The Macroeconomic Impact of Firm's Defensive Practices*

Iacopo Varotto

Banco de España

April 7, 2023

Abstract

Policy-makers are concerned that firms at the technological frontier are proactively blocking the entry and distorting knowledge diffusion for defensive purposes, leading to further market concentration and a productivity slowdown. Should these defensive practices be banned? To answer this question, I consider a heterogeneous firm model of technology adoption, wherein firms strategically compete both in the production and innovation activities. Importantly, firms can invest in two types defensive practices, those that deter the entry in the industry and those that slow down the catch-up of the laggards. I calibrate my model to the 2015 U.S. economy. My quantitative analysis suggests that cracking down on firms' defensive practices reduces endogenous productivity by 1.27 percentage points. Conversely, the model predicts that competition policies that only tackle defensive practices that reduce the catch-up improves the endogenous productivity by 0.61 percentage points.

Key Words: Endogenous Productivity, Firm Heterogeneity, Competition Policy.

*Addresses: iacopo.varotto@bde.es.I am grateful to Tatsuro Senga for his constant and valuable advice, patience and support that allowed me to get through this project. I also thank Giulio Fella, Sang Yoon Lee, Omar Rachedi and Yu Zheng for their suggestions. I also benefited from comments and suggestions from Alessandra Bonfiglioli, Anthony Savagar and Petr Sedláček. All remaining errors are mine. The opinions in this paper are those of the authors and do not necessarily reflect the views of the Banco de España or the Eurosystem.

1 Introduction

> "*Success doesn't necessarily comes from breakthrough innovation but from flawless execution. ... the win comes from basic blocking and tackling.* " Jain Naveen[1]

Market concentration has been increasingly becoming a common feature across developed economies, with a small number of superstar firms taking up a substantial market share and charging high markups.[2] This has caused policy-makers to question the potentially anti-competitive nature of superstar firms, scrutinizing, for instance, their defensive practices that distort knowledge diffusion and make difficult for competitors to innovate against.[3] Should these defensive practices be banned? I argue otherwise in this paper.[4] I develop a quantitative model of oligopolistic competition with technology adoption and I calibrated to the U.S. economy. I show that cracking down on defensive practices decreases the endogenous productivity by 1.27 percentage points and it increases consumption and investment respectively by 0.2 and 4.68 percentage points. Where does this ambiguity come from? On the one hand, imposing restrictions on firms' ability to control their dominant position improves the technology adoption and knowledge spillover, allowing more laggard firms to catch-up and compete against frontier firms. This creates more industries that are characterized by a neck-to-neck at the technological frontier. On the other hand, more competition implies that firms' ability to appropriate the profits generated by innovation is limited, which in turn discourages firms to improve the technology and to develop new products. My model has the following features distinct in the literature. Firstly, each industry is populated with a finite number of leaders and followers who strategically compete both statically in the production (*à la Cournot*) and dynamically in the innovation activities, playing contests in the spirit of Loury (1979). Secondly, leaders' innovation generates technology spillovers toward followers to catch-up and toward entrants by improving their potential available technology. Neverthe-

[1] Jain Naveen, 10 Secrets of Becoming a Successful Entrepreneur, INC. (August 13, 2012).

[2] See, for example, De Loecker et al. (2020), Autor et al. (2020), for seminal works.

[3] Strategic patenting is a prime example of this and Akcigit and Ates (2019) provide evidence of the use of patents to shield from imitators. See the statement of the Congressman, David Cicilline, Chairman of the U.S. House Antitrust, Commercial, and Administrative Law Subcommittee, in 2019.

[4] See the statement of Senator Elizabeth Warren, in March 2019 for her argument advocating to break up the big tech companies and the article of Feiner, Lauren. *"Lina Khan's FTC takes a first step toward expanding antitrust enforcement."* CNBC. 10 August 2021.

less, leaders can also influence the extent of technology spillovers by conducting two types of defensive practices. First leaders can implement defensive practices that reduce the probability of catch-up of the followers. Second, leaders can implement defensive practices that reduce the probability of the entry of an additional follower. For them, productivity improving and defensive intangible investments pair with complementarity. Thirdly, the model features two types of entry of firms. A new firm can either enter the incumbent industry as a follower or create a new technology to produce a new product and enter the new marketplace as a leader[5].

The baseline model is calibrated to U.S. economy. The model performances well in matching both the elasticity of intangible investment intensity with respect firm size, the average and dispersion of firms' market share build on the measures of product similarity of Hoberg and Phillips (2016), as well as the defensive intensity proxied by the share of patent citation that cite patents already existing within firms as stated in Akcigit and Ates (2019). Within this framework, this paper draws five main contributions to the study on the firm-level and macroeconomic implications of firms' defensive investment.

First, the model shows that cracking down on firms' defensive practices affects endogenous productivity through two channels. First It raises within industry competition for two reason. Firstly, there are no anymore barriers on entry that make difficult for firms to enter the incumbent industries as additional followers. Secondly the number of firms at the industry technology frontier increases since followers can now easily copy the technology of the leaders. This creates more industries that are characterized by a neck-to-neck at the technological frontier. Furthermore it reduces firms' market power and the dispersion of the firm size distribution by respectively 3.73 and 50 percentage points. As a result, the productivity of the followers raises by 0.62 percentage points. On the other hand, the model shows that the lack of the appropriability reduces the innovation of both the leaders to improve their technology and potential entrepreneurs to create new products. More specifically the productivity of leaders and the product variety fall respectively by 0.75 and 9.78 percentage points. The model shows that the latter effect is stronger than the former. As a result, preventing firms from investing in defensive practices reduces firms' productivity by 0.5 percent-

[5] In the spirit of Hoberg and Phillips (2016), I refer to an industry as the set of firms that operate in the same marketplace, whose choices mutually affect each other decisions and pay-offs. In the paper I use the words industry and market-place interchangeably.

age points that together with the fall in the product variety translate into a loss of 1.27 percentage points in the endogenous productivity.

Second, the paper shows that cracking down firms defensive practices increases both aggregate consumption and investment by respectively 0.19 and 4.68 percentage points. Given the lower endogenous productivity of the economy without defensive practices, the raise in the aggregate output is explained by the large number of firms that populate the economy. However, the raise in consumption is not enough to compensate the raise in labor and the consumption equivalent utility falls by 0.63 percentage points.

Third, the paper reveals that a competition policy that only precludes the possibility of implementing defensive practices that reduce the probability of catch-up improves the endogenous productivity relative to the economy with both defensive practices. The reason is twofold. First, it does not cause a dramatic reduction in the product variety in the economy. Second, it improves the technology adoption of the firms. In particular, it incentivises the leaders' innovation in industries characterised by high productivity gap between leaders and followers. The raise in the degree of competition compensates the fall in the incentive of the leader's innovation due to the lack of appropriability. Intuitively, especially in industries where leaders have a high productivity advantage with respect to the followers, defensive practices that limit the catch-up generate a strong reduction in the competition at the technology frontier that reduces the innovation. As a results, competition policies that achieve the removal of such defensive practices increase the productivity of both leaders and followers respectively by 0.63 and 1.34 percentage points relative to the baseline economy. The improvement in the firms' technology adoption leads to an increase in the endogenous productivity and consumption respectively by 0.61 and 1.48 percentage points. Furthermore this competition policy increases the consumption equivalent welfare by 1.39 percentage points.

Why does defensive practices that govern entry improve endogenous productivity? The reason is that the higher market power due to the reduction in entry is compensated by the raise in the innovation of leaders and potential entrepreneurs. Furthermore, defensive practices that deter the entry do not affect the catch-up of the incumbent followers. In other words, defensive practices that limit the entry incentivise leader's innovation without preventing the catch-up of the incumbent followers.

Fourth, I show that models that do not take into account the channel of product variety would predict that the economy without defensive practices has the highest endogenous productivity. Intuitively, the fall in firms' market power more than compensates the worse firm' technology adoption relative to the economy with only defensive practices that deter the entry.

Finally, the defensive practices that limit the entry always lead to the highest endogenous productivity relative to the the economy without it. More specifically, in economies of low product substitutability limiting the entry leads to an improvement in the product variety that more than compensates the worse technology adoption and the higher market power relative to the economy without it. In case the economy is characterised by a high product substitutability, the competition policy that allows firms to deter the entry generates the highest endogenous productivity since it provides the best technology adoption.

Related literature The theoretical ambiguity of the macroeconomic effect of firms' defensive behavior is associated with the non-monotonic relationship between competition and innovation as shown in Aghion et al. (2005). The concept retraces to Schumpeter (1976) that argues that the innovation activities are driven by the expectation of monopolistic rents, and consequently, high competition may weaken the incentive to innovate. Hence, defensive behavior may promote innovation by spurring the introduction of more productive technologies in the incumbent industries and the development of new technologies to produce new products, because it reduces the *knowledge spillovers* and increases the innovator's ability to appropriate the profits generated by the innovation[6]. Conversely, Arrow (1962) argues that monopolists lack the incentive to innovate because they already enjoy supernormal profits. My model reproduces the inverted-U relationship between the industry's expected R&D intensity and the competition level approximated with the Herfindahl index, mimicking the evidence in Aghion et al. (2005). This quadratic relationship is built on the different correlations between the market share and concentration that leaders and followers have. Both leaders and followers show that firm's R&D intensity is negatively correlated with market share, supporting the studies of Akcigit and Kerr (2018) and Akcigit (2009), but leaders' market share is positively correlated with the concentration. Indeed, in high concentrated industries, leaders do not invest in R&D because the technology spillovers are high, and they massively invest in

[6]As in Acemoglu and Akcigit (2011), the *trickle-down* incentive is indispensable for designing intellectual policy right (IPR) system. Greater protection to firms that have greater leads improves the incentives of innovation both to technologically advanced firms and to firms with limited leads because of the prospect of reaching levels with higher technology protection.

defensive practices to maintain their competitive advantage.

This paper is inspired by the idea that firms compete concurrently in the product market and innovation activities, as well as in the implementation of practices to defend their advantage that retraces to Stigler (1971) that argues that regulation is acquired by the industry and is designed and operated primarily for its benefit[7]. Later, this concept was formalised by Krusell and Rios-Rull (1996) that develop a model of long cycle of stagnation and growth. Over the cycle, the incumbent producers use their political power to prevent new technologies from being adopted until the incumbents are phase out of the economy. Following this, Comin and Hobijn (2009) empirically show how lobbying activities affect legislative authorities to erect entry barriers that significantly slow the speed of diffusion of new technology and protect the incumbents. More recently, Zingales (2017) presents a political theory of firm, based on the concept of *Medici vicious circle*, in which market power is used to obtain political power inducing to additional market power. Gutiérrez and Philippon (2019) detect that the regulation driven by lobbying induces the fall in the entry elasticity with respect to the Tobin's Q and in the growth of small firms relative to large ones in the U.S. Akcigit et al. (2018) empirically study how political connections affect innovation activities and firm dynamics. Cavenaile et al. (2021) show how advertisement is welfare-improving because it enhances static allocative efficiency in the economy. Moreover, macroeconomic studies have been investigating another tool used by firms to defend their market position: the implementation of defensive and strategic patents to prevent rivals from leapfrogging the market leaders. Shapiro (2000) first considers this trend and described the patent thickets as a dense web of overlapping intellectual property rights that a company must overcome to actually commercialize new technology[8]. Abrams et al. (2013) empirically estimate an inverted-U relationship between citations and value of patents arguing that high value of innovations create a greater incentive to protect them, which deters downstream innovation and induces to a negative relationship between citations and value above a threshold. Consistent with the previous study, Argente et al. (2020) empirically show that market leaders are more patent intensive, but those

[7] For example Ota, Alan. *"Disney In Washington: The Mouse That Roars"* CNN. 10 August 2021.

[8] For example, a famous patent thicket of the car industry is the one built by Toyota around its hybrid technology (Murphy, John. *"Toyota Builds Thicket of Patents Around Hybrid To Block Competitors."* The Wall Street Journal. 10 August 2021.). This practice is also substantially rooted in the pharmaceutical industry (Lopez, Ian. *"Biden Vaccine IP Waiver Stance Offers Clue on Patent Office Pick."* Bloomberg Law. 10 August 2021.).

patents are associated with declining product introduction of the competitors, supporting the notion that market leaders use their patents to limit competition. Ledezma (2013) shows defensive practices reduce the incentives to innovate. My contribution consists in determining under which conditions the reduction of knowledge spillovers improves the aggregate productivity in the economy.

