Scaling versus Selling Startups: The Role of Foreign Acquirers in Entrepreneurial Ecosystems*

Thomas Hellmann[†] and Veikko Thiele[‡]

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Abstract

This paper analyzes the decision of growing startups to either scale up on their own or to sell to an established company. The model recognizes the intergenerational linkages that acquirers were startups themselves in the past who chose not to get acquired. The acquisition price depends on the demand from those established acquirers, as well as the willingness of startups to sell instead of scale themselves. The model shows that in a closed economy, the number of scaleups is efficient. In an open economy, foreign buyers increase demand and raise acquisition prices. This stimulates startup formation but also encourages too many growing startups to sell instead of scale. In a dynamic equilibrium without externalities, foreign acquirers are a net benefit to the domestic ecosystem. However, two model extensions identify conditions under which they can weaken it, namely (i) when there are intergenerational externalities in the accumulation of scaleup experience, and (ii) when there is significant brain drain of serial entrepreneurs.

Keywords: Startups, scaleup, acquisitions, serial entrepreneurs, brain drain.

JEL classification: L26, M13.

†University of Oxford, Saïd Business School, Park End Street, Oxford OX1 1HP, United Kingdom, e-mail: thomas.hellmann@sbs.ox.ac.uk.

‡Queen's University, Smith School of Business, Goodes Hall, 143 Union Street, Kingston, Ontario, Canada K7L 3N6, e-mail: thiele@queensu.ca.

1 Introduction

While entrepreneurs typically dream of building large independent companies – "the next Google" – reality is different. Many startups fail, and even the successful ones rarely scale up to become large established companies. Instead, most successful startups get acquired by large established companies.[1]

There is an intergenerational interdependence where at an earlier point those established companies themselves chose to scale their companies instead of selling out. For example, a 2018 report by Mind the Bridge (2018) lists Google, Apple, and Facebook as the top three global acquirers of startups. These three companies are frequently hailed as entrepreneurial success stories who refused to sell out early. From an ecosystem perspective, there is a delicate balance. On the one hand, there is a benefit of having a lively acquisition market that allows entrepreneurs and their investors to cash out and move on. On the other hand, there is a desire to create "scaleups" that grow into large independent companies. They become domestic anchor companies who can then make acquisitions, thereby keeping the acquired startups' activities at home.

Outside the US, a large number of startups get acquired by foreign buyers, especially US buyers. Consider the example of Israel.[2] While called the "startup nation", there is a serious concern that the fruits of its entrepreneurial efforts largely go to foreign acquirers (Senor and Singer, 2009, Bordo, 2018). Policy makers deplore the lack of large domestic anchor companies, and worry about brain drain and the loss of entrepreneurial talent. Similar debates apply to Europe and Canada, where some of the most promising startups are scooped by foreign acquirers (Duruflé, Hellmann, and Wilson, 2018).

What are the benefits of foreign acquisitions to the domestic ecosystem? Supporters argue that the presence of foreign acquirers invigorates the domestic entrepreneurial ecosystem. Their acquisition payments infuse resources and generate returns to domestic entrepreneurs and investors. This increases the incentives to start ventures, and even frees up successful entrepreneurs to move on and start their next ventures. Critics, however, worry that foreign acquirers tilt the incentives towards selling early, rather than building more ambitious domestic anchor companies. They also worry about the loss to the domestic scaleup ecosystem, not to mention entrepreneurial brain drain.

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[1]A recent study by Catalini, Guzman, and Stern (2019) suggests that 86.7 percent of all exits of VC-funded US companies happen via acquisitions. For non-VC-backed companies the fraction is even higher at 93.3 percent.

[2]In a study of Israeli startups, Conti, Guzman, and Rabi (2021) find that 85.7 percent of all VC-backed exits in Israel happen via acquisitions. Among those, half are acquired by US companies, and another 16.7 percent are acquired by non-US foreign companies.

In this paper we examine how entrepreneurial ecosystems balance the tension between having enough young companies scaling up versus selling out. We build an equilibrium model with the aforementioned intergenerational linkages. We ask what economic forces shape the equilibrium in a domestic economy, and whether it is socially efficient. We then introduce foreign buyers and ask how they affect acquisitions prices and scaling decisions. This gets us to the central question of the paper, namely whether, or under what circumstances, the presence of foreign acquirers is a net benefit to the domestic entrepreneurial ecosystem.

For this we consider an infinite horizon steady-state economy, where generations of entrepreneurs move through different stages of company development. We examine their decisions to scale up their ventures or to sell out to (domestic or foreign) buyers. The model endogenizes the number of acquisitions and scaleups, and also the number of domestic anchor companies that are in a position to make acquisitions themselves. We derive the equilibrium in the acquisition market, and how it is affected by the presence of foreign buyers. Two important model extensions look at the role of intergenerational scaleup externalities, and brain drain among serial entrepreneurs.

More specifically, our base model first considers a closed economy where in each period there is a new generation of startups. Some of them fail early, other grow into young companies. At that stage the entrepreneurs can either sell their companies in a competitive acquisition market, or they can try to scale up their companies in the hope of growing into large established companies. If scaling is successful, they join the ranks of mature anchor companies. In our model these established companies form the set of potential domestic acquirers. Thus, our model captures an important intergenerational linkage, namely that the very companies that refused to sell when they were young, end up acquiring future generations of young companies.

