Task-Specific Abilities in Multi-Task Principal-Agent Relationships*

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Abstract

This paper analyzes a multi-task agency framework where the agent exhibits task-specific abilities. It illustrates how incentive contracts account for the agent's task-specific abilities if contractible performance measures do not reflect the agent's contribution to firm value. This paper further sheds light on potential ranking criteria for performance measures in multi-task agencies. It demonstrates that the value of performance measures in multi-task agencies cannot necessarily be compared by their respective signal/noise ratios as in single-task agency relations. It is rather pivotal to take the induced effort distortion and measure-cost efficiency into consideration – both determined by the agent's task specific abilities.

Keywords: Task-specific human capital, performance measurement, distortion, multi-task agencies, congruence, incentives.

JEL classification: D23, D82, J24

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1 Introduction

Empirical investigations have offered an abundance of evidence suggesting that individuals are highly responsive to monetary incentives (see e.g. Asch [1990], Paarsch and Shearer [1999] and Lazear [2000]). Nevertheless, the specific effects of reward schemes are somewhat ambiguous when individuals are required to perform a collection of different tasks. In such situations, Kerr [1975] cautioned against the consequences of a reward system that inefficiently overemphasizes some tasks while underemphasizing others. An illustrative example cited by Kerr [1975] is the difficult trade-off between research and teaching responsibilities encountered by faculties at universities. Since teaching quality is harder to assess relative to research output, and prospective promotion decisions mainly hinge on research performance, it is a common phenomenon for faculty members to focus on research at the expense of teaching.[1] In general, inefficient effort allocations occur when available performance measures do not reflect employees' true contribution to firm value [Feltham and Xie, 1994]. In this case, employees focus on less or even non-valuable tasks, and disregarding more beneficial ones [Feltham and Xie, 1994].[2]

Previous multi-task agency literature such as Feltham and Xie [1994], Banker and Thevaranjan [2000], and Datar, Kulp, and Lambert [2001] focussed on performance measure congruity and its effects on the efficiency of incentive contracts, but these studies abstract from the possibility that agents may perform some tasks more efficiently than others.[3] Recent literature however, emphasizes the role of acquiring human capital for specific tasks (see e.g. Lindbeck and Snower [2000], Gibbons and Waldman [2006] and Gibbons and Waldman [2004]).[4] Since individuals differ substantially in their learning aptitudes, which inevitably lead to discrepancies in skills and abilities [Gibbons and Waldman, 2006], it is reasonable to infer that different individuals might perform different tasks with varying degrees of ease. For example, Sapienza and Gupta [1994] show in their study of principal-agent relations within venture capital-backed firms that the frequency of venture capitalist (principal) - CEO (agent) interaction is partially dependent on the CEOs' venture experience. They provide evidence that CEOs with prior expe-

[1] See Brickley and Zimmerman [2001] for an empirical study of this example.

[2] See as well the discussion in Gibbons [1998].

[3] Schnedler [2006] is an exception. However, his focus is different in the sense that he investigates the consequences of different marginal effort costs on the relative value of incongruent performance measures for the provision of incentives.

[4] For empirical evidence see Baker, Gibbs, and Holmström [1994].

riences (i.e. greater proficiency) in start-up ventures would have a lesser tendency of consulting with their venture capitalist.

In order to understand the nature of contracts in multi-task principal-agent relationships, it is essential to investigate whether and how task-specific abilities influence the agent's preferences for her effort allocation and thus, the optimal incentive provision in response to these abilities. This paper therefore analyzes a multi-task principal-agent relationship in order to gain new insights into the provision of incentives if available performance measures do not fully reflect the agent's contribution to firm value, and the agent exhibits different abilities for performing the relevant tasks. It further demonstrates how the value of performances measures can be compared in multi-task agencies. The analysis indicates that the signal/noise ratio – sufficient to rank performance measures in single-task agencies – can only be applied if all available measures provide the same information about the agent's *relative* effort allocation. In contrast, the proposed (more general) ranking criteria accounts for task-specific abilities of agents such that different agents may imply various orderings of performance measures. Put differently, the relative value of performance measures in multi-task agencies is closely tied to the characteristics of agents.

This paper proceeds as follows. In section 2, I give an overview of the model and derive the first-best contract in section 3. I provide in section 4 the second-best contract and focus on the relation between performance measure congruity and effort distortion in section 5. In section 6, I investigate how performance measures can be ranked in multi-task agencies, in particular when agents are characterized by task-specific abilities. Section 7 concludes.

2 The Model

Consider a single-period agency relationship between a risk-neutral principal and a risk-averse agent. The principal owns an asset and requires the agent's productive effort. Once employed, the agent is in charge of performing $n \geq 2$ tasks (multi-tasking). These tasks are tied together, i.e. the principal cannot split and allocate them to different agents.[5] The agent is thus in charge of implementing an effort vector $\boldsymbol{e} = (e_1, ..., e_n)^T$, $\boldsymbol{e} \in \mathbb{R}^{n+}$, where e_i is his effort allocated to

[5]For considerations on how multiple tasks are efficiently split among several agents, refer e.g. to Holmström and Milgrom [1991], Corts [2007], and Schöttner [2006].

task i.[6] Effort is non-verifiable and all activities e_i, $i = 1, ..., n$, are measured in the same unit.

