May the Force be With You: Investor Power and Company Valuations*

Thomas Hellmann† and Veikko Thiele‡

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Abstract

This paper re-examines the role of investor power in a model of staged equity financing. It shows how the usual effect where market power reduces valuations can be reversed in later rounds. Once they become insiders, powerful investors may use their market power to increase, not decrease valuations. Even though powerful investors initially lower valuations, companies prefer to bring them inside, to leverage their power in later financing rounds. The paper generates novel predictions about valuations and investor returns. It also explains why unrealized interim returns can be fundamentally misleading.

Keywords: Staged financing, valuation, inside investor, market power.

JEL classification: G32.

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†University of Oxford, Saïd Business School, Park End Street, Oxford OX1 1HP, United Kingdom, e-mail: thomas.hellmann@sbs.ox.ac.uk.

‡Queen's University, Smith School of Business, Goodes Hall, 143 Union Street, Kingston, Ontario, Canada K7L 3N6, e-mail: thiele@queensu.ca.

1 Introduction

Much of corporate finance assumes that investors are perfectly competitive, and the supply of capital is perfectly elastic. While this assumption may sometimes be suitable in public equity markets, private equity markets typically face an inelastic supply of capital, where a limited number of investors exercise market power. The standard implication of market power is that investors can take larger equity stakes, which drives down company valuations. This result is based on a single-round investment logic, where the investor is an outsider without a prior stake in the company. Many private equity markets, however, are characterized by staged financing where investors fund companies over several rounds. This is common with angel financing, venture capital, growth capital, or private placements. It also happens with publicly-listed companies that raise money in initial public offerings (IPOs), seasoned equity offerings (SEOs), or PIPEs (private investments in public equity). The fundamental difference with staged financing is that while an investor is an outsider the first time he invests, he becomes an insider thereafter. In this paper we pose two main research questions. First, in a staged financing context, what is the effect of investor power on the valuation of a company? Second, when does a company want to bring a powerful investor inside?

To answer these questions, we build a parsimonious theory of staged financing. The company needs two financing rounds. In the base model information is symmetric, and the company faces a supply of capital that is not perfectly elastic. Our focus is the effect of market power on the valuations of early and late rounds. As long as investors are competitive, the valuation of the late round is independent of the valuation that is obtained in the first round. However, with a powerful investor this is not true, because it matters what stake he obtained in the first round. If he is an outsider, the powerful investor always wants to push down the valuation of the second round. However, if he already has a stake in the company, there is a trade-off. On the one hand, the powerful investor invests new money and therefore wants a lower valuation just like an outsider. We call this the aggressive outsider logic. On the other hand, he already has a stake in the company, and prefers a higher valuation just like an insider. We call this the defensive insider logic. The net preference depends on the relative sizes of the existing stake versus the new investments. Our theory derives a simple condition that says that a powerful investor prefers a higher (lower) valuation whenever his second-round investment is below (above) the pro-rata threshold. In the next section we provide a simple numerical example to illustrate the logic of this result. The important novelty is that with staged financing, market power has an ambiguous effect on valuations. In particular, the model identifies a large range of parameters where market power leads to higher, not lower valuations in the second-round.

Turning to the first round, we ask about the effect of market power on valuations, and whether the company wants to bring the powerful investor into the first round. We distinguish three scenarios. The benchmark scenario is perfect competition, i.e., no powerful investor. We then look at a scenario where the company obtains first-round funding from the powerful investor, and finally the scenario where the company delays bringing in the powerful investor until the second round. We find that first-round valuations are highest with competition, lower when postponing the investment of the powerful investor, and lowest when the powerful insider participates in the first round. At first glance this result seems to suggest that bringing in the powerful investor up-front is a bad idea, because it generates the lowest first-round valuation. However, as noted above, having an insider increases second-round valuations. The question becomes whether the higher second-round valuations can justify the lower first-round valuation? We formally show that the company always prefers to bring in a powerful investor right from the start. The key intuition is that, while expensive, bringing in the powerful investor up-front allows the company to leverage his market power. Hence the title of this paper: "May the force be with you."

Our main model uses a simple setting with symmetric information, to demonstrate the basic economic forces at work in the most transparent manner. In a model extension we then show that asymmetric information reinforces our key insights. In particular we find that below pro-rata, a powerful inside investor would want to signal confidence in a good venture by investing larger amounts. In a separating equilibrium this leads to over-investments in the good state. Above pro-rata the signaling logic changes. Now a powerful inside investor wants to manage valuations downwards, and does so by underinvesting in the bad state. In both cases, higher investments by the insider signal better expected returns, which reinforces the intuition from the base model.

The model also generates some predictions about the returns to equity investors at different stages. Looking at the final returns of first-round investors, we note that powerful inside investors make the highest returns. However, powerful insiders have lower returns in the second round. Indeed, moderately powerful insiders who invest below pro-rata have even lower returns than the benchmark return of competitive investors. Powerful outsiders, by contrast, have the highest returns of all second-round investors. The analysis also shows that unrealized interim returns need to be interpreted with caution. For example, competitive investors experience low interim returns when the company is financed by a powerful outsider in the second round. However, these are not realized returns. In fact, these competitive investors achieve higher returns between the second round and the date where value is realized, fully recouping their benchmark returns. This last finding is a useful warning for empirical work that uses unrealized interim returns as a measure of performance. Our analysis suggests that market power invalidates the interpretation of unrealized returns as a proxy for expected realized returns.

Our paper builds on several prior literatures. Admati and Pfleiderer (1994) were the first to examine equity valuations in a staged financing context. Their model focuses on identifying robust financial contracts, and shows how an inside investor is neutral with respect to valuations when investing at pro-rata. However, their model artificially fixes the amount of insider funding at the pro-rata level, i.e., their model does not foresee inside investors investing below or above pro-rata. Moreover, market power is not directly accounted for in their model. The issue of market power does arise in models of hold-up, such as in the work of Grossman and Hart (1986), and Aghion and Tirole (1994). These models are based on incomplete contracting – no such assumption is needed in our model. In this line of research, Khanna and Mathews (2015) is probably closest to our paper. They use a staged financing model where the inside investor reduces hold-up power by using high prices that also provide a signal to third parties. In our model inside investors sometimes bring about higher but other times lower prices, depending on how the new investment compares against the existing ownership stake.[1]

Maintaining power across investment stages is also of practical importance in the venture capital industry. Many smaller investors worry about a lack of negotiating power in later rounds (see Hellmann and Veikko (2015)). Y Combinator, a famous Silicon Valley venture accelerator, recognized this problem and launched "Y Combinator Continuity". This is a $1B later stage fund that allows Y Combinator to continue investing in later rounds, to better protect its initial ownership stakes.[2] Along similar lines, Nanda, Sadun, and Hull (2018) report of a local Boston venture capital firm that aggregates capital from angel investors, in order to form more powerful investment syndicates for the later stage financing of its early stage portfolio.

In addition to the literature on staged equity financing, there is a closely related literature about staged debt financing. The seminal work by Rajan (1992) identifies advantages and disadvantages of inside lenders who have better information on the company at the time of refinancing, but can also use that informational advantage to extract some information rents. Berglöf and Von Thadden (1994) and Dewatripont and Tirole (1994) further extend this type of analysis to allow for optimal securities and multiple investors. In our model we assume symmetric information, and therefore abstract away from informational advantages of insiders. Instead we look at sized-based market power. This approach is related to Burkhart, Gromb, and Panunzi (1997)

[1]Further theories about staged financing include the work of Neher (1999), Cornelli and Yosha (2003), Bergemann and Hege (2005), Hellmann and Thiele (2015), and Nanda and Rhodes-Kropf (2017).

[2]According to their website: "YC Continuity is an investment fund dedicated to supporting founders as they scale their companies. Our primary goal is to support YC alumni companies by investing in their subsequent funding rounds [...]." (see `https://www.ycombinator.com/continuity/`).

who emphasize the importance of size for monitoring incentives. As an extension to our base model, we also consider the use of debt. We find a rationale for powerful insiders to use equity over debt. With equity they internalize valuation effects at the time of the second round, and therefore use their market power in a way that is more beneficial to the company. With debt, however, there is no need to protect prior stakes, and so market power is not used to defend the valuation.

The remainder of the paper is as follows. In the next section we provide a simple numerical example to illustrate how a powerful investor would like to change the company valuation. In Section 3 we provide our staged financing model with a powerful investor. Section 4 considers model extensions. We look at the structure of investor returns, discuss a model with debt, derive the model with asymmetric information, and further discuss the empirical predictions of the model. It is followed by a brief conclusion. All proofs are in the Appendix.

2 A Simple Numerical Example

To illustrate a central insight, we briefly consider a simple numerical example. Suppose a company needs to raise \$10M in a second-round financing. There is a powerful investor who already owns 20% of the company from the first round. Suppose that there are only two valuations under consideration. Either the second-round investors receive 50% of the company for their \$10M investment, which implies a post-money valuation of \$20M (= \$10/50%). Or the second-round investors receive 40% of the company for their \$10M investment, which implies a valuation of \$25M (= \$10/40%). What deal would the powerful investor prefer?

Suppose first that the powerful investor plans to invest \$2M in the new round. In this case he provides 20% of the new money and receives 20% from the stake of the new investors. However, if he already owns 20% of the existing stake, and now receives 20% of the new stake, his overall stake simply remains at 20%. This is called investing at pro-rata, i.e., investing in the new round an amount that is proportional to the existing stake. It allows the existing investor to exactly maintain his ownership stake.

An insight going back to Admati and Pfleiderer (1994) is that if an investor invests at pro-rata, then he is indifferent between lower and higher valuations, precisely because he always maintains the same ownership stake. While Admati and Pfleiderer fix the investment of the insider at pro-rata, we consider what would happen if the insider invested above or below pro-rata. Suppose the powerful investor invested \$3M out of the \$10M in the second round. Under the low valuation he would get a stake of 30%*50% = 15%, and his existing stake would be worth 20%*50%=10%, so his overall stake would be 25%. Under the high valuation he would get a stake of 30%*40% = 12%, and his existing stake would be worth 20%*60%=12%, so his overall stake would be 24%. This shows that a powerful investor who invests above pro-rata prefers the low valuation. The outsider logic of investing at a low valuation dominates because the new outsider stake is relatively larger than the existing insider stake.

Consider next the case where the powerful investor invests below pro-rata. Suppose he only provides $1M out of the $10M in the second round. Under the low valuation he would get a stake of 10%*50% = 5%, and his existing stake would be worth 20%*50%=10%, so his overall stake would be 15%. Under the high valuation he would get a stake of 10%*40% = 4%, and his existing stake would be worth 20%*60%=12%, so his overall stake would be 16%. This shows that when the powerful investor invests below pro-rata, he prefers the high valuation. In this case the investment made in the second round is relatively small (below pro-rata) compared the existing stake. Consequently, the insider logic of preferring high valuations dominates.

This example illustrates the way that a powerful insider would like to influence the valuation of the company. Naturally, this simple example is built on several artificial assumptions. It also does not explain how the investor can influence the valuation, and what the equilibrium looks like. For all this we need a proper theory model. We turn to that in the next section.

3 Main Model

3.1 Base Assumptions

Consider a company, called A, which requires two rounds of financing for a project.[3] Specifically, A needs to raise an amount K_1 in the first round, and K_2 in the second round, with $K_1 < K_2$. The project generates an expected return $x > 0$. For parsimony we assume no discounting. There are three dates, date 1 for the first round, date 2 for the second round, and date 3 for the realization of returns.

For simplicity we assume that financing occurs in two stages. This is empirically the relevant case. From a theoretical lens, it is possible to derive the optimality of staging endogenously, although this comes at a cost of adding complexity to the model.[4] A prior corporate

[3]For simplicity we directly assume that financing comes in two stages. It is easy to extend the model to endogenously derive optimal staging. All that is required is to add some uncertainty, and the possibility that in case of failure the entrepreneur spends all remaining funds on private benefits.

[4]What is technically required is to add the possibility that in case of early failure the entrepreneur spends the remaining funds inefficiently. In that case, no investor is willing to provide all the capital upfront. For a simple illustration of this, suppose there is an observable (but not verifiable) milestone that the company can achieve early

finance literature, including the work of Rajan (1992), Berglöf and von Thadden (1994), Neher (1999), and Nanda and Rhodes-Kropf (2017), formally establishes conditions for the optimality of staged financing.

There are n risk neutral competitive investors, which we denote by C. In Section 3.2.2 we will introduce a powerful investor. The competitive investors are all price-takers, and choose their investments simultaneously. The prices (α, β, to be introduced shortly) clear the market in a standard Walrasian fashion.

We assume that the first round is relatively small and that a single investor provides the entire amount K_1. For simplicity we assume that all competitive investors have the same cost of providing K_1, given by $C_1(K_1) = \mu_1 K_1$ ($\mu_1 > 1$). In return the investor gets an equity share, denoted by α.[5]

Similarly, the second-round investors collectively invest K_2, and jointly receive a total equity share β. The post-money valuations are given by $V_1 = K_1/\alpha$ and $V_2 = K_2/\beta$. We assume that K_2 is large, requiring the company to raise funding from multiple investors; this is also known as a syndicated investment round. A competitive investor provides an amount k_2^j, $j \in \{1, ..., n\}$, incurring a cost $c_2^j = \mu_2 k_2^j + \frac{\gamma}{2}\left(k_2^j\right)^2$. For $\gamma > 0$ costs are convex, so that the supply of capital is not infinitely elastic. The increasing marginal costs of investing should be interpreted as increasing opportunity costs for investors who are allocating more capital into A (as opposed to all other investment opportunities). The more the investor allocates his portfolio to this deal, the more exposed (or less diversified) his portfolio becomes. Note that our model uses competitive but not atomistic investors. This means that the market has a finite number of investors, each facing increasing marginal investment costs. This approach will allow us to introduce market power in a simple and intuitive way, as discussed in Section 3.2.2.

3.2 Second-round Investments and Valuations

We start by deriving the equilibrium investments and company valuation in the second round. To illustrate the effect of market power in our staged financing model, we first consider the

on. If the milestone is achieved the company is ready for a second round of financing. However, if the milestone is not achieved, the company can spend any remaining funds $\kappa = K_2 - K_1$, and with a $\varepsilon(\kappa)$ probability (where $\varepsilon' > 0$ but $\varepsilon \to 0$) the company still achieves the milestone. In this setting, providing all the capital upfront is inefficient since it gives the company incentives to spend it all. Staged financing, by contrast, eliminates this incentive and thus ensures a more efficient capital utilization.

[5]For simplicity we assume that any competitive investor from the first round does not participate in the second round. Relaxing this would increase the complexity of the exposition, but would not impact the results. This is because individual competitive investors are too small to affect market prices.

competitive benchmark where no investor can influence the price (β). We then show how market power affects investments and valuation.

3.2.1 Competitive Benchmark

Suppose there are n competitive investors in the second round. We assume that the second-round price, as represented by β, clears the market in a Walrasian fashion. We call this the NP case, which stands for "No Powerful" investor.