This study is also related to the broader literature on the decline in business dynamism, increasing difference between firms and market concentration. Akcigit and Ates (2019) argue that the reduction business dynamism is due to the declining knowledge diffusion from market leaders to followers due to the implementation of the defensive behavior that would prevent rivals from leapfrogging the market leaders. Cavenaile et al. (2020) show the decline in productivity growth is largely driven by the increasing costs of innovation. Feijoo-Moreira (Feijoo-Moreira) show that the decline in the average size of innovations induces a growth slowdown in a quality ladder growth model with provider-driven complementarity. Based on a cross-country comparison, Andrews et al. (2015) provide evidence of the slow down of the technology diffusion between firms on the global and national frontiers probably caused by the nature of new technologies and the increasing importance of the use of data and tacit knowledge in production processes. Grullon et al. (2019) state that the rise in concentration can be partly explained by the weak anti–trust law enforcement toward large firms. Autor et al. (2020) argue that the rise of concentration supports the *Winner takes all* hypothesis where the most productive firms increase their market shares due to higher competition driven by globalization. This study introduces an endogenous mechanism to reveal the structural change may have raised the pervasiveness in defensive behavior.

This paper is also inspired by the empirical literature on the recent trend of the market power as De Loecker and Warzynski (2012), De Loecker et al. (2016), De Loecker and Eeckhout (2018), Bank (2018), Van Reenen (2018) and Calligaris et al. (2018). Focusing on the U.S. economy, seminal papers on the rising market power are Grullon et al. (2019), De Loecker et al. (2020), Autor et al. (2020), Baqaee and Farhi (2020) and Kwon et al. (2021). My contribution is to show how the defensive behavior contributes to the rise in concentration and dispersion of the firms' size and markup distribution.

This paper contributes to the literature that examines the macroeconomic implications of the entry dynamics such as Sterk et al. (2021).

Finally this study is also related with the literature that studies the dynamics of competitive industries such as, Jovanovic and MacDonald (1994a), Jovanovic and MacDonald (1994b), Eeckhout and Jovanovic (2002) and Klepper (2015), and it introduces the strategic interaction and the defensive behavior. In addition, the model is inspired by the empirical evidences provided by Bloom et al. (2013) that showed that the technology spillovers are increasing in firms' size.

The remainder of this study is organized as follows: Sections 2 and 4 present the theoretical framework and the calibrated model, respectively; Section 5 examines the effect of the competition policies , respectively; Section 6, I examine the role of product variety; finally, Section **??** concludes the study.

2 Theoretical Framework

Time is discrete in infinite horizon. There is a continuum of heterogeneous industries. Each industry is populated by an endogenous finite number of oligopolistic firms, leaders and followers, that embody different brands and compete *à la Cournot*.

Each period comprises different sessions where oligopolistic firms implement intangible investments to improve their idiosyncratic productivities and to prevent competitors from entering the industry and the catch-up.

The model timing can be summarized as follows:

- *Early-Morning*: Creation of new products by the potential entrepreneurs that enter the new market-places as leaders.
- *Mid-Morning*: Firms invest to enter the incumbents industries as followers; meanwhile the leaders implement defensive practices to reduce the probability that a new firm enters the industry.
- *Late-Morning*: followers invest in intangible goods to catch up the leaders of the industry; meanwhile, leaders implement defensive practices to prevent it.
- *Early-Afternoon*: production.
- *Mid-Afternoon*: leaders invest to improve their productivities.
- *Late-Afternoon*: exit of the firms.

The model economy is presented as follows.

2.1 Environment

Preference. There is a representative household in the economy that maximizes its lifetime expected utility $U(C, L)$ where C is consumption and L labor. The household invests in physical capital, K, that depreciates at rate δ, and and she owns all the firms in the economy.

Final Output. The final good, Y, is produced by a competitive firm using the output of a continuum of industries y_j for $j \in [0, M]$ as inputs subject to a CES production function:

$$Y = \left(\int_0^M y_j^{\frac{(\eta-1)}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}, \tag{1}$$

where $\eta > 1$ is the elasticity of substitution between industries.

Industry Output Each industry is populated by an endogenous finite number of oligopolistic firms N_j. The oligopolistic firms compete *à la Cournot* by choosing the quantities to produce and they face an elastic demand curve.

Similarly, the industry output, y_j, is produced by a competitive firm that uses the output produced by the firms in the industries subject to a CES production function:

$$y_j = \left(\sum_{i=1}^{N_j} y_{i,j}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{2}$$

where $y_{i,j}$ is the output produced by the oligopolistic firm i in industry j. $\rho > \eta$ is the elasticity of substitution between the firms in the industry.

Production in the Industries. In each industry j, two types of oligopolistic firms exist: leaders, the most productive firms, and followers. Firms rent capital stock, k, and hire labor, l and produce using a constant-returns-to-scale Cobb-Douglas production function:

$$y_{i,j} = \begin{cases} \varepsilon_{i,j}^L k_{i,j}^{\alpha} l_{i,j}^{1-\alpha}, & \text{if } i \text{ is a leader,} \\ \varepsilon_{i,j}^F k_{i,j}^{\alpha} l_{i,j}^{1-\alpha}, & \text{if } i \text{ is a follower} \end{cases} \tag{3}$$

where $\varepsilon_{i,j} \in \{\varepsilon_1, \varepsilon_2, \ldots\ldots, \varepsilon_{N^\varepsilon}\}$, $\varepsilon^L > \varepsilon^F$ is the relative productivity. So each industry j is defined by:

$$(\varepsilon_j^L, \varepsilon_j^F, n_j^L, n_j^F)$$

where n^L and n^F are the number of leaders and followers.

2.2 The day in the economy

Each period comprises different sessions where the oligopolistic firms implement intangible investment to improve the productivity and to reduce the probability of entry or catch-up of the competitors. The technology frontier of the economy deterministically grows at the rate $\bar{\varepsilon}$. The firm that fails to adopt newer technologies will experience a decline in the relative productivity.

Early Morning. There is a mass Σ of potential entrepreneurs that invest x^V units of intangible good subject to the cost function $C(x^{V)})$. In case the potential entrepreneur succeeds, she creates a new technology to produce a new good with the associated productivity ε^V.

I can define the probability of succeed as:

$$p^V(x^V) = \frac{x^V}{x^V + \bar{Q}}, \tag{4}$$

where $\bar{Q}$ is a constant.

I assume that creating these new technologies allows the potential entrants to enter this new industry as followers with a technology whose productivity is lowest available in the economy.

Mid-Morning. In each industry $N^{Max} - N_j$ potential entrants invest x_0^E units of intangible good subject to the cost function $C(x_0^{E)})$ in order to enter the industry as an additional follower. Meanwhile, n_j^L leaders implement defensive practices to prevent the entry of an additional competitor. In particular, leaders play a public good game by choosing $x_{i,j,0}^L$ units of defensive practices to reduce the probability of entry of an additional follower subject to the cost $C(x_{i,j,0}^L)$.

The contest probability that an additional competitor enters as follower the industry j is:

$$\hat{\pi}_j^E(X_{j,0}^L, X_{j,0}^E) = \frac{\bar{Q}}{X_{j,0}^L + \bar{Q}} \times \frac{X_{j,0}^E}{X_{j,0}^E + \bar{T}} \tag{5}$$

with $X_{j,0}^L = \sum_{i=1}^{n_j^L} x_{i,j,0}^L$, $X_{j,0}^E = \sum_{i=1}^{N^{Max}-N_j} x_{i,j,0}^E$, $\bar{Q}$ and $\bar{T}$ constant.

Late-Morning. n_j^F followers play an innovation race by employing $x_{i,j,1}^F$ units of intangible good subject to the cost function $C(x_{i,j,1}^F)$. Winning the contest allows the follower to become an additional leader in the afternoon. Meanwhile, n_j^L leaders implement defensive practices to prevent the firms' catch-up. In particu-

lar, leaders play a public good game by choosing $x^L_{i,j,1}$ units of defensive practices to reduce the probability that a follower becomes an additional leader subject to the cost function $C(x^L_{i,j,1})$.
The contest probability that a follower moves up to the frontier of the industry j is:

$$\hat{\pi}^F_j(X^L_{j,1}, X^F_{j,1}) = \frac{\bar{Q}}{X^L_{j,1} + \bar{Q}} \times \frac{X^F_{j,1}}{X^F_{j,1} + \bar{T}} \tag{6}$$

with $X^L_{j,1} = \sum_{i=1}^{n^L_j} x^L_{i,j,1}$, $X^F_{j,1} = \sum_{i=1}^{n^F_j} x^F_{i,j,1}$, $\bar{T}$ and $\bar{Q}$ constant.
Consequently, I can define the contest probability that the follower i investing $x^F_{i,j,1}$ units of intangible good wins as:

$$\hat{\pi}^{FS}_{i,j}(x^F_{i,j,1}, X^L_{j,1}, X^F_{j,1}) = \hat{\pi}^F_j(X^L_{j,1}, X^F_{j,1}) \times \frac{x^F_{i,j,1}}{X^F_{j,1}} \tag{7}$$

and I can finally define the contest probability that a follower $\neq i$ succeeds as:

$$\hat{\pi}^{FF}_{i,j}(x^F_{i,j,1}, X^L_{j,1}, X^F_{j,1}) = \hat{\pi}^F_j(X^L_{j,1}, X^F_{j,1}) \times \frac{X^F_{j,1} - x^F_{i,j,1}}{X^F_{j,1}} \tag{8}$$

Early-Afternoon. Oligopolistic firms produce the output using a constant return to scale production function:

$$y_{i,j} = \varepsilon^{l(f)}_{i,j} k^{\alpha}_{i,j} l^{1-\alpha}_{i,j} \quad \text{with } y_{i,j},\ l_{i,j} \text{ and } k_{i,j} \in \mathcal{R}_+ \tag{9}$$

where $l_{i,j}$ and $k_{i,j}$ denote the labor and capital inputs of the leader (follower) i in the industry j.

Mid-Afternoon. After the production takes place, leaders play another contest by employing $x^L_{i,j,2}$ units of intangible good to improve their productivity subject to the cost function $C(x^L_{i,j,2})$. The winner can enjoy a one-step increase of its productivity. The implementation of the new technology allows the winner leader to become the only leader in the industry the next period, to convert the other leaders into followers, and to force the current followers out of the industry. In case there is just one leader in the industry, the implementation of the new technology simply allows the leader to increase its productivity gap with respect to the followers.
Similarly, I define the probability that a leader wins the contest in industry j as:

$$\hat{\pi}^L(X^L_{j,2}) = \frac{X^L_{j,2}}{X^L_{j,2} + \bar{Q}}, \tag{10}$$

with $X^L_{j,2} = \sum_{i=1}^{n^L} x^L_{i,j,2}$ and R_2 constant.
The probability of success for the leader i:

$$\hat{\pi}^{ls}(x^L_{i,j,2}, X^L_{j,2}) = \hat{\pi}^L(X^L_{j,2})\frac{x^L_{i,j,2}}{X^L_{j,2}}, \tag{11}$$

Finally, the probability of success for the leader $\neq i$:

$$\hat{\pi}^{ll}(x^1_{i,j,2}, X^L_{j,2}) = \hat{\pi}^L(X^L_{j,2})\frac{X^L_{j,2} - x^L_{i,j,2}}{X^L_{j,2}}, \tag{12}$$

Late-Afternoon. In each period, firms leave the economy for three different reasons in the late-afternoon. First, firms exit the industry when the productivity of the leaders of the industry depreciates below a specific productivity threshold. Second, in industries with multiple leaders, followers are forced to leave the industry in case there is one leader innovates in the contest of mid-afternoon. Third, each industry faces an exogenous probability of destruction ϕ that shut down the entire industry.