Let $\boldsymbol{\Psi} = \operatorname{diag}(\psi_1, ..., \psi_n)$, $\psi_i > 0$, be a diagonal $n \times n$ matrix representing the agent's task-specific abilities. The agent's quadratic effort costs are contingent on $\boldsymbol{\Psi}$ and take the form $C(\boldsymbol{e}) = \boldsymbol{e}^T \boldsymbol{\Psi} \boldsymbol{e}/2$. Hence, a higher ability for performing task i is characterized by a lower ψ_i, $i = 1, ..., n$, and vice versa.[7]

The agent's preferences are represented by the negative exponential utility function

$$U(w, \boldsymbol{e}) = -\exp\left[-\rho\left(w - C(\boldsymbol{e})\right)\right], \tag{1}$$

where ρ denotes the Arrow-Pratt measure of absolute risk-aversion and w as the agent's wage. For parsimony, let $\bar{w} = 0$ be his reservation wage implying a reservation utility $\bar{U} = -1$.

By implementing effort $\boldsymbol{e}$, the agent contributes to the principal's non-verifiable gross payoff $V(\boldsymbol{e}) = \boldsymbol{\mu}^T \boldsymbol{e} + \varepsilon_V$, where ε_V is a normally distributed random component with zero mean and variance σ_V^2, representing firm-specific and economy wide risk. The n-dimensional vector $\boldsymbol{\mu} = (\mu_1, ..., \mu_n)^T$, $\boldsymbol{\mu} \in \mathbb{R}^{n+}$, characterizes the marginal effect of $\boldsymbol{e}$ on gross payoff $V(\boldsymbol{e})$. Since $V(\boldsymbol{e})$ is non-verifiable, it cannot be part of an explicit single-period incentive contract. The only verifiable information about $\boldsymbol{e}$, however, is provided by the performance measure

$$P(\boldsymbol{e}) = \boldsymbol{\omega}^T \boldsymbol{e} + \varepsilon, \tag{2}$$

where $\boldsymbol{\omega} = (\omega_1, ..., \omega_n)^T$, $\boldsymbol{\omega} \in \mathbb{R}^{n+}$, is the vector of performance measure sensitivities. The random component ε is normally distributed with zero mean and variance σ^2, and represents potential effects on the performance measure beyond the agent's control.

As pointed out by Feltham and Xie [1994], a performance measure can be *incongruent*, i.e. it does not necessarily capture the agent's true contribution to firm value. In this framework, performance measure $P(e)$ is incongruent, if there exists no constant $\lambda \neq 0$ satisfying $\boldsymbol{\mu} = \lambda \boldsymbol{\omega}$. Baker [2002] derived a geometric measure for performance measure congruity. Since his result is fundamental to the subsequent analysis, it is summarized in the following definition.

[6] All vectors are column vectors where 'T' denotes the transpose.

[7] A similar approach is used by MacLeod [1996], where ψ_i, $i = 1, ..., n$, are random variables. However, his work is different in the sense that he focuses on the relationship between explicit and implicit incentive contracts rather than on the effort distortion induced by incongruent performance measurement.

Definition 1. *The congruence of performance measure $P(\boldsymbol{e})$ is measured by $\Upsilon^C(\varphi) = \cos\varphi$, where φ is the angle between the vector of payoff sensitivities $\boldsymbol{\mu}$ and the vector of performance measure sensitivities $\boldsymbol{\omega}$.*

Accordingly, as long as vector $\boldsymbol{\mu}$ and vector $\boldsymbol{\omega}$ are linearly independent ($\varphi \neq 0$), performance measure $P(e)$ is incongruent. Moreover, a more congruent performance measure implies a smaller angle φ and hence, leads to a higher measure of congruity $\Upsilon^C(\varphi)$ due to the definition of the cosine. Finally note that $\varphi \in [0, \pi/2]$ since $\mu_i, \omega_i \geq 0$, $i = 1, ..., n$.[8]

In line with previous multi-task agency literature, I restrict my analysis to a compensation scheme w which is linear in the performance measure $P(\boldsymbol{e})$:

$$w(\boldsymbol{e}) = \alpha + \beta P(\boldsymbol{e}). \tag{3}$$

The fixed payment α is utilized to split the surplus between the principal and the agent, whereas the incentive parameter β is used to motivate the agent to implement effort. Since the compensation scheme is linear, the agent's utility is exponential, and the error term is normally distributed, maximizing the agent's expected utility is analogous to maximizing her certainty equivalent

$$CE(\boldsymbol{e}) = \alpha + \beta\boldsymbol{\omega}^T\boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^T\boldsymbol{\Psi}\boldsymbol{e} - \frac{\rho}{2}\beta^2\sigma^2, \tag{4}$$

where $\rho\beta^2\sigma^2/2$ represents the agent's risk premium.

The timing of this problem is as follows. First, the principal offers the agent a contract (α^*, β^*). If this contract guarantees the agent at least the same expected utility as his best alternative, he accepts. After the agent implemented $\boldsymbol{e}$ and the random variables ε and ε_V are realized, all payments take place.

3 The First-Best Effort Allocation

Before I move on to the second-best contract, it is necessary to characterize the first-best effort allocation as a benchmark for the subsequent analysis. Suppose the principal can specify a desired effort allocation and intensity in an enforceable contract. The optimal (first-best) effort vector $\boldsymbol{e}$ maximizes the difference between the expected gross payoff $V(\boldsymbol{e})$ and effort costs

[8] All angles are represented in radian measures.