Our first finding is that there exists a unique steady-state equilibrium. If initially there are too few companies scaling up, then acquisition prices remain low, discouraging young companies from selling, and thereby encouraging them to scale. This ensures a well-behaved dynamic feedback mechanism that ensures a unique steady-state equilibrium. The second finding is that this unique equilibrium is socially efficient. For this we use the standard utilitarian social welfare criterion of maximizing the sum of all domestic utilities.

We then introduce foreign buyers. The higher the number of potential foreign buyers, the higher the equilibrium acquisition price, and the higher the number of young companies selling out. This implies fewer companies scaling up, which in the long run also means that the number of domestic anchor companies remains lower.

The model generates two central insights that address some of the popular debates about the merits of foreign acquirers. First, relative to the first best, the presence of foreign acquirers leads

to too many young companies selling, and too few companies scaling. This finding echoes the concerns of critics who worry about the lack of domestic scaleups and anchor companies. Our second finding, however, is that the domestic welfare increases with the number of potential foreign buyers. This echoes the supporters' argument that foreign acquirers bring resources into the domestic economy. In a model extension with endogenous entry, we also show that the presence of foreign buyers increases incentives to start companies, thereby enlarging the overall size of the domestic entrepreneurial ecosystem. Thus, even though foreign acquirers tilt incentives from scaling to selling, their net contribution to the domestic ecosystem is positive.

Critics of foreign start-up acquisitions typically object to how they take resources away from the domestic ecosystem. In our base model foreign acquirers pay for what they take out of the ecosystem, i.e., the acquisition price adequately compensates for the loss of a domestic scaleup. The interesting question is thus whether their contribution remains net positive in the presence of ecosystem externalities. Naturally there is a long list of potential externalities to consider, we focus on two that both involve some form of intergenerational knowledge transfer. The first concerns the accumulation of scaleup expertise, the second serial entrepreneurs and brain drain.

One of the key reasons why Silicon Valley is widely considered the mecca not just for startups but increasingly for scaleups, is the accumulation of people who have successfully scaled ventures before (Hoffman and Yeh, 2018). There is an ample supply of venture investors, experienced managers (such as CFOs, CMOs, etc.), mentors, and specialized service providers (such as lawyers, consultants, etc.), who all support ambitious entrepreneurs who want to scale their ventures instead of selling early. We provide a simple "reduced-form" model extension that captures the benefits of such accumulated scaleup experiences in the ecosystem, and generate two main insights. First, unlike in our base model, the closed economy equilibrium is no longer efficient. There are too few scaleups because entrepreneurs do not internalize the ecosystem externalities they create for future generations. Second, foreign acquirers can now make a net negative contribution to the domestic ecosystem, because they further discourage young companies from scaling.

Our second model extension looks at the potential loss to the domestic ecosystem from entrepreneurial brain drain. Whereas the first extension focused on intergenerational knowledge accumulation at the scaleup stage, this second extension focuses on the startup stage. Serial entrepreneurs can play an important role for the ecosystem because they carry their accumulated experiences from one venture to the next. We therefore extend our base model to examine what happens when a fraction of entrepreneurs, who just sold their companies, proceed to launch their next startups. Again we begin with a closed economy. Serial entrepreneurship leads to more acquisitions and lowers their prices. It increases the size of the domestic ecosystem and

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domestic welfare. The equilibrium choice of young companies between selling versus scaling, however, remains socially efficient (baring any scaleup externalities, not considered in this extension). The final step then is to examine the effect of foreign buyers in the model with serial entrepreneurs. This brings us to the issue of entrepreneurial brain drain. Specifically, we focus on the decisions of serial entrepreneurs who sold their companies to foreign acquirers. In practise, these entrepreneurs are frequently required to spend some time in the foreign buyer's location after the acquisition. When they leave, they have a choice to build their next venture either in the new foreign location or in the old domestic location. The former implies "brain drain", the latter may be dubbed "brain regain". We find that a higher fraction of serial entrepreneurs succumbing to brain drain shrinks the domestic ecosystem and lowers domestic welfare. In the steady state, there are both fewer acquisitions and fewer scaleups. Most important, if brain drain is sufficiently strong, the introduction of foreign buyers lowers domestic welfare. The reason is that entrepreneurial brain drain is an uncompensated loss to the domestic ecosystem. Interestingly, the foreign buyer does not capture the value of serial entrepreneurship either, as it only pays for the value of the acquired company. The net beneficiary of entrepreneurial brain drain is actually the foreign ecosystem.

Overall, our analysis sheds new light on the debate about the ecosystem impact of foreign companies acquiring domestic entrepreneurial ventures. Our base model identifies a fundamental trade-off between the resources they contribute to the acquisition market (which also encourages entrepreneurial entry), versus the distortions they create discouraging young companies to scale. Our two extensions further explain that foreign acquisitions can deprive the domestic ecosystem of scaleup externalities, and lead to entrepreneurial brain drain.

