Partner Uncertainty and the Dynamic Boundary of the Firm

Thomas Hellmann and Veikko Thiele

— ONLINE APPENDIX —

Long-term Contracts.

We now briefly show that the use of a long-term contract cannot improve the outcome. Suppose that Alice and Bob retain individual asset ownership at date 0, but commit to a contract that specifies the price at which they transact at date 4. If the contract is only structured as an option without commitment, then it has no effect at all. However, if the contract is binding, then the two partners face a similar renegotiation game as under joint asset ownership: If they want to switch partners, they cannot do so without the consent of their original trading partner. This in turn implies that a long-term contract cannot prevent retention externalities.

Moreover, the division of surplus in equilibrium is determined by the bargaining outcome. The only difference between a long-term contract and joint asset ownership is *how* the surplus is split between the two partners. Under joint asset ownership, Alice and Bob each get a constant fraction of the profits, as defined by α and β. With a long-term contract the partners agree on a pre-specified price (or pricing formula) that determines the division of surplus. What matters for the model is not the actual distribution at date 4, but the expected distribution at dates 2 and 3. Suppose Bob (upstream) sells the input to Alice (downstream). Let $\widetilde{v}$ be the value of the input for the buyer (Alice) at date 4, and $\widetilde{c}$ be the cost of the seller (Bob). Their joint surplus is then given by $y = \widetilde{v} - \widetilde{c}$. We conveniently denote the joint distribution of $\widetilde{v}$ and $\widetilde{c}$ by $\Omega_{vc}(\widetilde{v}, \widetilde{c})$, so that $\int y d\Omega_{in}(y) = \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$. Let λ be the transfer price specified in the long-term contract. This price can only be made contingent on verifiable information, i.e., on the realizations of $\widetilde{v}$ and $\widetilde{c}$ at date 4.

With a constant inside prospect π, it is easy to see that the two partners agree on a unique transfer price λ^* that allows them to split the expected surplus according to the bargaining outcome.[1] Alternatively, they can define a flexible pricing schedule that implements the bargaining outcome. The flexible pricing schedule must then satisfy $\alpha^* y = \widetilde{v} - \lambda^*$, which requires $\lambda^* = (1 - \alpha^*)\widetilde{v} + \alpha^* \widetilde{c}$. However, a long-term contract only affects the means through which the total surplus is split, and not the division of surplus itself. Thus, long-term contracts gen-

[1] Specifically, we have $\alpha^* \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c}) = \int (\widetilde{v} - \lambda^*) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$, or equivalently, $\lambda^* = \int \widetilde{v} d\Omega_{vc}(\widetilde{v}, \widetilde{c}) - \alpha^* \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$.

erate exactly the same ex-ante utilities as joint asset ownership. This also implies that leaving a partner, and thus breaking the long-term contract, requires the same compensation as under joint asset ownership, thus leading to the same retention externalities.

Alternative Ownership Structures.

We focus on individual and joint asset ownership as the only possible ownership structures. We now briefly explain why we can safely ignore all other ownership structures.

The main alternative ownership structure is full asset ownership in the hands of one of the two partners. With ex-ante symmetric partners, it does not matter which partner owns both assets; w.l.o.g. we assume it is Alice. It is easy to verify that whenever Alice finds an alternative partner and Bob does not, then the model behaves just like under individual asset ownership. And if Bob finds an alternative partner and Alice does not, then the model behaves just like under joint asset ownership. We show that either individual or joint asset ownership is optimal (see Proposition 1); thus, mixing individual and joint ownership is never optimal.

In fact, asymmetric asset ownership creates additional inefficiencies when Alice controls the two assets. If both Alice and Bob find alternative partners, then Alice can hold up Bob before releasing his asset. Thus, Bob would have to share some of the profits from his new partnership with Dora, which is clearly inefficient. At the ex-ante stage, Bob would also not relinquish his asset for free. In fact, Alice would have to give Bob a larger profit share ex-ante, which would lead to further inefficiencies. Asymmetric ownership is therefore never optimal within the present context.

From Proposition 1 it is immediate that randomization among the symmetric ownership structures is also suboptimal. Giving ownership to outsiders is not optimal in our model either, since owner-managers want to retain maximal effort incentives. Moreover, the type of outside ownership suggested by Gans (2005) is not an equilibrium, because we allow asset owners to coordinate on joint asset ownership. Finally, because there are no sequential investments in our model, there is no role for options on ownership as in Nöldeke and Schmidt (1998).

Joint Production.

The optimal effort levels, denoted $e_A^*(\alpha)$ and $e_B^*(\beta)$, are characterized by the first-order conditions

$$\alpha\mu'(e_A e_B)e_B\pi = c'(e_A) \tag{A.1}$$

$$\beta\mu'(e_A e_B)e_A\pi = c'(e_B). \tag{A.2}$$

Using $\beta = 1 - \alpha$, the joint surplus $V \equiv U_A + U_B$ is given by

$$V(\alpha;\pi) = \mu(e_A^*(\alpha)e_B^*(\alpha))\pi - c(e_A^*(\alpha)) - c(e_B^*(\alpha)).$$

The jointly optimal profit share α^J satisfies the first-order condition

$$\pi\mu'(e_A^* e_B^*)\left[\frac{de_A^*}{d\alpha}e_B^* + \frac{de_B^*}{d\alpha}e_A^*\right] = c'(e_A^*)\frac{de_A^*}{d\alpha} + c'(e_B^*)\frac{de_B^*}{d\alpha}. \tag{A.3}$$

Symmetry implies $de_A^*/d\alpha = -de_B^*/d\alpha$ and $e_A^* = e_B^*$ at $\alpha = 1/2$. Thus, (A.3) is satisfied for $\alpha = \beta = 1/2$. The solution $\alpha^J = \beta^J = 1/2$ is also unique due to the convexity of $c(e_i)$, $i = A, B$. Thus,

$$\left.\frac{dU_A}{d\alpha}\right|_{\alpha=1/2} = -\left.\frac{dU_B}{d\alpha}\right|_{\alpha=1/2} > 0.$$

Moreover, $e_A^*(0) = e_B^*(1) = 0$. This implies $1/2 < \alpha^{max} = \beta^{max} < 1$.

Now consider the bargaining at date 0, and suppose that Alice gets the profit share $\alpha > 1/2$ with probability $1/2$, and $1 - \alpha < 1/2$ otherwise. Alice's expected utility at date 0 is then given by $U_A(\alpha;\pi)/2 + U_A(1-\alpha;\pi)/2$. However, from the above we note that her expected utility is maximized when $\alpha = 1/2$ (symmetric for Bob). Thus, at date 0 both partners agree on splitting the expected joint surplus in half: $\alpha = \beta = 1/2$. □

Renegotiation under Individual Asset Ownership.

W.l.o.g. suppose that only Alice found an alternative partner, Charles. Consider first the case without wealth ($w = 0$). Alice is indifferent between staying (and renegotiating her profit share α) and leaving, if

$$U_A(\alpha_I^*;\pi) = U_A(\widehat{\alpha}_I;\sigma). \tag{A.4}$$

Recall that Bob and Charles both have zero outside options. The bargaining protocol à la Hart and Mas-Colell (1996) then implies that (A.4) is satisfied for $\pi = \sigma$. Thus, $\sigma \leq \pi$ implies $U_A(\alpha_I^*;\pi) \geq U_A(\widehat{\alpha}_I;\sigma)$. For $\sigma > \pi$ we have $U_A(\alpha_I^*;\pi) < U_A(\widehat{\alpha}_I;\sigma)$. We define $\widehat{\pi}_I(\sigma) = \sigma$ as the threshold below which Alice is better off leaving Bob ($\pi < \widehat{\pi}_I(\sigma)$).