$C(\boldsymbol{e})$:

$$\max_{\boldsymbol{e}} \ \Pi(\boldsymbol{e}) = \boldsymbol{\mu}^T\boldsymbol{e} - \frac{1}{2}\boldsymbol{e}^T\boldsymbol{\Psi}\boldsymbol{e}. \tag{5}$$

Let $\boldsymbol{\phi} \equiv \boldsymbol{\Psi}^{-1}\boldsymbol{\mu} = (\mu_1/\psi_1, ..., \mu_n/\psi_n)^T$ be the vector of the payoff-cost sensitivity ratios. Then, the first-best effort vector is characterized by

$$\boldsymbol{e}^{fb} = \boldsymbol{\phi}. \tag{6}$$

The principal maximizes her expected profit by inducing the agent to perform each activity e_i in accordance to its payoff-cost sensitivity ratio μ_i/ψ_i, $i = 1, ..., n$. Activities with high ratios are consequently more intensively conducted relative to activities with low ratios. For the subsequent analysis keep in mind that any implemented effort vector $\boldsymbol{e}^*$ characterizes a distorted effort allocation, if $\boldsymbol{e}^*$ and $\boldsymbol{e}^{fb}$ are linearly independent. Formally, $\boldsymbol{e}^*$ is distorted if there exists no constant $\lambda > 0$ satisfying $\boldsymbol{e}^* = \lambda\boldsymbol{e}^{fb}$.

4 The Second-Best Contract

If the principal cannot directly contract over $\boldsymbol{e}$, she faces an incentive problem for motivating the agent to implement appropriate effort. Since the gross payoff $V(\boldsymbol{e})$ is non-verifiable, the incentive contract must be based upon the contractible performance measure $P(\boldsymbol{e})$. However, the application of $P(\boldsymbol{e})$ in an incentive contract may cause two inefficiencies. First, the performance measure – and therefore the agent's compensation – is uncertain such that the risk-averse agent requires a risk premium for accepting a contract dependent on $P(\boldsymbol{e})$. Second, the performance measure can be incongruent and, therefore, motivates the agent to inefficiently allocate his effort across relevant tasks. The subsequent analysis focuses on the latter inefficiency since the trade-off between incentive provision and the agent's desire for insurance has intensively been analyzed by previous literature.[9]

In a second-best environment, the principal's problem is to design a contract (α^*, β^*) that maximizes her expected profit $\Pi = \mathrm{E}[V(\boldsymbol{e}) - w(\boldsymbol{e})]$ while ensuring the agent's participation. The optimal linear contract therefore solves

$$\max_{\alpha,\beta,\boldsymbol{e}} \ \Pi \equiv \boldsymbol{\mu}^T\boldsymbol{e} - \alpha - \beta\boldsymbol{\omega}^T\boldsymbol{e} \tag{7}$$

[9]For a detailed analysis in a LEN-setting, see e.g. Spremann [1987], Baker [1992], and Prendergast [1999]; and for a general approach Shavell [1979], Holmström [1979], Grossman and Hart [1983], and Rees [1985].

s.t.

$$\boldsymbol{e} = \arg\max_{\tilde{\boldsymbol{e}}} \alpha + \beta \boldsymbol{\omega}^T \tilde{\boldsymbol{e}} - \frac{1}{2} \tilde{\boldsymbol{e}}^T \boldsymbol{\Psi} \tilde{\boldsymbol{e}} - \frac{\rho}{2} \beta^2 \sigma^2 \tag{8}$$

$$\alpha + \beta \boldsymbol{\omega}^T \boldsymbol{e} - \frac{1}{2} \boldsymbol{e}^T \boldsymbol{\Psi} \boldsymbol{e} - \frac{\rho}{2} \beta^2 \sigma^2 \geq 0, \tag{9}$$

where (8) is the agent's incentive condition and (9) his participation constraint.

For the subsequent analysis, let $\boldsymbol{\Gamma} \equiv \boldsymbol{\Psi}^{-1} \boldsymbol{\omega} = (\omega_1/\psi_1, ..., \omega_n/\psi_n)^T$ be the vector of measure-cost sensitivity ratios. We can infer from (8) that the agent implements

$$\boldsymbol{e}^* = \boldsymbol{\Gamma} \beta. \tag{10}$$

In contrast to the first-best scenario, the agent's effort e_i for performing task i depends on the measure-cost sensitivity ratio ω_i/ψ_i and the incentive parameter β. Observe that the effort intensity can be influenced by adjusting β. The effort allocation however, is exogenously determined by the performance measure sensitivities relative to the agent's task-specific abilities.[10]

To maximize her expected profit, the principal sets α such that the agent's participation constraint binds. Solving (9) for α and substituting the resulting expression together with $\boldsymbol{e}^*$ in the principal's objective function yield an unconstrained maximization problem:

$$\max_{\beta} \;\; \Pi = \boldsymbol{\mu}^T \boldsymbol{\Gamma} \beta - \frac{\beta^2}{2} \left[\boldsymbol{\omega}^T \boldsymbol{\Gamma} + \rho \sigma^2 \right]. \tag{11}$$

The first-order condition identifies the optimal incentive parameter β^*:

$$\beta^* = \frac{\boldsymbol{\mu}^T \boldsymbol{\Gamma}}{\boldsymbol{\omega}^T \boldsymbol{\Gamma} + \rho \sigma^2}. \tag{12}$$

Besides the precision of the performance measure $1/\sigma^2$ with the agent's risk tolerance $1/\rho$, the optimal incentive parameter β^* is a function of the payoff sensitivities $\boldsymbol{\mu}$, the performance measure sensitivities $\boldsymbol{\omega}$, and the measure-cost sensitivity ratios $\boldsymbol{\Gamma}$. Recall that $\boldsymbol{\Gamma} = \boldsymbol{\Psi}^{-1} \boldsymbol{\omega}$. Hence, β^* incorporates the agent's task-specific abilities $\boldsymbol{\Psi}$ in two ways: (i) by their relation