The remainder of this paper is structured as follows. Section 2 situates our analysis in the prior literature. Section 3 introduces the base model. Section 4 derives the equilibrium in a closed economy without foreign buyers. Section 5 then examines the role of foreign acquirers in an open economy. Section 6 considers scaleup ecosystem externalities. Section 7 looks at serial entrepreneurship and brain drain. Section 8 discusses our findings and concludes. All formal proofs are in the Appendix.

2   Related Literature

Our paper builds on a diverse set of prior literatures. Clearly there is a very large literature on mergers and acquisitions, we are specifically interested in acquisitions of young companies by established incumbents. We build on the seminal work of Aghion and Tirole (1994), Gans

and Stern (2000, 2003) and Gans, Hsu, and Stern (2002), which explains why startups initially launch as independent entities but get acquired by incumbents at a later point. Our model is closely related to Arora, Fosfuri, and Rønde (2021) who develop a theory about the timing of acquisitions. They identify conditions where there are too few early acquisitions, due to incumbents underinvesting in absorption capabilities. Norbäck and Persson (2009) consider a similar trade-off where incumbents decide between early acquisitions of basic innovations versus later acquisitions of more developed ventures. They focus the role of venture capital to invest in the development of these basic innovations. In a related vein, Bena and Li (2018) identify innovation capabilities as a key driver for acquisition activities. Chondrakis, Serrano, and Ziedonis (2021) further argue that private acquisition markets lack transparency and provide evidence that better information disclosures lead to more private acquisitions. Cunningham, Ederer, and Ma (2021) find that acquisitions are used by incumbent firms to eliminate potential competitors. Bryan and Hovenkamp (2019) discuss the role of antitrust in shaping the acquisitions of startups. There is also a related line of research on corporate venture capital investments, and how they can lead to strategic acquisitions (see, e.g., Benson and Ziedonis (2010)).

From the perspective of a young company, the option of getting acquired has to be evaluated against its alternative. The prior literature often assumes that the alternative is an initial public offering (IPO). For example, the work of Ozmel, Robinson, and Stuart (2013) empirically examines strategic investments of incumbents into startups, and traces their effects on the exit outcome for startups in terms of acquisitions versus IPOs. Chemmanur, He, and Nandy (2018) also examine these exit outcomes and relate them to underlying company performance metrics such as sales growth and total factor productivity. This trade-off between acquisitions and IPOs implicitly adopts the perspective of investors who consider them as two alternative mechanisms for obtaining liquidity on their investments. From the perspective of a young company, however, this is a trade-off between getting acquired versus remaining independent and attempting to scale up.

To keep the model manageable there are no investors, implicitly assuming that all ventures are bootstrapped. However, the structure of the base model builds on the staged financing literature. The work of Admati and Pfleiderer (1994), Neher (1999), Cornelli and Yosha (2003), and Hellmann and Thiele (2022) explain the contractual foundations, whereas Nanda and Rhodes-Kropf (2013, 2017) and Hellmann and Thiele (2015) examine market dynamics of the staged financing process. The key trade-off in this literature is whether a company gets further funding or not. Obtaining further funding can be equated with scaling up, whereas receiving no further funding can be interpreted either as failure, or as an early acquisition.

An interesting question is what scaling up actually entails. Shepherd and Patzelt (2020) argue that the literature on scaleups is still underdeveloped. In their literature survey, Demir, Wennberg, and McKelvie (2017) identify five drivers of high growth, namely human capital, strategy, human resource management, innovation, and capabilities. Coad, Cowling, and Siepel (2017) empirically disentangle alternative sources of venture growth. Nielsen (2018) focuses on business model scalability to explain scaling. Coviello (2019) further argues that there is more to scaling up than merely growing fast. Scaling up occurs at certain stages of company development, leverages economies of scale, and involves the transformation of a company's processes, people, and places. Gulati and De Santola (2016) also provide a detailed discussion of the specific challenges of scaling up. Finally, List (2022) provides a behavioral economics perspective on scaling processes.

Parallel to this nascent academic literature on scaling, several industry reports discuss the scaleup challenge. Coutu (2014) and Hellmann and Kavadias (2016) identify the hurdles that need to be overcome to improve the UK scaleup ecosystem. Duruflé, Hellmann, and Wilson (2018) look more broadly at Europe and Canada. All of these reports recognize the trade-off between scaling up versus getting acquired, and especially the attraction of selling to large US buyers. The US, especially Silicon Valley, is commonly recognized as the most powerful ecosystem for scaling young companies. Among others, Reid Hoffman, founder of LinkedIn, makes this argument in his book on "Blitzscaling" (Hoffman and Yeh, 2018).

Our analysis specifically looks at the ecosystem impact of foreign acquisitions. This is related to the broader literature on foreign direct investments. For example, Brandstetter and Saggi (2011) examine the role of foreign direct investments and intellectual property rights in a North-South equilibrium model. Closer to our context, Bertoni and Groh (2014) study venture capital financed startups in Europe and examine the likelihood they get acquired by foreign buyers. They find that prior investments by foreign venture capitalists increase the likelihood of foreign acquisitions. Conti, Guzman, and Rabi (2021) provide an analysis of Israeli startups. They find informational spillovers where the foreign acquisition of one Israeli startup triggers the interest of other foreign buyers in related industries to also acquire Israeli startups. There is also a related literature on cross border investing, including the work of Devigne et al. (2018) and Bradley et al. (2019). Finally, Jones, Coviello, and Tang (2011) and Monaghan, Tippmann, and Coviello (2020) provide comprehensive overviews of the literature on the internationalization of startups.