3 Equilibrium

3.1 Household

I assume that the representative household's period utility is the result of indivisible labor (Rogerson (1988)) $U(C, 1-L) = \log C + \kappa(1-L)$ with $\kappa > 0$, as the labour disutility parameter and the preference specification allows for balanced growth. The household discounts the future utility by a subjective discount factor, β.
I can now derive the following optimality conditions:

$$\frac{1}{C} = \beta\frac{1}{C'}(R' + 1 - \delta) \tag{13}$$

$$\frac{1}{C}w = \kappa, \tag{14}$$

3.2 Production

I can formalize the problem of the competitive firm producing the final good as follows:

$$\max_{y_j} Y - \int_0^M p_j y_j dj \tag{15}$$

$$\text{subject to} \quad Y = \left(\int_0^M y_j^{\frac{(\eta-1)}{\eta}} dj\right)^{\frac{\eta}{\eta-1}}.$$

The firm maximization problem induces the inverse demand function for the output of the industry j:

$$p_j = \left(\frac{y_j}{Y}\right)^{\frac{-1}{\eta}} \tag{16}$$

Similarly, the maximization problem of the representative firm producing the industry output can be formalised as follows:

$$\max_{y_{i,j}} \quad p_j y_y - \sum_{i=1}^{N_j} p_{i,j} y_{i,j} \tag{17}$$

$$\text{subject to} \quad y_j = \left(\textstyle\sum_1^{N_j} y_{i,j}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$

$$, \qquad p_j = \left(\textstyle\sum_1^{N_j} p_{i,j}^{1-\rho} \right)^{\frac{1}{1-\rho}}.$$

The inverse demand functions for the firm i in industry j:

$$p_j = \left(\frac{y_j}{Y}\right)^{\frac{-1}{\eta}} \left(\frac{y_{i,j}}{y_j}\right)^{\frac{-1}{\rho}} \tag{18}$$

The oligopolistic firms compete *à la Cournot* in the industry. Each firm i in industry j solves the following maximization problem:

$$\pi_{i,j}^{l(f)}(\varepsilon^L, \varepsilon^F, n^L, n^F) = \max_{y_{i,j}, k_{i,j}, l_{i,j}} \left(p_{i,j} y_{i,j} - w l_{i,j} - R k_{i,j} \right) \tag{19}$$

$$\text{subject to} \quad y_{i,j} = \varepsilon_{i,j}^{l(f)} k_{i,j}^{\alpha} l_{i,j}^{1-\alpha} \text{ with } y_{i,j},\ l_{i,j} \text{ and } k_{i,j} \in \mathcal{R}_+,$$

$$p_{i,j} = \left(\tfrac{y_j}{Y}\right)^{\frac{-1}{\eta}} \left(\tfrac{y_{i,j}}{y_j}\right)^{\frac{-1}{\rho}}.$$

3.3 Contests

I assume that firms use labor to produce units of intangible good. In particular, I assume that l units of labor can produce $\sqrt{2l}$ units of x units of intangible good, such that the cost functions have the following form:

$$C(x) = \frac{w}{2} x^2$$

Early-Morning. Let V^V and V_0^L be the value of creating a new product and the value of being leader in a industry in the mid-morning. The potential entrepreneur's optimization problem can be formalised as follows:

$$V^V = \max_{x^V} -\frac{w}{2}(x^V)^2 + \frac{x_V}{x_V + \bar{Q}} V_0^L(\varepsilon^V, \varepsilon_0^F, 1, 0) \tag{20}$$

Mid-Morning. Let V_1^L and V_1^F be respectively the value of being leader and the value of being follower in the late-morning.

The potential entrant optimization problem can be formalised as follows:

$$V_0^E(\varepsilon^L, \varepsilon^F, n^L, n^F) = \max_{x_0^E} -\frac{w}{2}(x_0^E)^2 + \hat{\pi}^E(X_0^L, X_0^E, x_0^E) V_1^F(\varepsilon^L, \varepsilon^F, n^L, n^F + 1) \tag{21}$$

The last term of the equation 20 term reflects the value of being an additional follower of the industry in the late-morning.

The leader optimization problem can be formalised as follows:

$$V_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F) = \max_{x_0^L} -\frac{w}{2}(x_0^L)^2 + [\hat{\pi}^E(X_0^L, X_0^F, x_0^L) V_1^L(\varepsilon^L, \varepsilon^F, n^L, n^F + 1) + (1 - \hat{\pi}^E(X_0^L, X_0^F, x_0^L)) V_1^L(\varepsilon^L, \varepsilon^F, n^L, n^F)] \tag{22}$$

The first term within the square brackets represents the value of continuing to be a leader in the late-morning without the entry of an additional follower. The second term indicates the value of being a leader in the in the late-morning in the case an entrant becomes an additional competitor.

Late Morning. Let V_2^L and V_2^F be respectively the value of being leader and follower in the early afternoon.

The leader's optimization problem can be recursively formalised as follows:

$$V_1^L(\varepsilon^L, \varepsilon^F, n^L, n^F) = \max_{x_1^L} -\frac{w}{2}(x_1^L)^2 + [\hat{\pi}_1^F(x_1^L, X_1^F, x_1^L) V_2^L(\varepsilon^L, \varepsilon^F, n^L + 1, n^F - 1) + (1 - \hat{\pi}_1^F)(x_1^L, X_1^F, x_1^L)) V_2^L(\varepsilon^L, \varepsilon^F, n^L, n^F)] \tag{23}$$

The first term within the square brackets represents the value of continuing to be a leader in the afternoon with the unchanged industry. The second term indicates the value of being a leader in the afternoon in the case a follower succeeds and becomes an additional leader of the industry.

Proposition 1. *Given X_1^F, let $x_1^{L\star}$ be a symmetric equilibrium of the game represented by the recursive maximization problem 23, then*

- $x_1^{L\star}$ *is increasing in* $[V_2^L(\varepsilon^L, \varepsilon^F, n^L, n^F) - V_2^L(\varepsilon^L, \varepsilon^F, n^L + 1, n^F - 1)]$
- $x_1^{L\star}$ *is increasing* x_1^F
- $x_1^{L\star}$ *is unique.*

Proof*: see appendix A.1.*

Proposition 1 shows that the incentive of the leaders in implementing defensive practices has two drivers: *protecting rent* and *credible threat* . The first driver highlights the fact that the equilibrium quantity of defensive practices, $x_1^{L\star}$, is increasing in the return that leaders have from preventing the catch-up: leaders with large market shares produce more defensive practices. The *credible threat effect* shows as the equilibrium quantity of defensive practices is increasing in the level of effort of the followers: when the effort of the followers is high, leaders increase their investment in defensive practices.
I can now recursively define the optimization problem of the follower as:

$$\begin{aligned} V_1^F(\varepsilon^L, \varepsilon^F, n^L, n^F) = \max_{X_1^F \in [0,\infty)} -\frac{w}{2}(X_1^F)^2 + \Big\{ (1-\hat{\pi}^F) V_2^F(\varepsilon^L, \varepsilon^F, n^L, n^F) \\ + \hat{\pi}^{FF} V_2^F(\varepsilon^L, \varepsilon^F, n^L+1, n^F-1) + \hat{\pi}^{FS} V_2^L(\varepsilon^L, \varepsilon^F, n^L+1, n^F-1) \Big\} \end{aligned} \tag{24}$$

The first term inside the brace represents the value of continuing to be a follower in the afternoon when no follower succeeds. The second term indicates the value of continuing to a be a follower in the afternoon when another follower becomes an additional leader. The third term represents the value of winning the contests and becoming an additional leader in the afternoon.

Proposition 2. *Given x_1^L, let $x^{F\star}$ be a symmetric equilibrium of the game represented by the recursive maximization problem 24, then*

- $x^{F\star}$ *is increasing in* $V_2^L(\varepsilon^L, \varepsilon^F, n^L + 1, n^F - 1) - V_2^F(\varepsilon^L, \varepsilon^F, n^L, n^F)$
- $X_1^{F\star}$ *is increasing in* $V_2^L(\varepsilon^L, \varepsilon^F, n^L + 1, n^F - 1) - V_2^F(\varepsilon^L, \varepsilon^F, n^L + 1, n^F - 1)$
- $x^{F\star}$ *is decreasing* x_1^L
- *given* x_1^L*,* $x^{F\star}$ *is unique.*

Proof*: see appendix A.2.*

Proposition 2 shows that the incentive of the followers in investing in intangible goods is affected by the *return of being leader* and the *probability of being blocked*. First, the equilibrium quantity of intangible goods chosen by followers,

X_1^F, is increasing in the return of becoming leader: if the productivity at the technology frontier is high, then the followers increase their effort. Moreover, the *probability of being blocked* indicates that the equilibrium quantity of intangible goods chosen by followers is decreasing in the total units of defensive practices implemented by leaders: if the probability of being blocked is high, the marginal return of the investment is low.

Proposition 3. *The game, represented by the recursive maximization problems 23 and 24, admits a unique symmetric equilibrium* $(x_l^{L\star\star}, x_l^{F\star\star})$.
Proof*: see appendix A.3.*

The intuition of proposition 3 is straightforward. The leaders' symmetric equilibrium of defensive practices is increasing in the total intangible investment chosen by the followers. Conversely, the symmetric equilibrium of the followers is decreasing in the quantity of defensive practices implemented by leaders.
Finally Proposition 1, 2 and 3 can be directly applied to show that the contest of the mid-morning has a unique symmetric equilibrium taking into account that the entrant that fails the entry leaves the economy forever.

Afternoon. Then, the leader's optimization problem can be recursively formalised as follows:

$$V_2^L(\varepsilon^L, \varepsilon^F, n^L, n^F) = \max_{y^L, x_2^L} \pi^L(\varepsilon^L, \varepsilon^F, n^L, n^F) - c(x_2^L) + \hat{\beta}(1-\phi)$$
$$\Big\{ \hat{\pi}^{LS}(x_2^L, x_2^L) V_0^L(\varepsilon^{L\prime}, \varepsilon^{F\prime}, n^L, n^F) + \hat{\pi}^{LF}(x_2^L, x_2^L) V_0^F(\varepsilon^{L\prime}, \varepsilon^{F\prime}, n^L, n^F) + \tag{25}$$
$$(1 - \hat{\pi}^L(x_2^L, x_2^L)) V_0^L(\varepsilon^{L\prime}, \varepsilon^{F\prime}, n^L, n^F) \Big\}$$

The leader optimally chooses the level of output to produce, the units of intangible good to maximize the sum between the firm's current profit and the expected discounted continuation value of the early-morning conditional on the fact that the industry does not exogenously disappear from the economy with $(1-\phi)$ probability. The first term of the expected continuation value represents the value of winning the contest. After winning the contest, the leader improves his productivity by one step. The second term of the expected continuation value indicates the value of loosing the contest. The third term represents the value of being leader the next morning when no leader succeeds. Leaders that do not innovate such as followers will experience a one step decline in their relative productivity.

Proposition 4. *Let $x_2^{L\star}$ be a symmetric equilibrium of the game represented by the recursive maximization problem 25, then*

- $x_2^{L\star}$ *is increasing in* $[\tilde{V}_0^L(\varepsilon^{L\prime}, \varepsilon^{F\prime}, n^L, n^F) - V_0^L(\varepsilon^L, \varepsilon^L, n^L, n^F)]$
- $x_2^{L\star}$ *is increasing in* $[\tilde{V}_0^L(\varepsilon^{L\prime}, \varepsilon^{F\prime}, n^L, n^F) - V_0^F(\varepsilon^L, \varepsilon^F, n^L, n^F)]$
- $x_2^{L\star}$ *is unique.*

Proof*: see appendix A.4.*

Proposition 4 shows that the incentives of the leaders in investing in innovation has two drivers: *escape from competition* and *loss of competitiveness*. The *loss of competitiveness* indicates that the equilibrium quantity of the intangible investment, x_2^L, is decreasing in the value of continuing to be a leader with a one period depreciated technology. The *escape from competition* reveals that the choice of the intangible investment is decreasing in the value of becoming a follower.