[10] To illustrate the difference between effort intensity and effort allocation, let two arbitrary activities e_k and e_j vary to $\hat{e}_k$ and $\hat{e}_j$, respectively. If the ratio between both activities remains identical such that $e_k/e_j = \hat{e}_k/\hat{e}_j$, $k, j = 1, ..., n$, $k \neq j$, the relative effort allocation remains the same. In contrast, if $e_k/e_j \neq \hat{e}_k/\hat{e}_j$ for at least one pair $(k, j) \in \{1, ..., n\}$, $k \neq j$, the relative effort allocation varies. The overall effort intensity, however, changes without affecting the effort allocation, if there exists a constant $\lambda > 0$ satisfying $\boldsymbol{e} = \lambda \hat{\boldsymbol{e}}$, where $\hat{\boldsymbol{e}}$ is the modified effort vector.

to the payoff sensitivities $\boldsymbol{\mu}$ in the numerator; and (ii), by their relation to the performance measure sensitivities $\boldsymbol{\omega}$ in the numerator and denominator. It can therefore be inferred that agents with different task-specific abilities obtain diverse incentive contracts despite being in charge of performing an identical set of tasks and evaluated by the same information system.

Substituting β^* in (11) and using geometric representations give the principal's expected profit

$$\Pi^* = \frac{\|\boldsymbol{\mu}\|^2 \|\boldsymbol{\Gamma}\|^2 \cos^2\theta}{2(\|\boldsymbol{\omega}\|\|\boldsymbol{\Gamma}\|\cos\xi + \rho\sigma^2)}, \tag{13}$$

where θ denotes the angle between the vector of payoff sensitivities $\boldsymbol{\mu}$ and the vector of measure-cost sensitivity ratios $\boldsymbol{\Gamma}$. The angle between the vector of performance measure sensitivities $\boldsymbol{\omega}$ and vector $\boldsymbol{\Gamma}$ is denoted by ξ. These two angles (as will be shown in the subsequent section) characterize the efficiency of the agent's effort allocation, and as a logical consequence, affect the optimal incentive contract (α^*, β^*) and the principal's expected profit Π^*.

5 Performance Measure Congruity and Effort Distortion

In this section, I focus more intensively on performance measure congruity and its effect on effort distortion if the agent performs different tasks with varying degrees of ease. To do so, it is helpful to first clarify the distinction between performance measure congruity and effort distortion. Performance measure congruity refers to the degree of alignment between the agent's marginal effect on his performance measure and on the expected payoff for the firm [Feltham and Xie, 1994]. Performance measure congruity can thus be characterized by the angle φ between the vector of payoff sensitivities $\boldsymbol{\mu}$ and the vector of performance measure sensitivities $\boldsymbol{\omega}$, as emphasized by Baker [2002] and summarized by definition 1 in section 2.

In contrast, effort distortion refers to the relation between an implemented effort vector $\boldsymbol{e}^*$ and the first-best effort vector $\boldsymbol{e}^{fb}$. Formally, as previously emphasized, the agent's effort allocation is not distorted if there exists a constant $\lambda > 0$ satisfying $\boldsymbol{e}^{fb} = \lambda \boldsymbol{e}^*$. Recall that $\boldsymbol{e}^{fb} = \boldsymbol{\Psi}^{-1}\boldsymbol{\mu}$ and $\boldsymbol{e}^* = \beta \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}$. Thus, we can immediately infer that only a congruent performance measure ($\boldsymbol{\mu} = \lambda\boldsymbol{\omega}$, $\lambda \in \mathbb{R}^*$) motivates the implementation of non-distorted effort. Notice that this inference is independent of the agent's task-specific abilities. Consequently, Feltham and Xie's [1994] observation that only congruent performance measures induce non-distorted effort holds even for a more general setting where the agent is allowed to exhibit task-specific abilities.

However, if the applied performance measure is incongruent, we can conclude that the agent is motivated to implement an inefficient effort allocation across relevant tasks. The objective of the subsequent analysis is to characterize the degree of effort distortion, and to investigate how it is affected by the agent's task-specific abilities. To do so, it is first necessary to discuss the economic interpretation of the two angles, θ and ξ, which clearly affect the principal's expected profit and thus, can expected to be rooted in the agent's effort choice.

Proposition 1. *If* $\psi_k \neq \psi_j$ *for at least one pair* $(k, j) \in \{1, ..., n\}$, $k \neq j$, *then* $\Upsilon^D(\theta) = \cos\theta$ *measures effort distortion.*

Proof All proofs are given in the Appendix.

Note that the measure $\Upsilon^D(\theta)$ is negatively related to effort distortion. The less distorted the agent's effort allocation with respect to $\boldsymbol{\mu}$, the smaller is θ, and consequently, the higher is $\Upsilon^D(\theta)$. If $\theta = 0$, the application of performance measure $P(\boldsymbol{e})$ motivates non-distorted effort.

Now suppose that the available performance measure $P(\boldsymbol{e})$ changes such that the agent is motivated to implement a less distorted effort allocation. Formally, θ decreases. This implies, *ceteris paribus*, a higher expected profit Π^*. Note, however, that there is a second effect on Π^* captured by ξ as the angle between $\boldsymbol{\omega}$ and $\boldsymbol{\Gamma}$. To illustrate this effect, we can re-formulate the agent's effort costs by substituting $\boldsymbol{e}^*$:

$$C(\cdot) = \frac{1}{2}\beta^2 \|\boldsymbol{\omega}\| \|\boldsymbol{\Gamma}\| \cos\xi. \tag{14}$$

The properties of the agent's task-specific abilities affect her effort costs in two ways. The first effect is a result of the effort cost intensity over all tasks. For illustrative purposes, assume that the effort costs take the form $C(\boldsymbol{e}) = \boldsymbol{e}^T \lambda \boldsymbol{\Psi} \boldsymbol{e}/2$ with $\lambda > 0$. Increasing λ implies that all tasks become more costly to perform, thereby leading to a higher $\|\boldsymbol{\Gamma}\|$ without affecting $\cos\xi$. The second effect is caused by the relation between the performance measure sensitivities $\boldsymbol{\omega}$ and the agent's task-specific abilities $\boldsymbol{\Psi}$. The relative abilities across tasks thereby affect $\|\boldsymbol{\Gamma}\|$ and $\cos\xi$. Recall that $\|\boldsymbol{\Gamma}\|$ determines the effort intensity without affecting the allocation. In contrast, $\cos\xi$ measures the agent's effort costs (in utility terms) for a particular effort allocation motivated by $P(\boldsymbol{e})$. This observation leads to the next corollary.

