E}}_x - \bar{U}_z \partial_z \bar{B}_y + \bar{B}_z \partial_z \bar{U}_y.$$

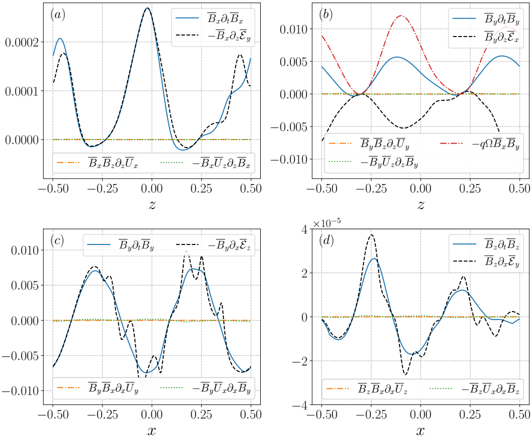
The mean field equations in y–z averaging are given by

$$\partial_t \bar{B}_y = -\partial_x \bar{\mathscr{E}}_z - \bar{U}_x \partial_x \bar{B}_y + \bar{B}_x \partial_x \bar{U}_y,$$
$$\partial_t \bar{B}_z = \partial_x \bar{\mathscr{E}}_y - \bar{U}_x \partial_x \bar{B}_z + \bar{B}_x \partial_x \bar{U}_z.$$

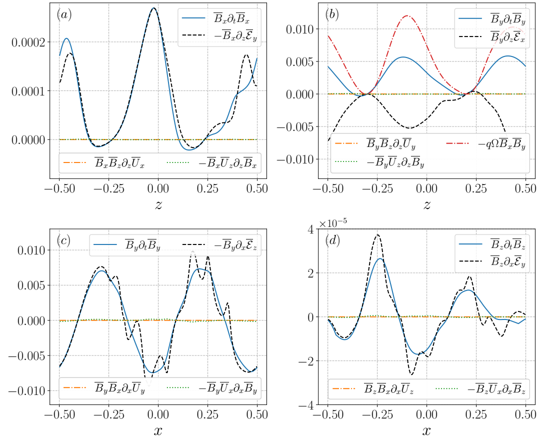
Here, the contributions arise from

- the shear term $(-q\Omega \bar{B}_x)$
- different components of the EMF $\bar{\mathscr{E}}_i$
- the advection term $(\bar{\mathbf{U}} \cdot \nabla \bar{\mathbf{B}})$
- the stretching term $(\bar{\mathbf{B}} \cdot \nabla \bar{\mathbf{U}})$

Mean-field Dynamo



Mean-field Dynamo



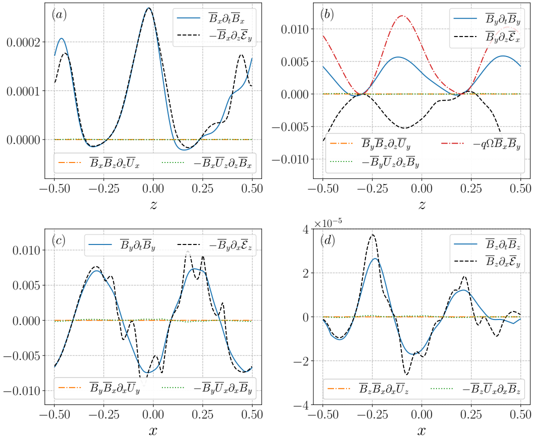
For x–y averaging (upper panels)

- The mean field $\bar{B}_x(z)$ arises from the vertical variation of the azimuthal EMF, $\partial_z\bar{\mathscr{E}}_y$.
- For $\bar{B}_y(z)$, shear acts as a source (the Ω-effect), whereas the radial EMF acts as a sink.

For y–z averaging (lower panels)

- Large-scale fields originate entirely from the respective EMF.