When investing k_2^j, $j = 1, ..., n$, in the second round, investor j gets the equity share $\frac{k_2^j}{K_2}\beta$. The objective function for each competitive investor is therefore given by

$$\max_{k_2^j} \frac{k_2^j}{K_2}\beta x - \mu_2 k_2^j - \frac{\gamma}{2}\left(k_2^j\right)^2.$$

Consequently, each competitive investor invests the amount $k_2^j(\beta)$ so that the price per unit of capital equals the marginal cost:

$$\frac{1}{K_2}\beta x = \mu_2 + \gamma k_2^j. \tag{1}$$

To ensure interior solutions, we assume that the relative return x/K_2 is sufficiently large and/or the cost of capital is sufficiently low, so that $k_2^j(\beta) > 0$. Moreover, the equilibrium equity share for all second-round investors, β, is defined by the market clearing condition $nk_2^j(\beta) = K_2$.

Solving the system of two equations we find that the investment by a competitive investor j is $k_2^{j|NP} = K_2/n$. Moreover, the equilibrium equity share β^{NP} is given by

$$\beta^{NP} = \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n}K_2\right]. \tag{2}$$

This implies the following company valuation:

$$V_2^{NP} = \frac{K_2}{\beta^{NP}} = \frac{x}{\mu_2 + \frac{\gamma}{n}K_2}. \tag{3}$$

For later comparison, we note that the first-round price α does not affect the second-round valuation V_2^{NP}. This is because second-round investors take the first-round price α as given. We will see that this is not true for the powerful investor.

It easy to see some additional comparative statics results from the expression of V_2^{NP}. A higher cost of capital forces A to issue more equity, which implies a lower second-round valuation (i.e., $dV_2^{NP}/d\mu_2$, $dV_2^{NP}/d\gamma < 0$). By contrast, a higher expected return implies a lower price β, and therefore a higher valuation (i.e., $dV_2^{NP}/dx > 0$). Likewise, if A requires a larger amount in the second round (K_2), then each investor needs to provide more capital (i.e., $dk_2^{j|NP}/dK_2 > 0$). Naturally the round can then only close if A issues more equity, which implies a lower valuation (i.e., $dV_2^{NP}/dK_2 < 0$). We find the opposite with respect to the number of investors n: The presence of more investors induces every single investor to invest less (so that $dk_2^{j|NP}/dn < 0$). This implies a lower total cost of financing across all investors (due to the convexity of c_2^j). The company can then raise the amount K_2 with less equity, which in turn improves the company valuation ($dV_2^{NP}/dn > 0$).

3.2.2 Powerful Investor

We now introduce a powerful investor, called P. For this we use the classic dominant firm model (DFM). The DFM is commonly used in industrial economics to analyze markets where there is one large firm with substantial market power, and many small firms (aka the "competitive fringe") that are price takers. The DFM dates back to the work of Forchheimer (1908), and was further developed by Knight (1921), Stackelberg (1934), Stigler (1940), and later Fudenberg and Tirole (1984). Schenzler, Siegfried, and Thweatt (1992) provide a comprehensive overview. The competitive firms always take the equilibrium price as given, and make their optimal investments accordingly. The dominant firm (P), however, calculates how its own investment quantity affects the equilibrium price (α or β). For each price, it anticipates the investment quantities of the competitive firms. It then chooses its own investment quantity to achieve the equilibrium price that maximizes its own profits. In equilibrium the quantities of the dominant firm plus the small firms add up to a total quantity that generates the equilibrium price.

To obtain a simple measure of market power, we assume that out of the n investors, m investors coordinate their investments, and therefore act as a single powerful investor (P). Thus, m will be our measure of market power. The rationale behind this way of modelling market power is as follows. In order to compare market constellations with more or less market power, we want to maintain an overall cost structure in the market. That is, we want to vary market power while holding constant the overall elasticity of capital supply. It would be tempting to simply add to the n competitive investors one powerful investor P, and give him his own convex cost function. However, this would increase the overall funds available, and thus alter the overall cost structure in the market. The approach chosen here allows us to preserve the overall cost structure. Specifically, we retain n “pockets” of financing, each with its convex cost

$c_2^j = \mu_2 k_2^j + \frac{\gamma}{2}\left(k_2^j\right)^2$. Market power in our model is the ability to coordinate the action of a subset m of these pockets of capital. One way of thinking about this is that m investors in the market manage to collude and act as a single decision maker. This kind of coordination can be interpreted as a financial intermediary who invests as a single actor on behalf of the m "pockets" of capital. This would be a stylized description of venture capital, where the venture capital firm (aka the general partner) acts on behalf of the ultimate investors (aka limited partners). It can also be thought of as an investment syndicate (such as an angel group or a closely-knit group of venture capitalists), where one leader acts on behalf of the group to negotiate the deal.

We assume that the powerful investor makes investments, but cannot make any transfer payments. Due to our sparse set of assumptions, there is a theoretical possibility in this model that a powerful investor makes a transfer payment to the company, and then owns and runs the entire company. This is clearly unrealistic, and it is easy to augment the model to exclude this possibility.[6]

The model with a powerful investor distinguishes two cases. First, there is the case of a powerful inside investor, who participated in the first financing round, and therefore already holds a stake in the company. We denote this stake by α^P. We call this the PI case, which stands for "Powerful Insider". Second, there is the case of the powerful outside investor. He only arrives at the second round, and is assumed to be unavailable at the time of the first round. In this case a competitive outcome occurs at the first stage investment K_1. At the second stage we can simply set $\alpha^P = 0$. We call this the PO case, which stands for "Powerful Outsider". We can think of two possible reasons why P is not around in the first round. One is that some investors do not like to invest in early stage deals, or find it too costly to do so (i.e., μ_1^P is very large). Another reason is that A does not yet want to approach P, i.e., the company wants to avoid facing the powerful investor. In the former case, P's absence is exogenous, in the latter case it is endogenous. In Section 3.3.3 we analyze the latter case.

[6]The assumption of no transfer payments is standard in the literature (see Rajan (1992), or Hellmann (2002) for a discussion). One way to derive this constraint endogenously is to add a simple adverse selection problem. Suppose that in addition to the honest companies described so far, there are dishonest companies that can pretend to have a business that looks identical to the honest ones. The dishonest companies take the transfer payment and disappears, leaving investors with a total loss. If for every honest company there are enough dishonest ones, no investor would ever make a transfer payment, simply because the probability of investing in a dishonest company is too high. Other ways of justifying the absence of transfer payments are based on moral hazard models, where the owner/manager needs to retain as much equity as possible to continue providing effort for the success of the company.

3.2.3 The Case of a Powerful Insider

Consider first the case of a powerful insider. P provides the amount $K_2^P = mk_2^i$, $i = 1, ..., m$, in the second round, while each of the competitive investors C invest k_2^j, $j = m+1, ..., n$. The total investment cost for the powerful investor P is[7]

$$C^P = m\left[\mu_2 k_2^i + \frac{\gamma}{2}\left(k_2^i\right)^2\right].$$

When choosing his investment K_2^P, P takes the effect on the investments the competitive investors (C), and therefore on the equilibrium price β, into account. The price β is defined by the market clearing condition

$$K_2^P + (n-m)\, k_2^j(\beta) = K_2, \quad j = m+1, ..., n, \tag{4}$$

where $k_2^j(\beta) = \frac{1}{\gamma}\left[\frac{1}{K_2}\beta x - \mu_2\right]$ is the amount provided by each competitive investor (which can be derived from (1) in Section 3.2.1).

P chooses K_2^P to maximize his expected net return

$$\pi_2^{P|PI}(K_2^P) = (1-\beta(K_2^P))\alpha^P x + \frac{K_2^P}{K_2}\beta(K_2^P)x - C^P(K_2^P), \tag{5}$$

where $\beta(K_2^P)$ is the total equity issued to the second-round investors as a function of P's investment (as defined by the market clearing condition (4)), and K_2^P/K_2 is P's share in the round. Intuitively P's expected net payoff depends on three components. The second term represent his second-round equity stake ($\frac{K_2^P}{K_2}\beta(K_2^P)$). The third term is his cost of capital. The most interesting component is the first term. P holds an equity stake α^P from his first-round investment K_1. This gets diluted as A needs to issue equity to new investors (including to P). This component therefore captures the fact that P is an inside investor who also cares about preserving his existing stake.

We informally note that the first component is decreasing in β. This means that as an insider P prefers a higher valuation (i.e., lower β). We call this the "defensive" insider logic. The second component, however, is increasing in β. As an outsider P prefers a lower valuation (i.e., higher β). We call this the "aggressive" outsider logic. The relative strength of these two

[7] We make a simplifying assumption that the convex cost function is based only on the second round investment amount, and does not account for any first round investment amounts. Relaxing this assumption would complicate the analysis by creating a linkage between the investment amount in the first round and the marginal investment cost in the second round. As long as K_1 is small, however, this additional effect is also small.

logics will play a key role in determining how P exercises his market power. We discuss this in Section 3.3.

In the Appendix we show that in equilibrium P invests

$$K_2^{P|PI} = \frac{m}{n+m}\left[1+\alpha^P\right] K_2. \tag{6}$$

We can immediately see that P's investment $K_2^{P|PI}$ depends on his first-round equity stake α^P. This also applies to the second-round valuation V_2^{PI}, given by

$$V_2^{PI} = \frac{K_2}{\beta^{PI}} = \frac{x}{\mu_2 + \gamma\dfrac{n-\alpha^P m}{n^2-m^2}K_2}. \tag{7}$$

Note that for $m = 0$, this simplifies back to the competitive valuation V_2^{NP} (see (3) in Section 3.2.1).

An important question is how P's investment $K_2^{P|PI}$ relates to the pro-rata investment. In other words, when does P invest below pro-rata and accept a dilution of his stake, and when does P invest above pro-rata to increase his stake? The next lemma provides a simple condition.

Lemma 1 *P invests above (below) pro-rata whenever $m > (<)\ \widehat{m} \equiv \alpha^P n$.*

Lemma 1 says that P invests above (below) pro-rata whenever his power (m), is above (below) a critical value $\widehat{m}$. This critical value varies with his prior equity stake (α^P). With this we can examine how P's market power (as measured by m), and his first-round equity stake α^P, affect $K_2^{P|PI}$ and V_2^{PI}.

Proposition 1 *Consider the PI case. The investment by P, $K_2^{P|PI}$, is increasing in both m and α^P. Moreover, there exists a threshold m', with $0 < m' < \widehat{m}$, such that the second-round valuation of A, V_2^{PI}, is increasing in m for $m < m'$, and decreasing thereafter. The valuation V_2^{PI} is also increasing in α^P.*

Figures 1 and 2 illustrate the main insights from Proposition 1 (these two figures also compare the PI case with the other cases, which we discuss in more detail in Section 3.2.5). More market power (m) encourages P to provide a larger share of the required second-round investment K_2. The effect on valuation is non-monotonic. To get an intuition, we first note that P behaves just like a competitive investor in two cases: when m is small, i.e. when there is not

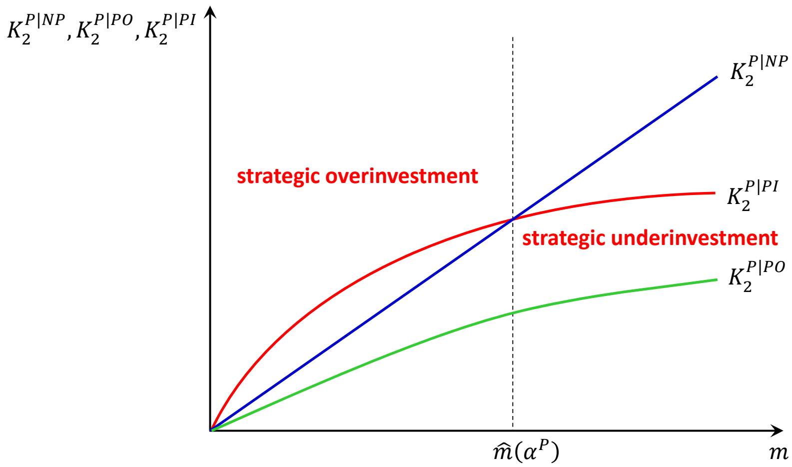

Figure 1: Second Round Investments

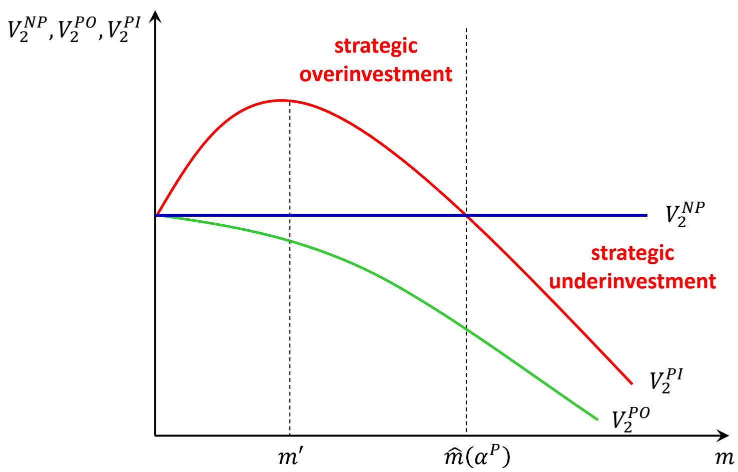

Figure 2: Second Round Valuations

much concentration of power, and when $m = \widehat{m}$, i.e., when P maintains exactly his pro-rata. Between these critical values, we find that the valuation first rises and then falls in m. The intuition for the valuation rising initially in m is that P is driven by the defensive insider logic that comes from the first component of equation (5). The intuition why it falls again is that as P invests larger amounts, he is increasingly driven by the aggressive outsider logic from the second component of (5). For $m = \widehat{m}$, the insider and outsider logic exactly cancel out. For $m > \widehat{m}$, the aggressive outsider logic always dominates. Note also that the intuition for why the valuation is always increasing in α^P is that a higher α^P increases the first component, and therefore strengthens the defensive insider logic.

3.2.4 The Case of a Powerful Outsider

So far we assumed that the powerful investor is an insider, with some ownership stake $\alpha^P > 0$. We now turn to the case of a powerful outside investor. Technically this is a special case of the model from the previous section with a powerful insider where we set $\alpha^P = 0$ in (6) and (7).

Proposition 2 *Consider the PO case. The investment by P, $K_2^{P|PO}$, is increasing m. The second-round valuation, V_2^{PO}, is always decreasing in m.*

In the PO case, P does not hold any equity when participating in the second financing round ($\alpha^P = 0$). This means that P can only invest above pro-rata (since $m > \alpha^P n = 0$). P is not concerned about equity dilution, and only uses his market power to drive the valuation down (see also Figures 1 and 2).

In the Appendix we derive some additional comparative statics results with respect to equilibrium investments and valuations for the PI and PO cases. The cost of capital (μ_2, γ), as well as the expected return (x), do not affect P's investment ($K_2^{P|PI}$, $K_2^{P|PO}$). However, the cost of capital has a negative effect on valuation (V_2^{PI}, V_2^{PO}), while the expected return has a positive effect on valuation. Furthermore, a higher capital requirement in the second round (K_2) results in P making a larger investment ($K_2^{P|PI}$, $K_2^{P|PO}$), leading to a lower company valuation (V_2^{PI}, V_2^{PO}). The presence of more investors (n) also implies a lower equilibrium investment by P, and a higher company valuation. Finally, we show in the Appendix that P's participation constraint is always satisfied.[8]

3.2.5 Comparing Second-round Constellations

We now compare the second-round investments and company valuations when P is an insider versus outsider. We also compare both cases against the competitive benchmark (NP case). For this it is useful to define $K_2^{P|NP} \equiv mk_2^i = \frac{m}{n}K_2$. This is the total amount that P would provide in a (counterfactual) competitive benchmark.[9] All results derived in this section are illustrated in Figures 1 and 2.