Similarly, the follower optimization problem can be recursively defined:

$$V_2^F(\varepsilon^L, \varepsilon^F, n^L, n^F) = \max_{y_1^F} \pi_1^F(\varepsilon^{L\prime}, \varepsilon^{F\prime}, n^L, n^F) + \hat{\beta}(1-\phi)$$
$$\left\{\hat{\pi}^L(x_2^L)\hat{V}_0^F(\varepsilon^L, \varepsilon^F, n^L, n^E) + (1-\hat{\pi}^L(x_2^L))V_0^F(\varepsilon^L, \varepsilon^F, n^L, n^F)\right\} \tag{26}$$

In 26, the follower optimally chooses the level of output to maximize the its profit. The follower does not participate in the contest of the afternoon.

If $\varepsilon^F > \varepsilon_0$, the follower will experience a decline in productivity. If the productivity of the leaders ε^L falls below the ε_1 then the industry endogenously disappears. Finally, I assume that if $\varepsilon^F = \varepsilon_0$ and $\varepsilon^{L\prime} \geq \varepsilon_1$, the follower will benefit from a positive externality of the leaders' technology and she continues to maintain the same relative productivity ε_0.

3.4 Balanced Growth and Equilibrium

I focus on the Balanced Growth Path (BGP) Markov perfect equilibrium, where equilibrium strategies depending only on the payoff-relevant state variable $(\varepsilon^L, \varepsilon^F, n^L, n^F)$ and all aggregate variables growing at the same rate $\bar{\varepsilon}$.

At the beginning of each period, an industry is defined by its state vector $(\varepsilon^L, \varepsilon^F, n^L, n^F)$; productivity of leaders $\varepsilon^L \in E$, productivity of followers $\varepsilon^F \in E$, number of leaders $n^L \in \mathbb{N}^+$, and number of followers $n^F \in \mathbb{N}^0$. I summarize the distribution of firms by a probability measure, $\mu(\varepsilon^L, \varepsilon^F, n^L, n^F)$, which is defined on a Borel algebra $\mathbb{S} \equiv E \times E \times \mathbb{N}^{+} \times \mathbb{N}^{0}$.Let $\mu^2(\varepsilon^L, \varepsilon^F, n^L, n^F)$ be the time invariant distributions of industries in the late-afternoon[9].

A stationary equilibrium is a set of prices (R, w, p^L, p^F), a set of allocations (Y, C, K, y^L, y^F), policies $(x^V, x_0^L, x_0^E, x_1^L, x_1^F, x_2^L)$, such that:

1. Given prices, the competitive final and industry good producers maximize their profits.
2. Given $(\varepsilon^L, \varepsilon^F, n^L, n^F)$, y^L and y^F maximize profits of the oligopolistic firms in the industries.
3. x^V maximizes the valuefo of the potential entrepreneur in the morning V^V.
4. Given X_0^L and X_0^E, x_0^E maximizes the mid-morning value of the potential entrant V_1^E.
5. Given X_0^E and X_0^L, x_0^L maximizes the mid-morning value of the leader V_0^L.
6. Given X_1^L and X_1^F, x_1^F maximizes the value of the follower in the late-morning V_1^F.
7. Given X_1^F and X_1^L, x_1^L maximizes the value of the leader in the late-morning V_1^L.
8. Given XS_2^L, x_2^L maximizes the value of the leader in the afternoon V_2^L.
9. Given prices, C satisfies 14.
10. The real interest rate $R = \frac{1+\bar{\varepsilon}}{\beta} + \delta - 1$.
11. Resource constraint is satisfied:

$$Y = C + \delta K,$$

    where:

    - $K = \int_{\mathbb{S}} (n^L \frac{y^L}{\varepsilon^L} + n^F \frac{y^F}{\varepsilon^F})[\frac{\alpha}{1-\alpha}\frac{w}{r}]^{1-\alpha} \mu^2(d[\varepsilon^L \times \varepsilon^F \times n^L \times n^F])$
12. The distribution of firms, $\mu^2(\varepsilon^L, \varepsilon^F, n^L, n^F)$, is the fixed point where their transition follow the policy functions, $(x^V, x_0^L, x_0^E, x_1^L, x_1^F, x_2^L)$.

4 Calibration

The model is calibrated to the U.S. economy at annual frequency. The discount factor, β, is set to have an annual interest rate of 3% as in Sedláček (2020). I set

[9] Note that the technology frontier is the only source of growth. Only firm-level and aggregate employment are stationary. All other variables can be stationarized by dividing them with $\varepsilon_{N^\varepsilon}$.

Table 1: Calibrated Parameters.

Parameter	Value	Description	Target & Source	Data	*Model*
β	0.97	Discount factor	Annual Real interest rate, Sedláček (2020)	3%	3%
κ	2.2	Leisure utility	Total hours worked, Sedláček (2020)	0.33	0.33
δ	0.1	Depreciation rate	Sedláček (2020)	10%	10%
ϕ	0.04	Exogenous exit probability	Entry Rate, Sedláček (2020)	11%	11.90%
Δg	0.018	Growth of Technology Frontier	Average productivity growth, Sedláček (2020)	0.018	0.018
α	0.5	El. of output w.r.t. capital	Capital/Output, Senga(2018)	2.30	2.30
η	3.8	El. sub. across industries	Bilbiie et al. (2012)		
$\bar{T}$	0.27	Productivity of follower's Intangible Investment	St.Dev. of Intangible Investment, Compustat 2001-2018	1.56	1.85
$\bar{Q}$	0.13	Productivity of leader's Intangible Investment	St.Dev. of Firms' Size, Compustat 2001-2018	2.41	2.24
ϵ^V	ϵ_{20}	Productivity of the new Variety	Share of Investment Investment over Output, Compustat 2001-2018	7.73	7.50

the preference parameter of labor disutility, κ, to get the average hours worked of 0.33. The depreciation rate, δ, is set to 0.1, as in Sedláček (2020). The production parameter, α, is set to be consistent with the average capital to output ratio of 2.3 in the postwar US economy, as in Senga (2015).

I assume almost perfect substitutability between the varieties within the industries and I set the parameters that governs the elasticity of substitution, ρ, to 100. The destruction probability, ϕ, is set to match the entry rate as in Sedláček (2020). I set the parameters that governs the substitutability between industries, η, to 3.8 following Bilbiie et al. (2012).I set the mass of potential entrants Σ to have the total mass of industries M equals to 1. The growth of the frontier technology $\bar{\varepsilon}$ targets the average labor productivity as in Sedláček (2020).

To set the remaining parameters I construct a panel dataset of firm-level intangible investment and sale based on data from Compustat databases. Using Standard Industry Classification (SIC) codes, I exclude firms in the oil, energy and financial sectors.[10]. I eliminate sample firms with missing data items to ensure that the data are valid for all the sample. In particular, I construct an annual panel of US public firms for the period 2001-2018 with the measure of intangible investment built by following Peters and Taylor (2017). I set the parameter that governs the productivity of followers' intangible investment, $\bar{Q}$, to match the standard deviation of the logarithm of the share of intangible investment over sale. I set the parameter that governs the productivity of the leaders' intangible investment productivity, $\bar{Q}$, by targeting the standard deviation of the firm size distribution defined as log-sale. Finally, I set the productivity of the new varieties, ϵ^V, to match the share of intangible investment over total sale.

Table 2 shows that the performances well in matching both the elasticity of intangible investment intensity with respect firm size. In addition the model can

[10]Specifically, we exclude oil and oil-related firms with SIC codes 2911, 5172, 1311, 4922, 4923, 4924, and 1389; energy firms with SIC code between 4900 and 4940; financial firms with SIC code between 6000 and 6999.

replicate the average and dispersion of firms' market share build on the measures of product similarity of Hoberg and Phillips (2016). In this regard, I use the Compustat dataset and the product similarity scores build by Hoberg and Phillips (2016) to estimate the firm market share of firm i in the following ways:

$$Market\_share_i = \frac{Sale_i}{\sum_{j=1}^{N_i} s_{j,i} Sale_j + Sale_i}$$

where $s_{j,i}$ is the simuilarity score between firm i and j. Finally, the model seems to replicate well the defensive intensity of the U.S. proxied by the share of patent citation that cite patents already existing within firms as stated in Akcigit and Ates (2019). In the model I define the defensive intensity as the share of defensive expenditure over the total leaders' intangible investment expenditure.

Table 2: Un-targeted Moments.

Description & Source	Data	*Model*
Elasticity of Intangible Investment Intensity w.r.t. Size, Compustat (2001-2018)B.1	−0.61	−0.89
Weighted Sale Market Share, Compustat (2001-2018)	75.09%	50.11%
Dispersion Market Share, Compustat (2001-2018)	0.35	0.24
Share of Leaders' Defensive Intangible Investment, Akcigit and Ates (2019)	0.38	0.42

5 Isolating the Effect of Defensive Practices

In this section, I first present the results from the counterfactual analysis in which the leaders are prevented from implementing defensive practices. Following, I study the effect of the different types of defensive practices and I investigate the mechanisms through which the defensive behavior affects the aggregate quantities. Finally I explore the effect of the defensive practices on endogenous productivity in economies without considering the channel of the product variety.

5.1 The Effect of Defensive behavior

The counterfactual policy exercise consists in comparing the pre-intervention equilibrium where firms are allowed to implement defensive practices, benchmark equilibrium, and the post-intervention equilibrium, which is the new equilibrium (ND) reached by the economy after an omnipotent competition authority detects and prevents any implementation of defensive practices. That is, I measure the equilibrium changes in the aggregate economy under each policy relative to the benchmark.

As in Table 3, preventing firms from implementing defensive practices increases aggregate consumption and investment in the economy. Specifically, the policy

Table 3: Cracking Down on Defensive Practices.

	No Defensive (ND)	Defensive in Entry (DE)	Defensive in Catch-up (DC)
Entry Rate as Followers $E^\star$	3.40%	0.97%	1.67%
Firms per Variety	14.40%	−5.86%	21.66%
Leaders per Variety	21.32%	12.91%	5.70%
Followers per Variety	7.03%	−28.72%	35.29%
Variety	−9.78%	−1.82%	−6.96%
Mass of Firms	4.63%	−7.65%	14.7%
Sale-weighted Markup	−3.73%	−1.12%	−2.66%
Markup Dispersion	−45.64%	−17.36%	−21.68%
Size Dispersion	−53.0%	−61.79%	−2.36%
Intangible Investment Intensity*	−4.46%	3.71%	−11.31%
Consumption	0.19%	1.48	−1.25%
Investment	4.68%	2.81	2.06%
Output	1.27%	1.83	−0.44%
Labor	1.09%	0.17%	0.47%
Labor*	4.46%	1.05%	3.27%
Variety	−9.78%	−1.82%	−6.96%
Ratio Capital over Output $\frac{K}{Y}$	0.19%	1.48	−1.25%
Endogenous productivity	−1.27%	0.61%	−1.57%
Endogenous productivity*	−3.31%	0.07%	−3.25%
Consumption Equivalent Welfare (CEV)	−0.63%	1.39%	−1.58%

Note: The table reports the effect of preventing firms from implementing defensive practices. In particular the first column on the left, ND, reports the effects of preventing leaders from investing in defensive practices that reduce probability of catch-up and entry. The second column, DE, reports the effects of precluding leaders from investing in defensive practices that prevent the catch-up of the followers. Finally the last column, DC, reports the effects of precluding leaders from implementing defensive practices that make more difficult the entry in the industry. I define $E^\star$ as the entry rate that takes into account only the entry in incumbent industries. Analogously, I define Intangible Investment Intensity* as the ratio between the total intangible investment, defensive expenditure excluded, over the total output. I obtain the Endogenous Productivity as the Solow residual, $Y/(K^\alpha L^{1-\alpha})$, given aggregate output (Y), capital (K), and labor (L). Labor* as the labor that is allocated to the production.

Table 4: Intangible Investment and Productivity.