Mean-field Dynamo



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- The mean field $\bar{B}_x(z)$ arises from the vertical variation of the azimuthal EMF, $\partial_z \bar{\mathscr{E}}_y$.
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For y–z averaging (lower panels)

- Large-scale fields originate entirely from the respective EMF.

Our findings align well with those obtained from DNS.
(Lesur & Ogilvie 2008; Bhat, Ebrahimi, & Blackman 2016)

Mean-field Dynamo: Construction of the EMF

Is it possible to construct a general form of the EMF, inclusive of inhomogeneities and anisotropies, self-consistently?

What is the explicit form of the transport coefficients?

Mean-field Dynamo: Construction of the EMF

- **The EMF is defined as $\bar{\mathscr{E}}_i = (\overline{\mathbf{u}' \times \mathbf{b}'})_i = \epsilon_{ijk}\bar{F}_{jk}$.**

$$\mathscr{D}_t \bar{F}_{ij} + \bar{U}_k \partial_k \bar{F}_{ij} - \bar{F}_{ik}\partial_k \bar{U}_j + \bar{F}_{kj}\partial_k \bar{U}_i + \boxed{2\epsilon_{ikl}\Omega_k \bar{F}_{lj} + \bar{S}^F_{ij}} - \frac{1}{\rho}(\bar{M}_{jk}\partial_k \bar{B}_i - \bar{R}_{ik}\partial_k \bar{B}_j)$$
$$= -\frac{1}{\rho}\langle b'_j \partial_i \Pi' \rangle + \bar{B}_k \langle u'_i \partial_k u'_j \rangle + \frac{\bar{B}_k}{\mu_0 \rho}\langle b'_j \partial_k b'_i \rangle + \bar{\mathscr{T}}^F_{ij} + \eta \langle u'_i \partial_{kk} b'_j \rangle + \nu \langle b'_j \partial_{kk} u'_i \rangle.$$

Here, $\bar{S}^F_{ij} = -\bar{F}_{ik}\partial_k \bar{U}^0_j + \bar{F}_{kj}\partial_k \bar{U}^0_i$, with $\bar{U}^0 = -q\Omega_0 x\hat{y}$ is the background shear.

- We utilize the interaction terms arising from the Coriolis force and background shear in the evolution equations for the Faraday tensors to construct the EMF.

Mean-field Dynamo: Construction of the EMF

- **Traditional mean-field theory**

$$\partial_t \overline{\mathbf{B}} = \nabla \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}}) + \boxed{\nabla \times \overline{\mathcal{E}}} + \eta \nabla^2 \overline{\mathbf{B}},$$

$$\overline{\mathcal{E}}_i = (\overline{u' \times b'})_i = \epsilon_{ijk} \bar{F}_{jk} = \alpha_{ij} \overline{B}_j - \eta^t_{ij} \epsilon_{jkl} \partial_k \overline{B}_l.$$

Mean-field Dynamo: Construction of the EMF

- **Traditional mean-field theory**

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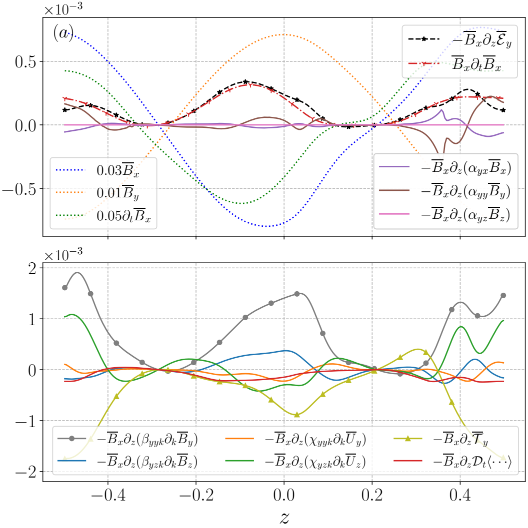
- **Poloidal magnetic fields, $\bar{B}_x(z)$ and $\bar{B}_z(x)$, appear through the azimuthal EMF**