Corollary 1. *If* $\psi_k \neq \psi_j$ *for at least one pair* $(k, j) \in \{1, ..., n\}$, $k \neq j$, *then* $\Upsilon^{M/C}(\xi) = \cos\xi$ *characterizes the measure-cost efficiency.*

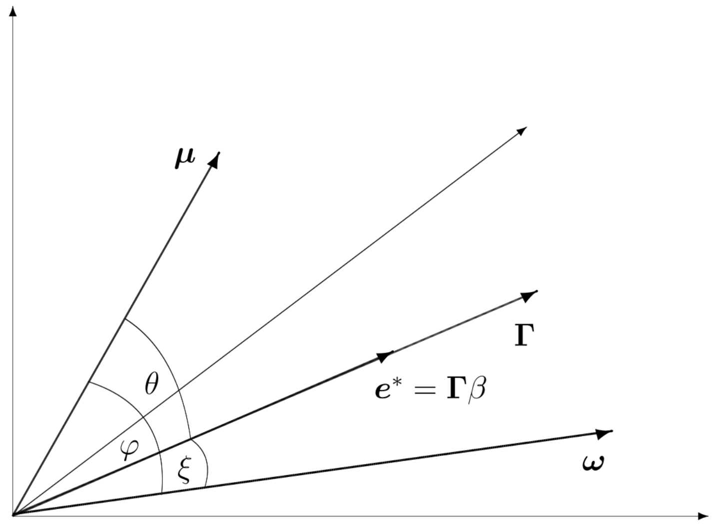

Figure 1: Performance Measure Congruity and Effort Distortion for $n = 3$

The previous results are illustrated in figure 1 for the three-dimensional case ($n = 3$). Besides the second-best effort vector $\boldsymbol{e}^*$, it depicts the vectors of the gross payoff sensitivities $\boldsymbol{\mu}$, performance measure sensitivities $\boldsymbol{\omega}$, and measure-cost sensitivity ratios $\boldsymbol{\Gamma}$. The effort vector $\boldsymbol{e}^*$ has the same direction as $\boldsymbol{\Gamma}$, only their lengths differ, depending on β. Observe that $\boldsymbol{e}^*$ is not necessarily on the plane spanned by $\boldsymbol{\mu}$ and $\boldsymbol{\omega}$. The location of $\boldsymbol{e}^*$ relative to $\boldsymbol{\mu}$ characterizes the induced effort distortion (angle θ), whereas the relation between $\boldsymbol{\mu}$ and $\boldsymbol{\omega}$ measures the congruity of performance measure $P(\boldsymbol{e})$ (angle φ). Finally, the measure-cost efficiency is characterized by the relation of $\boldsymbol{\Gamma}$ to $\boldsymbol{\omega}$ (angle ξ).

If vector $\boldsymbol{\mu}$ and vector $\boldsymbol{\omega}$ point in the same direction, then $\boldsymbol{e}^{fb} = \lambda \boldsymbol{e}^*$, $\lambda > 0$, i.e. the incentive contract motivates the agent to implement the first-best effort allocation, see corollary **??**. Nevertheless, inducing a first-best effort intensity by adjusting β can only be optimal if the agent is either risk-neutral or the performance measure is perfectly precise. Otherwise, the principal imposes too much incentive risk on the agent which requires the payment of a higher risk premium to ensure her participation.

Now consider the case where the agent has identical abilities for all tasks, i.e. $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$. As a consequence, $\boldsymbol{\Gamma} = \boldsymbol{\omega}/\hat{\psi}$ so that vector $\boldsymbol{\Gamma}$ and vector $\boldsymbol{\omega}$ point in the same direction. This additionally implies that $\boldsymbol{e}^* = \boldsymbol{\omega}\beta/\hat{\psi}$ and $\xi = 0$. Thus, $\boldsymbol{e}^*$ and $\boldsymbol{\omega}$ are identical with respect to their direction, only their lengths differ, depending on β and $\hat{\psi}$. Accordingly, the measure of congruity is now identical to the measure of distortion. This observation is summarized and proofed by the next proposition.

Proposition 2. *If* $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, *then* $\Upsilon^D(\varphi) = \Upsilon^C(\varphi) = \cos\varphi$.

If agents do not exhibit different task-specific abilities, performance measure congruity and effort distortion are captured by the same measure. However, if we allow the agent to possess different abilities across tasks, it becomes pivotal to distinguish between both concepts. The application of incongruent performance measures in incentive contracts leads to inefficient effort allocations, but the extent of these inefficiencies are further determined by the agent's relative abilities for performing the relevant tasks.

To summarize, consider again the expected second-best profit Π^* from section 4. According to the previous observations, it depends on three components: (i) the measure of distortion $\Upsilon^D(\theta)$ in the numerator; (ii) the measure-cost efficiency $\Upsilon^{M/C}(\xi)$ in the denominator; and (iii), the agent's risk aversion ρ in conjunction with the variance σ^2 of the applied performance measure in the denominator. It is common knowledge that the trade-off between incentive risk and the agent's desire for insurance affects optimal incentive contracts. Moreover, as demonstrated by Feltham and Xie [1994] and Baker [2002], incentive contracts in multi-task agency relations are adjusted to the congruity of applied performance measures. However, the previous analysis indicates that the measure-costs efficiency is a third crucial factor whenever the agent performs some tasks more efficiently than others due to task-specific abilities.