Suppose that Alice and Bob have each initial wealth $w > 0$. Let $V_I(\pi, w) \equiv U_A(\alpha_I^*;\pi,w) + U_B(\beta_I^*;\pi,w)$ denote Alice's and Bob's joint surplus under individual asset ownership when staying together. Recall that their joint surplus is maximized in case of joint production when $\alpha_I^* = \beta_I^* = 1/2$. The joint surplus is then given by $2U(\pi)$. Thus, the minimum value of wealth w required to eliminate displacement externalities under individual asset ownership, denoted $\overline{w}_I$, satisfies $V_I(\pi, w) = 2U(\pi)$. Next we characterize the minimum amount of wealth $\underline{w}_I$, which changes the renegotiation outcome. For $\pi \geq \widehat{\pi}_I(\sigma)$ we know that Alice stays with Bob, but profit shares are unbalanced. Bob can then use even small amounts of wealth to buy back some profit shares from Alice, which improves their joint surplus. Thus, $\underline{w}_I = 0$ for $\pi \geq \widehat{\pi}_I(\sigma)$. For $\pi < \widehat{\pi}_I(\sigma)$, Alice leaves the partnership with Bob. For $w \to 0$ Bob cannot retain Alice, and therefore cannot change the renegotiation outcome. Thus, $\underline{w}_I > 0$, where $\underline{w}_I$ satisfies $U_A(\alpha_I^*; \pi, \underline{w}_I) = U(\widehat{\alpha}_I, \sigma)$.

Proof of Lemma 1.

It follows directly from our previous derivations (see Section "Renegotiation under Individual Asset Ownership" in the Appendix) that the threshold $\widehat{\pi}_I(\sigma, w)$ is defined by

$$U_A(\alpha_I^*; \pi, w) = U_A(\widehat{\alpha}_I; \sigma). \tag{A.5}$$

Using (A.5) we can implicitly differentiate $\widehat{\pi}_I(\sigma, w)$ w.r.t. w:

$$\frac{d\widehat{\pi}_I(\sigma, w)}{dw} = -\frac{\frac{dU_A(\alpha_I^*;\pi,w)}{dw}}{\frac{dU_A(\alpha_I^*;\pi,w)}{d\pi}}.$$

Recall that $dU_A(\alpha_I^*; \pi, w)/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$. Moreover, applying the Envelope Theorem we find that $dU_A(\alpha_I^*; \pi, w)/d\pi > 0$. Thus, $d\widehat{\pi}_I(\sigma, w)/dw < 0$ for $\underline{w}_I \leq w < \overline{w}_I$. □

Profit Shares under Joint Ownership with Asymmetric Outside Options.

W.l.o.g. suppose that only Alice found an alternative partner, Charles. We denote a coalition by S, with $S \subset 3$. Let $\kappa = (\kappa_A, \kappa_B, \kappa_C)$ be a vector which measures the rate at which utility can be transferred. Moreover, $\eta_T \in V(T)$ denotes the payoff vector for the subcoalition T.

According to Hart (2004), the Maschler-Owen consistent NTU value can be derived by the following procedure: First, for all $i \in S$, let the payoff vector $\mathbf{z} \in \mathbb{R}^S$ satisfy

$$\kappa_i z_i = \frac{1}{|S|}\left[v_\kappa(S) - \sum_{j \in S \setminus i} \kappa_j \eta_{S\setminus i}(j) + \sum_{j \in S \setminus i} \kappa_i \eta_{S \setminus j}(i)\right],$$

where the maximum possible value $v_\kappa(S)$ is defined by

$$v_\kappa(S) = \sup\left\{\sum_{i \in S} \kappa_i U_i : (U_i)_{i \in S} \in V(S)\right\}.$$

Second, if $\mathbf{z}$ is feasible, then the payoff vector is given by $\eta_S = \mathbf{z}$.

The coalition functions for our setting are as follows:

$$\begin{aligned}
V_{\{A\}} &= V_{\{B\}} = V_{\{C\}} = 0 \\
V_{\{A,B\}} &= \left\{(U_A(\alpha;\pi), U_B(\beta;\pi)) \in \mathbb{R}^{\{A,B\}} : \alpha + \beta \leq 1; \alpha, \beta \geq 0\right\} \\
V_{\{A,C\}} &= \{0, 0\} \\
V_{\{B,C\}} &= \{0, 0\} \\
V_{\{A,B,C\}} &= \left\{(U_A(\alpha;\sigma), U_B(\beta;\sigma), U_C(\gamma;\sigma)) \in \mathbb{R}^{\{A,B,C\}} : \alpha + \beta + \gamma \leq 1; \alpha, \beta, \gamma \geq 0\right\},
\end{aligned}$$

where $V_{\{A,C\}} = \{0, 0\}$ follows from the fact that Alice cannot leave without Bob's consent under joint ownership. Note that the bargaining outcome must satisfy $\alpha^* \in (0, \alpha^{\max})$ and $\beta^* \in (0, \beta^{\max})$ for the Alice-Bob coalition, and $\alpha^* \in (0, \alpha^{\max})$, $\beta^* \in (0, \beta^{\max})$, and $\gamma^* \in (0, \gamma^{\max})$ for the grand coalition (Alice, Bob, and Charles). Thus, $dU_A/d\alpha > 0$, $dU_B/d\beta > 0$, and $dU_C/d\gamma > 0$ for the relevant values of α, β, and γ. This implies that the inverse of each utility function exists. We define $\alpha(U_A) \equiv U_A^{-1}(\alpha)$, $\beta(U_B) \equiv U_B^{-1}(\beta)$, and $\gamma(U_C) \equiv U_C^{-1}(\gamma)$. Pareto efficiency then requires

$$\begin{aligned}
\alpha(U_A) + \beta(U_B) &= 1 \quad \text{for } \overline{V}_{\{A,B\}} \\
\alpha(U_A) + \beta(U_B) + \gamma(U_C) &= 1 \quad \text{for } \overline{V}_{\{A,B,C\}}.
\end{aligned}$$

The payoffs for the single-player coalitions are given by $\eta_1(A) = \eta_1(B) = \eta_1(C) = 0$. For the two-player coalitions, the equilibrium payoffs satisfy the Nash bargaining solution. Due to symmetry, the payoffs are given by

$$\begin{aligned}
\eta_2(A,B) &= (U(\pi), U(\pi)) \\
\eta_2(A,C) &= (0, 0) \\
\eta_2(B,C) &= (0, 0)\,.
\end{aligned}$$

It remains to derive the payoff vector $\eta_3(A, B, C)$ for the hyperplane game. For a vector $\mathbf{z} = (z_A, z_B, z_C)$ the equation of the hyperplane is

$$\alpha'(U_A)z_A + \beta'(U_B)z_B + \gamma'(U_C)z_C = r, \tag{A.6}$$

where

$$r = \alpha'(U_A)U_A + \beta'(U_B)U_B + \gamma'(U_C)U_C. \tag{A.7}$$

Using the payoffs for the two-player coalitions, we can now define the equilibrium payoffs for the grand coalition:

$$\begin{aligned}
\eta_3(A) &= U_A(\alpha; \sigma) = \frac{1}{3}\left[U(\pi) + z_A\right] \\
\eta_3(B) &= U_B(\beta; \sigma) = \frac{1}{3}\left[U(\pi) + z_B\right] \\
\eta_3(C) &= U_C(\gamma; \sigma) = \frac{1}{3}z_C,
\end{aligned}$$

where, using (A.7),

$$\begin{aligned}
z_A &= \frac{1}{\alpha'(U_A)}\left[r - \beta'(U_B)\cdot 0 - \gamma'(U_C)\cdot 0\right] = \frac{r}{\alpha'(U_A)} \\
z_B &= \frac{1}{\beta'(U_B)}\left[r - \alpha'(U_A)\cdot 0 - \gamma'(U_C)\cdot 0\right] = \frac{r}{\beta'(U_B)} \\
z_C &= \frac{1}{\gamma'(U_C)}\left[r - \alpha'(U_A)U(\pi) - \beta'(U_B)U(\pi)\right].
\end{aligned}$$