	No Defensive (ND)	Defensive in Entry (DE)	Defensive in Catch-up (DC)
Potential Entrant's Intangible Investment Intensity	23.18%	12.31%	9.17%
Potential Entrepreneur's Intangible Investment Intensity	−48.52%	−9.37%	−31.68%
Follower's Intangible Investment Intensity	12.52%	13.49%	−5.81%
Leader's Intangible Investment Intensity *	−38.23%	−9.17%	−28.04%
Follower's Sale-weighted Exogenous Productivity	0.62%	1.34%	−0.90%
Leader's Sale-weighted Exogenous Productivity	−0.75%	0.63%	−1.40%
Sale-weighted Exogenous Productivity	−0.50%	0.99%	−1.61%

Note: The table reports the percentage changes in the aggregate quantities relative to the benchmark economy that follow the implementation of the different competition policies. I define Leader's Intangible Investment Intensity * as the ratio leaders' total intangible investment in the mid-afternoon over total sale.

raises aggregate consumption and capital, respectively, by 0.19 and 4.68 percent from the benchmark, while endogenous productivity falls by -1.27 percent. In the following, I explain that there are two main channels that contribute to the reduction in the endogenous productivity that are related with the change both in the entry composition and leaders' innovation. In addition, the welfare measurement (CEV) decreases by more than 0.63 percent following the implementation of this new competition policy. This is mainly driven by the rise in the equilibrium labor. Because we abstract from real feasibility of the policy intervention, the aggregate effects in Table 3 can be interpreted as the lower bounds of the impacts of such competition policy.

The above results reflect the impact of a change in the degree of competition in the new economy that is now characterised by lower markups and dispersion in the firm size distribution as documented in Table 3.

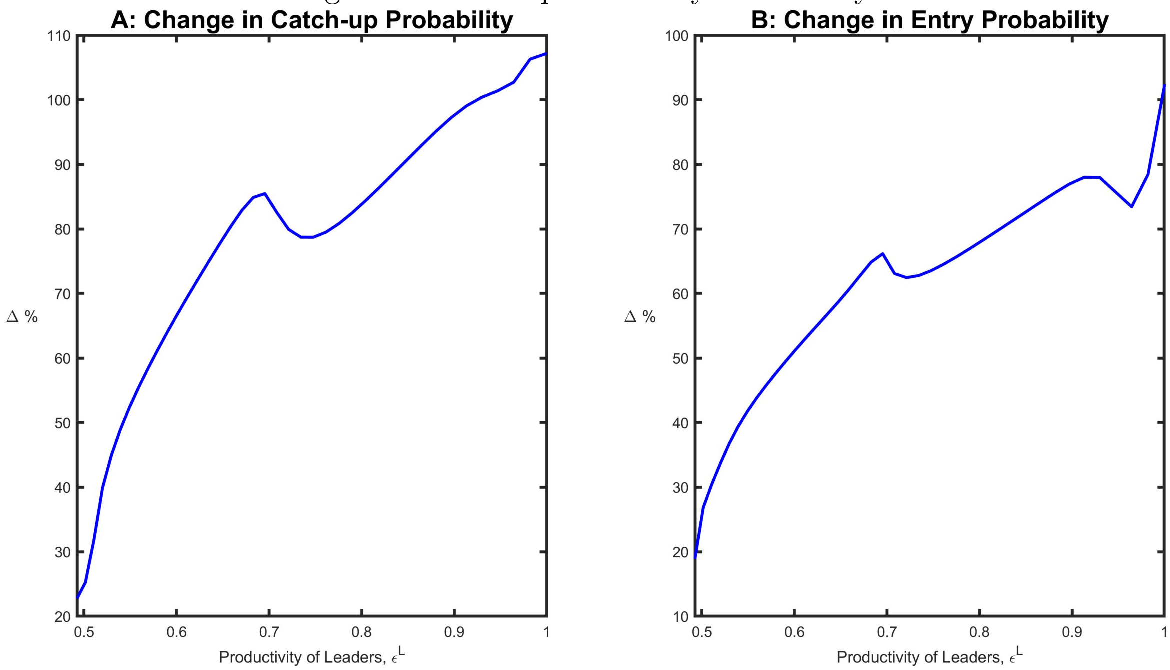
First I start by showing the effect of the competition policy on the entry composition. In particular, preventing firms from implementing defensive practices has two countervailing effects on entry. On the one hand it promotes the entry in the incumbent industries. Table 3 shows that cracking down on firms' defensive practices raises the number of firms per industry by 14.40 percentage points since potential entrants do not face barriers anymore. More specifically, preventing leaders from investing in defensive practices raises the probability of entry especially in those industries characterised by high productive leaders as shown in Plot B 1. The reason is twofold. Firstly Potential entrants have the highest incentive in entering and copying high productive technology. Secondly, high productive leaders had the highest incentive in avoiding the catch-up. On the other hand, the higher degree of competition that potential entrepreneurs may eventually face in the future reduces their effort to create new products, as documented in Table 4, that is reflected in a drop of almost 10 percentage points in the product variety in the economy. This latter effect negatively contributes to the reduction in the endogenous productivity.

Following I explain how this new environment characterised by a low appropriability affects the endogenous productivity also by reducing the leader's innovation incentive. The model shows that implementing this new competition policy has two countervailing effects. On the one hand, imposing restrictions on leaders' ability to control their dominant position improves the technology adoption and knowledge spillover, allowing more laggard firms to catch-up and compete against frontier firms. This creates more industries that are characterized by a neck-to-neck at the technological frontier. On the other hand, more competition implies that firms' ability to appropriate the profits generated by the innovation is limited, which in turn discourages leaders to improve their productivity. The model shows that the former effect dominates the latter and the leaders' average productivity shrinks by 0.75 percentage points.

All in all, preventing firms from implementing defensive practices worsens the endogenous productivity and it increases consumption and investment. As expected, the implementation of this competition policy reduces the concentration and firm size dispersion since it increases followers' productivity and the number of firms at the industry technology frontier. In addition, preventing firms from investing in defensive practices increases the mass of firms in economy. This leads to an increase in the consumption and investment. However, the higher degree of competition has two unintended effects on firms' innovation. First, it discourages leaders' innovation. Second, it reduces the incentive of the potential entrepreneurs to invest and create new products. As a result, the adoption of less productive technologies by leaders and the drop in the product varieties reduce the endogenous productivity. Finally the new equilibrium embeds a lower utility for the representative household. Intuitively, the raise in the aggregate consumption can not compensate the raise in the aggregate labor due to the fall in the firms' productivity.

Together, these effects jointly quantify the aggregate outcome of the competition policy. The result suggest that the unintended effects are quantitatively important to determine the impacts on productivity and welfare.

Figure 1: Catch-up and Entry Probability.



Note: The Plot A shows the percentage change in the catch-up probability relative to the benchmark economy calculated among industries that have the same leaders' productivity. The Plot B shows the percentage change in the entry probability relative to the benchmark economy calculated among industries that have the same leaders' productivity.

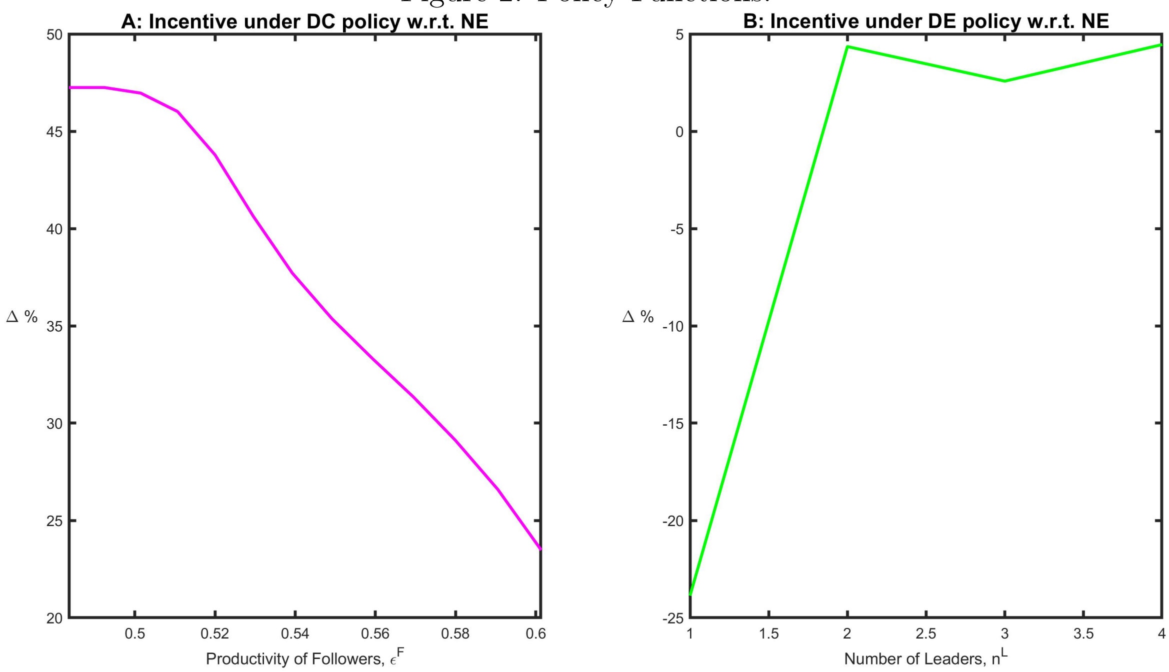
5.2 Inspecting the mechanisms: Catch-up VS Entry

The unintended effects of the reduction of the appropriability on endogenous productivity and welfare are quantitatively important. In this sub-subsection, I isolate the effects of each type of defensive practices on productivity and welfare. In the following analysis I consider two counterfactual policy exercises. First, I consider the competition policy (DE) that only precludes the implementation of defensive practices that make more difficult the entry in the incumbent industries. Second, I consider the policy intervention (DC) that only prevents the implementation of defensive practices that slow down the catch-up of the followers.

In the last two columns of Table 3 and 4, I report the results of the comparison between the benchmark equilibrium and the two new equilibria associated with the DE and DC competition policies. In the following, I show that the implementation of the DE competition policy improves both endogenous productivity and welfare relative to the benchmark economy.

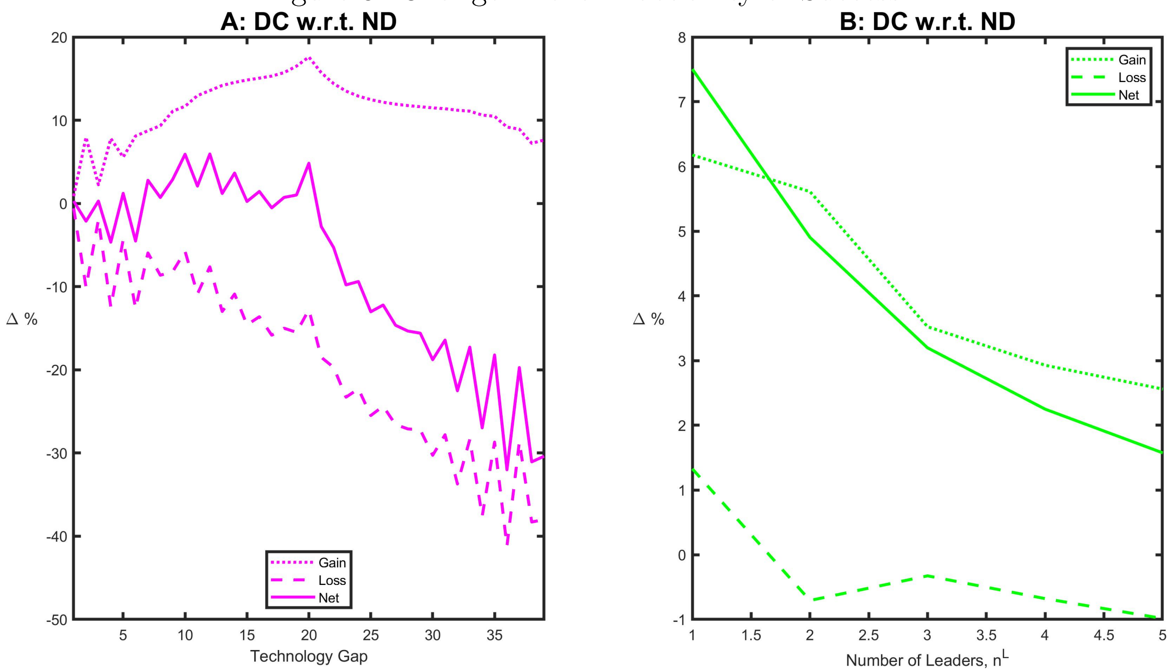
Table 3 shows that preventing only defensive practices that slow down the catch-up increases aggregate consumption and investment in the economy. Specifically,

Figure 2: Policy Functions.