$$\begin{aligned}
\bar{\mathscr{E}}_y \quad = \quad & (\bar{F}_{zx} - \bar{F}_{xz}) = \frac{-1}{q(2-q)\Omega} \Big[- q \mathscr{D}_t \bar{F}_{yz} + (2-q) \mathscr{D}_t \bar{F}_{zy} \\
& + \left\{ q \bar{F}_{yk} + (2-q) \bar{F}_{ky} \right\} \partial_k \bar{U}_z - \left\{ q \bar{F}_{kz} + (2-q) \bar{F}_{zk} \right\} \partial_k \bar{U}_y \\
& + \frac{1}{\rho} \left\{ q \bar{M}_{zk} + (2-q) \bar{R}_{zk} \right\} \partial_k \bar{B}_y - \frac{1}{\rho} \left\{ q \bar{R}_{yk} + (2-q) \bar{M}_{yk} \right\} \partial_k \bar{B}_z \\
& + \frac{\bar{B}_k}{\mu_0 \rho} \left\{ q \left(\langle u_y \partial_k u_z \rangle + \langle b_z \partial_k b_y \rangle \right) - (2-q) \left(\langle u_z \partial_k u_y \rangle + \langle b_y \partial_k b_z \rangle \right) \right\} + \mathscr{T}_y \Big]
\end{aligned}$$

- ◇ mean magnetic fields
- ◇ gradient of mean velocity fields
- ◇ gradient of mean magnetic fields
- ◇ nonlinear three-point terms

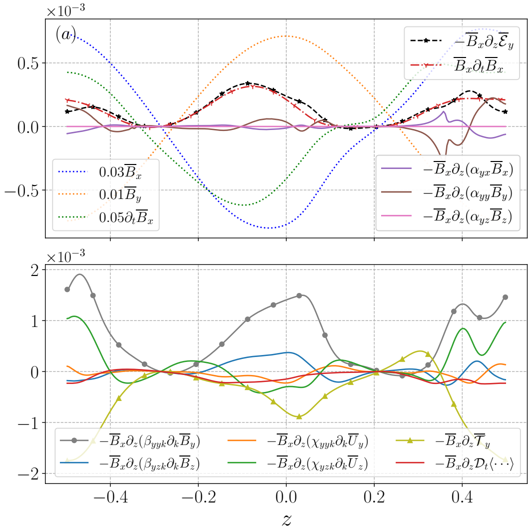
Generation of radial magnetic fields



$$\partial_t \bar{B}_x = \boxed{-\partial_z \bar{\mathscr{E}}_y} - \bar{U}_z \partial_z \bar{B}_x + \bar{B}_z \partial_z \bar{U}_x,$$

- To assess the contribution of each term, we multiply $\bar{B}_x$ on both sides of $-\partial_z \bar{\mathscr{E}}_y$ equation.

Generation of radial magnetic fields



$$\partial_t \bar{B}_x = \boxed{-\partial_z \bar{\mathscr{E}}_y} - \bar{U}_z \partial_z \bar{B}_x + \bar{B}_z \partial_z \bar{U}_x,$$

- To assess the contribution of each term, we multiply $\bar{B}_x$ on both sides of $-\partial_z \bar{\mathscr{E}}_y$ equation.
- The term proportional to $\partial_z \bar{B}_y$ plays a significant role in the growth of $\bar{B}_x(z)$, while nonlinear term acts as a sink.
- The associated turbulent transport coefficient:

$$\boxed{\eta_{yx} = -\frac{1}{\Omega}\left[\frac{1}{2-q}\bar{M}_{zz} + \frac{1}{q}\bar{R}_{zz}\right]}$$

Mean-field Dynamo: Generation of radial magnetic fields

$$\boxed{\partial_t \bar{B}_x = -\partial_z \bar{\mathscr{E}}_y = -\partial_z(\eta_{yx} \bar{J}_x)}, \quad \boxed{\eta_{yx} = -\frac{1}{\Omega}\left[\frac{1}{2-q}\bar{M}_{zz} + \frac{1}{q}\bar{R}_{zz}\right]}$$