We begin with a Proposition focussed on the powerful outsider (PO) case.

[8] Note that this also implies that a constellation where P participates in the first round, but not the second round, can never arise in equilibrium.

[9] Note that the investment amount of an individual investment "pocket", $k_2^{j|NP}$, does not depend on m. However, the more individual pockets belong to P, the higher the total investment $K_2^{P|NP}$.

Proposition 3 *For all m, and all $\alpha^P > 0$, a powerful outsider makes the smallest second-round investment, i.e., $K_2^{P|PO} < \{K_2^{P|PI}, K_2^{P|NP}\}$. This also results in the lowest second-round company valuation, i.e., $V_2^{PO} < \{V_2^{PI}, V_2^{NP}\}$.*

Figures 1 and 2 illustrate how the investment and valuation in the PO case compare to the other scenarios. In the PO case, P does not have a stake in A ($\alpha^P = 0$). It is then optimal for P to exploit his market power to drive down the valuation of the company. This can be achieved by investing a smaller amount compared to the competitive benchmark (i.e., $K_2^{P|PO} < K_2^{P|NP}$, so that $V_2^{PO} < V_2^{NP}$). This result is consistent with the standard economics result that market power leads to smaller quantities (here: smaller investment) and higher prices (here: lower valuation, which means higher cost of capital for the company).

To further understand why the powerful insider behaves differently from the powerful outsider, we note that having a stake in the company (PI case with $\alpha^P > 0$) curbs P's temptation to drive valuations down in the second round. That is, when choosing his second-round investment K_2^P, P trades off the positive effect on his second-round equity stake ($\frac{K_2^P}{K_2}\beta(K_2^P)$), and the negative effect on his first-round stake (α^P). A larger stake from the first financing round (α^P) makes it optimal for P to invest a larger amount in the second round, resulting in a higher valuation (see Proposition 1).

Proposition 3 shows that having a powerful outsider leads to the lowest possible second-round valuation. We also just discussed how the powerful insider increases second-round valuations, relative to the powerful outsider. It remains to be seen how the powerful insider compares to the competitive benchmark (the NP case).

Proposition 4 *Comparing the powerful insider against the competitive benchmark, we distinguish two cases:*

(i) ***Moderately powerful insider:*** *Suppose $m \leq \widehat{m}$. P invests more in the second round compared to the competitive benchmark (i.e., $K_2^{P|PI} \geq K_2^{P|NP}$). The second-round valuation is higher compared to the competitive benchmark (i.e., $V_2^{PI} \geq V_2^{NP}$).*

(ii) ***Highly powerful insider:*** *Suppose $m > \widehat{m}$. P invests less in the second round compared to the competitive benchmark (i.e., $K_2^{P|PI} < K_2^{P|NP}$). The second-round valuation is lower compared to the competitive benchmark (i.e., $V_2^{PI} < V_2^{NP}$).*

The insights from Proposition 4 can be seen in Figures 1 and 2. At the critical level of market power ($m = \widehat{m}$), we know that P invests exactly at pro-rata, and therefore maintains his stake in the company. In this case the outcome is identical to the competitive benchmark, i.e., P provides the same amount of capital in the second round ($K_2^{P|PI} = K_2^{P|NP}$), leading to the same company valuation ($V_2^{PI} = V_2^{NP}$).

If the inside investor is highly powerful ($m > \widehat{m}$), he invests more in absolute terms, but strategically reduces his investment compared to the competitive benchmark ($K_2^{P|PI} < K_2^{P|NP}$), as shown in Figure 1. In doing so P accepts a dilution of his first-round equity stake, but obtains a lower company valuation in the second round ($V_2^{PI} < V_2^{NP}$), as shown in Figure 2. This captures the problem of having an insider that is too powerful. At the same time we note that it is better for A to have the powerful investor as an insider, than as an outsider. This follows from the fact that valuations are even lower with the powerful outsider.

Maybe the most interesting insight pertains to the investments of a moderately powerful insider ($m < \widehat{m}$). Such an inside investor strategically overinvests ($K_2^{P|PI} > K_2^{P|NP}$); see Figure 1. This increases the second-round valuation, and helps to preserve P's stake from the first-round investment, as shown in Figure 2. The moderately powerful insider is therefore beneficial for the company, in the sense that his market power is now used to the benefit of the company: "May the force be with you!"

3.3 First-round Investments and Valuations

We now close our staged financing model by deriving the equilibrium investments and company valuation in the first financing round. Ideally we could use the same cost structure in the first as in the second round. Unfortunately, the model becomes too complex to analyze, so we have to resort to some simplifying assumptions. Our main interests are the interactions between inside and outside investors in the second round. We therefore simplify the model by assuming that it is either the powerful investor or the competitive fringe that provides the first round financing, but not both. Moreover, we assume that investment costs are not quadratic but linear. One way of thinking about this is that the first round is relatively small, and that the cost function can be represented by its first order Taylor approximation. In a dominant firm model with such linear costs, it is easy to show (see Appendix) that the powerful investor either has a sufficient cost advantage to take the entire first round, or else he takes none of it at all. Put differently, the first order Taylor approximation simplifies the first round to a linear cost structure that generates a binary outcome where either the powerful investor or the fringe take away the entire investment. With that, the analysis simplifies to three scenarios: (i) only competitive investors participate in the first and second round (the NP case), (ii) the powerful investor only invests in the second round (the PO case), and (iii) the powerful investor participates in both financing rounds (the PI case). With that simplification, we ask how first-round valuations differ across these three scenarios. In addition, we derive whether in equilibrium the powerful investor enters in the first or second round.[10]

3.3.1 Investment by Competitive Investor

Suppose the required amount K_1 in the first round is provided by a competitive investor. For parsimony we derive the outcome for the NP and PO case jointly (they are very similar, the only difference being β^{NP} versus β^{PO}). The winning investor receives an equity stake α^i ($i \in \{NP, PO\}$). To derive the valuation we consider the zero-profit condition for a competitive investor:

$$\left(1-\beta^i\right)\alpha^i x - \mu_1 K_1 = 0 \quad \Leftrightarrow \quad \alpha^i = \frac{\mu_1 K_1}{\left(1-\beta^i\right)x}. \tag{8}$$

We note that α^i is increasing in β^i because a competitive investor knows that his stake will get diluted in the second round when A issues β^i to new investors. The equilibrium first-round price α^i is such that the investor's diluted share of the expected payoff equals his cost of investing K_1.

Using the expressions for β^{NP} and β^{PO}, we get the following first-round competitive equilibrium valuations:

$$\begin{aligned} V_1^{NP} &= \frac{K_1}{\alpha^{NP}} = \frac{1}{\mu_1}\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n}K_2\right]\right)x \\ V_1^{PO} &= \frac{K_1}{\alpha^{PO}} = \frac{1}{\mu_1}\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma n}{n^2 - m^2}K_2\right]\right)x. \end{aligned}$$

The next proposition summarizes how P's market power (m) as an outsider affects the company valuation when the first-round investment is provided by a competitive investor.

[10] It is worth explaining briefly what we miss out on with our simplifying assumption. The main loss is that in the first round we do not allow for joint funding between the powerful investor P and the competitive fringe. However, it is easy to see that such joint funding would generate second round dynamics that are very similar to the PI case examined in Section 3.2.3. We already explained that the fringe investors from the first round do not participate in the second round (although this assumption can easily be relaxed). With joint funding, P can again manipulate the second round valuation along the lines described in Section 3.2.3. The only difference is that P has a slightly lower prior ownership stake α, because he only provided part but not all of the first round funding.

Proposition 5 *Suppose a competitive investor provides the amount K_1 in the first round. If a powerful outsider provides capital in the second round, then the valuation V_1^{PO} is decreasing in m.*

This result is very intuitive, and can be seen from the expression of V_1^{PO}. As noted above, the difference between the company valuations is rooted in the amount of equity issued to the second-round investors (β^{NP} versus β^{PO}). The presence of a more powerful outsider P implies that in equilibrium more equity is being issued to the second-round investors (i.e., $d\beta^{PO}/dm > 0$). This implies that the equity stake of the competitive first-round investor gets more diluted. Thus the company needs to provide a higher α^{PO} to the competitive investors, to compensate for their subsequent dilution. This means that the first-round valuation V_1^{PO} is lower.

From the expressions of V_1^{NP} and V_1^{PO} it is easy to derive that the first and second-round costs of capital (μ_1, μ_2, γ) have a negative effect on the first-round valuation, while the effect of the expected return (x) is positive. The required amount in the second round (K_2) has a negative effect on the first-round valuation. The presence of more second-round investors (n) implies a higher first-round valuation. This is because it requires issuing less equity to second-round investors, and therefore curbs the dilution of the first-round investor's equity stake.[11]

3.3.2 Investment by Powerful Investor

Now suppose that P wants to finance the first round. For parsimony we assume that company A accepts P's offer as long as A is indifferent between having P investing K_1, and a competitive investor providing K_1.

We know from Section 3.2.2 that P always wants to participate in the second round. Consequently, P makes the first-round investment K_1 (PI case) as long as it leads to a higher expected profit compared to waiting for the second round (PO). Formally, P's participation constraint for the first round is given by $\pi^{P|PI}(\alpha^P) \geq \pi^{P|PO}$.

The next proposition characterizes P's offer in the first round.

Proposition 6 *P's offer in the first round is given by α^P, where α^P satisfies*

$$\alpha^P \left(x - K_2 \left[\mu_2 + \frac{\gamma \left(n + \left(1 - \alpha^P \right) m \right)}{n^2 - m^2} K_2 \right] \right) = \mu_1 K_1. \tag{9}$$

[11] Not surprising we find equivalent effects on company A's expected profit, which we denote by $\pi^{A|i}$, $i \in \{NP, OP\}$. The only exception concerns K_1, which has a negative effect on $\pi^{A|i}$. We provide more details in the Appendix (see Proof of Proposition 8).

The resulting first-round valuation $V_1^{PI} = K_1/\alpha^P$ is decreasing in m.

When submitting a bid to A for the first financing round, P is competing with all the other competitive investors. The equity stake α^P, defined in Proposition 6, makes A just indifferent between accepting the bid α^P and taking the bid α^{PO} from a competitive investor. And by assumption, A then chooses P to make the first-round investment K_1.[12]

Proposition 6 also shows that a more powerful investor asks for a higher equity stake α^P, which leads to a lower company valuation in the first round (i.e., $dV_1^{PI}/dm < 0$). We know that the powerful outsider fully exploits his market power in the second period to maximize his economic rents. In addition, we know from Propositions 2 and 3 that more market power implies a lower second-round valuation (see also Figure 2). It follows that the powerful outsider is very unattractive for A. This situation improves for A when P becomes an insider. Knowing this, P can ask for a higher equity stake α^P in the first round. This implies a lower valuation V_1^{PI}.

In the Appendix we derive additional comparative statics results. Specifically, we show that the first-round valuation V_1^{PI} is decreasing in the costs of capital (μ_1, μ_2, γ), and increasing in the expected return (x) and the number of second-round investors (n). Moreover, we find that the required financing amount in the first round (K_1) has a positive effect on V_1^{PI}, while the effect of the required amount in the second round (K_2) is negative.[13]

3.3.3 Comparing First-round Constellations

We can now compare the first-round valuations for the different investor constellations.

Proposition 7 *The first-round valuation is lowest with a powerful insider (PI case), higher with a powerful outsider (PO case), and highest when there is no powerful investor (NP case). Formally, $V_1^{PI} < V_1^{PO} < V_1^{NP}$ for all $m > 0$.*

The company valuation in the first round is lowest in the PI case. To see why, we first note that P's offer in the first period makes A marginally better off, compared to having a competitive investor in the first round and P only participating in the second round (PO case). According to Proposition 3, the second-round valuation is the lowest for the PO constellation. P then

[12]Note that A would strictly prefer P's bid if P asks for a slightly lower equity share $\alpha^P - \epsilon$, with $\epsilon \to 0$, in exchange for K_1.

[13]The effects on company A's expected profit, $\pi^{A|PI}$, are equivalent; see Proof of Proposition 8 in the Appendix for more details.

exploits the lower outside option for A, by asking for a higher equity stake, which implies a lower first-round valuation. This also explains why the equilibrium valuation V_1^{PI} in the first financing round is below the competitive benchmark.

A key insight so far is that the presence of a powerful investor can have opposite effects on company valuations across different financing rounds. For example, with a powerful insider (PI case) the first-round valuation is always below the competitive benchmark, while the second-round valuation may be above the benchmark level (see Proposition 4). It therefore remains to compare A's expected profits under the different investor constellations.

Proposition 8 *A's expected profits for the different investor constellations are as follows: $\pi^{A|NP} > \pi^{A|PI} > \pi^{A|PO}$ for all $m > 0$. Furthermore, A's expected profit in the PO case ($\pi^{A|PO}$) and the PI case ($\pi^{A|PI}$) is decreasing in m.*

It is intuitive that the presence of a powerful investor always leads to reduced profits for the company. The important insight pertains to the question whether a company should bring in a powerful investor early, thereby making him an insider in later financing rounds. Having an early stake encourages P to use his market power to defend the company valuation in later rounds. However, bringing in P in at the beginning comes at the price of a lower valuation. While A gets a higher valuation in the second round, it pays for it up-front. Still, Proposition 8 shows that "having the force with you" leads to a higher expected profit for the company.

4 Extensions and Discussions

4.1 Investor Returns

So far, our analysis looks at equilibrium valuations at date 1 and date 2. The model therefore provides some insights into the structure of investor returns. In particular, we distinguish three types of returns. Define R_{13} (R_{23}) as the return that a date 1 (date 2) investment generates at date 3 when the company realizes its value. Moreover, define R_{12} as the unrealized interim return (at date 2) for a date 1 investment. In the model these three returns are defined as follows:

$$R_{12} = \frac{V_2 - K_2}{V_1} \qquad R_{23} = \frac{x}{V_2} \qquad R_{13} = \frac{(1-\beta)x}{V_1} = \frac{x}{V_1}\frac{V_2 - K_2}{V_2}.$$

These returns differ across the various model permutations. We distinguish four scenarios. First we look at the benchmark case where there is no powerful investor (NP case). Second, we look at the case with a powerful outsider (PO case). Third, we consider a moderately powerful insider where $m < \widehat{m}$. We call this the MPI case. Finally, there is the highly powerful insider with $m > \widehat{m}$, which we call the HPI case.

We immediately state the main result.