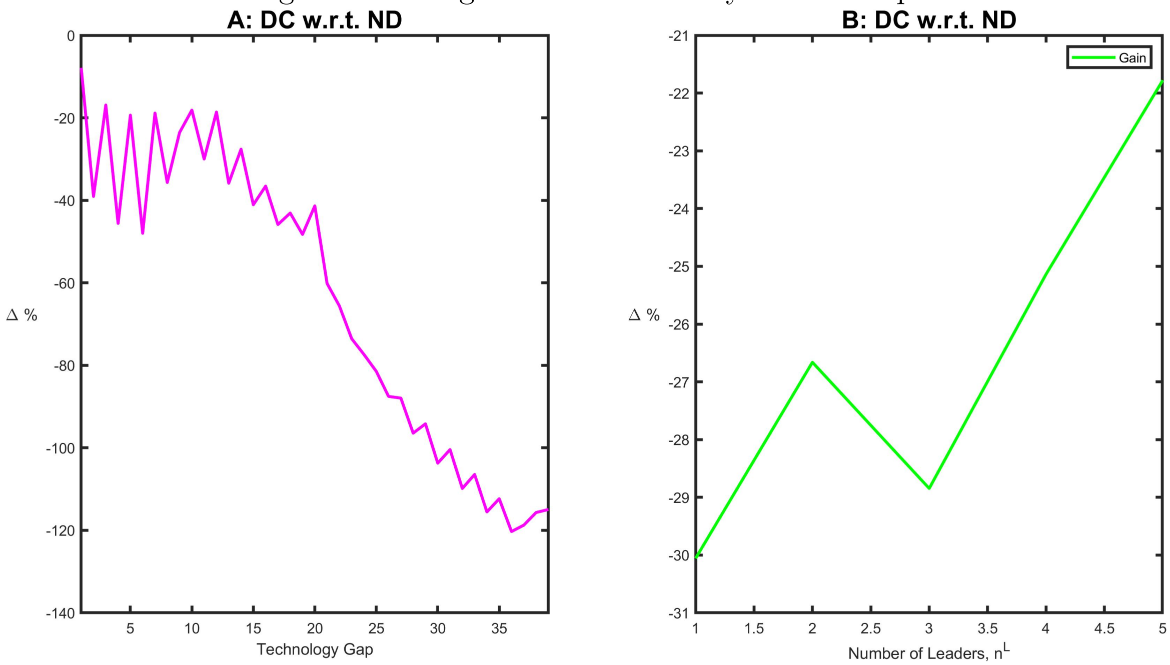
Note: The Plot A and B show the percentage change in the leader's innovation effort due to the two different defensive practices. In particular, Plot A shows how the policy function of a representative leader changes from the DE to the DC economy. This policy function is a function of the follower's productivity. Plot B shows how the policy function of a representative leader changes from the DC to the DE economy. This policy function is a function of the number of leaders in the industry.

Figure 3: Change in the Probability of Success.



Note: The dotted line of Plot A shows the incentive effect of the defensive practices that limit the catch-up among industries that have the same productivity gap among leaders and followers. In particular, it shows how the average probability that a leader wins the contest would change relative to the ND economy if leaders had the policy function of the DC economy. The dashed line of Plot A shows distributional effect of the defensive practices that limit the catch-up. In particular, it shows how the average probability that a leader wins the contest would change relative to the ND economy if there were the distribution of industries of the DC economy.
The dotted line of Plot B shows the incentive effect of the defensive practices that limit the entry among industries that have the same number of leaders. In particular, it shows how the average probability that a leader wins the contest would change relative to the ND economy if leaders had the policy function of the DE economy . The dashed line of Plot B shows distributional effect of the defensive practices that limit the catch-up. In particular, it shows how the average probability that a leader wins the contest would change relative to the ND economy if there were the distribution of industries of the DE economy. The solid lines represent the net effect.

Figure 4: Change in the Probability of Catch-up.



Note: The Plot A shows the percentage change in the probability of catch-up between economy ND and DC calculated among industries that have the same productivity gap between leaders and followers. The Plot B shows the percentage change in the probability of catch-up between economy ND and DE calculate among industries that have the same number of leaders.

the DE policy raises aggregate consumption, investment and endogenous productivity respectively by 1.48, 2.81 and 0.61 percentage points. Conversely, the implementation of the DC policy reduces both consumption and endogenous productivity respectively by 1.25 and 1.57 percentage points. In the following, I explain that the different aggregate effects of the two competition policies reflect the different impacts that the two defensive practices have on the leader's innovation and entry.

I first start by comparing the DE and the DC equilibria relative to the economy without defensive practices, ND, by focusing on the endogenous productivity. As documented in Table 3, the reason of the higher endogenous productivity of DE economy is twofold. First, in contrast to the DC policy, the DE policy leads to a larger product variety. Intuitively, the leaders of the new industries are monopolists and preventing competitors from entering the industry is more than enough to protect their market shares. Second, the implementation of the DE competition policy improves the technology adoption of the firms. What is the intuition behind? First of all it is important to note that the two types of defensive practices benefit two different kinds of leaders. Figure 2 compares how the policy function of a representative leader changes between the economy DC and DE. In particular, Plot A of Figure 2 shows how the policy function of a representative leader changes from the DE to the DC economy. This policy function is a function of the follower's productivity. Plot B of Figure 2 shows how the policy function of a representative leader changes from the DC to the DE economy. This policy function is a function of the number of leaders in the industry. Leaders with a large productivity gap with respect to the followers care more in preventing the catch-up. In contrast, preventing the entry of additional competitors benefits more the leader that operate in competitive industries that are populated by more than one leader. What is the intuition? In case of success, leaders want to prevent that an additional follower enter the industry with its former productivity.

How does these incentives impact on the technology adoption? To answer to this question I decompose the two countervailing effects of the two defensive practices. Indeed, on the one hand, the leaders' ability to implement defensive practices prevents more firms from competing against at the industry frontier. On the other hand, less competition implies that firms' ability to appropriate the profits generated by innovation is higher, which in turn encourages leaders to improve their productivity. Figure 3 shows how the countervailing effects affect the probability that a leader wins the contest relative to the ND economy. The dotted line of Plot A shows the incentive effect of the defensive practices that limit the catch-up among industries that have the same productivity gap between leaders and followers. The dotted line of Plot B shows the incentive effect of the defensive practices that limit the entry among industries that have the same number of leaders. In particular, they show how the probability that a leader wins the contest would change relative to the ND economy if leaders had the policy function of the DC or DE economy. Conversely, the dashed lines of Plot A and B show distributional effect. More specifically, they show how the probability that a leader wins the contest would change relative to the ND economy if there were the industry distributions of the economy DC or DE.

Figure 3 leads two important considerations. First defensive practices that limit the catch-up generate a stronger incentive on leader's innovation than the defensive practices that limit the entry. Second, the former defensive practices also generate a stronger negative distributional effect. In other words, the fall in the competition generated by defensive practices that slow down the catch-up causes a reduction in leaders' innovation especially in those industries characterised by a high productivity gap between the leader and follower as documented in Figure 4.

Why does not the catch-up probability dramatically fall under DE policy?. The intuition is that the increase in the effort of the followers can partially compensate the fall in the number of followers that are competing to copy the leader's technology. Indeed, defensive practices that prevent the entry encourage leaders' innovation without negatively affecting the probability of catch-up of the incumbent followers.

All in all, the competition policy that only tackles defensive practices that prevent the catch-up improves the endogenous productivity with respect to the benchmark economy. More specifically the model shows that precluding leaders from preventing the catch-up affects aggregate productivity through two effects. On the one hand it reduces the leaders and potential entrepreneurs' incentive to innovate even though not so strongly as in the case the competition authority decided to prevents both defensive investments. On the other hand, it raises the probability of catch-up especially in the high technology gap industries. The model suggests that this latter effect is quantitatively stronger than the former and the endogenous productivity improves.

Finally the DE policy also raises the utility of the representative household. The raise in the endogenous productivity is driven by the improvement in the technology adoption to a substantial increase in the aggregate consumption that requires a slightly increase in labor.

5.2.1 Competition Policies without Product Variety

Table 5: Endogenous Productivity without Product Variety.

	No Defensive (ND)	Defensive in Entry (DE)	Defensive in Catch-up (DC)
Follower's Sale-weighted Exogenous Productivity	0.66%	1.35%	−0.74%
Leader's Sale-weighted Exogenous Productivity	−0.62%	0.67%	−1.33%
Sale-weighted Exogenous Productivity	−0.41%	1.02%	−1.52%
Sale-weighted Markup	−3.77%	−1.16%	−2.76%
Endogenous Productivity	2.21%	1.26%	0.97%

Note: The table reports the effect of preventing firms from implementing defensive practices. Each economy has the same product variety of the benchmark economy.

The counterfactual policy exercise consists in comparing the pre-intervention equilibrium where firms are allowed to implement defensive practices, benchmark, and the post-intervention equilibria, which are ND, DE and DC. Differently from the previous counterfactual exercises, I impose that the post-competition policy intervention equilibria had the same product variety of the benchmark economy. More specifically, I draw the additional mass of industries from a distribution of industry that resembles the equilibrium distribution of the economy post-intervention. This exercise allows to disentangle the relative contribution of the different channels that determine the effects of the competition policies on endogenous productivity. Table 5 the results of the exercise that leads three main considerations.

First the product variety channel plays an important role in determining the effect of competition policies on aggregate productivity. It negatively contributed to the post-intervention endogenous productivity. In particular, I can infer that the reduction of product variety implies a loss in the endogenous productivity for the ND, DE and DC economies respectively by 2.48, 0.65 and 2.54 percentage points.

Second, the model that does not take into account the product variety channel predicts that the ND competition policy delivers the highest endogenous productivity. The reason is that there would be only two channels that contribute to endogenous productivity: market power and technology adoption. In particular, looking at table 5, the ND economy delivers a higher endogenous productivity than the DE economy since the worse technology adoption is more than compensated by the stronger reduction in the market power.

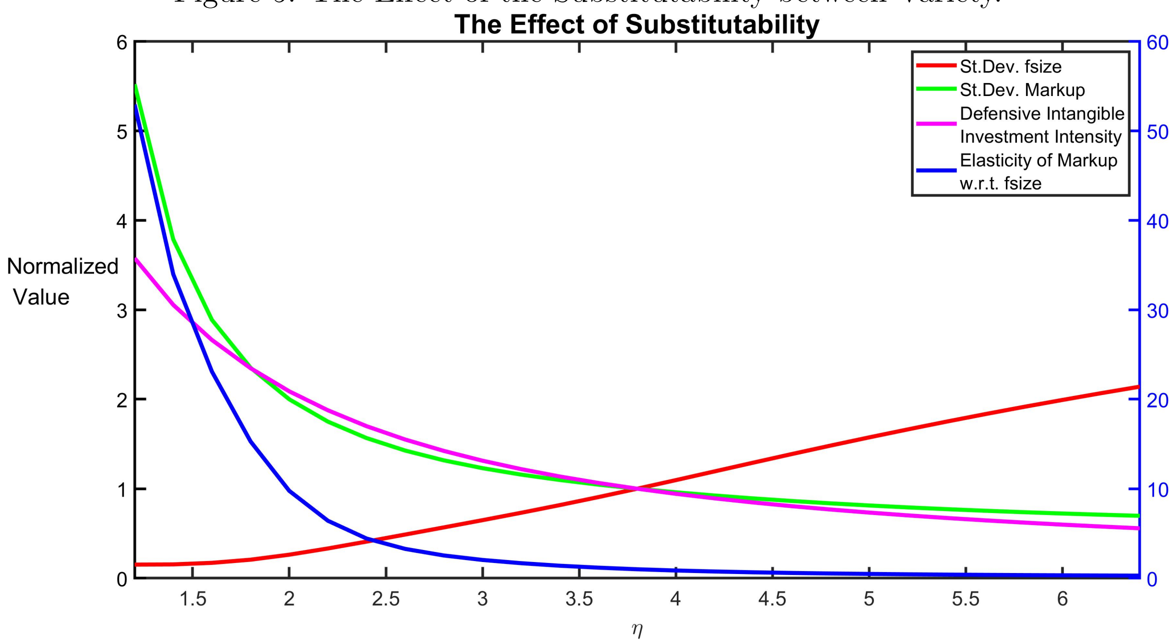
All in all, the analysis delivers an important result. The DE competition policy delivers the highest endogenous productivity compared to the ND and DC economies. The reason is that in the DE economy the larger product variety and the better technology adoption more than compensate the highest market power of the firms.