While a prior literature examines the drivers of foreign acquisitions, it rarely addresses the anxieties of domestic policy makers to let go of their country's most promising startups.

For example, Phil Hogan, EU trade commissioner, provided the following guidance on FDI screening regulations on April 16, 2020:[3]

> *The first and immediate [concern] is about protecting the EU's strategic assets.*
> *[...] Economic vulnerability could result in a sell-off of critical infrastructure or*
> *technologies [...] Today more than ever, the EU's openness to foreign investment*
> *needs to be balanced by appropriate screening tools.*

Lunden (2015) describes how the French government prevented the sale of Dailymotion, one of France's most prominent startups that directly competes with YouTube. The government intervened in the company's proposed sale twice, first to Yahoo, and then to Hong Kong-based PCCW. Eventually the government supported its acquisition by Vivendi, a French company, at a disappointingly low price. In the UK, Herman Hauser, the founder of ARM, which is arguably the most successful UK tech startup to date, openly called it a "sad day" when ARM was acquired by Japanese Softbank (BBC, 2016).

In this paper we look at foreign acquisitions from an ecosystem perspective. This recognizes that entrepreneurship is embedded in its local environment and puts an emphasis on understanding the complex interconnections within ecosystems. Ács, Autio and Szerb (2014) discuss how innovations occur within national systems. Autio et al. (2018) provide an overview of the recent research on entrepreneurial ecosystems. The work of Glaser, Kerr and Ponzetto (2010) and Delgado, Porter, and Stern (2010) identify fundamental agglomeration drivers that underlie these entrepreneurial ecosystems.

As part of the ecosystem perspective, our paper emphasizes intergenerational linkages, notably the fact that young companies that choose to scale (rather than getting acquired) become the potential acquires of the future. Hellmann and Thiele (2019) discuss the broader importance of intergenerational linkages in entrepreneurial ecosystems. Their model focusses specifically on the transition of successful entrepreneurs from one generation, to becoming the mentors and angel investors for the next generation of entrepreneurs. In a similar spirit, this paper considers the accumulation of experience within an ecosystem. This builds on the seminal work of Saxenian (1994) which describes the human capital and networks of expertise that underlie the success of Silicon Valley. Departing employees of successful ventures that got acquired, can become an important source of talent for startups and scaleups alike, the so-called "PayPal Mafia" being a famous example (Forrest, 2014). Of particular note are serial entrepreneurs who

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[3]https://ec.europa.eu/commission/commissioners/2019-2024/hogan/announcements/introductory-statement-commissioner-phil-hogan-informal-meeting-eu-trade-ministers_en

recycle their experience and knowledge from one venture to the next. Gompers et al. (2010) find performance persistence across the ventures started by serial entrepreneurs. Lafontaine and Shaw (2016) examine the role of learning for serial entrepreneurs. Hsu (2007) finds that serial entrepreneurs are more successful raising funds from venture capitalists.

Our analysis links the phenomenon of serial entrepreneurship to brain drain. For this we focus on the question whether the founders of companies acquired by foreign buyers, start a new company in the foreign location (brain drain) or return to their home location (brain regain). We build on the prior brain drain literature, surveyed by Docquier and Rapoport (2012). The work of Kerr et al. (2017) and Kerr (2018) further documents the interrelationship between migration and agglomeration. Saxenian (2006) provides an ethnographic study of how Silicon Valley is connected to international networks of entrepreneurs. Nanda and Khanna (2010) document how in India diaspora entrepreneurs differ from their domestic counterparts. Agrawal et al. (2011) examine the trade-off between the loss of innovative talents through brain drain versus the access to advanced knowledge networks, what they call the brain bank. Finally, Conti and Guzman (2021) empirically trace the location decision of Israeli entrepreneurs. They argue that the comparative advantage of moving to the US is explained by the better availability of investors, as well as large consumer and acquisition markets.

3   Base Model

We consider an infinite horizon economy with discrete dates. Figure 1 provides an overview of the model dynamics, which we explain below.

Each date represents the start of a new generation of startup companies. The number of startups at date t is denoted by $n_{S,t}$, where the subscript S refers to startups. The number of startups is exogenous in our base model, but we will endogenize entry of startups in Section 5.3.

At date $t+1$ there is a new generation of startups ($n_{S,t+1}$). Moreover, the startups from generation t have either failed, which happens with probability $(1-\rho)$, or they have succeeded and progressed to become "young companies", which happens with probability ρ. Using the subscript Y for young companies, there are $n_{Y,t+1} = \rho n_{S,t}$ young companies in the economy at date $t+1$.