Using the Inverse Function Theorem we get $\alpha'(U_A) = (dU_A/d\alpha)^{-1}$, $\beta'(U_B) = (dU_B/d\beta)^{-1}$, and $\gamma'(U_C) = (dU_C/d\gamma)^{-1}$. The equations for the fixed point for the grand coalition are thus given by

$$U_A(\alpha; \sigma) = \frac{1}{3}\left[U(\pi) + r\frac{dU_A(\alpha; \sigma)}{d\alpha}\right] \tag{A.8}$$

$$U_B(\beta; \sigma) = \frac{1}{3}\left[U(\pi) + r\frac{dU_B(\beta; \sigma)}{d\beta}\right] \tag{A.9}$$

$$U_C(\gamma; \sigma) = \frac{1}{3}\frac{dU_C(\gamma; \sigma)}{d\gamma}\left[r - U(\pi)\left[\left(\frac{dU_A(\alpha; \sigma)}{d\alpha}\right)^{-1} + \left(\frac{dU_B(\beta; \sigma)}{d\beta}\right)^{-1}\right]\right], \tag{A.10}$$

where, using (A.7),

$$r = U_A(\alpha;\sigma)\left(\frac{dU_A(\alpha;\sigma)}{d\alpha}\right)^{-1} + U_B(\beta;\sigma)\left(\frac{dU_B(\beta;\sigma)}{d\beta}\right)^{-1} + U_C(\gamma;\sigma)\left(\frac{dU_C(\gamma;\sigma)}{d\gamma}\right)^{-1}.$$

The equilibrium payoff vector $\eta_3(A,B,C) = (\widehat{U}_A(\alpha;\sigma), \widehat{U}_B(\beta;\sigma), \widehat{U}_C(\gamma;\sigma))$ thus satisfies the system of three equations, (A.8), (A.9), and (A.10), which also defines the equilibrium profit shares $\widehat{\alpha}_J$, $\widehat{\beta}_J$, and $\widehat{\gamma}_J$.

Renegotiation under Joint Asset Ownership.

W.l.o.g. suppose that only Alice found an alternative partner (Charles). We first consider the case without wealth ($w = 0$). Alice will then stay with Bob under joint asset ownership with an equal split of profits if

$$U_A(\pi) \geq U_A(\widehat{\alpha}_J;\sigma). \tag{A.11}$$

Note that (A.11) is never satisfied when $\pi = 0$ and $\sigma > 0$. Using the Envelope Theorem one can show that $dU_A(\pi)/d\pi > 0$. Moreover, $\lim_{\pi\to\infty} U_A(\pi) = \infty > U_A(\widehat{\alpha}_J;\sigma)$ for any finite σ. Thus, there exists a threshold $\widehat{\pi}_J(\sigma)$ such that (A.11) is satisfied for $\pi \geq \widehat{\pi}_J(\sigma)$. Now consider briefly the case where both Alice and Bob found alternative partners (symmetric outside options). They then stay together if $U(\pi) \geq U(\sigma)$, which is equivalent to $\pi \geq \sigma$. Recall that $U(\sigma) > U_A(\widehat{\alpha}_J;\sigma)$ for all $\sigma > 0$ because $\widehat{\beta}_J > 0$ and $e_B^* = 0$ in case of asymmetric outside options. Thus, $\widehat{\pi}_J(\sigma) < \sigma$.

We can now consider the case where Alice and Bob have each some initial wealth $w > 0$. The minimum value of wealth w required to eliminate retention externalities under joint asset ownership, denoted $\overline{w}_J$, ensures that Alice can fully compensate Bob without offering him an equity stake in the new partnership with Charles. Thus, $\overline{w}_J$ satisfies $\widehat{\beta}_J(w) = 0$. It remains to characterize the minimum amount of wealth $\underline{w}_J$, which changes the renegotiation outcome. For $\pi \geq \widehat{\pi}_J(\sigma)$ we know that Alice stays with Bob. For $w \to 0$ Alice cannot buy herself free, so the renegotiation outcome does not change. Thus, $\underline{w}_J = 0$ for $\pi \geq \widehat{\pi}_J(\sigma)$. For $\pi < \widehat{\pi}_J(\sigma)$, Alice leaves Bob, but needs to offer him an equity stake in the new partnership with Charles. Alice can then use even small amounts of wealth to buy back some equity from Bob, which improves Alice's expected utility when partnering with Charles. Consequently, $\underline{w}_J > 0$ for $\pi < \widehat{\pi}_J(\sigma)$.

Proof of Lemma 2.

From our previous derivations (see Section "Renegotiation under Joint Asset Ownership" in the Appendix), we can immediately infer that the threshold $\widehat{\pi}_J(\sigma, w)$ is defined by

$$U(\pi) = U_A(\widehat{\alpha}_J; \sigma, w). \tag{A.12}$$

Using (A.12) we can implicitly differentiate $\widehat{\pi}_J(\sigma, w)$ w.r.t. w:

$$\frac{d\widehat{\pi}_J(\sigma, w)}{dw} = \frac{\frac{dU_A(\widehat{\alpha}_J;\sigma,w)}{dw}}{\frac{dU(\pi)}{d\pi}}.$$

We know that $dU_A(\widehat{\alpha}_J; \sigma, w)/dw > 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Furthermore, using the Envelope Theorem it is straightforward to show that $dU(\pi)/d\pi > 0$. Consequently, $d\widehat{\pi}_J(\sigma, w)/dw > 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Finally, recall that Alice and Bob agree on $\alpha^* = \beta^* = 1/2$ at date 0 under joint ownership. And because $\alpha^* = \beta^* = 1/2$ also maximizes total surplus, renegotiation does not change the equity allocation when the partner with the better outside option stays with his original partner. □

Proof of Proposition 1.

We focus on the case with asymmetric outside options because only then the ownership structure matters. Moreover, maximizing a partner's expected utility at date 0 is equivalent to maximizing the joint surplus of Alice and Bob.

We first derive the cutoff $\widehat{\pi}_V(\sigma)$, so that staying together with $\alpha^* = \beta^* = 1/2$ is jointly efficient for $\pi \geq \widehat{\pi}_V(\sigma)$, and dissolving the partnership is jointly efficient for $\pi < \widehat{\pi}_V(\sigma)$. W.l.o.g. suppose that only Alice found an alternative partner at date 2 (the case where only Bob found an alternative partner is symmetric). The joint surplus in case of joint production with $\alpha^* = \beta^* = 1/2$, is given by $2U(\pi)$. When Alice leaves, the joint surplus of Alice and Bob is maximized when Bob, as unproductive partner, does not get a stake in the new Alice-Charles partnership. The joint surplus is then given by $U_A(\widehat{\alpha}; \sigma)$, where $\widehat{\alpha}$ is Alice's equity share in the new partnership with Charles. Thus, staying together (with $\alpha^* = \beta^* = 1/2$) and dissolving the partnership are both jointly efficient if

$$2U(\pi) = U_A(\widehat{\alpha}; \sigma). \tag{A.13}$$

Recall that $dU(\pi)/d\pi > 0$. Moreover, note that $U(0) = 0$ and $\lim_{\pi\to\infty} U(\pi) = \infty > U_A(\widehat{\alpha}; \sigma)$ for any finite σ. Thus, there exists a threshold $\widehat{\pi}_V(\sigma)$, defined by (A.13), such that $2U(\pi) \geq$

$U_A(\widehat{\alpha};\sigma)$ for $\pi \geq \widehat{\pi}_V(\sigma)$, and $2U(\pi) < U_A(\widehat{\alpha};\sigma)$ for $\pi < \widehat{\pi}_V(\sigma)$. Using (A.13) we can implicitly differentiate $\widehat{\pi}_V(\sigma)$ w.r.t. σ:

$$\frac{d\widehat{\pi}_V(\sigma)}{d\sigma} = \frac{\frac{dU_A(\widehat{\alpha};\sigma)}{d\sigma}}{\frac{dU(\pi)}{d\pi}}.$$

Using the Envelope Theorem we can show that $dU_A(\widehat{\alpha};\sigma)/d\sigma > 0$ and $dU(\pi)/d\pi > 0$. Thus, $d\widehat{\pi}_V(\sigma)/d\sigma > 0$.