6 Markups and Defensive Behavior

Section 5.1 documented that the preventing firms from implementing defensive practices affects endogenous productivity through three channels: technology adoption, the creation of new products and market power. In this section, I study how the relative importance of these two channels may have different implications related to the effects of the implementation of the different competition policies. To implement the analysis I compare the effects of the different competition policies on endogenous productivity between economies that are characterised by a different product substitutability that determines the love for variety. In particular I first show that economies with a lower product substitutability are characterised by a higher dispersion of mark-up distribution and by a higher pervasiveness of defensive practices. Following I show that the implementation of the DE policy always generate the highest endogenous productivity even though this results can be based on different channels.

6.1 Steady State Comparison

Figure 5: The Effect of the Substitutability between Variety.



Note: The Figure shows the effect of a change in the product substitutability aggregate variables in economy where firms are allowed to implement defensive practices. The values are normalised to the values of the benchmark economy.

Figure 5 compares the dispersion of markup and firm size distributions and defensive intensity between economies that are characterised by a different degree of substitutability between products. I consider economies in which firms can implement both types of defensive practices. I report the values normalised respect to the values of the benchmark economy[11].

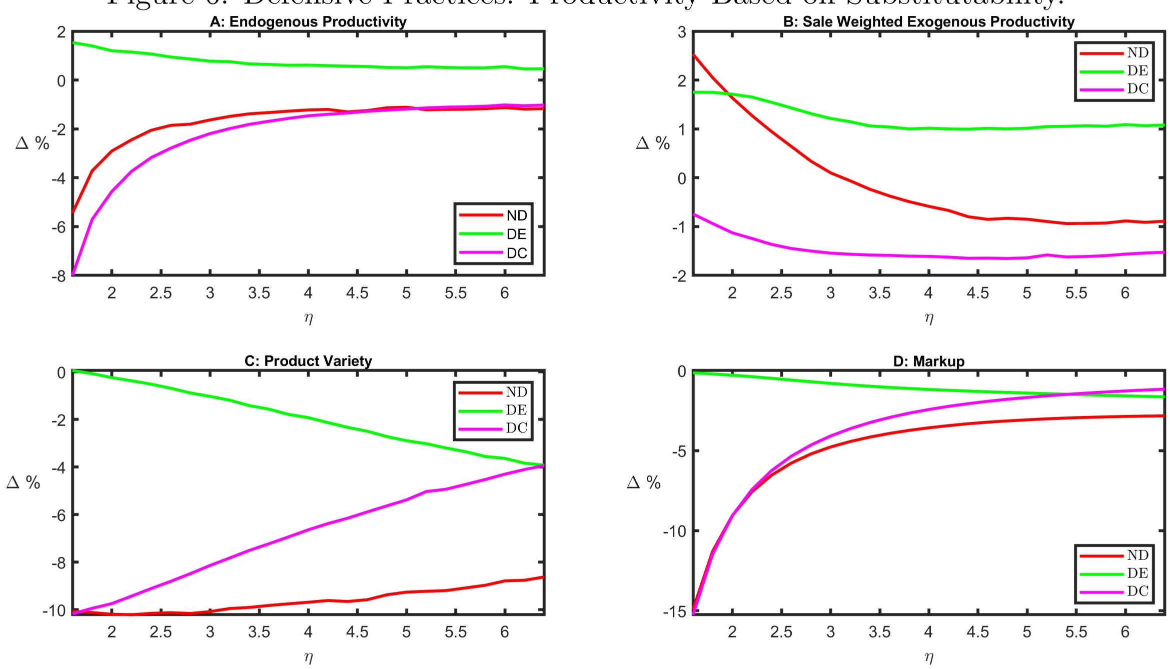
Figure 5 leads three considerations. First economies characterised by a low product substitutability embed a high pervasiveness of defensive practices. The reduction in the substitutability between products increases the competition between firms within industries. The low substitutability increases the incentive of the leader to maintain its dominant position toward the direct competitors.

[11]I define the defensive intensity as the share of the total expenditure in defensive intangible investment on the total output.

Intuitively, leaders predominantly compete against the followers within their industry and not against the other leaders belonging to different industries who produce different looking products.
Second, the dispersion of markup distribution is decreasing in the product substitutability in the economy. The dispersion of the markup distribution decreases in the product substitutability due to the exponential relation between markup and within market share.
Third, the dispersion of firm size distribution is increasing in the product substitutability. Intuitively, in economies with low product substitutability the firm size mainly depends on the within industry market share such that low and high productive monopolistic firms share have the same firm size. In contrast, when the products are substitute the size of a firm depends also on its efficiency.

6.2 The effect of Defensive Behavior based on Substitutability

Figure 6: Defensive Practices: Productivity Based on Substitutability.



Note: The Figure compares the effect of a change in the competition policy on endogenous productivity relative to their benchmark economy between economies characterised by a different product substitutability. Plot A refers to the policies' effects on endogenous productivity, Plot B on technology adoption, Plot C on product variety and Plot D on market power.

Figure 6 reports the results from implementation of the counterfactual policy exercises applied to economies that differ for the degree of product substitutability. More specifically I evaluate the impact of the implementation of the ND, DE and DC competition policies to economies that have a different return from the expansion of the product variety in terms of endogenous productivity. The results of the exercise lead to four main considerations.

First, Plot C of Figure 6 shows that when the product substitutability is high the competition policies DC and DE generate the same degree of product variety. This result can be explained by the entry dynamics in the incumbent industries. As expected, the high product substitutability discourages the entry of low productive followers. Furthermore, followers enter the new created industries with the lowest productivity level available in the economy. As a result, it is less important for a leader of the new industry implementing defensive practices that deter the entry.

Second, as documented in plot B of figure 6, the ND policy leads to a better technology adoption with respect to the DE policy in low product substitutability economies. The reason is that the implementation of defensive practices is too high. The intuition is the following. Low product substitutability incentivises also low productive leaders to deter the entry.

Finally, the implementation of the DE policy always delivers the highest endogenous productivity. This result is due to the different relative importance of the channels that determine the endogenous productivity. More specifically, in economies of low product substitutability the DE policy provides the highest endogenous productivity since it generates the highest product variety that more than compensates the worse technology adoption and the highest market power relative to the ND economy. In case the economy is characterised by a high product substitutability, the DE policy has the highest endogenous productivity since it provides the best technology adoption.

7 Conclusion

Aggregate impact of the firms' defensive practices crucially depend on their effects on market power, technology adoption and product variety. To study the quantitative implication of the defensive practices on endogenous productivity, I build a heterogeneous firm model of technology adoption in which industry leaders can invest to deter the entry or slow down the catch-up of the followers. The model shows that a competition policy that only prevents firms from implementing defensive practices that slow down the catch-up improves the endogenous productivity. Although defensive practices that deter the entry maintain a high market power of the firms in the economy, they incentivise innovation of the incumbent firms and the creation of new products.

References

Abrams, D. S., U. Akcigit, and J. Grennan (2013). Patent value and citations: Creative destruction or strategic disruption? Technical report, National Bureau of Economic Research.

Acemoglu, D. and U. Akcigit (2011). Intellectual property rights, competition and innovation. Technical report, mimeo.

Aghion, P., N. Bloom, R. Blundell, R. Griffith, and P. Howitt (2005). Competition and innovation: An inverted-u relationship. *The quarterly journal of economics 120*(2), 701–728.

Akcigit, U. (2009). Firm size, innovation dynamics and growth.

Akcigit, U. and S. T. Ates (2019). What happened to us business dynamism? Technical report, National Bureau of Economic Research.

Akcigit, U., S. Baslandze, and F. Lotti (2018). Connecting to power: political connections, innovation, and firm dynamics. Technical report, National Bureau of Economic Research.

Akcigit, U. and W. R. Kerr (2018). Growth through heterogeneous innovations. *Journal of Political Economy 126*(4), 1374–1443.

Andrews, D., C. Criscuolo, and P. N. Gal (2015). Frontier firms, technology diffusion and public policy: Micro evidence from oecd countries.

Argente, D., S. Baslandze, D. Hanley, and S. Moreira (2020). Patents to products: Product innovation and firm dynamics.

Arrow, K. (1962). Economic ilfare and the allocation of resources for invention. In *The Rate and Direction of Inventive Activity: Economic and Social Factors*, pp. 609–626. Princeton University Press.

Autor, D., D. Dorn, L. F. Katz, C. Patterson, and J. Van Reenen (2020). The fall of the labor share and the rise of superstar firms. *The Quarterly Journal of Economics 135*(2), 645–709.

Bank, W. (2018). World development report 2019: The changing nature of work.

Baqaee, D. R. and E. Farhi (2020). Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics 135*(1), 105–163.

Bilbiie, F. O., F. Ghironi, and M. J. Melitz (2012). Endogenous Entry, Product Variety, and Business Cycles. *Journal of Political Economy 120*(2), 304–345.

Bloom, N., M. Schankerman, and J. Van Reenen (2013). Identifying technology spillovers and product market rivalry. *Econometrica 81*(4), 1347–1393.

Calligaris, S., C. Criscuolo, and L. Marcolin (2018). Mark-ups in the digital era.

Cavenaile, L., M. A. Celik, P. Roldan-Blanco, and X. Tian (2021). Style over substance? advertising, innovation, and endogenous market structure. *Advertising, Innovation, and Endogenous Market Structure (March 13, 2021)*.

Cavenaile, L., M. A. Celik, and X. Tian (2020). Are markups too high? competition, strategic innovation, and industry dynamics.

Comin, D. and B. Hobijn (2009). Lobbies and technology diffusion. *The Review of Economics and Statistics 91*(2), 229–244.

De Loecker, J. and J. Eeckhout (2018). Global market power. Technical report, National Bureau of Economic Research.

De Loecker, J., J. Eeckhout, and G. Unger (2020). The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics 135*(2), 561–644.

De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik (2016). Prices, markups, and trade reform. *Econometrica 84*(2), 445–510.

De Loecker, J. and F. Warzynski (2012). Markups and firm-level export status. *American economic review 102*(6), 2437–71.

Eeckhout, J. and B. Jovanovic (2002). Knowledge spillovers and inequality. *American economic review 92*(5), 1290–1307.

Feijoo-Moreira, S. Provider-driven complementarity and firm dynamics.

Grullon, G., Y. Larkin, and R. Michaely (2019). Are us industries becoming more concentrated? *Review of Finance 23*(4), 697–743.

Gutiérrez, G. and T. Philippon (2019). The failure of free entry. Technical report, National Bureau of Economic Research.

Hoberg, G. and G. Phillips (2016). Text-based network industries and endogenous product differentiation. *Journal of Political Economy 124*(5), 1423–1465.

Jovanovic, B. and G. M. MacDonald (1994a). Competitive diffusion. *Journal of Political Economy 102*(1), 24–52.

Jovanovic, B. and G. M. MacDonald (1994b). The life cycle of a competitive industry. *Journal of Political Economy 102*(2), 322–347.

Klepper, S. (2015). *Experimental capitalism: The nanoeconomics of America high-tech industries*. Princeton University Press.

Krusell, P. and J.-V. Rios-Rull (1996). Vested interests in a positive theory of stagnation and growth. *The Review of Economic Studies 63*(2), 301–329.

Kwon, S. Y., Y. Ma, and K. Zimmermann (2021). 100 years of rising corporate concentration. *Available at SSRN 3936799*.

Ledezma, I. (2013). Defensive strategies in quality ladders. *Journal of Economic Dynamics and Control 37*(1), 176–194.

Loury, G. C. (1979). Market structure and innovation. *The quarterly journal of economics*, 395–410.

Peters, R. H. and L. A. Taylor (2017). Intangible capital and the investment-q relation. *Journal of Financial Economics 123*(2), 251–272.