Proposition 9 *The returns compare as follows:*

- R_{13}*:* $R_{13}^{HPI} > R_{13}^{MPI} > R_{13}^{NP} = R_{13}^{PO} = \mu_1$
- R_{23}*:* $R_{23}^{PO} > R_{23}^{HPI} > R_{23}^{NP} > R_{23}^{MPI}$
- R_{12}*:* $R_{12}^{MPI} > R_{12}^{NP} > R_{12}^{PO}$ *and* $R_{12}^{HPI} > R_{12}^{PO}$*.*

Proposition 9 contains several intuitive results. The realized returns of date 1 investors (R_{13}) can be ranked according to the amount of market power exercised. Returns are highest for highly powerful insiders, followed by moderately powerful insiders, followed by competitive investors in the PO and NP case. Note that $R_{13}^{PO} = R_{13}^{NP} = \mu_1$ because these are the returns of the competitive first-round investors, not the powerful investor.

The realized returns of date 2 investors (R_{23}) are also influenced by market power, but rank entirely differently. In particular we find that returns are highest in the PO case. This is because a powerful investor depresses the valuation to generate the highest possible returns to himself. The powerful insider moderates the use of his market power, because of his prior stake in the company, as discussed in Proposition 4. In fact, the realized returns of date 2 investors are lowest in the MPI case, which is when the moderately powerful investor boosts valuation above the competitive benchmark.

The unrealized returns of date 1 investors at date 2 (R_{12}^{PO}) follow yet another pattern. They rank lowest in the PO case. This is not because date 1 investors made bad investments, but because the powerful outside investor depresses the valuation. The final realized returns of these date 1 investors (R_{13}^{PO}) are in fact the same as the returns in the benchmark NP case (i.e., $R_{13}^{PO} = \mu_1$). This generates an important insight about the dangers of looking at unrealized interim returns: In the presence of market power, unrealized interim returns can be a misleading indicator of expected realized returns.[14]

[14]For completeness, note also that date 1 investors in the MPI case achieve both higher unrealized interim returns (R_{12}^{MPI}) and higher realized final returns (R_{13}^{MPI}), compared to the benchmark returns (R_{12}^{NP} and R_{13}^{NP}), and also compared to the PO case (R_{12}^{PO} and R_{13}^{PO}). Finally, comparisons for the HPI case cannot always be signed unambiguously.

4.2 The Role of Debt

So far our model considers equity investors. In this section we ask whether our results change in the presence of debt. The main question is whether it would be more efficient for the powerful investor to hold debt instead of equity? To examine this, suppose A wants to raise capital by issuing debt claims to investors. The (endogenous) interest rates are denoted by r_1 and r_2. The expected return from the project is uncertain, and given by $x = \rho x_H + (1-\rho)x_L$, with $x_H > x_L$. For brevity's sake we focus on the case where debt is safe, i.e., $x_L \geq (1+r_1)K_1 + (1+r_2)K_2$.[15]

Consider the second financing round, and assume for a moment that A issued equity to P in the first round – this allows us to focus exclusively on the effect of debt financing in the second round. In the Appendix we formally derive the equilibrium investments in the second round under debt financing, and derive the equilibrium cost of capital r_2^*. We find that the costs of debt and equity financing in the second round are identical, i.e., $(1 + r_2^*)K_2 = \beta^{PI}x$. This is essentially the Modigliani-Miller theorem. In the second round it is irrelevant whether A raises the required amount K_2 through debt or equity, because the financial structure doesn't affect any behaviors.

Now consider the first financing round, and suppose that A issues a debt claim to raise the amount K_1. In the Appendix we establish the following two results. First, when K_1 is raised from a competitive investor (PO case), then debt and equity financing are again equivalent. Second, when the powerful investor provides K_1 (PI case), then equity is optimal. The intuition is that issuing equity in the first round makes the powerful investor defend the valuation in the second round, to limit the dilution of his first-round equity stake. This strategic effect is not present when P holds debt, because in this case the value of his existing stake is not affected by the valuation in the second round. Our model therefore identifies a new reason for the use of equity, which is similar but not identical to the traditional incentive argument. The traditional argument is that investors need equity to have incentives for providing more value-adding services. This is typically modelled as a two-sided moral hazard problem (see, e.g., Da Rin et al. 2012, and Hong, Serfes, and Thiele (2020)). The reason for using equity in our model is also related to incentives, but not to moral hazard. Instead the argument is that by giving equity to the powerful investor, he takes the value of insiders into consideration when exercising his market power. This argument is reminiscent of Da Rin and Hellmann (2002) who look at how market power allows a bank to finance "big push" investments.

[15] In this simple model with only two states, any risky debt is equivalent to equity. Hence the combination of safe debt and equity covers all relevant combinations.

4.3 Asymmetric Information and Insider Signaling

So far our model assumes symmetric information. As a model extension we briefly consider the possibility of asymmetric information where inside investors have better information about the company than outside investors (Rajan, 1992). Let us assume that at date 2 there are two possible states for the expected returns x, denoted by x_L and x_H, with $0 < x_L < x_H$. We assume that only the company (A) and the powerful insider (P) can observe the true state. However, outside investors (powerful or not) cannot observe the true state and instead rely on a rational equilibrium belief.[16] To focus on the most interesting case, we assume a date 2 investment is always worthwhile, even at $x = x_L$.

The main question of interest concerns the possibility of insider signaling: can P signal information by investing different amounts in different states? In the Appendix we derive sufficient conditions for a separating equilibrium to exist. In such an equilibrium P invests one amount for the low and a different amount for the high signal. The most interesting insight is that P's signaling strategy depends on whether P wants to invest below or above pro-rata. Specifically below pro-rata P wants to signal high expected returns (x_H) in order to increase valuations. In contrast, above pro-rata P wants to signal low expected returns (x_L) in order to decrease valuations. The next proposition sheds more light on P's investments in the separating equilibrium.

Proposition 10 *The investment amounts in the separating equilibrium are as follows:*

- ***Below pro-rata case*** ***(**$m < \widehat{m}$**):*** *P overinvests in the high state ($K_2^{P*}(x_H) > K_2^{P|PI}$). In the low state, P invests as before ($K_2^{P*}(x_L) = K_2^{P|PI}$).*
- ***Above pro-rata case*** ***(**$m > \widehat{m}$**):*** *P underinvests in the low state ($K_2^{P*}(x_L) < K_2^{P|PI}$). In the high state, P invests as before ($K_2^{P*}(x_H) = K_2^{P|PI}$).*

The key intuition is that signaling incentives differ below versus above pro-rata. Below pro-rata the binding truth-telling constraint is in the high state: P wants to exaggerate company returns in order to trigger a higher valuation that benefits the insiders. However, signaling incentives switch above pro-rata, where the temptation is to understate true company returns in the low state, in order to lower the valuation to the benefit of outside investors. Still, in all cases higher investments by P signal higher company returns.

[16]The competitive investors from date 1 do not reinvest at date 2, and therefore do not matter here.

Overall we note that the asymmetric information model reinforces the key messages from the symmetric information model. With asymmetric information, powerful insiders can use investment amounts to signal their private information. Below pro-rata we get the intuitive result that insiders signal their confidence by investing higher amounts in the good state. This echoes our main finding that below pro-rata powerful insiders want to push for higher valuations. However, above pro-rata powerful insiders do the opposite. They underinvests in the low state, in order to convince the market that the company deserves a lower valuation. This is again consistent with the insights from the main model where, above pro-rata, powerful insiders push for lower valuations.

4.4 Empirical Predictions

Our theory generates several testable empirical predictions. The most important empirical predictions concern the relationship between investor power and company valuations. Our first empirical prediction is related to the initial investments of a powerful investor. Specifically, the model generates an unambiguous prediction that powerful investors drive valuations down when they invest in a company for the first time. The second empirical prediction concerns the follow-up investments of powerful investors. Here the model generates a richer pattern of predictions. Specifically, powerful investors drive valuations down if they invest above pro-rata. However, if they invest below pro-rata, the model predicts that they actually drive valuations up. The model also predicts that the ability to drive valuations up or down depends on the convexity of the powerful investor's costs.

In addition to making empirical predictions about valuations, our model also generates predictions about investor returns. Powerful investors achieve higher investment returns, compared to the competitive benchmark. The interesting exception concerns follow-on investments of moderately powerful insiders who invest below pro-rata. They obtain lower returns on their follow-on investments, although this helps to obtain higher returns on their initial investments. The model thus produces some useful guidance to empirical research, showing the importance of decomposing investor returns by stages of investment, and generating differential predictions for initial versus follow-on investment rounds.

Our theory also provides a warning against the use of unrealized interim returns. Empirical studies of investor returns sometimes makes use of unrealized interim valuations to estimate investor portfolio returns (see Harris, Jenkinson, and Kaplan (2014)). However, our model shows that this can be misleading. For example, the presence of a powerful outsider may reduce unrealized interim returns. However, if we go forward and only measure realized returns, the model predicts that powerful investors obtain higher realized returns. More generally, the theory suggests that if unrealized interim returns are used to estimate expected realized returns, they would need to be adjusted to take market power into consideration.

Another interesting empirical prediction concerns the presence of venture debt. Our analysis suggests that adding venture debt weakens the beneficial effects of bringing a powerful investor inside. The stylized fact that venture debt gets used mostly in later rounds is broadly consistent with this prediction.[17] More specifically, our model predicts that in later rounds venture debt should become more attractive when there are no new powerful investors. Empirically, raising venture debt and adding powerful investors should be negatively correlated.

Let us briefly consider how these empirical predictions might be tested, and what challenges occur. The theory generates predictions about two main dependent variables: company valuations (V_1 and V_2) and investor returns (R_{13}, R_{23}, and R_{12}).[18] In public markets this information is readily available, but in private markets this data is more difficult to obtain. Even if valuations can be found, it is often difficult to find data on realized returns. The most important dependent variables from the theory are market power (m) and prior equity stakes (α). Measuring prior equity stakes is straightforward in principle, provided the data is disclosed. Measuring market power, however, is considerably more challenging. Our theory suggests looking at investor fund sizes. One complicating factor is that different investors specialize in different deals, and face different competitors at different stages. For example, a venture capital fund might be a large powerful investor in an early stage deal, when all others are smaller angel investors. However, that same venture capital fund might be a small player in later stage deals where there are much larger investors, such as private equity funds, mutual funds, hedge funds, or institutional investors. Alternative measures of market power might relate to an investor's reputation and track record. Hsu (2004), for example, establishes that venture capitalists with higher reputations offer lower valuations. In addition to these data measurement problem, there is the empirical challenge of finding exogenous variation in the independent variables. This requires finding exogenous shocks to investor power and market structures. The work of Nanda and Rhodes-Kropf (2013) contains some useful ideas for that.

[17]See Tykvová (2017), Hochberg, Serrano, and Ziedonis (2018), and Fulghieri, García, and Hackbarth (2020).

[18]The model also makes predictions about the share of investments provided by powerful inside investors in later rounds (K_2^P/K_2). The data required for this is a breakdown of round investments by individual investors.

5 Conclusion

In this paper we develop a model of staged equity financing with investor market power. Standard economic reasoning suggests that a powerful investor obtains lower valuations, and thereby achieves higher returns. We show that while this result holds the first time a powerful investor invests, it may not hold in subsequent financing rounds. As an insider, a powerful investor faces dual motives. One is that higher valuations preserve his existing stake, the other is that lower valuations are more attractive for his new investments. We show that the former motive dominates when the investor is moderately powerful and invests below pro-rata in later financing rounds. In this case the effect of market power is reversed, i.e., the investor uses his market power to increase, not decrease valuations. We explain how this is an equilibrium behavior, and describe the circumstances under which this result obtains.

Our model also asks whether the company prefers to have the powerful investor up-front, or delay him to a later round. Even though the powerful investor can extract a lower valuation in the first round, the company prefers to bring him in up-front. This is because once he becomes an insider, the company can leverage his power to defend its valuation. Hence the title of the paper: "May the force by with you."

Appendix

PI **Case: Equilibrium Investment and Valuation.**

Using $k_2^j(\beta) = \frac{1}{\gamma}\left[\frac{1}{K_2}\beta x - \mu_2\right]$ we can write the market clearing condition (4) as

$$K_2^P + (n-m)\,\frac{1}{\gamma}\left[\frac{1}{K_2}\beta x - \mu_2\right] = K_2.$$

Solving for β we get

$$\beta(K_2^P) = \frac{K_2}{x}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2\right]. \tag{10}$$

Using the expression for $\beta(K_2^P)$ and $k_2^i = K_2^P/m$, $i = 1, ..., m$, we can write the objective function of P as follows:

$$\begin{aligned}\max_{K_2^P} \pi_2^{P|PI}(K_2^P) \;&=\; \alpha^P x - \alpha^P K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2\right] + K_2^P\left[\frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2\right]\\ &\quad - \left[\mu_2 K_2^P + \frac{\gamma}{2m}\left(K_2^P\right)^2\right].\end{aligned}$$

The optimal investment, $K_2^{P|PI}$, is then defined by the first-order condition:

$$\alpha^P K_2\frac{\gamma}{n-m} + \frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2 = K_2^P\frac{\gamma}{n-m} + \mu_2 + \frac{\gamma}{m}K_2^P.$$

Solving for K_2^P we get $K_2^{P|PI} = \frac{m}{n+m}\left[1 + \alpha^P\right]K_2$. Substituting $K_2^{P|PI}$ in (10) then yields the equilibrium equity share for all investors:

$$\beta^{PI} = \frac{K_2}{x}\left[\mu_2 + \frac{\gamma\left(n - \alpha^P m\right)}{n^2 - m^2}K_2\right]. \tag{11}$$

Thus, the equilibrium valuation, V_2^{PI}, is given by

$$V_2^{PI} = \frac{K_2}{\beta^{PI}} = \frac{x(n^2 - m^2)}{\gamma\left(n - \alpha^P m\right)K_2 + \mu_2(n^2 - m^2)}.$$

Proof of Lemma 1.

Note that P invests above (below) pro-rata when

$$(1-\beta^{P-P})\alpha^P + \frac{K_2^{P|P-P}}{K_2}\beta^{P-P} > (<)\alpha^P.$$

Using $K_2^{P|PI} = \frac{m}{n+m}\left[1+\alpha^P\right] K_2$, we can write this condition as

$$\frac{m}{n+m}\left[1+\alpha^P\right]\beta^{P-P} > (<)\alpha^P\beta^{P-P},$$

which can be simplified to $\frac{m}{n} > (<)\alpha^P$. $\square$

Proof of Proposition 1.