Suppose that $\pi < \widehat{\pi}_V(\sigma)$, so that dissolving the partnership is jointly optimal. Under individual asset ownership, Alice would leave if $\pi < \widehat{\pi}_I(\sigma, w)$, where according to Lemma 1, $\widehat{\pi}_I(\sigma, w)$ is defined by

$$U_A(\alpha_I^*;\pi, w) = U_A(\widehat{\alpha}_I;\sigma). \tag{A.14}$$

Note that $2U(\pi) > U_A(\alpha_I^*;\pi, w)$ for $w < \overline{w}_I$, whereas the right-hand sides of (A.13) and (A.14) are identical. Thus, $\widehat{\pi}_V(\sigma) < \widehat{\pi}_I(\sigma)$. This implies that individual asset ownership is optimal for $\pi < \widehat{\pi}_V(\sigma)$ as it always ensures the jointly efficient dissolution of the partnership in case of asymmetric outside options.

Now suppose that $\pi \geq \widehat{\pi}_V(\sigma)$, so that staying together with $\alpha^* = \beta^* = 1/2$ is jointly optimal in case of asymmetric outside options. Under joint asset ownership, Alice stays (with $\alpha^* = \beta^* = 1/2$) if $\pi \geq \widehat{\pi}_J(\sigma)$. Recall from Lemma 2 that $\widehat{\pi}_J(\sigma, w)$ is defined by

$$U(\pi) = U_A(\widehat{\alpha}_J;\sigma, w). \tag{A.15}$$

To show that $\widehat{\pi}_J(\sigma, w) < \widehat{\pi}_V(\sigma)$ for $w < \overline{w}_J$, we define $\widehat{\pi}_J^V(\sigma)$ as the value of π under joint asset ownership where staying together (with $\alpha^* = \beta^* = 1/2$) and dissolving the partnership (with $\widehat{\beta}_J > 0$) lead to the same joint surplus:

$$2U(\pi) = U_A(\widehat{\alpha}_J;\sigma, w) + U_B(\widehat{\beta}_J;\sigma, w). \tag{A.16}$$

Note that $U_A(\widehat{\alpha};\sigma) > U_A(\widehat{\alpha}_J;\sigma, w) + U_B(\widehat{\beta}_J;\sigma, w)$ for $w < \overline{w}_J$, whereas the left-hand sides of (A.13) and (A.16) are identical. Thus, $\widehat{\pi}_J^V(\sigma, w) < \widehat{\pi}_V(\sigma)$. Moreover, we can write (A.16) as

$$U(\pi) + \underbrace{U(\pi) - U_B(\widehat{\beta}_J;\sigma, w)}_{\equiv \chi} = U_A(\widehat{\alpha}_J;\sigma, w), \tag{A.17}$$

where, according to the Maschler-Owen consistent NTU value, $\chi < 0$ (otherwise Bob would not release his asset). Thus, the left-hand side of (A.17) is smaller than the left-hand side of

(A.15), while their right-hand sides are identical. Hence, $\widehat{\pi}_J(\sigma, w) < \widehat{\pi}_J^V(\sigma, w)$. This implies that $\widehat{\pi}_J(\sigma, w) < \widehat{\pi}_V(\sigma)$. Thus, joint asset ownership is optimal for $\pi \geq \widehat{\pi}_V(\sigma)$ as it always preserves the partnership with $\alpha^* = \beta^* = 1/2$.

We can now identify the optimal asset ownership for different values of $\pi \in \{\pi_L, \pi_H\}$ and $w < \max\{\overline{w}_I, \overline{w}_J\}$. From the above we can immediately infer that choosing individual asset ownership at date 0 is always optimal when $\pi_L, \pi_H < \widehat{\pi}_V(\sigma)$. Likewise, joint asset ownership is always optimal when $\pi_L, \pi_H \geq \widehat{\pi}_V(\sigma)$.

Next we derive the optimal asset ownership for $\pi_L < \widehat{\pi}_V(\sigma) < \pi_H$. For this we first derive the expected utilities at date 2 when both partners observe the inside prospect $\pi \in \{\pi_L, \pi_H\}$. Consider individual asset ownership. Let α_I^+ denote the profit share of the partner with the only outside option (asymmetric case), and α_I^- the profit share for the partner without outside option, where $\alpha_I^- = 1 - \alpha_I^+$. Moreover, let $\widehat{\alpha}_I$ denote the equilibrium profit share of the partner with outside option when he leaves. The expected utility of a partner a date 2 is then given by

$$EU_I(\pi, \sigma, w) = q^2 \max\{U(\pi), U(\sigma)\} + (1-q)^2 U(\pi) + q(1-q)V_I(\pi, \sigma, w), \tag{A.18}$$

where

$$V_I(\pi, \sigma, w) = \begin{cases} U(\widehat{\alpha}_I; \sigma) & \text{if } \pi < \widehat{\pi}_I(\sigma) \\ U(\alpha_I^+; \pi, w) + U(\alpha_I^-; \pi, w) & \text{if } \pi \geq \widehat{\pi}_I(\sigma) \end{cases}$$

is the total expected utility of a partner in case of asymmetric outside options.

Now consider joint asset ownership. Let $\widehat{\alpha}_J$ denote the new profit share of the partner with the only outside option when leaving the partnership, and $\widehat{\beta}_J$ the profit share of his former partner as compensation. The expected utility of a partner at date 2 is then given by

$$EU_J(\pi, \sigma) = q^2 \max\{U(\pi), U(\sigma)\} + (1-q)^2 U(\pi) + q(1-q)V_J(\pi, \sigma, w), \tag{A.19}$$

where

$$V_J(\pi, \sigma, w) = \begin{cases} U(\widehat{\alpha}_J; \sigma, w) + U(\widehat{\beta}_J; \sigma, w) & \text{if } \pi < \widehat{\pi}_J(\sigma) \\ 2U(\pi) & \text{if } \pi \geq \widehat{\pi}_J(\sigma) \end{cases}$$

is the total expected utility of a partner in case of asymmetric outside options.

We can now write the expected utility of a partner at date 0 under individual asset ownership ($EU_I(p)$) and joint asset ownership ($EU_J(p)$) as

$$EU_k(p) = pEU_k(\pi_H, \sigma, w) + (1-p)EU_k(\pi_L, \sigma, w), \quad k = I, J$$

where $EU_I(\pi, \sigma, w)$ and $EU_J(\pi, \sigma, w)$ are defined by (A.18) and (A.19), respectively. Thus, both partners agree on joint asset ownership at date 0 when $EU_J(p) \geq EU_I(p)$, which is equivalent to

$$p \geq \widehat{p} \equiv \frac{V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w)}{V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w) + V_J(\pi_H, \sigma, w) - V_I(\pi_H, \sigma, w)}.$$

where $V_J(\pi_H, \sigma, w) - V_I(\pi_H, \sigma, w) > 0$ and $V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w) > 0$ for $\pi_L < \widehat{\pi}_V(\sigma) < \pi_H$. □

Alternative Bargaining Protocols.

If both partners have zero outside options, they are perfectly symmetric. Any reasonable bargaining solution then suggests an equal split of surplus. Similarly, if Alice and Bob both found alternative partners, then we have two pairs of symmetric partners. Again we note that an equal split of surplus is the most reasonable bargaining outcome. Alternative bargaining protocols therefore only matter for the case of asymmetric outside options. We distinguish between the bargaining games under individual versus joint asset ownership.