Young companies face a choice between scaling as an independent company (indicated by the superscript sc), or selling out in an acquisition as described below (indicated by the superscript a). The cost of scaling is given by c_{t+1}, which is a random variable drawn from a
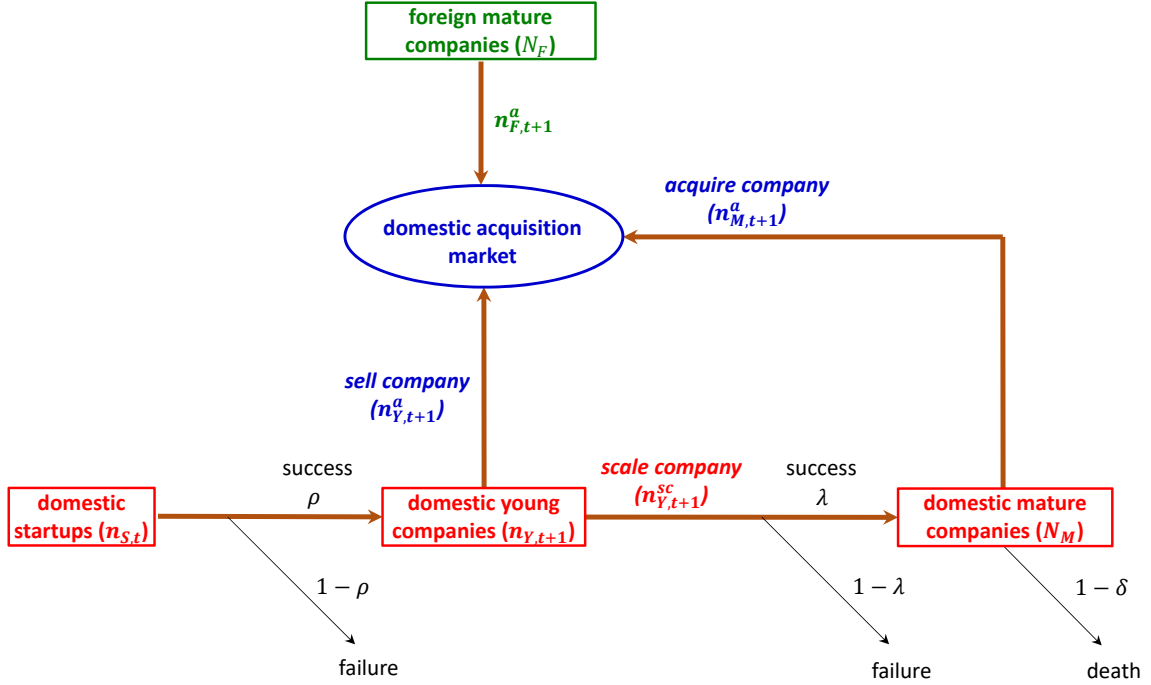
Figure 1: Model Dynamics

uniform distribution over the interval $[0, \bar{c}]$. If a young company chooses to scale, it succeeds with probability λ and becomes a mature company at date $t + 2$. With probability $(1 - \lambda)$ the scaling fails.

A mature company survives the next period with probability δ, and dies with probability $(1 - \delta)$. While alive, a mature company earns an exogenous core profit Ψ in every period. In addition, it may enter the acquisition market to buy a young company, as described below. Using the subscript M for mature companies, we denote the total number of mature companies at date $t + 2$ by $N_{M,t+2}$, given by the following dynamic equation:

$$N_{M,t+2} = \delta N_{M,t+1} + \lambda n_{Y,t+1}^{sc}.$$

Any company (startup, young, or mature) that fails, is assumed to be worthless, i.e., its payoff is zero. Moreover, for parsimony we assume zero discounting.

We now describe the acquisition market in more detail. In every period t the market for acquisitions consists of an endogenous number of young companies as potential sellers $n_{Y,t}^{a}$, and an endogenous number of mature companies as potential buyers. To begin with we consider a closed economy where all potential acquirers N_M are domestic. The equilibrium number of

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actual acquirers is denoted by $n^a_{M,t}$. We assume that the market for acquisitions is perfectly competitive and denote the acquisition price in period t by P_t.

We assume that each potential acquirer can make one acquisition at most. A domestic acquirer's strategic benefit from an acquisition is π^d. Moreover, each potential acquirer has an acquisition cost θ_t, which is a random variable drawn from a uniform distribution over the interval $[0, \overline{\theta}]$. We can think of this as the cost of making the transaction and absorbing the acquired unit. In addition, θ_t captures variations in how well the acquired company strategically fits with the buyer.

In Section 5 we examine an open economy with foreign acquirers. We model them in a similar way to domestic acquirers but with two important differences. First, we assume that the stock of foreign potential acquirers, denoted by N_F, is exogenous. This follows the long tradition of studying "small open economies". It generates a comparative statics analysis that sheds light on the effect of foreign acquirers on the domestic market equilibrium. Second, we allow foreign acquirers to have their distinct model parameters. Specifically, let π^f be the strategic benefit for foreign acquirers, and ϕ_t their acquisition costs, where ϕ_t is a random variable drawn from a uniform distribution over the interval $[0, \overline{\phi}]$. Finally, while the stock of potential acquirers N_F is exogenous, the number of actual acquirers, denoted by $n^a_{F,t}$, is endogenous.

4   Closed Economy

In this section we examine the benchmark case of a closed economy where only domestic mature companies can acquire young companies. We defer the question of how foreign acquirers affect the domestic acquisition market, until Section 5.