6 Ranking Performance Measures

As Feltham and Xie [1994] emphasized, performance measures may differ with respect to their congruity and precision. The previous analysis additionally indicates that task-specific abilities play a crucial role for the contract efficiency. This section therefore focuses on how the attributes of performance measures and agents eventually determine the relative value of measures in multi-task agencies.

Consider a set $\mathbf{P}$ of $m \geq 2$ performance measures $P_i(\boldsymbol{e}) = \boldsymbol{\omega}_i^T \boldsymbol{e} + \varepsilon_i$, with $P_i(\boldsymbol{e}) \in \mathbf{P} \subseteq \mathbb{R}^m$ and $\varepsilon_i \sim N(0, \sigma_i^2)$.[11] To illustrate the relative value of individual performance measures, we can compare the expected profits each of them would induce if applied in the agent's incentive contract. Then, performance measure $P_k(\boldsymbol{e})$ is referred to be strictly superior, if it provides the principal a strictly higher expected profit than all other available measures $P_i(\boldsymbol{e}) \in \mathbf{P}$, $i \neq k$.

[11] Subscript i refers henceforth to performance measure $P_i(\boldsymbol{e}) \in \mathbf{P}$.

For single-task agency relations, Kim and Suh [1991] have shown that the value of performance measures can be compared by their respective signal/noise ratio. Schnedler [2006] generalized their signal/noise ratio to a setting, where the agent is in charge of conducting multiple tasks. By applying the formulation proposed by Schnedler [2006] (see Definition 2), the signal/noise ratio of performance measures $P_i(\boldsymbol{e})$ is

$$\Lambda_i = \frac{\left(\nabla P_i(\boldsymbol{e}^*)\right)^T \left(\nabla P_i(\boldsymbol{e}^*)\right)}{\sigma_i^2}, \tag{15}$$

where $\nabla P_i(\boldsymbol{e}^*)$ is the gradient of performance measure $P_i(\boldsymbol{e})$ with respect to $\boldsymbol{e}$. In single-task agencies, performance measures with higher signal/noise ratios provide more precise information about the implemented effort and are therefore strictly preferred to measures with lower ratios. In this multi-task setting, the signal/noise ratio of performance measures $P_i(\boldsymbol{e})$ is

$$\Lambda_i = \frac{\|\boldsymbol{\omega}_i\|^2}{\sigma_i^2}. \tag{16}$$

One can immediately infer from the previous analysis that signal/noise ratios are not necessarily sufficient to rank performance measures in multi-task agencies, especially, when agents differ in their task-specific abilities. This deduction is supported by the next proposition.

Proposition 3. *Performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $j \neq k$, if and only if,*

$$\frac{\|\boldsymbol{\omega}_k\|}{\|\boldsymbol{\Gamma}_k\|}\frac{\Upsilon^{M/C}(\xi_k)}{(\Upsilon^D(\theta_k))^2} + \frac{\rho\sigma_k^2}{\|\boldsymbol{\Gamma}_k\|^2(\Upsilon^D(\theta_k))^2} < \frac{\|\boldsymbol{\omega}_j\|}{\|\boldsymbol{\Gamma}_j\|}\frac{\Upsilon^{M/C}(\xi_j)}{(\Upsilon^D(\theta_j))^2} + \frac{\rho\sigma_j^2}{\|\boldsymbol{\Gamma}_j\|^2(\Upsilon^D(\theta_j))^2}, \tag{17}$$

where $\Upsilon^D(\theta_i)$ is the measure of distortion induced by $P_i(\boldsymbol{e})$, and $\Upsilon^{M/C}(\xi_i)$ is the related quantification for the measure-cost efficiency, $i = \{j, k\}$.

Proof Follows directly by rearranging $\Pi^*(P_k(\boldsymbol{e})) > \Pi^*(P_j(\boldsymbol{e}))$ and substituting $\Upsilon^{M/C}(\xi_i) = \cos\xi_i$ and $\Upsilon^D(\theta_i) = \cos\theta_i$, $i = k, j$.

The value of a performance measure in comparison to any other measure is contingent on two ratios: (i) the normalized ratio between the measure-cost efficiency $\Upsilon^{M/C}(\cdot)$ and the induced effort distortion $\Upsilon^D(\cdot)$; and, (ii) the normalized inverse of the distortion measure $\Upsilon^D(\cdot)$ with the precision $1/\sigma_k^2$ of the performance measure and the agent's risk tolerance $1/\rho$. Observe finally that performance measure congruity does not directly enter into this ranking criteria. It, however, affects indirectly the measure of effort distortion $\Upsilon^D(\theta_i)$ and the measure-cost efficiency characterized by $\Upsilon^{M/C}(\xi_i)$.

Clearly, the value of performance measures in multi-task agencies cannot necessarily be compared by their respective signal/noise ratios. It is rather pivotal to take the induced effort distortion and measure-cost efficiency into consideration – both determined by the performance measure sensitivities $\boldsymbol{\omega}_i$ *and* the agent's task specific abilities $\boldsymbol{\Psi}$. Therefore, comparing the value of performance measures requires specific knowledge about the agent's characteristics, which is not necessary for ranking performance measures in single-task agencies.