Consider first the bargaining game under joint asset ownership with binding wealth constraints, where Alice wants to leave Bob to partner with Charles. Because the agreement of all three parties is required, any reasonable bargaining involves trilateral bargaining. While there may be many bargaining protocols that affect the distribution of rents between the three parties, the key insight is that the critical threshold $\widehat{\pi}_J(\sigma, w)$ from Lemma 2 does not depend on the specific distribution of these rents. This threshold only depends on the feasibility of obtaining an agreement between Alice, Bob and Charles that satisfies all three participation constraints. Specifically, at $\pi = \widehat{\pi}_J(\sigma, w)$ both Alice and Bob are indifferent between dissolving their partnership and staying together (each getting $U(\pi)$), while Charles receives the minimum equity stake $\gamma = 1 - \alpha^{\max}$. For any $\pi > \widehat{\pi}_J(\sigma, w)$ it is impossible to get a tripartite agreement, and for any $\pi \leq \widehat{\pi}_J(\sigma, w)$ it is always possible get such an agreement. As a consequence, the specific bargaining protocol actually does not matter for the partners' decision to stay together or to do a buyout.

Under individual asset ownership with binding wealth constraints we know from Lemma 1 that there exists a critical threshold $\widehat{\pi}_I(\sigma, w)$, such that Alice leaves Bob whenever $\pi < \widehat{\pi}_I(\sigma, w)$, and stays whenever $\pi \geq \widehat{\pi}_I(\sigma, w)$. Again we argue that reasonable alternative bargaining protocols may generate different utilities, but the critical threshold remains unaffected. One important restriction of the bargaining protocol by Hart and Mas-Colell (1996) is that at any point in time only one party can make an offer. Consider relaxing this assumption, and suppose that there can be simultaneous offers. In particular assume that the unique partner (Alice) can hold an auction for offers from the non-unique partners (Bob and Charles). Such an auction game results in a standard Bertrand pricing. It is easy to show that these Bertrand offers are more favorable to Alice than the bargaining outcome under the Hart and Mas-Colell protocol. However, since the auction is always won by the player with the highest valuation, it continues to be true that Alice teams up with Bob whenever $\pi \geq \widehat{\pi}_I(\sigma, w)$, and with Charles whenever $\pi < \widehat{\pi}_I(\sigma, w)$. Again we find that the critical threshold $\widehat{\pi}_I(\sigma, w)$ remains unaffected by the specific bargaining protocol.

Proof of Proposition 2.

Consider individual asset ownership. At date 1 partner $i = A, B$ chooses his specific investment r_i to maximize his expected utility:[2]

$$\begin{aligned} EU_I(r_i, r_j) &= p(r_i, r_j)\left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)V_I(\pi_H, \sigma, w)\right] \\ &\quad +(1-p(r_i, r_j))\left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_I(\pi_L, \sigma, w)\right] - \psi(r_i), \end{aligned}$$

where $j \in \{A, B\}$ and $j \neq i$. The equilibrium investment levels $r^*_{A(I)}(w)$ and $r^*_{B(I)}(w)$ under individual asset ownership are then characterized by the first-order conditions:

$$\frac{\partial p(r_A, r_B)}{\partial r_i}\Phi_I(w) = \psi'(r_i), \quad i = A, B,$$

where, using $V_I(\pi_L, \sigma, w) = U(\sigma)$,

$$\begin{aligned} \Phi_I(w) &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)V_I(\pi_H, \sigma, w)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)U(\sigma)\right]. \end{aligned}$$

Because Alice and Bob are symmetric at date 1, their investment levels $r^*_{A(I)}(w)$ and $r^*_{B(I)}(w)$ must be also symmetric in equilibrium. We define $r^*_I(w) \equiv r^*_{A(I)}(w) = r^*_{B(I)}(w)$ as the equilibrium relation-specific investment of a partner under individual asset ownership.

[2]Note that $U(\sigma) > U(\pi_L)$ when Alice and Bob each found an alternative partner; thus, $\max\{U(\pi_L), U(\sigma)\} = U(\sigma)$.

Likewise, the expected utility of partner $i = A, B$ at date 1 under joint asset ownership is given by

$$
\begin{aligned}
EU_J(p_i, p_j) \;=\;& p(r_i, r_j)\left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)V_J(\pi_H, \sigma, w)\right] \\
&+(1 - p(r_i, r_j))\left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_J(\pi_L, \sigma, w)\right] - \psi(r_i).
\end{aligned}
$$

The following first-order conditions define the equilibrium investment levels $r^*_{A(J)}(w)$ and $r^*_{B(J)}(w)$ under joint asset ownership:

$$
\frac{\partial p(r_A, r_B)}{\partial r_i}\Phi_J(w) = \psi'(r_i) \quad i = A, B,
$$

where, using $V_J(\pi_H, \sigma, w) = 2U(\pi_H)$,

$$
\begin{aligned}
\Phi_J(w) \;=\;& \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)2U(\pi_H)\right] \\
&- \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_J(\pi_L, \sigma, w)\right].
\end{aligned}
$$

Again, the Nash equilibrium is symmetric; we thus define $r^*_J(w) \equiv r^*_{A(J)}(w) = r^*_{B(J)}(w)$ as the equilibrium relation-specific investment of a partner under joint asset ownership.

Next, we define

$$
\begin{aligned}
F \;\equiv\;& \frac{\partial p(r_A, r_B)}{\partial r_A}\Phi_k(w) - \psi'(r_A) = 0 \\
G \;\equiv\;& \frac{\partial p(r_A, r_B)}{\partial r_B}\Phi_k(w) - \psi'(r_B) = 0,
\end{aligned}
$$

where $k \in \{I, J\}$. Applying Cramer's Rule we get

$$
\frac{dr^*_{A(k)}(w)}{dw} = \frac{\det(X_1)}{\det(X_2)},
$$

where

$$
X_1 = \begin{pmatrix} -\frac{\partial F}{\partial w} & \frac{\partial F}{\partial r_B} \\ -\frac{\partial G}{\partial w} & \frac{\partial G}{\partial r_B} \end{pmatrix} \qquad X_2 = \begin{pmatrix} \frac{\partial F}{\partial r_A} & \frac{\partial F}{\partial r_B} \\ \frac{\partial G}{\partial r_A} & \frac{\partial G}{\partial r_B} \end{pmatrix}.
$$

Because $U_i(\cdot)$, $i = A, B$, is concave, X_2 must be negative definite, so that $\det(X_2) > 0$. Thus, $dr^*_{A(k)}(w)/dw > 0$ if

$$\det(X_1) = -\frac{\partial F}{\partial w}\frac{\partial G}{\partial r_B} + \frac{\partial G}{\partial w}\frac{\partial F}{\partial r_B} > 0.$$

The second-order condition for $r^*_{B(k)}(w)$ implies $\partial G/\partial r_B < 0$. Moreover,

$$\frac{\partial F}{\partial r_B} = \frac{\partial^2 p(\cdot)}{\partial r_A \partial r_B}\Phi_k(w)$$

and

$$\frac{\partial F}{\partial w} = \frac{\partial p(\cdot)}{\partial r_A}\frac{d\Phi_k(w)}{dw} \qquad \frac{\partial G}{\partial w} = \frac{\partial p(\cdot)}{\partial r_B}\frac{d\Phi_k(w)}{dw},$$

where

$$\begin{aligned}\frac{d\Phi_I(w)}{dw} &= q(1-q)\frac{dV_I(w,\pi_H,\sigma)}{dw} \\ \frac{d\Phi_J(w)}{dw} &= -q(1-q)\frac{dV_J(w,\pi_L,\sigma)}{dw}.\end{aligned}$$

For individual asset ownership, recall that $dV_I(w,\pi_H,\sigma)/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$, which implies that $\partial F/\partial w > 0$ and $\partial G/\partial w > 0$ for $\underline{w}_I \leq w < \overline{w}_I$. Thus, $dr^*_{A(I)}(w)/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$ and $\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa$, where κ is the lower bound of the cross-partial satisfying $\det(X_1) = 0$. Symmetry implies $dr^*_{A(I)}(w)/dw = dr^*_{B(I)}(w)/dw$. For joint asset ownership, recall that $dV_J(w,\pi_L,\sigma)/dw > 0$ for $\underline{w}_J \leq w < \overline{w}_J$, so that $\partial F/\partial w < 0$ and $\partial G/\partial w < 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Thus, $dr^*_{A(J)}(w)/dw < 0$ for $\underline{w}_J \leq w < \overline{w}_J$ and $\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa$. Due to symmetry, $dr^*_{A(J)}(w)/dw = dr^*_{B(J)}(w)/dw$.