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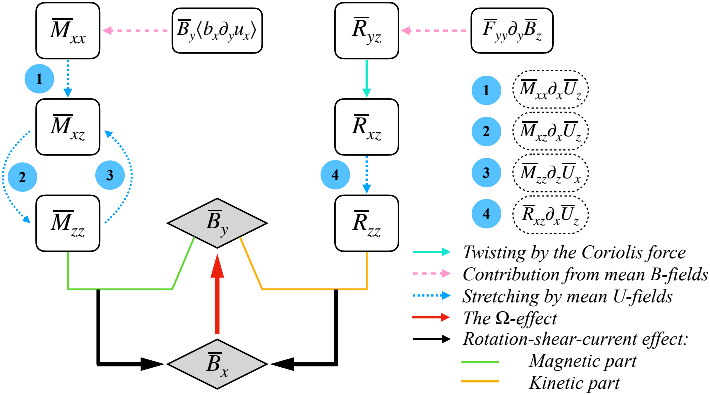

Mean-field Dynamo: Generation of radial magnetic fields

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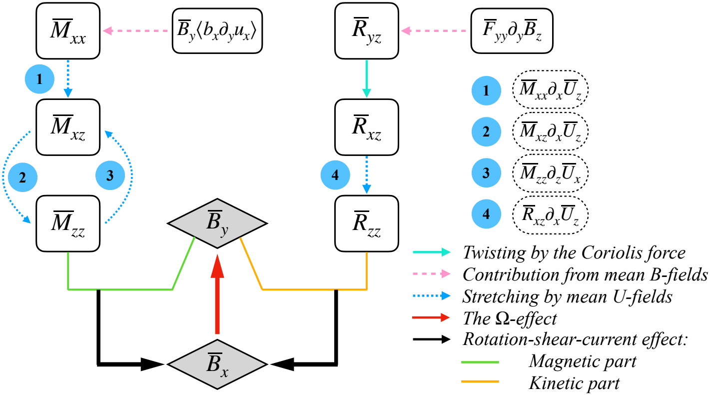

We uncover a significant revelation: the presence of a large-scale vorticity dynamo is essential to operate the "**rotation-shear-current effect**."

Discussion: conflict associated with the sign of η_{yx}

$$\boxed{\eta_{yx} = -\frac{1}{\Omega}\left[\frac{1}{2-q}\bar{M}_{zz} + \frac{1}{q}\bar{R}_{zz}\right]} \xrightarrow{\text{without rotation}} \boxed{\eta_{yx} = -\frac{1}{\Omega}\left[-\frac{1}{q}\bar{M}_{zz} + \frac{1}{q}\bar{R}_{zz}\right]}$$

In the absence of rotation, relevant to the traditional
shear-current effect: kinetic contribution η^{u}_{yx} has a favorable negative sign for dynamos.
magnetic shear-current effect: magnetic contribution η^{b}_{yx} has wrong sign for dynamos.

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magnetic shear-current effect: magnetic contribution η_{yx}^{b} has wrong sign for dynamos.

Kinetic contribution: In kinetically forced simulations

- Using projection method, it has been found that η_{yx}^{u} is positive with only shear, and negative when a Keplerian rotation is added (Squire and Bhattacharjee 2015).
- Nonlinear test-field method in MHD burgulence has reported a negative η_{yx}^{u} for the non-rotating case, but did not explore the case including rotation (Kapyla et al. 2020).