We can immediately see that $dK_2^{P|PI}/d\alpha^P > 0$ and $dV_2^{PI}/d\alpha^P > 0$. Moreover,

$$\begin{aligned}
\frac{dK_2^{P|PI}}{dm} &= \frac{n}{[n+m]^2}\left[1+\alpha^P\right] K_2 > 0, \\
\frac{dV_2^{PI}}{dm} &= \frac{-2mx\left[\gamma\left(n-\alpha^P m\right)K_2 + \mu_2(n^2-m^2)\right] + x(n^2-m^2)\left[\gamma\alpha^P K_2 + 2m\mu_2\right]}{\left[\gamma\left(n-\alpha^P m\right)K_2 + \mu_2(n^2-m^2)\right]^2}.
\end{aligned}$$

We have $dV_2^{PI}/dm > 0$ when

$$\begin{aligned}
(n^2-m^2)\left[\gamma\alpha^P K_2 + 2m\mu_2\right] &> 2m\left[\gamma\left(n-\alpha^P m\right)K_2 + \mu_2(n^2-m^2)\right] \\
\Leftrightarrow \quad \gamma\alpha^P n^2 K_2 + 2\mu_2 m n^2 - \gamma\alpha^P m^2 K_2 &> 2\gamma m n K_2 - 2\gamma\alpha^P m^2 K_2 + 2\mu_2 m n^2 \\
\Leftrightarrow \quad \underbrace{\alpha^P\left(n^2+m^2\right) - 2mn}_{\equiv Z} &> 0
\end{aligned}$$

Note that this condition is satisfied for $m = 0$. Thus, $dV_2^{PI}/dm > 0$ for $m \to 0$. Moreover, for $m = n$ this condition simplifies to $\alpha^P - 1 > 0$, which is clearly violated. Therefore, $dV_2^{PI}/dm < 0$ for $m \to n$. Next, note that $dZ/dm = 2\alpha^P m - 2n < 0$. Consequently, there exists a unique $m' > 0$ so that $dV_2^{PI}/dm > 0$ for $m < m'$, and $dV_2^{PI}/dm \leq 0$ for $m \geq m'$. Finally, evaluating Z at $m = \widehat{m} = \alpha^P n$ we get $\alpha^P n^2\left(\alpha^{P2}-1\right) < 0$. Thus, $\left.\frac{dV_2^{PI}}{dm}\right|_{m=\widehat{m}} < 0$, which implies that $m' < \widehat{m}$. $\square$

Proof of Proposition 2.

Setting $\alpha^P = 0$ in (6) and (7) we get

$$K_2^{P|PO} = \frac{m}{n+m} K_2 \tag{12}$$

$$V_2^{PO} = \frac{x(n^2 - m^2)}{\gamma n K_2 + \mu_2(n^2 - m^2)}. \tag{13}$$

Thus,

$$\begin{aligned} \frac{dK_2^{P|PO}}{dm} &= \frac{n}{[n+m]^2} K_2 > 0 \\ \frac{dV_2^{PO}}{dm} &= \frac{-2mx\left[\gamma n K_2 + \mu_2(n^2 - m^2)\right] + x(n^2 - m^2)2\mu_2 m}{\left[\gamma n K_2 + \mu_2(n^2 - m^2)\right]^2} \\ &= -\frac{2\gamma m n x K_2}{\left[\gamma n K_2 + \mu_2(n^2 - m^2)\right]^2} < 0. \end{aligned}$$

$\square$

Second Round: Additional Comparative Statics.

Consider first $K_2^{P|PI}$. We can immediately see that $dK_2^{P|PI}/d\mu_2 = dK_2^{P|PI}/d\gamma = dK_2^{P|PI}/dx = 0$, $dK_2^{P|PI}/dK_2 > 0$, and $dK_2^{P|PI}/dn < 0$. Likewise, for V_2^{PI} we can see that $dV_2^{PI}/d\mu_2$, dV_2^{PI}/dK_2, $dV_2^{PI}/dK_2 < 0$, and $dV_2^{PI}/dx > 0$. Moreover,

$$\begin{aligned} \frac{dV_2^{PI}}{dn} &= \frac{2xn\left[\gamma\left(n - \alpha^P m\right) K_2 + \mu_2(n^2 - m^2)\right] - x(n^2 - m^2)\left[\gamma K_2 + 2n\mu_2\right]}{\left[\gamma\left(n - \alpha^P m\right) K_2 + \mu_2(n^2 - m^2)\right]^2} \\ &= \frac{\overbrace{\left[n^2 - 2\alpha^P mn + m^2)\right]}^{\equiv Z(\alpha^P)} x\gamma K_2}{\left[\gamma\left(n - \alpha^P m\right) K_2 + \mu_2(n^2 - m^2)\right]^2}. \end{aligned}$$

Note that $Z(\alpha^P)$ is decreasing in α^P. Moreover, evaluating $Z(\alpha^P)$ at $\alpha^P = 1$ we get $Z(1) = (n-m)^2 > 0$. This implies that $Z(\alpha^P) > 0$ for all $\alpha^P \in [0, 1]$. Thus, $dV_2^{PI}/dn > 0$.

Next consider $K_2^{P|PO}$ (see (12) in Proof of Proposition 2). Note that $dK_2^{P|PO}/d\mu_2$, $dK_2^{P|PO}/d\gamma$, $dK_2^{P|PO}/dx = 0$, $dK_2^{P|PO}/dK_2 > 0$, and $dK_2^{P|PO}/dn < 0$. In addition, for V_2^{PO} (see (13) in

Proof of Proposition 2) we can immediately see that $dV_2^{PO}/d\mu_2$, $dV_2^{PO}/d\mu_2$, $dV_2^{PO}/dK_2 < 0$, and $dV_2^{PO}/dx > 0$. Furthermore,

$$\begin{aligned}\frac{dV_2^{PO}}{dn} &= \frac{2xn\left[\gamma n K_2 + \mu_2(n^2-m^2)\right] - x(n^2-m^2)\left[\gamma K_2 + 2\mu_2 n\right]}{\left[\gamma n K_2 + \mu_2(n^2-m^2)\right]^2} \\ &= \frac{\left[n^2+m^2)\right]\gamma x K_2}{\left[\gamma n K_2 + \mu_2(n^2-m^2)\right]^2} > 0.\end{aligned}$$

Second Round Participation of Powerful Investor.

Note that $K_2^{P|P-P} > 0$ for all $\alpha^P \in [0,1]$; see (6). Given the structure of P's profit function (see (5)), this implies that P always chooses to invest in the second round for all $\alpha^P \in [0,1]$. Consequently, his participation constraint is always satisfied in the second round.

Proof of Proposition 3.

We can immediately see that $K_2^{P|PO} < K_2^{P|PI}$ for all $\alpha^P > 0$, and $K_2^{P|PO} < K_2^{P|NP}$ for all $m > 0$. Moreover, note that $V_2^{PO} = V_2^{PI}$ at $\alpha^P = 0$, and recall from Proposition 1 that $dV_2^{PI}/d\alpha^P > 0$. Thus, $V_2^{PO} < V_2^{PI}$ for all $\alpha^P > 0$. Finally, $V_2^{PO} < V_2^{NP}$ because

$$\begin{aligned}\frac{x(n^2-m^2)}{\gamma n K_2 + \mu_2(n^2-m^2)} &< \frac{x}{\mu_2 + \frac{\gamma}{n}K_2} \\ \Leftrightarrow \quad (n^2-m^2)\left(\mu_2 + \frac{\gamma}{n}K_2\right) &< \gamma n K_2 + \mu_2(n^2-m^2) \\ \Leftrightarrow \quad n^2 - m^2 &< n^2.\end{aligned}$$

□

Proof of Proposition 4.

We have $K_2^{P|PI} \geq K_2^{P|NP}$ when

$$\frac{m}{n+m}\left[1+\alpha^P\right]K_2 \geq \frac{m}{n}K_2 \quad \Leftrightarrow \quad \alpha^P \geq \widehat{\alpha}^P(m) = \frac{m}{n},$$

which is equivalent to $m \leq \widehat{m} = \alpha^P n$. Moreover, note that $V_2^{PI} \geq V_2^{NP}$ when

$$\begin{aligned}
\frac{x(n^2-m^2)}{\gamma\left(n-\alpha^P m\right)K_2+\mu_2(n^2-m^2)} &\geq \frac{x}{\mu_2+\frac{\gamma}{n}K_2} \\
\Leftrightarrow \quad (n^2-m^2)\frac{\gamma}{n}K_2 &\geq \gamma\left(n-\alpha^P m\right)K_2 \\
\Leftrightarrow \quad \alpha^P &\geq \widehat{\alpha}^P(m)=\frac{m}{n},
\end{aligned}$$

which, again, is equivalent to $m \leq \widehat{m} = \alpha^P n$. □

First-round Investments.

When investing k_1^j, $j = 1, ..., n$, in the first round, investor j gets the equity share $\frac{k_1^j}{K_1}\alpha$. Thus, the objective function for each competitive investor is

$$\max_{k_1^j} \frac{k_1^j}{K_1}\left(1-\beta^i\right)\alpha^i x - \mu_1 k_1^j.$$

The first-order condition is

$$\left(1-\beta^i\right)\alpha^i x = \mu_1 K_1.$$

Note that the LHS and RHS are both constant in k_1^j. Thus, we get a bang-bang solution: (i) $k_1^j(\alpha^i) = K_1$ if $(1-\beta^i)\,\alpha^i x > \mu_1 K_1$, and (ii) $k_1^j(\alpha^i) = 0$ otherwise.

Likewise, P's objective function in the first round is

$$\max_{k_1^{PI}} = \frac{k_1^{PI}}{K_1}(1-\beta^{PI})\alpha^P x + \frac{K_2^{P|PI}}{K_2}\beta^{PI}x - \mu_1 k_1^{PI} - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right].$$

Note that $K_2^{P|PI}$ and β^{PI} do not depend on k_1^{PI}. Thus, the first-order condition is

$$(1-\beta^{PI})\alpha^P x = \mu_1 K_1.$$

Again, we get a bang-bang solution: (i) PI chooses $k_1^{PI}(\alpha^P) = K_1$ if $(1-\beta^{PI})\alpha^P x > \mu_1 K_1$, and (ii) $k_1^{PI}(\alpha^P) = 0$ otherwise.

Participation Constraint of Powerful Investor (First Round).

Using the expressions for $K_2^{P|PI}$ and β^{PI} (see (6) and (11)) we get

$$
\begin{aligned}
\pi^{P|PI}(\alpha^P) &= (1-\beta^{PI})\alpha^P x + \frac{K_2^{P|PI}}{K_2}\beta^{PI}x - \mu_1 K_1 - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right] \\
&= \left(x - K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right]\right)\alpha^P + \frac{m}{n+m}\left[1+\alpha^P\right]\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2^2 \\
&\quad -\mu_1 K_1 - \frac{\gamma m}{2}\frac{1}{[n+m]^2}\left[1+\alpha^P\right]^2 K_2^2. \qquad (14)
\end{aligned}
$$

Likewise, by setting $\alpha^P = 0$ and $\mu_1 K_1 = 0$, we get

$$
\pi^{P|PO} = \frac{m}{n+m}\frac{\gamma n}{n^2-m^2}K_2^2 - \frac{\gamma m}{2}\frac{1}{[n+m]^2}K_2^2
$$

We then find that $\pi^{P|PI}(\alpha^P) \geq \pi^{P|PO}$ is equivalent to $Z \geq \mu_1 K_1$, where

$$
\begin{aligned}
Z &= \left(x - K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right]\right)\alpha^P - \frac{m}{n+m}\frac{\gamma\alpha^P m}{n^2-m^2}K_2^2 + \frac{m}{n+m}\alpha^P\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2^2 \\
&\quad -\frac{\gamma m}{2}\frac{1}{[n+m]^2}\left[2\alpha^P + \left[\alpha^P\right]^2\right]K_2^2 \\
&= \left(x - K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right]\right)\alpha^P - \frac{1}{2}\alpha^P\frac{\gamma m}{(n+m)^2}\frac{\alpha^P}{(n^2-m^2)}K_2^2\left[m^2+2mn+n^2\right] \\
&= \left(x - K_2\left[\mu_2 + \frac{\gamma\left(n-\frac{1}{2}\alpha^P m\right)}{n^2-m^2}K_2\right]\right)\alpha^P.
\end{aligned}
$$

Thus, P's participation constraint can be written as

$$
\alpha^P\left(x - K_2\left[\mu_2 + \frac{\gamma\left(n-\frac{1}{2}\alpha^P m\right)}{n^2-m^2}K_2\right]\right) \geq \mu_1 K_1. \qquad (15)
$$

Proof of Proposition 6.

Let α^P denote the first round equity share which makes A indifferent between accepting α^P from P, and α^{PO} from a competitive investor. Formally, α^P is defined by $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$. Using (11) we get

$$\pi^{A|PI}(\alpha^P) = \left(1-\alpha^P\right)\left(1-\beta^{PI}(\alpha^P)\right)x = \left(1-\alpha^P\right)\left(1-\frac{K_2}{x}\left[\mu_2+\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right]\right)x.$$

Moreover, recall from the comparative statics analysis for the PO case (see (21)) that

$$\pi^{A|PO} = \left(1-\frac{K_2}{x}\left[\mu_2+\frac{\gamma n}{n^2-m^2}K_2\right]\right)x-\mu_1 K_1.$$

Using these two expressions we then find that $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$ is equivalent to

$$\alpha^P\left(x-K_2\left[\mu_2+\frac{\gamma\left(n+\left(1-\alpha^P\right)m\right)}{n^2-m^2}K_2\right]\right) = \mu_1 K_1, \tag{16}$$

which defines α^P.

Next we need to show that α^P satisfies P's participation constraint (15). Using (16) we can write (15) as

$$\alpha^P\left(x-K_2\left[\mu_2+\frac{\gamma\left(n-\frac{1}{2}\alpha^P m\right)}{n^2-m^2}K_2\right]\right) \;\geq\; \alpha^P\left(x-K_2\left[\mu_2+\frac{\gamma\left(n+\left(1-\alpha^P\right)m\right)}{n^2-m^2}K_2\right]\right)$$

$$\Leftrightarrow \quad 0 \;\leq\; 1-\frac{1}{2}\alpha^P,$$

which is clearly satisfied for all $\alpha^P \in [0,1]$.

Finally, using (16), we can implicitly differentiate α^P w.r.t. m:

$$\frac{d\alpha^P}{dm} = \frac{\alpha^P\gamma K_2^2\left(\frac{\left(1-\alpha^P\right)\left(n^2-m^2\right)+2m\left(n+\left(1-\alpha^P\right)m\right)}{\left[n^2-m^2\right]^2}\right)}{\underbrace{x-K_2\left[\mu_2+\frac{\gamma\left(n+\left(1-\alpha^P\right)m\right)}{n^2-m^2}K_2\right]}_{>0}+\alpha^P\frac{\gamma m}{n^2-m^2}K_2^2} > 0.$$

Consequently,

$$\frac{dV_1^{PI}}{dm} = \frac{d}{dm}\left[\frac{K_1}{\alpha^P}\right] < 0.$$

Powerful Investor in First Round – Comparative Statics.