For $w \geq \max\{\overline{w}_I, \overline{w}_J\}$ we know that $V_I(w,\pi_H,\sigma) = 2U(\pi_H)$ (individual ownership), and $V_J(w,\pi_L,\sigma) = U(\sigma)$ (joint ownership). Thus, we have $\Phi_I(w) = \Phi_J(w)$ for $w \geq \max\{\overline{w}_I, \overline{w}_J\}$, so that $r^*_I(w) = r^*_J(w)$. Furthermore, because $dr^*_I/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$, and $dr^*_J/dw < 0$ for $\underline{w}_J \leq w < \overline{w}_J$, we can infer that $r^*_J(w) > r^*_I(w)$ for $w < \max\{\overline{w}_I, \overline{w}_J\}$. □

Proof of Lemma 3.

Under individual asset ownership the expected utility of Alice at date 0 is given by

$$\begin{aligned}EU^A_I(p^*, w) &= p^*\left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)V_I(\pi_H,\sigma,w)\right] \\ &\quad +(1-p^*)\left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_I(\pi_L,\sigma,w)\right] - \psi(r^*_{A(I)}),\end{aligned}$$

with $p^* \equiv p(r^*_{A(I)}, r^*_{B(I)})$ and $V_I(\pi_L, \sigma, w) = U(\sigma)$. The expected utility of Bob is symmetric. Applying the Envelope Theorem we get

$$\frac{dEU_I^A(p^*, w)}{dw} = \frac{\partial EU_I^A(p^*, w)}{\partial r_{B(I)}} \frac{dr^*_{B(I)}}{dw} + p^* q(1-q) \frac{\partial V_I(\pi_H, \sigma, w)}{\partial w}.$$

Note that $\partial EU_I^A(\cdot)/\partial r_{B(I)} > 0$. We need to consider three cases: (i) $w \le \underline{w}_I$, (ii) $w > \overline{w}_I$; and (iii), $\underline{w}_I < w \le \overline{w}_I$. For the first two cases we know that $dr^*_{B(I)}/dw = 0$ and $\partial V_I/\partial w = 0$; thus, $dEU_I^A(p^*, w)/dw = 0$. For $\underline{w}_I < w \le \overline{w}_I$ we know that $dr^*_{B(I)}/dw > 0$ and $\partial V_I/\partial w > 0$; thus, $dEU_I^A(p^*, w)/dw > 0$. This also implies that $EU_I^A(p^*, w)$ is maximized for $w \ge \overline{w}_I$. □

Proof of Lemma 4.

Under joint asset ownership the expected utility of Alice at date 0 is given by

$$\begin{aligned} EU_J^A(p^*, w) &= p^* \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) V_J(\pi_H, \sigma, w)\right] \\ &\quad + (1-p^*) \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) V_J(\pi_L, \sigma, w)\right] - \psi(r^*_{A(I)}), \end{aligned}$$

with $p^* = p(r^*_{A(J)}, r^*_{B(J)})$ and $V_J(\pi_H, \sigma, w) = 2U(\pi_H)$. The expected utility of Bob is symmetric. Applying the Envelope Theorem yields

$$\begin{aligned} \frac{dEU_J^A(p^*, w)}{dw} &= \frac{\partial EU_J^A(p^*, w)}{\partial r_{B(J)}} \frac{dr^*_{B(J)}}{dw} + (1-p^*) q(1-q) \frac{\partial V_J(\pi_L, \sigma, w)}{\partial w} \\ &= \underbrace{\Phi_J(w) \frac{\partial p(\cdot)}{\partial r_{B(J)}} \frac{dr^*_{B(J)}}{dw}}_{\equiv \psi_1} + \underbrace{(1-p^*) q(1-q) \frac{\partial V_J(\pi_L, \sigma, w)}{\partial w}}_{\equiv \psi_2}, \end{aligned}$$

where

$$\begin{aligned} \Phi_J(w) &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) 2U(\pi_H)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) V_J(\pi_L, \sigma, w)\right] > 0. \end{aligned}$$

By definition, $\partial p(\cdot)/\partial r_{B(J)} > 0$. Moreover, recall from Proposition 2 that $dr^*_{B(J)}/dw < 0$ for $\underline{w}_J \le w < \overline{w}_J$. Thus, $\psi_1 < 0$ for $\underline{w}_J \le w < \overline{w}_J$. Furthermore, $\partial V_J(\pi_L, \sigma, w)/\partial w > 0$ for $\underline{w}_J \le w < \overline{w}_J$, so that $\psi_2 > 0$ for $\underline{w}_J \le w < \overline{w}_J$. We define w_J^* as the wealth level which satisfies $dEU_J^A(p^*, w)/dw = 0$ for $\underline{w}_J \le w < \overline{w}_J$, and thus maximizes Alice's expected utility at date 0. Note that $\underline{w}_J \leq w_J^* < \overline{w}_J$ because $dr_{B(J)}^*/dw = 0$ and $\partial V_J(\pi_L, \sigma, w)/\partial w = 0$ for $w < \underline{w}_J$ and $w \geq \overline{w}_J$. To summarize, (i) $dEU_J^A(\cdot)/dw = 0$ for $w \leq \underline{w}_J$, $w \geq \overline{w}_J$, and $w = w_J^*$ (as $\psi_1 + \psi_2 = 0$), (ii) $dEU_J^A(\cdot)/dw > 0$ for $\underline{w}_J < w < w_J^*$ (as $\psi_1 + \psi_2 > 0$); and (iii), $dEU_J^A(\cdot)/dw < 0$ for $w_J^* < w < \overline{w}_J$ (as $\psi_1 + \psi_2 < 0$).

Finally note that $\lim_{\pi_H \to \infty} \Phi_J(w) = \infty$ as $dU(\pi_H)/d\pi_H > 0$ with $\lim_{\pi_H \to \infty} U(\pi_H) = \infty$. This implies that $\lim_{\pi_H \to \infty} \psi_1 = -\infty$ for $\underline{w}_J \leq w < \overline{w}_J$, while $\sup(\psi_2) < \infty$. Thus, there exists a threshold $\widehat{\pi}_H$ such that $dEU_J^A(\cdot)/dw < 0$ for all $\pi_H \geq \widehat{\pi}_H$ and $w \in (\underline{w}_J, \overline{w}_J)$, which implies a corner solution with $w_J^* \leq \underline{w}_J$. □

Proof of Proposition 3.

Suppose $w \geq \overline{w} = \max\{\overline{w}_I, \overline{w}_J\}$. Under individual asset ownership, $V_I(\pi_H, \sigma, w) = 2U(\pi_H)$ for $w \geq \overline{w}_I$. Under joint asset ownership, $V_J(\pi_L, \sigma, w) = V_I(\pi_L, \sigma, w) = U(\widehat{\alpha}_I; \sigma)$ for $w \geq \overline{w}_J$. Moreover, recall from Proposition 2 that $r_I^*(w) = r_J^*(w)$ for all $w \geq \overline{w}$. Thus, $EU_I(r_I^*, w) = EU_J(r_J^*, w)$ for $w \geq \overline{w}$.

Next, recall from Lemma 3 that $dEU_I(\cdot)/dw > 0$ for $\underline{w}_I < w \leq \overline{w}_I$, where $EU_I(\cdot)$ is maximized for $w \geq \overline{w}_I$. Moreover, we know from Lemma 4 that $dEU_J(\cdot)/dw > 0$ for $\underline{w}_J < w < w_J^*$, and $dEU_J(\cdot)/dw < 0$ for $w_J^* < w \leq \overline{w}_J$, where $EU_J(\cdot)$ is maximized when $w = w_J^*$. This implies that $EU_J(\cdot) > EU_I(\cdot)$ for $w \in [w_J^*, \overline{w})$.