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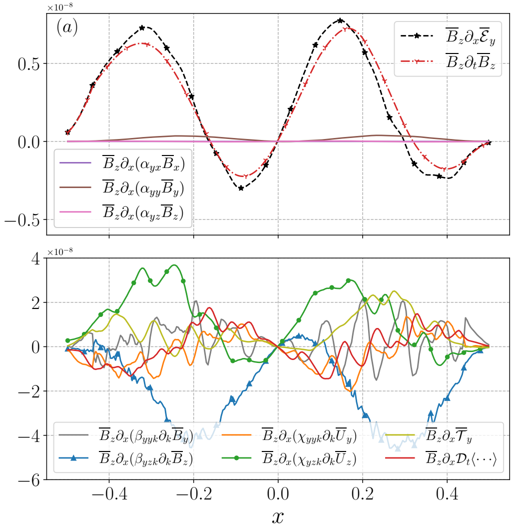
Magnetic contribution

- Magnetically forced quasi-linear simulations using projection method found that $\eta^b_{yx} < 0$ either with or without Keplerian rotation (Squire and Bhattacharjee 2015).
- Nonlinear test-field method in MHD burgulence with magnetic foring found that $\eta^b_{yx} > 0$ for non-rotating shearing cases (Kapyla et al. 2020).

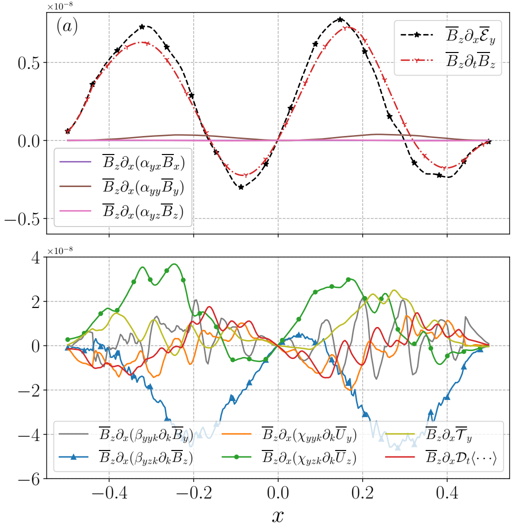
Generation of vertical magnetic fields, $\bar{B}_z(x)$



$$\partial_t \bar{B}_z = \boxed{\partial_x \bar{\mathcal{E}}_y} - \bar{U}_x \partial_x \bar{B}_z + \bar{B}_x \partial_x \bar{U}_z$$

- To assess the contribution of each term, we multiply $\bar{B}_z(x)$ on both sides of $\partial_x \bar{\mathcal{E}}_y$ equation.

Generation of vertical magnetic fields, $\bar{B}_z(x)$



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- The associated turbulent transport coefficient:

$$\boxed{-\frac{1}{\Omega}\left[\frac{1}{(2-q)}\bar{F}_{yx} + \frac{1}{q}\bar{F}_{xy}\right]}$$

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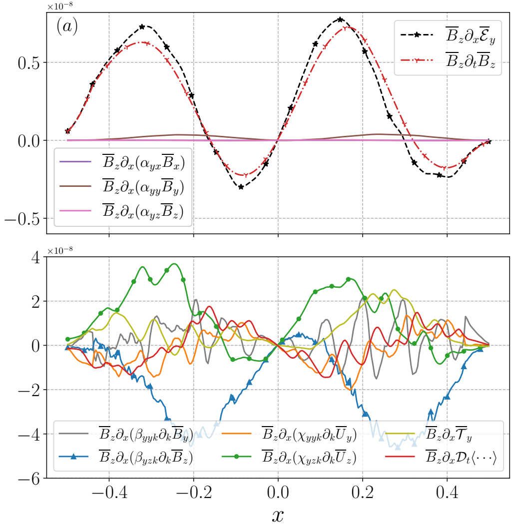

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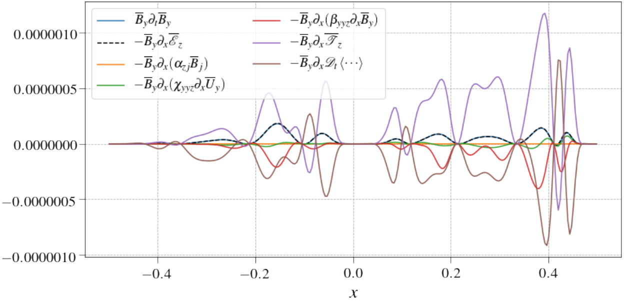
This dynamo mechanism is named as the **rotation-shear-vorticity effect**.