Recall that α^P is defined by

$$Z \equiv \alpha^P \left(x - K_2 \left[\mu_2 + \frac{\gamma\left(n + \left(1-\alpha^P\right)m\right)}{n^2 - m^2} K_2 \right] \right) - \mu_1 K_1 = 0.$$

Moreover, note that

$$\frac{\partial Z}{\partial \alpha^P} = \underbrace{x - K_2 \left[\mu_2 + \frac{\gamma\left(n + \left(1-\alpha^P\right)m\right)}{n^2 - m^2} K_2 \right]}_{>0} + \alpha^P \frac{\gamma m}{n^2 - m^2} K_2^2 > 0. \tag{17}$$

Using Z we can then implicitly differentiate α^P and get

$$\begin{aligned}
\frac{d\alpha^P}{d\mu_1} &= \frac{K_1}{\frac{\partial Z}{\partial \alpha^P}} > 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{d\mu_1} = \frac{d}{d\mu_1}\left[\frac{K_1}{\alpha^P}\right] < 0 \\
\frac{d\alpha^P}{d\mu_2} &= \frac{\alpha^P K_2}{\frac{\partial Z}{\partial \alpha^P}} > 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{d\mu_2} = \frac{d}{d\mu_2}\left[\frac{K_1}{\alpha^P}\right] < 0 \\
\frac{d\alpha^P}{d\gamma} &= \frac{\alpha^P \frac{n+\left(1-\alpha^P\right)m}{n^2-m^2} K_2^2}{\frac{\partial Z}{\partial \alpha^P}} > 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{d\gamma} = \frac{d}{d\gamma}\left[\frac{K_1}{\alpha^P}\right] < 0 \\
\frac{d\alpha^P}{dx} &= -\frac{\alpha^P}{\frac{\partial Z}{\partial \alpha^P}} < 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{dx} = \frac{d}{dx}\left[\frac{K_1}{\alpha^P}\right] > 0 \\
\frac{d\alpha^P}{dn} &= -\frac{\alpha^P K_2^2 \frac{\gamma\left(n^2+m^2\right)+2\gamma\left(1-\alpha^P\right)mn}{\left[n^2-m^2\right]^2}}{\frac{\partial Z}{\partial \alpha^P}} < 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{dn} = \frac{d}{dn}\left[\frac{K_1}{\alpha^P}\right] > 0.
\end{aligned}$$

Likewise,

$$\frac{d\alpha^P}{dK_1} = \frac{\mu_1}{\frac{\partial Z}{\partial \alpha^P}}, \tag{18}$$

so that

$$\frac{dV_1^{PI}}{dK_1} = \frac{d}{dK_1}\left[\frac{K_1}{\alpha^P}\right] = \frac{\overbrace{\alpha^P - K_1\frac{d\alpha^P}{dK_1}}^{\equiv T}}{\left[\alpha^P\right]^2}.$$

Using (18) with (17), we can write T as

$$\begin{aligned} T &= \frac{\overbrace{\alpha^P\left[x - K_2\left[\mu_2 + \frac{\gamma\left(n+\left(1-\alpha^P\right)m\right)}{n^2-m^2}K_2\right]\right] - \mu_1 K_1}^{=Z=0} + \left(\alpha^P\right)^2\frac{\gamma m}{n^2-m^2}K_2^2}{x - K_2\left[\mu_2 + \frac{\gamma(n+(1-\alpha^P)m)}{n^2-m^2}K_2\right] + \alpha^P\frac{\gamma m}{n^2-m^2}K_2^2} \\ &= \frac{\left(\alpha^P\right)^2\frac{\gamma m}{n^2-m^2}K_2^2}{\underbrace{x - K_2\left[\mu_2 + \frac{\gamma\left(n+\left(1-\alpha^P\right)m\right)}{n^2-m^2}K_2\right]}_{>0} + \alpha^P\frac{\gamma m}{n^2-m^2}K_2^2} > 0. \end{aligned}$$

Thus, $dV_1^{PI}/dK_1 > 0$.

Finally note that α^P is defined by $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$. Consequently, we get the same comparative statics results for $\pi^{A|PI}$ as for $\pi^{A|PO}$.

Proof of Proposition 7.

Recall that α^P is defined by (9) (see Proposition 6). For $m = 0$ we can write condition (9) as

$$\alpha^P = \frac{\mu_1 K_1}{\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n}K_2\right]\right)x}.$$

Clearly, $\alpha^P = \alpha^{NP}$ for $m = 0$. This implies that $V_1^{PO} = V_1^{NP}$ for $m = 0$. Moreover, recall from Proposition 6 that $dV_1^{PI}/dm < 0$. Consequently, $V_1^{PI} < V_1^{NP}$ for all $m > 0$.

Next, note that the condition $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$, which defines α^P, can be written as

$$\left(1-\alpha^P\right)\left(1-\beta^{PI}\right) - \left(1-\alpha^{PO}\right)\left(1-\beta^{PO}\right) = 0. \tag{19}$$

Moreover, recall that β^{PI} and β^{PO} are given by

$$\beta^{PI} = \frac{K_2}{x}\left[\mu_2 + \frac{\gamma\left(n - \alpha^P m\right)}{n^2 - m^2} K_2\right] \qquad \beta^{PO} = \frac{K_2}{x}\left[\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2\right].$$

We can immediately see that $\beta^{PI} = \beta^{PO}$ for $m = 0$. This implies that $\alpha^P = \alpha^{PO}$ for $m = 0$, and therefore $\alpha^P = \alpha^{PO}$ for $m = 0$. Hence, $V_1^{PI} = V_1^{PO}$ for $m = 0$. Moreover, it is easy to see that $\beta^{PI} < \beta^{PO}$ for all $m > 0$. And using (19) we get

$$\frac{\partial \alpha^P}{\partial \beta^{PI}} = -\frac{1 - \alpha^P}{1 - \beta^{PI}} < 0.$$

This implies that $\alpha^P > \alpha^{PO}$ for all $m > 0$. Consequently, $V_1^{PI} < V_1^{PO}$ for all $m > 0$. $\square$

Proof of Proposition 8.

The expected profit for A is given by

$$\pi^{A|i} = \left(1 - \alpha^i\right)\left(1 - \beta^i\right) x = \left(1 - \beta^i\right) x - \mu_1 K_1,$$

with $i \in \{NP, PO, PI\}$. Using the expressions for β^{NP} (see (2)) and β^{PO} (see (11) with $\alpha^P = 0$) we get

$$\pi^{A|NP} = \left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n} K_2\right]\right) x - \mu_1 K_1 \tag{20}$$

$$\pi^{A|PO} = \left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2\right]\right) x - \mu_1 K_1. \tag{21}$$

It is easy to see that $\pi^{A|NP} = \pi^{A|PO}$ when $m = 0$. Moreover, note that $d\pi^{A|PO}/dm < 0$. Thus, $\pi^{A|NP} > \pi^{A|PO}$ for all $m > 0$.

Likewise, we can immediately see that $d\pi^{A|i}/d\mu_1$, $d\pi^{A|i}/d\mu_2$, $d\pi^{A|i}/d\gamma$, $d\pi^{A|i}/dK_1$, $d\pi^{A|i}/dK_2 < 0$, and $d\pi^{A|i}/dx > 0$, $i \in \{NP, PO\}$. Moreover, $d\pi^{A|NP}/dn > 0$, and

$$\frac{d\pi^{A|PO}}{dn} = -\frac{\gamma\left[n^2 - m^2\right] - 2\gamma n^2}{\left[n^2 - m^2\right]^2} K_2^2 = \frac{\gamma\left[n^2 + m^2\right]}{\left[n^2 - m^2\right]^2} K_2^2 > 0.$$

Next, recall from Proof of Proposition 7 that $\alpha^P = \alpha^{PO}$ when $m = 0$. For $m > 0$ we have $\alpha = \alpha^P$, where α^P is defined by $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$. This implies that $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$. Consequently, $\pi^{A|NP} > \pi^{A|PI} > \pi^{A|PO}$ for all $m > 0$.

We can immediate see from (20) that $d\pi^{A|NP}/dm = 0$. Moreover, it is easy to see from (21) that $d\pi^{A|PO}/dm < 0$. And because $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$, this also implies that $d\pi^{A|PI}/dm < 0$.

Finally recall that α^P is defined by $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$. Consequently, we get the same comparative statics results for $\pi^{A|PI}$ as for $\pi^{A|PO}$. □

Proof of Proposition 9.

We first derive and compare R_{23} for the different investor constellations. Using the expressions of V_2^{NP}, V_2^{PO}, and V_2^{PI}, we get

$$R_{23}^{NP} = \mu_2 + \frac{\gamma}{n} K_2 \quad R_{23}^{PO} = \mu_2 + \gamma \frac{n}{n^2 - m^2} K_2 \quad R_{23}^{PI} = \mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2.$$

Recall from Propositions 3 and 4 that $V_2^{PO} < V_2^{HPI} < V_2^{NP} < V_2^{MPI}$. Thus, $R_{23}^{PO} > R_{23}^{HPI} > R_{23}^{NP} > R_{23}^{MPI}$.

Next consider R_{12}. Using the expressions of V_1^{PO} and V_2^{PO} we get

$$R_{12}^{PO} = \frac{V_2^{PO} - K_2}{V_1^{PO}} = \frac{\frac{x}{\mu_2 + \gamma \frac{n}{n^2 - m^2} K_2} - K_2}{\frac{1}{\mu_1}\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2\right]\right) x} = \frac{\mu_1}{\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2}.$$

Likewise, using the expressions of V_1^{NP} and V_2^{NP} we find

$$R_{12}^{NP} = \frac{V_2^{NP} - K_2}{V_1^{NP}} = \frac{\frac{\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n} K_2\right]\right) x}{\mu_2 + \frac{\gamma}{n} K_2}}{\frac{1}{\mu_1}\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n} K_2\right]\right) x} = \frac{\mu_1}{\mu_2 + \frac{\gamma}{n} K_2}.$$

Finally, using V_1^{PI} and V_2^{PI} we get

$$R_{12}^{PI} = \frac{V_2^{PI} - K_2}{V_1^{PI}} = \frac{x - K_2\left[\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2\right]}{\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2} \frac{\alpha^P}{K_1},$$

where α^P satisfies

$$\alpha^P \left(x - K_2 \left[\mu_2 + \frac{\gamma \left(n + \left(1 - \alpha^P \right) m \right)}{n^2 - m^2} K_2 \right] \right) = \mu_1 K_1. \tag{22}$$

Note that we can rewrite R_{12}^{PI} as

$$R_{12}^{PI} = \frac{\alpha^P \left(x - K_2 \left[\mu_2 + \gamma \frac{n + \left(1 - \alpha^P \right) m}{n^2 - m^2} K_2 \right] \right) \frac{1}{K_1} + \left[\gamma \frac{m}{n^2 - m^2} K_2^2 \right] \frac{\alpha^P}{K_1}}{\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2}.$$

Using (22) we then get

$$R_{12}^{PI} = \frac{\mu_1 + \gamma \frac{m}{n^2 - m^2} K_2^2 \frac{\alpha^P}{K_1}}{\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2}.$$

We know from Propositions 3 and 4 that $V_2^{PO} < V_2^{HPI} < V_2^{NP} < V_2^{MPI}$. And according to Proposition 7, $V_1^{PI} < V_1^{PO} < V_1^{NP}$. Moreover, we know from Proposition 6 that $dV_1^{PI}/dm < 0$. Consequently, $V_1^{HPI} < V_1^{MPI} < V_1^{PO} < V_1^{NP}$. This implies that (i) $R_{12}^{HPI} > R_{12}^{PO}$ because $V_2^{PO} < V_2^{HPI}$ and $V_1^{HPI} < V_1^{PO}$, (ii) $R_{12}^{PO} < R_{12}^{MPI}$ because $V_2^{PO} < V_2^{MPI}$ and $V_1^{MPI} < V_1^{PO}$, and (iii) $R_{12}^{NP} < R_{12}^{MPI}$ because $V_2^{NP} < V_2^{MPI}$ and $V_1^{MPI} < V_1^{PO} < V_1^{NP}$. Moreover, it is straightforward to show that $R_{12}^{PO} < R_{12}^{NP}$. Consequently, $R_{12}^{PO} < R_{12}^{NP} < R_{12}^{MPI}$ and $R_{12}^{PO} < R_{12}^{HPI}$.

Finally, using the above expressions for R_{12} and R_{23} we get

$$R_{13}^{PO} = R_{12}^{PO} R_{23}^{PO} = \frac{\mu_1}{\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2} \left[\mu_2 + \gamma \frac{n}{n^2 - m^2} K_2 \right] = \mu_1$$

$$R_{13}^{NP} = R_{12}^{NP} R_{23}^{NP} = \frac{\mu_1}{\mu_2 + \frac{\gamma}{n} K_2} \left[\mu_2 + \frac{\gamma}{n} K_2 \right] = \mu_1.$$

Likewise,

$$
\begin{aligned}
R_{13}^{PI} &= R_{12}^{PI} R_{23}^{PI} = \frac{\mu_1 + \gamma \dfrac{m}{n^2 - m^2} K_2^2 \frac{\alpha^P}{K_1}}{\mu_2 + \gamma \dfrac{n - \alpha^P m}{n^2 - m^2} K_2} \left[\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2 \right] \\
&= \mu_1 + \underbrace{\gamma \frac{m}{n^2 - m^2} K_2^2 \frac{\alpha^P}{K_1}}_{\equiv Z} .
\end{aligned}
$$

We can immediately see that $R_{13}^{PO} = R_{13}^{NP} = \mu_1$. And because $Z > 0$, we have $R_{13}^{PI} > R_{13}^{PO} = R_{13}^{NP} = \mu_1$. Moreover, recall from Proof of Proposition 6 that $d\alpha^P/dm > 0$ – this implies that $dZ/dm > 0$. Consequently, $R_{13}^{HPI} > R_{13}^{MPI}$. All this implies that $R_{13}^{HPI} > R_{13}^{MPI} > R_{13}^{PO} = R_{13}^{NP} = \mu_1$. □

The Role of Debt – Derivations.

Consider the second financing round. Given r_2 each competitive investor will invest k_2^j, $j = m+1, ..., n$, so that the price per unit of capital equals the marginal cost: $1 + r_2 = \mu_2 + \gamma k_2^j$. Consequently, $k_2^{j|D}(r_2) = [1 + r_2 - \mu_2] / \gamma$. Using $k_2^{j|D}(r_2)$ in the market clearing condition, $K_2^P + (n - m)\, k_2^{j|D}(r_2) = K_2$, and solving for $1 + r_2$, we get

$$
1 + r_2 = \frac{\gamma}{n - m} \left[K_2 - K_2^P \right] + \mu_2. \tag{23}
$$

P then chooses K_2^P to maximize his expected net return

$$
\pi_2^{P|D}(K_2^P) = \alpha^P \left(x - (1 + r_2) K_2 \right) + (1 + r_2) K_2^P - \left[\mu_2 K_2^P + \frac{\gamma}{2m} \left(K_2^P \right)^2 \right]. \tag{24}
$$

Using (23) we can write (24) as

$$
\pi_2^{P|D}(K_2^P) = \alpha^P x + \left(\frac{\gamma}{n - m} \left[K_2 - K_2^P \right] + \mu_2 \right) \left[K_2^P - \alpha^P K_2 \right] - \left[\mu_2 K_2^P + \frac{\gamma}{2m} \left(K_2^P \right)^2 \right].
$$

The optimal investment, $K_2^{P|D}$, is then defined by the first-order condition:

$$
-\frac{\gamma}{n - m} \left[K_2^P - \alpha^P K_2 \right] + \frac{\gamma}{n - m} \left[K_2 - K_2^P \right] + \mu_2 = \mu_2 + \frac{\gamma}{m} K_2^P .
$$

Solving for K_2^P we get

$$K_2^{P|D} = \frac{m}{n+m}\left[1+\alpha^P\right]K_2. \tag{25}$$

Next, using (25) we can rewrite (23) as

$$1+r_2^* = \frac{\gamma}{n-m}\left[K_2 - \frac{m}{n+m}\left[1+\alpha^P\right]K_2\right] + \mu_2 = \mu_2 + \left[\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}\right]K_2. \tag{26}$$

Thus, the cost of debt financing in the second round is given by

$$(1+r_2^*)K_2 = K_2\left[\mu_2 + \left[\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}\right]K_2\right].$$

Moreover, note that the cost of equity financing is given by

$$\beta^{PI}x = K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right].$$

Consequently, $(1+r_2^*)K_2 = \beta^{PI}x$.