Finally we examine whether $EU_I(\cdot) > EU_J(\cdot)$ for some $w < w_J^*$. Suppose $\pi_H \to \pi_L$. We can then immediately see that $r_I^*(w) = r_J^*(w) = 0$, and hence, $EU_I(\cdot) > EU_J(\cdot)$. We define w_0 as the critical wealth level so that $EU_J(\cdot) > EU_I(\cdot)$ for $w \in [w_0, \overline{w})$. Note that $w_0 < w_J^*$ because $EU_J(\cdot) > EU_I(\cdot)$ for $w_J^* \leq w < \overline{w}$. Moreover, $w_0 \geq 0$ because, when π_H is sufficiently high, $EU_J(\cdot) > EU_I(\cdot)$ even for $w = 0$. Thus, joint asset ownership is strictly optimal for $w_0 \leq w < \overline{w}$, with $w_0 \in [0, w_J^*]$. According to Lemma 4, the optimal wealth level is then $w_J^* \in [0, \overline{w})$, with $w_J^* \leq \underline{w}_J$ for all $\pi_H \geq \widehat{\pi}_H$. □

Outside Financing.

Consider date 2 and assume w.l.o.g. that only Alice found an alternative partner, Charles. Suppose the partners' wealth constraints are binding, i.e., they cannot fully eliminate the ex-post inefficiencies associated with asymmetric outside options. We now ask whether raising the amount $K > 0$ from an outside investor for (additional) transfer payments, can lead to a Pareto improvement (which is required for changing the renegotiation outcome).

Assume a competitive capital market, with cost of capital $r \geq 0$. Moreover, recall that the only payoff in our model is y, which is realized at date 4. Thus, the partners can only offer the outside investor, who does not contribute to the production process, a share δ on the return y in exchange for K, with expected values $\pi = \int_{\underline{y}}^{\overline{y}} y d\Omega_{in}(y)$ (inside prospect) and $\sigma = \int y d\Omega_{out}(y)$ (outside prospect).

Consider joint asset ownership. With insufficient wealth ($w < \overline{w}_J$), Alice would need to offer Bob the stake $\widehat{\beta}_J$ in the new partnership with Charles – in addition to the transfer w – to buy out her asset (note that $\widehat{\beta}_J$ already accounts for the transfer w, i.e., $\widehat{\beta}_J = \widehat{\beta}_J(w)$). This, however, compromises effort incentives for Alice and Charles. Alice can also raise the amount $K \in [0, \overline{w}_J - w]$ to mitigate (or even to eliminate) the inefficiency associated with the buy out. Alice can then offer Bob the payment K and the new stake $\widehat{\beta}_J(K) \in [0, \widehat{\beta}_J]$, with $d\widehat{\beta}_J(K)/dK < 0$ and $\widehat{\beta}_J(K = \overline{w}_J - w) = 0$. Bob accepts the amount K in exchange for a lower equity stake $\widehat{\beta}_J(K)$ when

$$K = \left[\widehat{\beta}_J - \widehat{\beta}_J(K)\right] \mu\left(e_A(K)e_C(K)\right)\sigma. \tag{A.20}$$

Furthermore, the zero profit condition for the outside investor, who gets the stake $\widehat{\delta}_J(K)$, implies

$$\widehat{\delta}_J(K)\mu\left(e_A(K)e_C(K)\right)\sigma = (1+r)\,K.$$

Combining the two conditions we get

$$\widehat{\delta}_J(K) = (1+r)\left[\widehat{\beta}_J - \widehat{\beta}_J(K)\right].$$

Thus, for $r \geq 0$, the equity stake $\widehat{\delta}_J(K)$ for the (unproductive) outside investor is at least as high as the equity stake that Bob relinquishes in exchange for the extra transfer K. This implies that $d\widehat{\alpha}_J/dK \leq 0$ and $d\widehat{\gamma}_J/dK \leq 0$, and therefore, $de_A(K)/dK \leq 0$ and $de_C(K)/dK \leq 0$.

For any transfer $K \in [0, \overline{w}_J - w]$, the expected joint utility for Alice and Bob under joint asset ownership is

$$U_A(K)+U_B(K) = \widehat{\alpha}_J(K)\mu\left(e_A(K)e_C(K)\right)\sigma - c\left(e_A(K)\right) - w + \widehat{\beta}_J(K)\mu\left(e_A(K)e_C(K)\right)\sigma + K + w.$$

Using (A.20) we can write this as

$$U_A(K) + U_B(K) = \widehat{\alpha}_J(K)\mu\left(e_A(K)e_C(K)\right)\sigma - c\left(e_A(K)\right) + \widehat{\beta}_J\mu\left(e_A(K)e_C(K)\right)\sigma.$$

Note that Alice chooses $e_A(K)$ such that $dU_A(K)/de_A = 0$; thus,

$$\begin{aligned} \frac{d}{dK}\left[U_A(K) + U_B(K)\right] &= \overbrace{\frac{d\widehat{\alpha}_J(K)}{dK}}^{\leq 0} \mu\left(e_A(K)e_C(K)\right)\sigma \\ &\quad + \widehat{\alpha}_J(K)\mu'\left(e_A(K)e_C(K)\right)e_A(K)\overbrace{\frac{de_C(K)}{dK}}^{\leq 0}\sigma \\ &\quad + \widehat{\beta}_J\mu'\left(e_A(K)e_C(K)\right)\sigma\left[\underbrace{\frac{de_A(K)}{dK}}_{\leq 0}e_C(K) + e_A(K)\underbrace{\frac{de_C(K)}{dK}}_{\leq 0}\right]. \end{aligned}$$

Consequently, $d\left[U_A(K) + U_B(K)\right]/dK \leq 0$, which implies that raising $K > 0$ for (additional) transfer payments is not Pareto improving under joint ownership. Hence, $K_J^* = 0$.

Now consider individual asset ownership, and suppose that Alice decided to stay but renegotiated a more favorable profit share. With insufficient wealth ($w < \overline{w}_I$), Bob can buy back some of his original profit share by paying Alice the amount w, but this is not enough to fully eliminate the ex-post inefficiency, as in equilibrium we still have $\alpha_I^* > \beta_I^*$ (note that α_I^* and β_I^* already account for the transfer w, i.e., $\alpha_I^* = \alpha_I^*(w)$ and $\beta_I^* = \beta_I^*(w)$). Bob can also raise the amount $K \in [0, \overline{w}_I - w]$ to buy back more profit shares, in order to better align team incentives. The new payoffs for Alice and Bob are then given by $(1 - \delta_I^*(K))\alpha_I^*(K)\pi$ and $(1 - \delta_I^*(K))\beta_I^*(K)\pi$, respectively, where $\delta_I^*(K)$ is the investor's profit share. Alice accepts the transfer K (in addition to the transfer w) in exchange for relinquishing some of her profit shares when

$$K = \alpha_I^*\mu(e_Ae_B)\pi - c_A(e_A) - \left[\widetilde{\alpha}_I(K)\mu(e_A(K)e_B(K))\pi - c_A(e_A(K))\right], \tag{A.21}$$

where $\widetilde{\alpha}_I(K) = (1 - \delta_I^*(K))\alpha_I^*(K)$ is Alice's net profit share. Because $d\widetilde{\alpha}_I(K)/dK < 0$, it is straightforward to show that $de_A(K)/dK < 0$. Moreover, the outside investor's stake $\delta_I^*(K)$ is defined by his zero profit condition:

$$\delta_I^*(K)\mu(e_A(K)e_B(K))\pi = (1 + r)K.$$

For any transfer $K \in [0, \overline{w}_I - w]$, the expected joint utility under individual asset ownership with Alice staying with Bob, is

$$\begin{aligned} U_A(K) + U_B(K) &= \widetilde{\alpha}_I(K)\mu(e_A(K)e_B(K))\pi - c_A(e_A(K)) + w + K \\ &\quad + \widetilde{\beta}_I(K)\mu(e_A(K)e_B(K))\pi - c_B(e_B(K)) - w, \end{aligned}$$

where $\widetilde{\beta}_I(K) = (1 - \delta_I^*(K))\beta_I^*(K)$ is Bob's net profit share. Using (A.21) we can write the expected joint utility as

$$U_A(K) + U_B(K) = \alpha_I^*\mu(e_A e_B)\pi - c_A(e_A) + \widetilde{\beta}_I(K)\mu(e_A(K)e_B(K))\pi - c_B(e_B(K)).$$

Note that $dU_A(K)/dK = 0$. Moreover, the total surplus that is split between Alice and Bob, $(1 - \delta_I^*(K))\pi$, is decreasing in K, where the profit share $\delta_I^*(K)\pi$ goes to an unproductive party (the investor). Thus, $d[U_A + U_B(K)]/dK < 0$. Consequently, raising $K > 0$ for (additional) transfer payments is not Pareto improving under individual asset ownership (case of staying and renegotiation), so that $K_I^* = 0$.