4.1   Market Equilibrium

We first derive the market equilibrium for the closed economy with $N_F = 0$. For the steady-state equilibrium we can omit the time subscript t. The equilibrium in the acquisition market is determined by two participation constraints. On the supply side, young companies want to sell whenever the acquisition price exceeds the expected utility of scaling. Let U_M be the expected utility of becoming a mature company (we derive U_M below). Young companies therefore sell whenever

$$P \geq \lambda U_M - c \quad \Leftrightarrow \quad c \geq c^* \equiv \lambda U_M - P.$$

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We call c^* the scaling cost ceiling, below which young companies prefer to scale, and above which young companies choose to sell. The number of young companies that prefer to scale, is therefore given by

$$n_Y^{sc} = \frac{c^*}{\overline{c}} n_Y = \frac{c^*}{\overline{c}} \rho n_S. \tag{1}$$

Likewise, the number of young companies that choose to sell is given by

$$n_Y^a = \left(1 - \frac{c^*}{\overline{c}}\right) n_Y = \left(1 - \frac{c^*}{\overline{c}}\right) \rho n_S. \tag{2}$$

A higher acquisition price P makes selling more attractive, and scaling less so. Formally, the scaling cost ceiling c^* is decreasing in P (below we show that U_M is decreasing in P). Consequently, the number of young companies that prefer to sell is increasing in P.

On the buyer side, mature companies choose to make an acquisition whenever

$$\pi^d - P - \theta \geq 0 \quad \Leftrightarrow \quad \theta \leq \theta^* \equiv \pi^d - P.$$

We call θ^* the acquisition cost ceiling below which buyers choose to acquire a young company.

The number of active buyers also depends on the stock of domestic mature companies N_M. In the steady state this does not change, i.e., the inflow (successful scaleups) must be equal to the outflow (death of some mature companies): $\lambda n_Y^{sc} = (1 - \delta)N_M$.[4] In steady state we obtain

$$N_M = \frac{1}{1 - \delta} \lambda n_Y^{sc}.$$

The number of active buyers in the acquisition market is therefore given by

$$n_M^a = \frac{\theta^*}{\overline{\theta}} N_M = \frac{1}{1 - \delta} \frac{\theta^*}{\overline{\theta}} \lambda n_Y^{sc}. \tag{3}$$

A higher price P reduces the net benefit of an acquisition ($\pi^d - P - \theta$), which reduces demand from mature companies. Moreover, we can see from (1) that a higher acquisition price results in less scaling, i.e., n_Y^{sc} is decreasing in P. As a result, the number of active buyers is also decreasing in the acquisition price P.

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[4]To ensure that some mature companies exist in the steady state ($N_M > 0$), we assume that at least some young companies choose to scale, even for the highest possible acquisition price $P = \pi^d$ (so that $n_M^a > 0$). Formally we assume that $\lambda U_M(P = \pi^d) > \pi^d$. Using the expression of U_M as derived below, it is easy to see that this condition is equivalent to $\frac{1}{1-\delta} \lambda \Psi > \pi^d$.

A key feature of the model is the presence of intergenerational linkages. The expected utility of scaling today depends on the future rents from making acquisitions as a mature company, and therefore also on the future price of acquisitions. Using the general time-varying specification, the expected utility of becoming a mature company is given by the following expression:

$$U_{M,t} = \Psi + \int_0^{\theta_t^*} \left(\pi^d - P_t - \theta_t\right) \frac{1}{\bar{\theta}} d\theta_t + \delta U_{M,t+1}.$$

In the steady state (where $P_t = P_{t+1} = ... = P_{t+n}$, $U_{M,t} = U_{M,t+1} = ... = U_{M,t+n}$, etc.) this simplifies to

$$U_M = \frac{1}{1-\delta} \left[\Psi + \frac{1}{\bar{\theta}} \left((\pi^d - P)\theta^* - \frac{1}{2}(\theta^*)^2\right)\right] = \frac{1}{1-\delta} \left[\Psi + \frac{1}{2}\frac{1}{\bar{\theta}} \left(\pi^d - P\right)^2\right]. \quad (4)$$

We can see that U_M is decreasing in P. This is because a higher acquisition price diminishes the rents from acquisitions ($\pi^d - P$).

Finally, the equilibrium condition for the acquisition market is given by

$$n_M^a = n_Y^a. \quad (5)$$

We already noted that n_M^a is decreasing in P, and n_Y^a is increasing in P. To ensure that at least the high-cost young companies (with $c \to \bar{c}$) choose to sell (so that $n_Y^a > 0$), even for $P \to 0$, we assume that $\lambda U_M(P = 0) < \bar{c}$, which is equivalent to $\bar{c} > \lambda \frac{1}{1-\delta} \left[\Psi + \frac{1}{2}\frac{1}{\bar{\theta}} \left(\pi^d\right)^2\right]$. We then have a well-behaved demand and supply system in the acquisition market. In other words, there exits a unique acquisition price which satisfies the market clearing condition (5). The next lemma formally states this.