Corollary 2. *Suppose $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$. Then, performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $j \neq k$, if and only if,*

$$\frac{1}{\Upsilon^C(\varphi_k)} \left[1 + \hat{\psi}\rho\Lambda_k^{-1}\right]^{\frac{1}{2}} < \frac{1}{\Upsilon^C(\varphi_j)} \left[1 + \hat{\psi}\rho\Lambda_j^{-1}\right]^{\frac{1}{2}}, \tag{18}$$

where Λ_i, $i = \{j, k\}$, is the signal/noise ratio of performance measure $P_i(\boldsymbol{e})$, and $\Upsilon^C(\varphi_i)$ its congruity measure.

According to Proposition 2, one can only use adjusted signal/noise ratios to rank performance measures in multi-task agencies if the agent has identical abilities for all relevant tasks and thus, her effort allocation depends only on the characteristics of her performance evaluation. Nevertheless, it is still necessary to know $\hat{\psi}$ and ρ in order to assess the relative value of available performance measures.

The next proposition offers a sufficient condition ensuring that performance measures can be ranked exclusively by their respective signal/noise ratios, which in turn allows to abstract from individual characteristics of agents.

Proposition 4. *Suppose there exist constants $\lambda_j \neq 0$ satisfying $\boldsymbol{\omega}_i = \lambda_j \boldsymbol{\omega}_j$ for all $i, j = 1, ..., m$, $i \neq j$. Then, performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $j \neq k$, if and only if, $\Lambda_k > \Lambda_j$.*

Accordingly, the signal/noise ratio is sufficient to rank performance measures in multi-task agencies if all measures provide the same information about the agent's relative effort allocation. In this case, observe that $\Upsilon^C(\varphi_i) = \Upsilon^C(\varphi_j)$, $i, j = 1, ..., m$, i.e., all performance measures share the same measure of congruity.[12] As a consequence, every available performance measure – if

[12]Note that the reversed inference cannot be made, i.e. if $\Upsilon^C(\varphi_i) = \Upsilon^C(\varphi_j)$, it is not necessarily true that $\boldsymbol{\omega}_i = \lambda_j \boldsymbol{\omega}_j$, $\lambda_j \neq 0$, $i, j = 1, ..., m$, $i \neq j$. In this case, the signal/noise ratio is not sufficient to rank performance measures in multi-task agencies.

applied in the agent's incentive contract – would imply the same effort distortion and measure-cost efficiency. Then, their relative value is defined by their precision and scale, which in turn is represented by their respective signal/noise ratio.

To investigate the effects of task-specific abilities on the ordering of performance measures, it is insightful to eliminate effects related to their precision. By setting $\rho = 0$, condition (17) simplifies to

$$\nu \, \frac{\cos^2 \theta_k}{\cos^2 \theta_j} > \frac{\cos \xi_k}{\cos \xi_j}, \qquad \nu = \frac{\|\boldsymbol{\omega}_j\|}{\|\boldsymbol{\omega}_k\|} \frac{\|\boldsymbol{\Gamma}_k\|}{\|\boldsymbol{\Gamma}_j\|}. \tag{19}$$

The value of performance measure $P_k(\boldsymbol{e})$ relative to $P_j(\boldsymbol{e})$ depends – besides on their precision and scaling as previously emphasized – on their relative effort distortion ($\cos \theta_i$) and relative measure-cost efficiency ($\cos \xi_i$) weighted by the multiplier ν, $i = k, j$. In order to make both measures comparable, it is essential to normalize their scale $\|\boldsymbol{\omega}_i\|$, and exclude their effect on $\|\boldsymbol{\Gamma}_i\|$, $i = k, j$. Accordingly, if either the agent is risk-neutral or the realization of performance measures is not influenced by random effects, the relative value of performance measures depends on two factors: (i) the motivated effort allocation and its contribution to gross payoff $V(\boldsymbol{e})$; and, (ii) the imposed costs to motivate this effort allocation.

7 Conclusion

Applying incongruent performance measures in incentive contracts motivates agents to implement an inefficient effort allocation across relevant tasks. This paper incorporates task-specific abilities in a multi-task agency framework and investigates their effects on the provision of effort incentives. As demonstrated, incentive contracts are tailored to agents' task-specific abilities and, particularly, depend on three factors: (i) the inefficiency of effort distortion as a result of applying incongruent performance measures in incentive contracts, relative to the agent's task-specific abilities (*distortion effect*), (ii) the agent's effort costs associated with the motivated effort allocation (*measure-cost efficiency*); and (iii), the precision of the information system with the agent's risk-aversion (*risk effect*).

This paper further investigates the relative value of performance measures in multi-task agencies. One important observation is that the signal/noise ratio, commonly used to assess performance measures in single-task agencies, is not a sufficient ranking criteria in multi-task principal-agent relationships. The relative value of performance measures depends – besides on their precision – on their congruity relative to the agent's task-specific abilities, thereby implying that their ranking is tied to agents' individual characteristics. Hence, we can infer that the selection of 'suitable' agents for a given information system provides the principal some latitude to improve the contract efficiency.

This paper is part of a larger research agenda. Previous multi-task literature focused primarily on performance measure congruity and its effect on incentive contracts. As this paper illustrates, we can shed more light on the nature of incentive contracts in multi-task agency relations, when we keep in mind that agents may differ in their skills and abilities to perform particular tasks.

8 Appendix

Proof of Proposition 1.