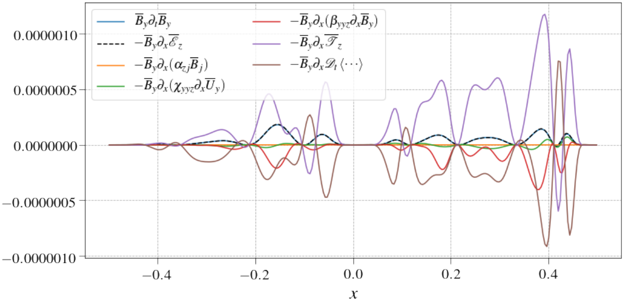
Generation of azimuthal magnetic fields, $\bar{B}_y(x)$



$$\partial_t \bar{B}_y = \boxed{-\partial_x \bar{\mathscr{E}}_z} - \bar{U}_x \partial_x \bar{B}_y + \bar{B}_x \partial_x \bar{U}_y$$

- To assess the contribution of each term, we multiply $\bar{B}_y(x)$ on both sides of $\partial_t \bar{B}_y$.

Generation of azimuthal magnetic fields, $\bar{B}_y(x)$

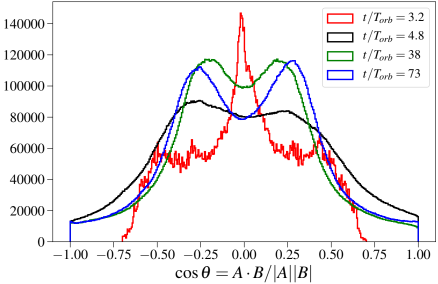


$$\partial_t \bar{B}_y = \boxed{-\partial_x \bar{\mathscr{E}}_z} - \bar{U}_x \partial_x \bar{B}_y + \bar{B}_x \partial_x \bar{U}_y$$

- To assess the contribution of each term, we multiply $\bar{B}_y(x)$ on both sides of $\partial_t \bar{B}_y$.
- The third-order correlator plays a significant role in the growth of $\bar{B}_y(x)$.

Magnetic Helicity

Magnetic helicity, $H = \int_V A \cdot B \; dV$**, measures linkage of flux.**



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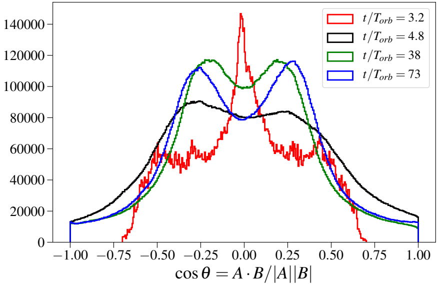
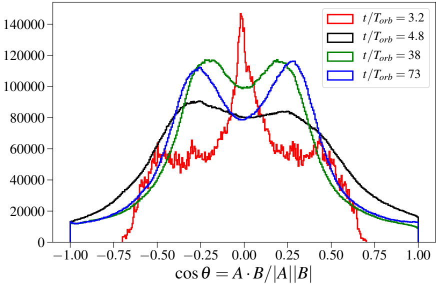
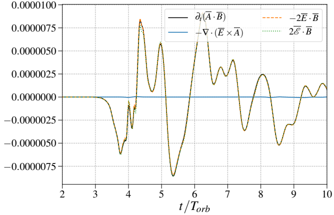


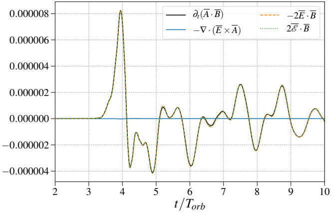
$$\partial_t(\bar{A} \cdot \bar{B}) = -\nabla \cdot (\bar{E} \times \bar{A}) - 2\bar{E} \cdot \bar{B},$$
$$\bar{E} = \bar{J}/\sigma - \bar{U} \times \bar{B} - \bar{\mathscr{E}}.$$

Magnetic Helicity

Magnetic helicity, $H = \int_V A \cdot B \; dV$, measures linkage of flux.