Now consider the case where Alice wants to leave Bob under individual asset ownership. Bob can then offer Alice a lump sum payment (equal to w) and a higher profit share to make her stay. Without sufficient wealth ($w < \overline{w}_I$), Bob's retention offer either is not enough to convince Alice to stay, or ensures that Alice stays but with her getting more than half of the surplus. Bob can also raise the amount $K \in [0, \overline{w}_I - w]$ to make Alice a more efficient retention offer.

First suppose that Bob's original offer (with $K = 0$) is enough to retain Alice, but with an unequal split of surplus, so that $\alpha_I^* > \beta_I^*$. Bob can then raise the amount $K > 0$ to buy additional profit shares from Alice, in order to better align team incentives. Alice's new payoff is then given by $\widetilde{\alpha}_I(K)\pi$, and Bob's by $\widetilde{\beta}_I(K)\pi$, where $\widetilde{\alpha}_I = (1 - \delta_I^*(K))\alpha_I^*(K)$ and $\widetilde{\beta}_I(K) = (1 - \delta_I^*(K))\beta_I^*(K)$. Alice accepts Bob's retention offer when

$$K = \widehat{\alpha}_I\mu(e_A e_C)\sigma - c_A(e_A) - \left[\widetilde{\alpha}_I(K)\mu(e_A(K)e_B(K))\pi - c_A(e_A(K))\right]. \tag{A.22}$$

Again we have $de_A(K)/dK < 0$ because $d\widetilde{\alpha}_I(K)/dK < 0$. Furthermore, the outside investor's stake $\delta_I^*(K)$ is defined by

$$\delta_I^*(K)\mu(e_A(K)e_B(K))\pi = (1 + r)K.$$

For any transfer $K \in [0, \overline{w}_I - w]$, the expected joint utility for Alice and Bob under individual asset ownership – with Bob's original retention offer ($K = 0$) being enough to keep Alice – is given by

$$\begin{aligned} U_A(K) + U_B(K) &= \widetilde{\alpha}_I(K)\mu(e_A(K)e_B(K))\pi - c_A(e_A(K)) + w + K \\ &\quad + \widetilde{\beta}_I(K)\mu(e_A(K)e_B(K))\pi - c_B(e_B(K)) - w. \end{aligned}$$

Using (A.22) we can write the expected joint utility as

$$U_A(K) + U_B(K) = \widehat{\alpha}_I\mu(e_Ae_C)\sigma - c_A(e_A) + \widetilde{\beta}_I(K)\mu(e_A(K)e_B(K))\pi - c_B(e_B(K)).$$

Again we note that $dU_A(K)/dK = 0$. In addition, the total surplus that is split between the two productive partners, $(1 - \delta_I^*(K))\pi$, is decreasing in K, where the profit share $\delta_I^*(K)$ goes to the (unproductive) outside investor. Thus, $d[U_A + U_B(K)]/dK < 0$. Consequently, $K_I^* = 0$ in case Bob's original retention offer ($K = 0$) can only retain Alice with an unbalanced split of surplus.

Next consider the case where Bob's original offer ($K = 0$) is not enough to retain Alice (in which case Bob's expected utility is zero). Raising $K > 0$ is then optimal for Bob when

$$U_B(K) = \widetilde{\beta}_I(K)\mu(e_A(K)e_B(K))\pi - c_B(e_B(K)) - w > 0.$$

Note that $U_B(0) = 0$ (in this case Bob cannot retain Alice, so he does not offer her the lump sum payment w). Moreover, from the above we know that $dU_A(K)/dK = 0$ and $d[U_A + U_B(K)]/dK < 0$, which implies that $dU_B(K)/dK < 0$. Thus, this condition is not satisfied for any $K > 0$, so that $K_I^* = 0$ in case Bob's original retention offer ($K = 0$) is insufficient to retain Alice.

The fundamental reason why outside investors cannot improve efficiency is that their return on investment must come from the profits of the venture. Giving the investors a share on those profits creates an inefficiency that is at least as large as the inefficiency that their investments are supposed to solve. It is worth noting that in our base model the venture does not generate any risk-free returns. A transfer of safe returns would not create those inefficiencies as they do not affect team incentives. It is possible to extend our base model to allow for some risk-free returns. This can be in the form of safe (interim or final) profits, or any fixed asset liquidation values. The main insight from such an extension is that we can add the risk-free returns to our measure of partner wealth. Specifically we can redefine a partner's effective wealth as the sum of exogenous wealth plus half of his/her share on the risk-free returns of the venture. In a renegotiation there are two ways that partners can use their respective shares of the risk-free returns. Suppose Bob wants to make a transfer to Alice. He can simply assign a part or all of his share on the safe returns to Alice, directly giving her a first claim on the safe returns. Alternatively, the two partners can raise some safe debt from an outside investor, and use the proceeds to pay Alice. In the latter case, outside investors can play a role, but again they cannot improve the outcomes that can be achieved by the two partners alone. In particular, the two partners prefer to internally reassign the safe returns without the help of an outside investor whenever there is a cost of raising outside funds (e.g. when $r > 0$).

Preferences – Ex-ante Asymmetric Outside Options.

W.l.o.g. we focus on Alice's preference for the allocation of control rights; Bob's preference is symmetric. Let $\overline{q}_i \equiv 1 - q_i$, $i = A, B$. Alice's expected utility at date 0 under joint asset ownership is

$$U_A^J = q_A q_B \frac{\sigma}{2} + (\overline{q}_A q_B + q_A \overline{q}_B + \overline{q}_A \overline{q}_B) \frac{\pi}{2}.$$

Likewise, Alice's expected utility at date 0 under individual asset ownership is

$$U_A^I = q_A q_B \frac{\sigma}{2} + q_A \overline{q}_B \frac{\sigma}{2} + \overline{q}_A \overline{q}_B \frac{\pi}{2}.$$

Alice prefers joint asset ownership if $U_A^J > U_A^I$, which is equivalent to

$$q_A < \widehat{q}_A = \left[\left(\frac{1 - q_B}{q_B} \right) \left(\frac{\sigma - \pi}{\pi} \right) + 1 \right]^{-1}.$$

Moreover, after some simplifications we find that

$$\begin{aligned} \frac{d\widehat{q}_A}{dq_B} &= \left[(1 - q_B) \left(\frac{\sigma - \pi}{\pi} \right) + q_B \right]^{-2} \left(\frac{\sigma - \pi}{\pi} \right) > 0 \\ \frac{d^2\widehat{q}_A}{dq_B^2} &= -2 \left[(1 - q_B) \left(\frac{\sigma - \pi}{\pi} \right) + q_B \right]^{-3} \left(\frac{\sigma - \pi}{\pi^2} \right) (2\pi - \sigma). \end{aligned}$$

Note that $d^2\widehat{q}_A/dq_B^2 < 0$ if $\pi > \sigma/2$, and $d^2\widehat{q}_A/dq_B^2 > 0$ if $\pi < \sigma/2$.