Next we derive the outcome for the first financing round. Consider first the PO case (where $\alpha^P = 0$), and let

$$T_2 \equiv (1+r_2^*)K_2 = \beta^{PO}x = K_2\left[\mu_2 + \frac{\gamma n}{n^2-m^2}K_2\right]$$

denote the cost of financing in the second round (which is the same for debt and equity financing). Under debt financing the competitive investor's zero profit condition is given by $(1+r_1)K_1 - \mu_1 K_1 = 0$. Consequently, the equilibrium return is $r_1^C = \mu_1 - 1$. The expected profit for A, denoted by $\pi^{A|D(PO)}$, is then given by

$$\pi^{A|D(PO)} = x - \left(1+r_1^C\right)K_1 - T_2 = \left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma n}{n^2-m^2}K_2\right]\right)x - \mu_1 K_1.$$

We can immediately see that $\pi^{A|D(PO)} = \pi^{A|PO}$ (see (21), i.e., for the PO case debt and equity financing are equivalent for A.

Now consider the PI case. The expected profit for A is then given by

$$\pi^{A|D(PI)} = x - (1 + r_1) K_1 - T_2 = x - (1 + r_1) K_1 - K_2 \left[\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right].$$

Let $\widehat{r}_2$ denote the return which makes A indifferent between accepting $\widehat{r}_2$ from P, and r_1^C from a competitive investor. Formally, $\widehat{r}_2$ is defined by $\pi^{A|D(PI)}(\widehat{r}_2) = \pi^{A|D(PO)}$, which immediately implies that $\widehat{r}_2 = r_1^C = \mu_1 - 1$. Consequently, P offers $r_1^P = \widehat{r}_2 = \mu_1 - 1$. The expected profit for A is then given by $\pi^{A|D(PI)} = \pi^{A|D(PO)}$. And because $\pi^{A|D(PO)} = \pi^{A|PO}$, we find again that debt and equity financing lead to the same expected profit for A.

It remains to show that P is strictly better off under equity financing. For this it is sufficient to compare the joint surplus of A and P under equity and debt financing, since $\pi^{A|D(PO)} = \pi^{A|PO}$. Note that P's expected profit under debt financing can be written as

$$\pi^{P|D(PI)} = \left(1 + r_1^P\right) K_1 + (1 + r_2^*) K_2^P - \mu_1 K_1 - \left[\mu_2 K_2^P + \frac{\gamma}{2m} \left(K_2^P\right)^2 \right].$$

Using $r_1^P = \mu_1 - 1$, (25) with $\alpha^P = 0$, and (26), we can write $\pi^{P|D(PI)}$ as

$$\pi^{P|D(PI)} = K_2^P \left[(1 + r_2^*) - \left[\mu_2 + \frac{\gamma}{2m} K_2^P \right] \right] = \frac{1}{2} \frac{\gamma m}{n^2 - m^2} K_2^2.$$

Thus, the joint surplus for A and P under debt financing, $\Pi^D \equiv \pi^{A|D(PI)} + \pi^{P|D(PI)}$, is given by

$$\begin{aligned} \Pi^D &= x - K_2 \left[\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right] - \mu_1 K_1 + \frac{1}{2} \frac{\gamma m}{n^2 - m^2} K_2^2 \\ &= x - \mu_1 K_1 - \mu_2 K_2 - \frac{\gamma \left(n - \frac{1}{2} m\right)}{n^2 - m^2} K_2^2. \end{aligned}$$

Next we derive the joint surplus for equity financing (with $\alpha^P > 0$). We first note that A's expected profit is given by

$$\pi^{A|PI} = \left(1 - \alpha^{PI}\right) \left(1 - \beta^{PI}\right) x = \left(1 - \alpha^{PI}\right) \left(1 - \frac{K_2}{x} \left[\mu_2 + \frac{\gamma \left(n - \alpha^P m\right)}{n^2 - m^2} K_2 \right] \right) x.$$

Moreover, the expected profit for P is given by (14). Thus, the joint surplus under equity financing, $\Pi^E \equiv \pi^{A|PI} + \pi^{P|PI}$, can be written as

$$\begin{aligned}\Pi^E &= x - K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right] - \mu_1 K_1 + \frac{\gamma\left(1+\alpha^P\right)m}{\left(n+m\right)^2}K_2^2\left[\frac{\left(n-\alpha^P m\right)}{\left(n-m\right)} - \frac{1}{2}\left[1+\alpha^P\right]\right] \\ &= x - K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right] - \mu_1 K_1 + \frac{1}{2}\frac{\gamma\left(1-\left(\alpha^P\right)^2\right)m}{n^2-m^2}K_2^2.\end{aligned}$$

Finally, we have $\Pi^E > \Pi^D$ if

$$\begin{aligned}-\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2^2 + \frac{1}{2}\frac{\gamma\left(1-\left(\alpha^P\right)^2\right)m}{n^2-m^2}K_2^2 &> -\frac{\gamma\left(n-\frac{1}{2}m\right)}{n^2-m^2}K_2^2 \\ \Leftrightarrow \quad \alpha^P m + \frac{1}{2}\left(1-\left(\alpha^P\right)^2\right)m &> \frac{1}{2}m \\ \Leftrightarrow \quad 2 &> \alpha^P,\end{aligned}$$

which is clearly satisfied. Thus, $\Pi^E > \Pi^D$. And because $\pi^{A|D(PO)} = \pi^{A|PO}$, we can infer that $\pi^{P|D(PO)} < \pi^{P|PO}$.

Proof of Proposition 10.

Let $\widetilde{x} = \widetilde{x}_i$, $i = L, H$, denote the market belief about the expected return x_i. Using $k_2^j(\beta) = \frac{1}{\gamma}\left[\frac{1}{K_2}\beta\widetilde{x} - \mu_2\right]$ we can write the market clearing condition (4) as follows:

$$K_2^P + (n-m)\frac{1}{\gamma}\left[\frac{1}{K_2}\beta\widetilde{x} - \mu_2\right] = K_2.$$

Solving for β we get the total equity issued to the second-round investors:

$$\beta(K_2^P, \widetilde{x}) = \frac{K_2}{\widetilde{x}}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2\right]. \tag{27}$$

We can infer from (27) that P's signaling strategy depends on whether he wants to invest below or above pro-rata. Below pro-rata ($m < \widehat{m}$), P wants to signal x_H to reduce $\beta(K_2^P, \widetilde{x})$, and therefore to increase valuations. Above pro-rata ($m > \widehat{m}$), P wants to signal x_L to increase $\beta(K_2^P, \widetilde{x})$, and therefore to reduce valuations.

The objective function of P can be written as

$$\begin{aligned}
\max_{K_2^P} \pi_2^{P|PI}(K_2^P) &= (1-\beta(K_2^P,\widetilde{x}))\alpha^P x + \frac{K_2^P}{K_2}\beta(K_2^P,\widetilde{x})x - \left[\mu_2 K_2^P + \frac{\gamma}{2m}\left(K_2^P\right)^2\right] \\
&= \alpha^P x - \alpha^P \frac{x}{\widetilde{x}} K_2 \left[\frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2\right] \\
&\quad + K_2^P \frac{x}{\widetilde{x}} \left[\frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2\right] - \left[\mu_2 K_2^P + \frac{\gamma}{2m}\left(K_2^P\right)^2\right].
\end{aligned}$$

The optimal investment, denoted $K_2^{P*}(x,\widetilde{x})$, is then defined by the first-order condition:

$$\alpha^P \frac{x}{\widetilde{x}} K_2 \frac{\gamma}{n-m} + \frac{x}{\widetilde{x}}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2\right] - K_2^P \frac{x}{\widetilde{x}} \frac{\gamma}{n-m} = \mu_2 + \frac{\gamma}{m} K_2^P.$$

Solving for K_2^P we get

$$K_2^{P*}(X) = \frac{m\left[XK_2\left[1+\alpha^P\right] + \frac{n-m}{\gamma}\left[X-1\right]\mu_2\right]}{\left(2X-1\right)m+n}, \tag{28}$$

where $X = x/\widetilde{x}$. Moreover,

$$\frac{dK_2^P(X)}{dX} = \frac{m\left[K_2\left(1+\alpha^P\right) + \Omega\mu_2\right]\left(\left(2X-1\right)m+n\right) - 2m^2\left[XK_2\left(1+\alpha^P\right) + \Omega\left[X-1\right]\mu_2\right]}{\left[\left(2X-1\right)m+n\right]^2},$$

where $\Omega = \left(n-m\right)/\gamma$. The derivative is positive if

$$\begin{aligned}
K_2\Theta\left(\left(2X-1\right)m+n\right) + \Omega\mu_2\left(\left(2X-1\right)m+n\right) &> 2mXK_2\Theta + 2m\Omega\left[X-1\right]\mu_2 \\
\Leftrightarrow \quad K_2\Theta\left(n-m\right) + \Omega\mu_2\left(m+n\right) &> 0.
\end{aligned}$$

This condition is satisfied because $m > n$. Thus, $dK_2^P(X)/dX > 0$.

Now consider the case where P wants to invest below pro-rata ($m < \widehat{m}$), and would therefore like to signal a high value x_H to increase the valuation. In a separating equilibrium, P will invest $K_2^P(x_L)$ in the low state $x = x_L$ (instead of $K_2^P(x_H)$), and $K_2^P(x_H)$ in the high state

$x = x_H$ (instead of $K_2^P(x_L)$). Thus, in a separating equilibrium the following two truth-telling constraints need to be satisfied:

$$\pi_2^{P|PI}\left(x_L, K_2^P(x_L), \widetilde{x}_L\right) \geq \pi_2^{P|PI}\left(x_L, K_2^P(x_H), \widetilde{x}_H\right) \tag{29}$$

$$\pi_2^{P|PI}\left(x_H, K_2^P(x_H), \widetilde{x}_H\right) \geq \pi_2^{P|PI}\left(x_H, K_2^P(x_L), \widetilde{x}_L\right). \tag{30}$$

Because $\widetilde{x}_L = x_L$ in the separating equilibrium, we can immediately infer from (28) that $K_2^{P*}(x_L)$ equals the equilibrium investment for the full information case, $K_2^{P|PI}$:

$$K_2^{P*}(x_L) = K_2^{P|PI} = \frac{mK_2\left[1+\alpha^P\right]}{m+n}.$$

The optimal investment in the high state $x = x_H$, $K_2^{P*}(x_H)$, is then defined by the binding truth-telling constraint (30).

Next we show that $\pi_2^{P|PI}\left(x_H, K_2^P(x_H) = K_2^{P|PI}, \widetilde{x}_H\right) > \pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right)$. This would imply that there exists a $K_2^P(x_H) \neq K_2^{P|PI}$ which satisfies the binding truth-telling constraint (30). Note that

$$\begin{aligned}
\pi_2^{P|PI}\left(x_H, K_2^P(x_H) = K_2^{P|PI}, \widetilde{x}_H\right) &= \left(1-\beta(K_2^{P|PI}, x_H)\right)\alpha^P x_H + \frac{K_2^{P|PI}}{K_2}\beta(K_2^{P|PI}, x_H)x_H \\
&\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right] \\
\pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right) &= \left(1-\beta(K_2^{P|PI}, x_L)\right)\alpha^P x_H + \frac{K_2^{P|PI}}{K_2}\beta(K_2^{P|PI}, x_L)x_H \\
&\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right].
\end{aligned}$$

Thus, $\pi_2^{P|PI}\left(x_H, K_2^P(x_H) = K_2^{P|PI}, \widetilde{x}_H\right) > \pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right)$ is equivalent to

$$\left(\frac{K_2^{P|PI}}{K_2} - \alpha^P\right)\beta(K_2^{P|PI}, x_H) > \left(\frac{K_2^{P|PI}}{K_2} - \alpha^P\right)\beta(K_2^{P|PI}, x_L). \tag{31}$$

Note that

$$\frac{K_2^{P|PI}}{K_2} - \alpha^P = \frac{1}{K_2}\frac{mK_2\left[1+\alpha^P\right]}{m+n} - \alpha^P = \frac{m - n\alpha^P}{m+n}.$$

This is negative if $\alpha^P > m/n$, which is the case when P wants to invest below pro-rata. Thus, condition (31) is equivalent to $\beta(K_2^{P|PI}, x_H) < \beta(K_2^{P|PI}, x_L)$. We can immediately see from (27) that this is always satisfied. Thus, $\pi_2^{P|PI}\left(x_H, K_2^P(x_H) = K_2^{P|PI}, \widetilde{x}_H\right) > \pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right)$. This implies that there exists a $K_2^{P*}(x_H)$, with $K_2^{P*}(x_H) > K_2^{P|PI}$, which is defined by the binding truth-telling constraint (30).

It remains to derive a sufficient condition so that the truth-telling constraint (29) is satisfied with strict inequality for $K_2^{P*}(x_H)$. We first note that $\pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_L\right)$ has an inverted U-shape, and is maximized at $K_2^P = K_2^{P|PI} = \frac{mK_2\left[1+\alpha^P\right]}{m+n}$. Moreover, for $x_H \to x_L$ the binding truth-telling constraint (30) becomes $\pi_2^{P|PI}\left(x_L, K_2^P(x_H), \widetilde{x}_L\right) = \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_L\right)$, which implies that $K_2^{P*}(x_H) = K_2^{P|PI}$ for $x_H \to x_L$. Consequently, the truth-telling constraint (29) is a strict equality for $x_H \to x_L$. Furthermore, it is easy to see that $d\pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_L\right)/dx_H = 0$. Thus, we need to show that $d\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right)/dx_H < 0$. This would imply that the truth-telling constraint (29) is a strict inequality for $x_H > x_L$ and $K_2^{P*}(x_H)$.