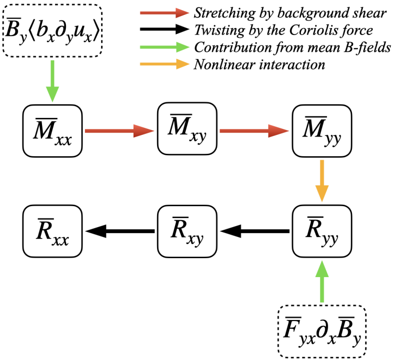




$$\partial_t(\bar{A}\cdot\bar{B}) = -\nabla\cdot(\bar{E}\times\bar{A}) - 2\bar{E}\cdot\bar{B},$$
$$\bar{E} = \bar{J}/\sigma - \bar{U}\times\bar{B} - \bar{\mathscr{E}}.$$

Summary

Angular Momentum Transport



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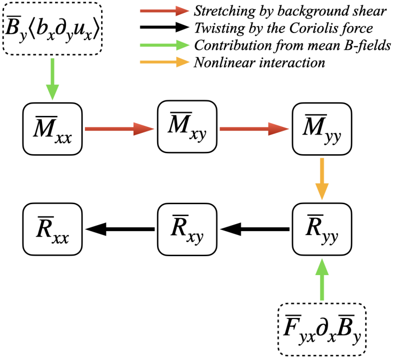
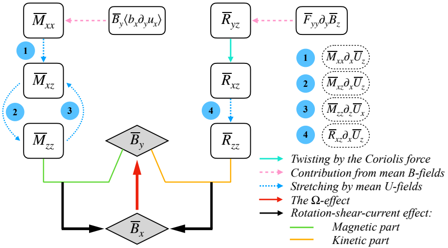


Mean-field Dynamo

- We identify two crucial dynamo mechanisms, the *rotation-shear-current effect* and the *rotation-shear-vorticity effect*, that are responsible for generating the radial and vertical magnetic fields, respectively.



Mondal & Bhat, Phys. Rev. E 108, 065201 (2023)

Next

Objective I: Modeling a unified mean-field theory for a stratified disk

- Understand the intricate network of interactions connecting different stress components and mean fields.
- MRI dynamo cycles for $\bar{B}_y(z)$, their origin and implications.

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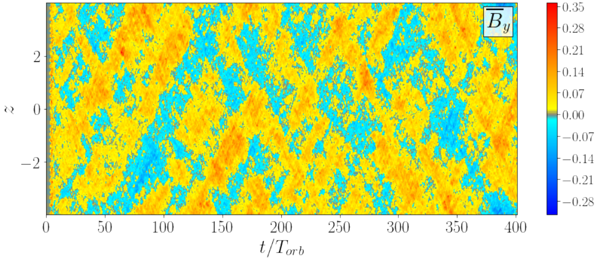

- Vertically elongted box: 1:1:8, **grid**: 128x128x1024; Pm = 8.
  Cubic box: 1:1:1, **grid**: 256x256x256; Rm = 1250, Pm = 4.
- The effects of vertical boundary instead of periodic one.

Next

Objective II: Interplay between large-scale vorticity & magnetic fields dynamos

- Identify the dominant mechanisms responsible for generating large-scale velocity and magnetic fields.