Rogerson, R. (1988). Indivisible labor, lotteries and equilibrium. *Journal of monetary Economics 21*(1), 3–16.

Schumpeter, J. A. (1976). Capitalism, socialism and democracy (1942). *J. Econ. Literature 20*, 1463.

Sedláček, P. (2020). Creative destruction and uncertainty. *Journal of the European Economic Association 18*(4), 1814–1843.

Senga, T. (2015). A new look at uncertainty shocks: Imperfect information and misallocation. Technical report, working paper.

Shapiro, C. (2000). Navigating the patent thicket: Cross licenses, patent pools, and standard setting. *Innovation policy and the economy 1*, 119–150.

Sterk, V., P. Sedláček, and B. Pugsley (2021, February). The nature of firm growth. *American Economic Review 111*(2), 547–79.

Stigler, G. J. (1971). The theory of economic regulation. *The Bell journal of economics and management science*, 3–21.

Van Reenen, J. (2018). Increasing differences between firms: market power and the macro-economy.

Zingales, L. (2017). toward a political theory of the firm. *Journal of Economic Perspectives 31*(3), 113–30.

A Propositions

A.1 Proposition 1

I can now describe the optimal choice of the defensive practices chosen by the leaders. In the case where $V_2^L(\varepsilon^L, \varepsilon^F, n^L, n^F) - V_2^L(\varepsilon^L, \varepsilon^F, n^L + 1, n^F - 1) < 0$, the unique symmetric equilibrium is $x_1^L = 0$.
Conversely, if $V^{l,2}(\varepsilon^L, \varepsilon^F, n^L, n^F) - V_2^L(\varepsilon^L, \varepsilon^F, n^L + 1, n^F - 1) > 0$, The first and second order conditions of the optimization problem 23:

$$\begin{aligned}\frac{\partial V_1^L(\varepsilon^L, \varepsilon^F, n^L, n^F)}{\partial x_1^L} = -w x_1^L + \frac{X_1^F}{X_1^F + \bar{T}} \frac{\bar{Q}}{(x_1^L + \bar{Q})^2} \Big[V_2^L(\varepsilon^L, \varepsilon^F, n^L, n^F) \\ - V_2^L(\varepsilon^L, \varepsilon^F, n^L + 1, n^F - 1) \Big]\end{aligned} \tag{A.1}$$

$$\begin{aligned}\frac{\partial^2 V_1^L(\varepsilon^L, \varepsilon^F, n^L, n^F)}{\partial x_1^{L2}} = -w - 2\frac{X_1^F}{X_1^F + \bar{T}} \frac{\bar{Q}}{(x_1^L + \bar{Q})^3} \Big[V_2^L(\varepsilon^L, \varepsilon^F, n^L, n^F) \\ - V_2^L(\varepsilon^L, \varepsilon^F, n^L + 1, n^F - 1) \Big] < 0\end{aligned} \tag{A.2}$$

Rearranging A.1, I derive the equation that describes the symmetric equilibria:

$$\begin{aligned}x_1^L = f(x_1^L, X_1^F) = \frac{1}{n^L} \sqrt{\frac{X_1^F}{X_1^F + \bar{T}} \frac{\bar{Q}}{w x_1^L}} \times \\ \sqrt{\Big[V_2^L(\varepsilon^L, \varepsilon^F, n^L, n^F) - V_2^L(\varepsilon^L, \varepsilon^F, n^L + 1, n^F - 1) \Big]} - \frac{\bar{Q}}{n^L}\end{aligned} \tag{A.3}$$

Given X_1^F, A.3 describes a self-map where:

- $\lim_{x \to 0} f(x_1^L, X_1^F) = +\infty$
- $\lim_{x \to \infty} f(x_1^L, X_1^F) = -\frac{\bar{Q}}{n^L}$
- $\frac{\partial f(x_1^L, X_1^F)}{\partial x_1^L} < 0 \quad \forall x_1^L \in R_+$

So there must exist a unique x_1^{L*} such that $x_1^{L*} = f(x_1^{L*}, X_1^F)$.
Moreover, I can verify that

$$\frac{\partial f(x_1^L, X_1^F)}{\partial X_1^F} > 0 \quad \forall X_1^F \in \mathcal{R}_+$$

that implies:

$$\frac{\partial x_1^{L*}}{\partial X_1^F} > 0 \quad \forall X_1^F \in \mathcal{R}_+ \tag{A.4}$$

A.2 Proposition 2

The first and second order conditions of the optimization problem 24:

$$\frac{\partial V_1^F(\varepsilon^L,\varepsilon^F,n^L,n^F)}{\partial X_1^F} = -wX_1^F + \frac{\bar{Q}}{x_1^L+\bar{Q}}\times \frac{[\bar{T}\Delta\tilde{V}_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F) + (X_1^F - X_1^F)\Delta\hat{V}_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F)]}{(X_1^F+\bar{T})^2} = 0 \tag{A.5}$$

$$\frac{\partial^2 V_1^F(\varepsilon^L,\varepsilon^F,n^L,n^F)}{\partial x^{F2}} = -w - 2\frac{\bar{Q}}{x_1^L+\bar{Q}}\times \frac{[\bar{T}\Delta\hat{V}_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F) + (X_1^F - X_1^F)\Delta\tilde{V}_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F)]}{(X_1^F+\bar{T})^3} < 0 \tag{A.6}$$

where $\Delta\hat{V}_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F) = [V_2^L(\varepsilon^L,\varepsilon^F,n^L+1,n^F-1) - V_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F)$ is the difference between the value of winning and the value when no follower succeeds, and, $\Delta\tilde{V}_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F) = [V_2^L(\varepsilon^L,\varepsilon^F,n^L+1,n^F-1) - V_2^F(\varepsilon^L,\varepsilon^F,n^L+1,n^F-1)]$ is the difference between the expected value of winning and the expected value when another followers wins the contest.

Rearranging the A.5, I derive the equation that describes the symmetric equilibria:

$$X_1^F = q(X_1^F,x_1^L) = -\frac{\bar{T}}{n^F} + \frac{1}{n^F}\sqrt{\frac{\bar{Q}}{x_1^L+\bar{Q}}}\times \sqrt{\left\{\frac{\bar{T}}{wX_1^F}\Delta\hat{V}_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F) + \frac{(n^F-1)}{w}\Delta\tilde{V}_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F)\right\}} \tag{A.7}$$

Given x_1^L, A.7 describes a self-map where:

- $\lim_{X_1^F\to 0} q(X_1^F,x_1^L) = +\infty$
- $\lim_{X_1^F\to\infty} q(X_1^F,x_1^L) = \frac{1}{n^F}\sqrt{\frac{\bar{Q}}{x_1^L+\bar{Q}}\frac{(n^F-1)}{w}\Delta\tilde{V}_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F)} - \frac{\bar{T}}{n^F}$
- $\frac{\partial q(x_1^L,X_1^F)}{\partial X_1^F} < 0 \quad \forall X_1^F \in \mathcal{R}_+$

So there must exist a unique X_1^{F*} such that $X_1^{F*} = f(X_1^{F*},x_1^L)$. The equilibrium quantities of $R\&D$ chosen by followers, X_1^{F*}, is increasing $\Delta\hat{V}_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F)$ and $\Delta\tilde{V}_2^F(\varepsilon^L,\varepsilon^F,n^L,n^F)$.

Moreover, I can verify that

$$\frac{\partial q(X_1^F,x_1^L)}{\partial x_1^L} < 0 \quad \forall x_1^L \in \mathcal{R}_+$$

that implies:

$$\frac{\partial X_1^{F*}}{\partial x_1^L} < 0 \quad \forall X^l \in \mathcal{R}_+ \tag{A.8}$$

A.3 Proposition 3

Let $f^*: X_1^F \to x_1^{L*}$ and $q^*: x_1^L \to X_1^{F*}$, by A.4 and A.8:

- $\frac{\partial f^*(X_1^F)}{\partial X_1^F} > 0 \quad \forall X_1^F \in \mathcal{R}_+$
- $\frac{\partial q^*(X_1^F)}{\partial x_1^L} < 0 \quad \forall X_1^F \in \mathcal{R}_+$

then it implies that there must exist a unique (x_1^{L**}, X_1^F) such that:

- $x_1^{L**} = f^*(q^*(x_1^{L**}))$
- $X_1^{F**} = q^*(f^*(X_1^{F**}))$

A.4 Proposition 4

Let $\Delta\hat{V}_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F) = [\tilde{V}_0^L(\varepsilon^{L\prime}, \varepsilon^{F\prime}, n^L, n^F) - V_0^L(\varepsilon^l, \varepsilon^f, n^L, n^F)]$ be the difference between the value of winning the contests and the value when no leader succeeds. For $\Delta\hat{V}_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F) < 0$, the unique symmetric equilibrium is $x_2^{L*} = 0$.

Conversely, if $\Delta\hat{V}_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F) > 0$, I take the first and second order conditions of the recursive optimization problem 25:

$$\frac{\partial V_2^L(\varepsilon^L, \varepsilon^F, n^L, n^F)}{\partial x_2^L} = -wx_2^L + \frac{[\bar{Q}\Delta\hat{V}_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F) + (x_2^L - x_2^L)\Delta\check{V}_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F)]}{(x_1^L + \bar{Q})^2} = 0 \tag{A.9}$$

$$\frac{\partial^2 V_2^L(\varepsilon^L, \varepsilon^F, n^L, n^F)}{\partial x_2^{L2}} = -w - 2\times \frac{[\bar{T}\Delta\hat{V}_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F) + (x_2^L - x_2^L)\Delta\check{V}_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F)]}{(x_2^L + \bar{Q})^3} < 0 \tag{A.10}$$

where $\Delta\check{V}_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F) = [\tilde{V}_0(\varepsilon^{L\prime}, \varepsilon^{F\prime}, n^L, n^F) - V_0^F(\varepsilon^{L\prime}, \varepsilon^{F\prime}, n^L, n^F)]$ is the difference between the value of winning contest and the value if another leader wins.

Rearranging A.9, I derive the equation that describes the symmetric equilibria:

$$x_2^L = \omega(x_2^L) = \frac{1}{n^L}\sqrt{\frac{\bar{Q}}{wx_1^L}\Delta\hat{V}_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F) + \frac{(n^L-1)}{w}\Delta\check{V}_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F)} - \frac{\bar{Q}}{n^L} \tag{A.11}$$

A.3 represents a self-map where:

- $\lim_{x_1^L \to 0} \omega(x_1^L) = +\infty$
- $\lim_{x_1^L \to \infty} \omega(x_1^L) = \sqrt{\frac{(n^L-1)}{w}\Delta\hat{V}_0^L(\varepsilon^L, \varepsilon^F, n^L, n^F)} - \frac{\bar{Q}}{n^L}$
- $\frac{\partial w(x_2^L)}{\partial x_2^L} < 0 \quad \forall x_2^L \in \mathcal{R}_+$

So, also in this game, there must exist a unique x_2^{L*} such that $x_2^{L*} = \omega(x_2^{L*})$.

B Size and Intangible Investment Intensity Relation: Robustness Checks

Table B.1: Intangible Investment and Productivity.

	(1) intangible_inv_int	(2) intangible_inv_int	(3) intangible_inv_int	(4) intangible_inv_int
size	-0.32***	-0.62***	-0.32***	-0.44***
	(0.01)	(0.02)	(0.01)	(0.06)
_cons	-0.24***	1.44***	-0.22***	0.45
	(0.06)	(0.11)	(0.06)	(0.45)
Observations	47006.00	46274.00	45944.00	47012.00
R2	0.73	0.96	0.74	0.47
Firm f.e.	N	Y	N	N
Industry f.e.	Y	Y	N	N
Year f.e.	Y	Y	N	N
Industry-Year f.e.	N	N	Y	N

Standard errors in parentheses

* p< 0.10, ** p<0.05, *** p<0.01

Note: The table reports the estimates of a firm panel regression at yearly frequency from 2001 to 2018. The dependent variable is the intangible investment intensity, defined as the logarithm of the share of intangible investment over the total sale. The independent variable is firm size that is defined as the logarithm of the sale.