Note that

$$\begin{aligned}
\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right) &= \left(1 - \beta(K_2^{P*}(x_H), x_H)\right)\alpha^P x_L + \frac{K_2^{P*}(x_H)}{K_2}\beta(K_2^{P*}(x_H), x_H)x_L \\
&\quad - \left[\mu_2 K_2^{P*}(x_H) + \frac{\gamma}{2m}\left(K_2^{P*}(x_H)\right)^2\right] \\
&= \left(x_L - \frac{x_L}{x_H}K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right]\right)\alpha^P \\
&\quad + K_2^{P*}(x_H)\frac{x_L}{x_H}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right] \\
&\quad - \left[\mu_2 K_2^{P*}(x_H) + \frac{\gamma}{2m}\left(K_2^{P*}(x_H)\right)^2\right].
\end{aligned}$$

Thus,

$$\frac{d\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right)}{dx_H} = \left.\frac{d\pi_2^{P|PI}(\cdot)}{dK_2^P}\right|_{K_2^P = K_2^{P*}(x_H)} \cdot \frac{dK_2^{P*}(x_H)}{dx_H} + \frac{\partial\pi_2^{P|PI}(\cdot)}{\partial x_H}.$$

We immediately get

$$\frac{\partial\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right)}{\partial x_H} = \left(\alpha^P K_2 - K_2^{P*}(x_H)\right)\frac{x_L}{(x_H)^2}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right].$$

Next, recall that $\pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_H\right)$ is maximized at $K_2^P = K_2^P(X)$, with $X = x/\widetilde{x}$; see (28). Moreover, recall that $dK_2^P(X)/dX > 0$. Clearly, $x_L/x_H < x_L/x_L = x_H/x_H = 1$. Hence, $K_2^P(x_L, \widetilde{x} = x_H) < K_2^{P|PI}$. And because $K_2^{P|PI} < K_2^{P*}(x_H)$, we have $K_2^P(x_L, \widetilde{x} = x_H) < K_2^{P*}(x_H)$. This implies that

$$\left.\frac{d\pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_H\right)}{dK_2^P}\right|_{K_2^P = K_2^{P*}(x_H)} < 0.$$

Next, recall that $K_2^{P*}(x_H)$ is implicitly defined by the binding truth-telling constraint (30). Implicitly differentiating $K_2^{P*}(x_H)$ w.r.t. x_H yields

$$\frac{dK_2^{P*}(x_H)}{dx_H} = -\frac{\frac{\partial\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_H), \widetilde{x}_H\right)}{\partial x_H} - \frac{\partial\pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right)}{\partial x_H}}{\left.\frac{\partial\pi_2^{P|PI}\left(x_H, K_2^P, \widetilde{x}_H\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_H)}}. \tag{32}$$

We have already shown that $K_2^{P*}(x_H) > K_2^{P|PI}$. Therefore,

$$\left.\frac{\partial\pi_2^{P|PI}\left(x_H, K_2^P, \widetilde{x}_H\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_H)} < 0.$$

Moreover, using (27) with $X = x_H/\widetilde{x}_H = 1$, we get

$$\begin{aligned}
\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_H), \widetilde{x}_H\right) &= \left(1 - \beta(K_2^{P*}(x_H), x_H)\right)\alpha^P x_H + \frac{K_2^{P*}(x_H)}{K_2}\beta(K_2^{P*}(x_H), x_H)x_H \\
&\quad - \left[\mu_2 K_2^{P*}(x_H) + \frac{\gamma}{2m}\left(K_2^{P*}(x_H)\right)^2\right] \\
&= \left(x_H - K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right]\right)\alpha^P \\
&\quad + K_2^{P*}(x_H)\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right] \\
&\quad - \left[\mu_2 K_2^{P*}(x_H) + \frac{\gamma}{2m}\left(K_2^{P*}(x_H)\right)^2\right].
\end{aligned}$$

Note that $\partial\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_H), \widetilde{x}_H\right)/\partial x_H = \alpha^P$. Moreover, we have

$$\begin{aligned}\pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right) &= \left(x_H - \frac{x_H}{x_L}K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right]\right)\alpha^P \\ &\quad + K_2^{P|PI}\frac{x_H}{x_L}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right] \\ &\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right].\end{aligned}$$

Consequently,

$$\frac{\partial\pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right)}{\partial x_H} = \alpha^P - \left(\alpha^P K_2 - K_2^{P|PI}\right)\frac{1}{x_L}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right].$$

Using all this we can write (32) as

$$\frac{dK_2^{P*}(x_H)}{dx_H} = \frac{\left(\alpha^P K_2 - K_2^{P|PI}\right)\frac{1}{x_L}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right]}{\underbrace{-\left.\frac{\partial\pi_2^{P|PI}\left(x_H, K_2^P, \widetilde{x}_H\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_H)}}_{>0}}.$$

Note that

$$\alpha^P K_2 - K_2^{P|PI} = \alpha^P K_2 - \frac{mK_2\left[1+\alpha^P\right]}{m+n} = \frac{n\alpha^P - m}{m+n}K_2.$$

This is positive if $\alpha^P > \frac{m}{n}$, which is the case when P invests below pro-rata ($m < \widehat{m}$). Consequently, $dK_2^{P*}(x_H)/dx_H > 0$.

Finally, we can write the total derivative as follows:

$$\frac{d\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right)}{dx_H} = \underbrace{\left.\frac{d\pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_H\right)}{dK_2^P}\right|_{K_2^P = K_2^{P*}(x_H)}}_{<0} \cdot \underbrace{\frac{dK_2^{P*}(x_H)}{dx_H}}_{>0} + Z_1,$$

where

$$Z_1 = \left(\alpha^P K_2 - K_2^{P*}(x_H)\right)\frac{x_L}{\left(x_H\right)^2}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right].$$

We have already shown that $\alpha^P K_2 - K_2^{P|PI} > 0$ when P invests below pro-rata. Moreover, recall that $K_2^{P*}(x_H) > K_2^{P|PI}$. Thus, we can either have $Z_1 \leq 0$ or $Z_1 > 0$. We can see that $d\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right)/dx_H < 0$ when x_L is sufficiently small. Thus, there exists a threshold $\widehat{x}_L$ so that we have a separating equilibrium for $x_L < \widehat{x}_L$ (which is a sufficient condition) when P invests below pro-rata (i.e., $m < \widehat{m}$).

Next consider the case where P wants to invest above pro-rata ($m > \widehat{m}$), and would therefore like to signal a low value x_L to reduce the valuation. Again, the two truth-telling constraints (29) and (30) must then be satisfied in a separating equilibrium.

In the separating equilibrium we have $\widetilde{x}_L = x_L$, so that $K_2^{P*}(x_H)$ equals the equilibrium investment for the full information case, i.e., $K_2^{P*}(x_H) = K_2^{P|PI}$; see (28) (equation K_2^{P*}). The optimal investment in the low state $x = x_L$ is then defined by the binding truth-telling constraint (29).

Next we show that $\pi_2^{P|PI}\left(x_L, K_2^P(x_L) = K_2^{P|PI}, \widetilde{x}_L\right) > \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right)$. We have

$$\begin{aligned}
\pi_2^{P|PI}\left(x_L, K_2^P(x_L) = K_2^{P|PI}, \widetilde{x}_L\right) &= \left(1 - \beta(K_2^{P|PI}, x_L)\right)\alpha^P x_L + \frac{K_2^{P|PI}}{K_2}\beta(K_2^{P|PI}, x_L)x_L \\
&\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right] \\
\pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right) &= \left(1 - \beta(K_2^{P|PI}, x_H)\right)\alpha^P x_L + \frac{K_2^{P|PI}}{K_2}\beta(K_2^{P|PI}, x_H)x_L \\
&\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right].
\end{aligned}$$

Note that $\pi_2^{P|PI}\left(x_L, K_2^P(x_L) = K_2^{P|PI}, \widetilde{x}_L\right) > \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right)$ is then equivalent to

$$\left(\frac{K_2^{P|PI}}{K_2} - \alpha^P\right)\beta(K_2^{P|PI}, x_L) > \left(\frac{K_2^{P|PI}}{K_2} - \alpha^P\right)\beta(K_2^{P|PI}, x_H). \tag{33}$$

Recall that $K_2^{P|PI}/K_2 - \alpha^P = \left(m - n\alpha^P\right)/(m+n)$. This is positive if $\alpha^P < m/n$, which is true when P wants to invest above pro-rata. Hence, condition (33) simplifies to $\beta(K_2^{P|PI}, x_L) > \beta(K_2^{P|PI}, x_H)$, which, according to (27), is always satisfied. Consequently, $\pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_L\right) > \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right)$. Moreover, it is straightforward to show that $\pi_2^{P|PI}\left(x_L, K_2^P = 0, \widetilde{x}_L\right) < \pi_2^{P|PI}\left(x_L, K_2^P = 0, \widetilde{x}_H\right)$ for all $\alpha^P > 0$. Thus, there exists a $K_2^{P*}(x_L)$, with $K_2^{P*}(x_L) < K_2^{P|PI}$, which is defined by the binding truth-telling constraint (29).

Next we derive a sufficient condition so that the truth-telling constraint (30) is satisfied with strict inequality for $K_2^{P*}(x_L)$. Recall that $\pi_2^{P|PI}\left(x_H, K_2^P(x_H), \widetilde{x}_H\right)$ has an inverted U-shape, and is maximized at $K_2^P = K_2^{P|PI}$. Suppose for a moment that $x_L \to x_H$. The truth-telling constraint (29) then becomes $\pi_2^{P|PI}\left(x_H, K_2^P(x_L), \widetilde{x}_H\right) = \pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_H\right)$, so that $K_2^{P*}(x_L) = K_2^{P|PI}$ for $x_L \to x_H$. Moreover, we can see that $d\pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_H\right)/dx_L = 0$. Consequently, we need to show that $d\pi_2^{P|PI}\left(x_H, K_2^P(x_L), \widetilde{x}_L\right)/dx_L > 0$. In this case the truth-telling constraint (30) is a strict inequality for $x_L < x_H$ and $K_2^{P*}(x_L)$.

Note that

$$\begin{aligned}
\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_L), \widetilde{x}_L\right) &= \left(1-\beta(K_2^{P*}(x_L), x_L)\right)\alpha^P x_H + \frac{K_2^{P*}(x_L)}{K_2}\beta(K_2^{P*}(x_L), x_L)x_H \\
&\quad - \left[\mu_2 K_2^{P*}(x_L) + \frac{\gamma}{2m}\left(K_2^{P*}(x_L)\right)^2\right] \\
&= \left(x_H - \frac{x_H}{x_L}K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right) + \mu_2\right]\right)\alpha^P \\
&\quad + K_2^{P*}(x_L)\frac{x_H}{x_L}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right) + \mu_2\right] \\
&\quad - \left[\mu_2 K_2^{P*}(x_L) + \frac{\gamma}{2m}\left(K_2^{P*}(x_L)\right)^2\right].
\end{aligned}$$

Consequently,

$$\frac{d\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_L), \widetilde{x}_L\right)}{dx_L} = \frac{d\pi_2^{P|PI}(\cdot)}{dK_2^P}|_{K_2^P = K_2^{P*}(x_L)} \cdot \frac{dK_2^{P*}(x_L)}{dx_L} + \frac{\partial \pi_2^{P|PI}(\cdot)}{\partial x_L}.$$

Note that

$$\frac{\partial \pi_2^{P|PI}\left(x_H, K_2^{P*}(x_L), \widetilde{x}_L\right)}{\partial x_L} = \left(\alpha^P K_2 - K_2^{P*}(x_L)\right)\frac{x_H}{(x_L)^2}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right) + \mu_2\right].$$

Recall that $\pi_2^{P|PI}\left(x_H, K_2^P, \widetilde{x}_L\right)$ is maximized at $K_2^P = K_2^P(X)$, with $X = x/\widetilde{x}$; see (28). Furthermore, we have shown that $dK_2^P(X)/dX > 0$. Notice that $x_H/x_L > x_L/x_L = x_H/x_H = 1$. This implies that $K_2^P(x_H, \widetilde{x} = x_L) > K_2^{P|PI}$. Moreover, recall that $K_2^{P*}(x_L) < K_2^{P|PI}$. Thus, $K_2^{P*}(x_L) < K_2^P(x_H, \widetilde{x} = x_L)$, which also implies that

$$\left.\frac{d\pi_2^{P|PI}\left(x_H, K_2^P, \widetilde{x}_L\right)}{dK_2^P}\right|_{K_2^P = K_2^{P*}(x_L)} > 0.$$

Next, using the binding truth-telling constraint (29), which defines $K_2^{P*}(x_L)$, we get

$$\frac{dK_2^{P*}(x_L)}{dx_L} = -\frac{\frac{\partial \pi_2^{P|PI}\left(x_L, K_2^{P*}(x_L), \widetilde{x}_L\right)}{\partial x_L} - \frac{\partial \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right)}{\partial x_L}}{\left.\frac{\partial \pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_L\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_L)}}.$$

Recall that $K_2^{P*}(x_L) < K_2^{P|PI}$. Consequently,

$$\left.\frac{\partial \pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_L\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_L)} > 0.$$

Moreover, note that

$$\begin{aligned}
\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_L), \widetilde{x}_L\right) &= \left(1 - \beta(K_2^{P*}(x_L), x_L)\right)\alpha^P x_L + \frac{K_2^{P*}(x_L)}{K_2}\beta(K_2^{P*}(x_L), x_L)x_L \\
&\quad - \left[\mu_2 K_2^{P*}(x_L) + \frac{\gamma}{2m}\left(K_2^{P*}(x_L)\right)^2\right] \\
&= \left(x_L - K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right) + \mu_2\right]\right)\alpha^P \\
&\quad + K_2^{P*}(x_L)\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right) + \mu_2\right] \\
&\quad - \left[\mu_2 K_2^{P*}(x_L) + \frac{\gamma}{2m}\left(K_2^{P*}(x_L)\right)^2\right].
\end{aligned}$$

Clearly, $\partial \pi_2^{P|PI}\left(x_L, K_2^{P*}(x_L), \widetilde{x}_L\right)/\partial x_L = \alpha^P$. Furthermore,

$$\begin{aligned}
\partial \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right) &= \left(x_L - \frac{x_L}{x_H}K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right]\right)\alpha^P \\
&\quad + K_2^{P|PI}\frac{x_L}{x_H}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right] \\
&\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right].
\end{aligned}$$

Thus,

$$\frac{\partial \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right)}{\partial x_L} = \alpha^P - \left(\alpha^P K_2 - K_2^{P|PI}\right)\frac{1}{x_H}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right].$$

Therefore we get

$$\frac{dK_2^{P*}(x_L)}{dx_L} = -\frac{\left(\alpha^P K_2 - K_2^{P|PI}\right)\frac{1}{x_H}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right)+\mu_2\right]}{\underbrace{\left.\frac{\partial \pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_L\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_L)}}_{>0}}.$$

It is straightforward to show that $\alpha^P K_2 - K_2^{P|PI} = \frac{n\alpha^P - m}{m+n}K_2$, which is negative if $\alpha^P < m/n$. Note that this is true when P invests above pro-rata ($m > \widehat{m}$). Hence, $dK_2^{P*}(x_L)/dx_L > 0$.

Using all this we can eventually write the total derivative as follows:

$$\frac{d\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_L), \widetilde{x}_L\right)}{dx_L} = \underbrace{\frac{d\pi_2^{P|PI}\left(x_H, K_2^P, \widetilde{x}_L\right)}{dK_2^P}|_{K_2^P = K_2^{P*}(x_L)}}_{>0}\underbrace{\frac{dK_2^{P*}(x_L)}{dx_L}}_{>0} + Z_2,$$

where

$$Z_2 = \left(\alpha^P K_2 - K_2^{P*}(x_L)\right)\frac{x_H}{(x_L)^2}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right)+\mu_2\right].$$

Recall that $\alpha^P K_2 - K_2^{P|PI} < 0$ when P invests above pro-rata. Moreover, we have shown that $K_2^{P*}(x_L) < K_2^{P|PI}$. Hence, it is possible that either $Z_2 \geq 0$ or $Z_2 < 0$. Clearly, we always have $d\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_L), \widetilde{x}_L\right)/dx_L > 0$ when x_H is sufficiently small. This implies that there exists a threshold $\widehat{x}_H$ so that we have a separating equilibrium for $x_H < \widehat{x}_H$ (which, again, is a sufficient condition) when P invests above pro-rata (i.e., $m > \widehat{m}$). □

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