Effort distortion refers to the relation of $\boldsymbol{e}^*$ to $\boldsymbol{\mu}$ and can be therefore measured by the vector product $\boldsymbol{\mu}^T\boldsymbol{e}^*$. Since $\boldsymbol{e}^* = \boldsymbol{\Gamma}\beta$,

$$\boldsymbol{\mu}^T\boldsymbol{e} = \beta \sum_{i=1}^{n} \mu_i \Gamma_i = \beta \|\boldsymbol{\mu}\| \|\boldsymbol{\Gamma}\| \cos\theta. \tag{20}$$

First note that $\|\boldsymbol{\mu}\|$ does not affect the relative importance of tasks for $V(\boldsymbol{e})$. Furthermore, $\beta\|\boldsymbol{\Gamma}\|$ determines the lengths of vector $\boldsymbol{e}^*$, but not its direction in the n-dimensional space. The length is arbitrary in the sense that it can be adjusted by β. Consequently, $\Upsilon^D(\theta) = \cos\theta \in [0, 1]$ measures the induced effort distortion under second-best.

Q.E.D.

Proof of Proposition 2.

To measure effort distortion, we can use the vector product $\boldsymbol{\mu}^T\boldsymbol{e}^*$. If $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, then $\boldsymbol{e}^* = \beta\boldsymbol{\omega}/\hat{\psi}$. This leads to

$$\boldsymbol{\mu}^T\boldsymbol{e} = \frac{\beta}{\hat{\psi}} \sum_{i=1}^{n} \mu_i \omega_i = \frac{\beta}{\hat{\psi}} \|\boldsymbol{\mu}\| \|\boldsymbol{\omega}\| \cos\varphi. \tag{21}$$

Again, $\|\boldsymbol{\mu}\|$ does not affect the relative importance of tasks for $V(\boldsymbol{e})$, and $\beta\|\boldsymbol{\omega}\|$ determines the lengths of vector $\boldsymbol{e}^*$ but not its direction in the n-dimensional space. Thus, $\bar{\Upsilon}^D(\varphi) = \cos\varphi \in [0, 1]$ measures distortion under second-best if $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$. Consequently, $\bar{\Upsilon}^D(\varphi) = \Upsilon^C(\varphi)$.

Q.E.D.

Proof of Corollary 2.

If $\psi_i = \hat{\psi} > 0$, $i = 1, ..., n$, then $\boldsymbol{\Gamma}_i = \boldsymbol{\omega}_i/\hat{\psi}$ and $\|\boldsymbol{\Gamma}_i\| = \|\boldsymbol{\omega}_i\|/\hat{\psi}$, $i = \{j, k\}$. Consequently, $\Upsilon^{M/C}(\xi = 0) = 1$ and $\bar{\Upsilon}^D(\varphi_i) = \Upsilon^C(\varphi_i)$, see proposition 2. By substituting $\Lambda_i = \|\boldsymbol{\omega}_i\|^2/\sigma_i^2$, $i = \{j, k\}$, the ranking criteria of proposition 3 can be reformulated to the one stated in the corollary.

Q.E.D.

Proof of Proposition 4.

Observe first that the expected profit on the basis of $P_i(\boldsymbol{e})$ can be written as

$$\Pi^* = \frac{\left(\boldsymbol{\mu}^T \boldsymbol{\Gamma}_i\right)^2}{2(\boldsymbol{\omega}_i^T \boldsymbol{\Gamma}_i + \rho\sigma_i^2)}. \tag{22}$$

Recall that $\boldsymbol{\Gamma}_i = \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_i$. Consequently, performance measure $P_k(\boldsymbol{e})$ is strictly superior to any other performance measure $P_j(\boldsymbol{e}) \in \mathbf{P}$, $\forall j \neq k$, if and only if,

$$\frac{\left(\boldsymbol{\mu}^T \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_k\right)^2}{2(\boldsymbol{\omega}_k^T \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_k + \rho\sigma_k^2)} > \frac{\left(\boldsymbol{\mu}^T \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_j\right)^2}{2(\boldsymbol{\omega}_j^T \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}_j + \rho\sigma_j^2)}. \tag{23}$$

If $\boldsymbol{\omega}_k = \lambda\boldsymbol{\omega}_j$, we can re-scale $P_j(\boldsymbol{e})$ such that it is characterized by the same sensitivity in $\boldsymbol{e}$ as $P_k(\boldsymbol{e})$. Accordingly,

$$\bar{P}_j(\boldsymbol{e}) = \boldsymbol{\omega}_j^T \boldsymbol{e} + \frac{\varepsilon_j}{\lambda}, \tag{24}$$

where $\text{Var}\left[\bar{P}_j(\boldsymbol{e})\right] = \sigma_j^2\lambda^{-2}$. Let $\boldsymbol{\omega} \equiv \boldsymbol{\omega}_i$, $i = j, k$. This leads to

$$\frac{\left(\boldsymbol{\mu}^T \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}\right)^2}{2(\boldsymbol{\omega}^T \boldsymbol{\Psi}^{-1}\boldsymbol{\omega} + \rho\sigma_k^2)} > \frac{\left(\boldsymbol{\mu}^T \boldsymbol{\Psi}^{-1}\boldsymbol{\omega}\right)^2}{2(\boldsymbol{\omega}^T \boldsymbol{\Psi}^{-1}\boldsymbol{\omega} + \rho\sigma_j^2\lambda^{-2})}, \tag{25}$$

which can be re-arranged to

$$\frac{1}{\sigma_k^2} > \frac{\lambda^2}{\sigma_j^2}. \tag{26}$$

Recall that after re-scaling, $\boldsymbol{\omega}_k = \boldsymbol{\omega}_j$. Thus, (26) can be written as

$$\frac{\|\boldsymbol{\omega}_k\|^2}{\sigma_k^2} > \frac{\lambda^2\|\boldsymbol{\omega}_j\|^2}{\sigma_j^2}, \tag{27}$$

which is identical to $\Lambda_k > \Lambda_j$.

Q.E.D.

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