Lemma 1 *There exists a unique steady-state equilibrium in the acquisition market with $P^* \in (0, \pi^d)$.*

Markets with intertemporal dynamics often have multiple steady-state equilibria. In this model, however, there is a single steady-state equilibrium. The intuition is as follows. Suppose that, to begin with, there are no mature companies, and therefore no buyers in the acquisition market. Young companies can only scale and not sell. Some of them grow to become mature companies, which kick-starts the acquisition market. Initially there are few buyers, so acquisition prices are still low. This makes it relatively unattractive for young companies to sell, so more companies scale and join the ranks of mature companies. This further grows the stock

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of potential buyers, and thus raises acquisition prices. Eventually the acquisition price reaches its steady state. To see this, consider the possibility that the acquisition price were to exceed its steady-state level. In this case many young companies prefer to sell rather than scale. Over time this reduces the number of mature companies and thus the number of potential buyers, thereby reducing the acquisition price. This explains how the economy converges to the unique steady-state equilibrium.

The next proposition examines some comparative statics of this unique steady-state equilibrium. We focus our attention on the scaleup parameter λ, which measure how conducive the external environment is towards scaleups.

Proposition 1 *A higher probability of scaleup success λ leads to*

- – *a higher equilibrium acquisition price P^*,*
- – *a higher scaleup cost ceiling c^*,*
- – *a larger number of scaleups n_Y^{sc*},*
- – *a smaller number of early acquisitions n_Y^{a*},*
- – *a lower acquisition cost ceiling θ^*, and*
- – *a larger number of domestic mature companies N_M^*.*

The main intuition for these results is as follows. A higher value of λ clearly makes scaling more attractive, and therefore increases the scaleup cost ceiling c^*. More young companies then choose to scale, increasing n_Y^{sc*}. Consequently, fewer young companies get acquired, i.e., n_Y^{a*} decreases. All this tilts the balance in the acquisition market towards fewer sellers (lower n_Y^{a*}) and more mature companies as potential buyers (where the higher N_M^* comes from the higher n_Y^{sc*}). As a result, the equilibrium acquisition price P^* rises, which also explains the buyer's lower acquisition cost ceiling θ^*.

4.2 Efficiency Analysis

We now examine the efficiency properties of the steady-state equilibrium. Specifically we ask whether the equilibrium scaleup cost ceiling of young companies (c^*) is equal to the socially optimal level, which we denote by c^{fb}. In other words, we want to know whether in equilibrium too many young companies decide to sell early, or choose to scale.

Suppose a social planer can force young companies with sufficiently low scaling costs $c \leq c^{fb}$ to scale (getting the expected utility $\lambda U_M - c$), and young companies with sufficiently high

scaling costs $c > c^{fb}$ to sell (getting the acquisition price P^*). In the Appendix we formally show that the steady-state domestic welfare is then given by

$$W^d(c^{fb}) = n_S U_S(c^{fb}),$$

where $U_S(c^{fb})$ is the expected steady-state utility of a startup company in the ecosystem given the social planer's choice of c^{fb}. Formally, $U_S(c^{fb})$ is defined as

$$U_S(c^{fb}) = \rho \left[\int_0^{c^{fb}} (\lambda U_M - c) \frac{1}{\bar{c}} dc + \int_{c^{fb}}^{\bar{c}} P \frac{1}{\bar{c}} dc \right]. \tag{6}$$

Our social welfare criterion maximizes the sum of all ex-ante expected utilities in the steady state, which is a standard (Bergson-type) utilitarian social welfare function. The next proposition compares the socially efficient scaling decision of young companies (as defined by c^{fb}) against the actual scaling decision in the market equilibrium (as reflected by c^*).

Proposition 2 *The scaling decisions in the steady-state equilibrium in a closed economy occur at the socially efficient level, i.e., $c^* = c^{fb}$.*

This is a significant result because it shows that despite its complex intergenerational linkages, a competitive acquisition market in a closed economy generates the socially efficient balance between acquisitions and scaleups of young companies. At first glance this result is surprising because when a young company decides to scale today and then succeeds, it will push up the acquisition price tomorrow (as more mature companies are now available to make acquisitions). The scaling decision today therefore benefits the next generation of young companies that choose to sell, as they now get a higher acquisition price. On the other hand, the decision to scale also costs the next generation of young companies that choose to scale, as they now have to pay higher prices when making acquisitions as mature companies. It turns out that the equilibrium scaling cost ceiling c^* exactly offsets these two effects. It therefore maximizes the ex-ante expected utility of a startup U_S. Domestic welfare, which is measured by the sum of all ex-ante expected utilities, is therefore also maximized at c^*.[5]

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[5]In the Appendix we also show that the domestic welfare, W^d, is an increasing function of λ. This simply says that a better scaleup environment has a positive effect on domestic welfare.

5 Open Economy with Foreign Acquirers

We now consider an open economy where foreign mature companies can also acquire domestic young companies (i.e., $N_F > 0$). Our main focus is on how the presence of foreign acquirers affects the domestic acquisition market, the scaling decision of young companies, and ultimately domestic welfare.