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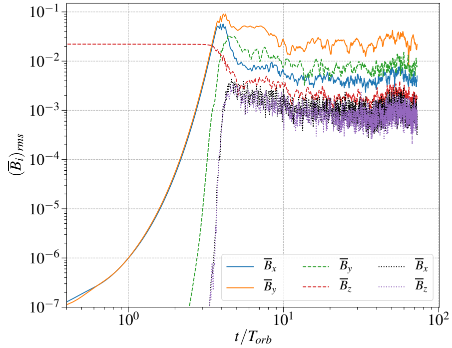

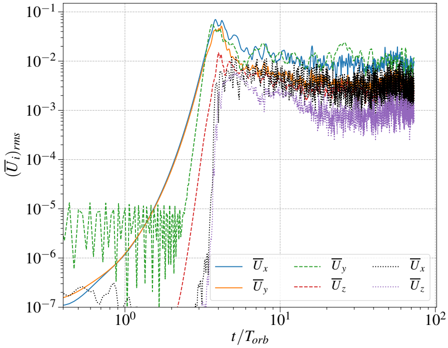

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Objective III: Unraveling dynamo mechanism in global simulations

- The in-situ origin of a large-scale poloidal magnetic flux which is essential for launching powerful jets.
- Role of α **(helicity)** effect, **rotation-shear-current** and **rotation-shear-vorticity** effects in generating large-scale poloidal magnetic fields.
- Developing reduced (subgrid) mean-field models to comprehend dynamo mechanisms.

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Thank you for your attention ..

Two-Scale Approach

Let us derive $\bar{Y}_{ij} = \bar{B}_m \langle u_i \partial_m u_j \rangle$. Employing Fourier transformation and two-scale approach:

$$\begin{aligned}\langle u_i(\mathbf{x}_1) u_j(\mathbf{x}_2) \rangle &= \int \langle \hat{u}_i(\mathbf{k}_1) \hat{u}_j(\mathbf{k}_2) \rangle \exp\{i(\mathbf{k}_1 \cdot \mathbf{x}_1 + \mathbf{k}_2 \cdot \mathbf{x}_2)\} \, d^3k_1 \, d^3k_2 \\ &= \int R_{ij}(\mathbf{k}, \mathbf{X}) \exp(i\mathbf{k} \cdot \mathbf{x}) \, d^3k \ ,\end{aligned}$$

where,

$$R_{ij}(\mathbf{k}, \mathbf{X}) = \int \langle \hat{u}_i(\mathbf{k} + \mathbf{K}/2) \hat{u}_j(-\mathbf{k} + \mathbf{K}/2) \rangle \exp(i\mathbf{K} \cdot \mathbf{X}) \, d^3K \ ,$$

and $\mathbf{X} = (\mathbf{x}_1 + \mathbf{x}_2)/2$, $\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2$, $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2$, $\mathbf{k} = (\mathbf{k}_1 - \mathbf{k}_2)/2$. Here, $\mathbf{X}$ and $\mathbf{K}$ correspond to the large scales, while $\mathbf{x}$ and $\mathbf{k}$ correspond to the small scales.
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Consider the computation of $\bar{W}_{ij} = \bar{B}_m \langle u_i \partial_m b_j \rangle = \bar{B}_m \bar{W}_{ijm}$, where $\bar{W}_{ijm} = \langle u_i \partial_m b_j \rangle$.

$$\begin{aligned} W_{ijm}(\mathbf{k}, \mathbf{X}) &= i \int (-k_m + K_m/2) \langle \hat{u}_i(\mathbf{k} + \mathbf{K}/2) \hat{b}_j(-\mathbf{k} + \mathbf{K}/2) \rangle \exp(i\mathbf{K} \cdot \mathbf{X})\, d^3K \ , \\ &= -ik_m F_{ij}(\mathbf{k}, \mathbf{X}) + \frac{1}{2} \nabla_m F_{ij}(\mathbf{k}, \mathbf{X}). \end{aligned}$$

$$\therefore \bar{W}_{ij}(x = 0) = \bar{B}_m \int W_{ijm}(\mathbf{k}, \mathbf{X})\, d^3k = -\bar{B}_m \int ik_m F_{ij}(k) d^3k + \frac{1}{2} (\bar{\mathbf{B}} \cdot \nabla) \bar{F}_{ij}.$$