5.1 Market Equilibrium

A mature foreign company wants to buy a domestic young company whenever

$$\pi^f - \phi - P \geq 0 \quad \Leftrightarrow \quad \phi \leq \phi^* \equiv \pi^f - P.$$

The threshold ϕ^* is the acquisition cost ceiling for foreign companies below which they choose to buy a young domestic company. The number of foreign buyers in the domestic acquisition market is thus given by

$$n_F^a = \frac{\phi^*}{\overline{\phi}} N_F. \tag{7}$$

To ensure that at least some foreign companies enter the domestic acquisition market, we assume that $\pi^f \geq \pi^d$ (so that $P^* < \pi^d, \pi^f$).

With domestic as well as foreign buyers, the equilibrium condition for the acquisition market is now given by

$$n_M^a + n_F^a = n_Y^a. \tag{8}$$

The finding of Lemma 1 about a unique steady-state equilibrium continues to apply in the open economy model. In the Appendix we also show that the comparative statics results for λ in Proposition 1 continue to apply. The main new comparative statics thus concerns N_F, the stock of potential foreign acquirers.

Proposition 3 *A higher number of potential foreign buyers N_F leads to*

  – *a higher equilibrium acquisition price P^*,*

  – *a lower scaleup cost ceiling c^*,*

  – *a smaller number of scaleups n_Y^{sc*},*

  – *a larger number of early acquisitions n_Y^{a*},*

  – *a lower acquisition cost ceiling θ^* for mature domestic companies, and*

– *a smaller number of domestic mature companies N_M^*.*

The key intuition is that foreign buyers tilt the trade-off between selling and scaling. Adding more buyers naturally raises the equilibrium acquisition price P^*. This lowers the scaleup cost ceiling c^* and leads to more acquisitions n_Y^{a*}, fewer scaleups n_Y^{sc*}, and thus also fewer mature domestic companies N_M^*. The higher price P^* also lowers the acquisition cost ceiling θ^* for mature domestic companies.

5.2 Efficiency Analysis

We now consider the efficiency properties of the equilibrium with foreign acquirers. We take a domestic perspective that ignores any value to foreign entities. This is not to endorse nationalistic thinking, but to focus on how domestic policy makers and ecosystem participants are likely to look at the market outcome.

We divide our efficiency analysis into two main steps. First we ask whether young companies still make the efficient scaling versus selling decision (as in the closed economy), or whether the presence of foreign acquirers alters this decision. Second we ask whether the presence of foreign acquirers enhances or diminishes domestic welfare.

Proposition 3 in Section 5.1 shows that foreign buyers boost the price for acquisitions. A higher acquisition price clearly benefits young companies that choose to sell. The question is whether the possibility of getting acquired by a foreign buyer distorts the efficiency of the scaling versus selling decision of domestic young companies.

Proposition 4 *Relative to the first-best scaleup cost ceiling c^{fb}, the competitive cost ceiling in the open economy, c^*, is too low, i.e., $c^* < c^{fb}$. This results in too few scaleups and too many acquisitions of young companies.*

This result is an important finding as it lends credibility to the popular criticism that many young companies sell too early, instead of pursuing more ambitious scaling strategies. The higher acquisition price, caused by additional foreign buyers, makes selling more attractive relative to scaling. Moreover, selling companies do not take into account the domestic social benefit of scaling, i.e., that they enrich the domestic ecosystem by becoming a mature company that generates value and makes acquisitions in its own right.[6]

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[6]The proof of Proposition 4 in the Appendix includes one additional step. It checks whether this result is driven by the focus on domestic welfare. Indeed, we find that the competitive market outcome c^* actually maximizes global welfare, i.e., the combined domestic and foreign welfare.

In light of Proposition 4, it may be tempting to conclude that foreign acquirers are the source of the problem of having too few scaleups, and that their access to the domestic market ought to be limited. It turns out that this logic is flawed – the next proposition explains why.

Proposition 5 *The presence of foreign buyers increases domestic welfare, i.e., W^d is an increasing function of N_F. Specifically, $\left.\frac{dW^d}{dN_F}\right|_{N_F=0} = 0$ and $\left.\frac{dW^d}{dN_F}\right|_{N_F>0} > 0$.*

Proposition 5 constitutes a central result for our analysis of acquisitions by foreign companies. Even though Proposition 4 identified an inefficiency resulting from foreign buyers, their net impact on the domestic economy remains unambiguously positive. The intuition is that foreign buyers pay the (inflated) market price for the acquisitions they make. The proceeds flow into the domestic economy. This benefit outweighs the cost of distorting the scaling decision of young companies.[7]

5.3   Endogenous Entry

The domestic welfare argument so far is based on a rent capture argument: more young companies prefer selling over scaling because the higher acquisition price is attractive. We now consider an important extension of this argument where higher acquisition prices not only increase the rents to the existing entrepreneurs, but also encourages more people to become entrepreneurs in the first place. This requires endogenizing the level of entrepreneurial activity in the economy, which we do in this section.

Suppose that at the beginning of each period there is a pool of potential entrepreneurs who consider starting a company. They face an entry cost l that is drawn from a uniform distribution over the interval $[0, \bar{l}]$. This represents the cost of launching the venture and/or the opportunity cost of foregoing alternative options. In the presence of such entry costs, the entry decision of entrepreneurs is governed by the following simple participation constraint:

$$U_S - l \geq 0 \quad \Leftrightarrow \quad l \leq l^* \equiv U_S.$$

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[7]It is worth noting that in the neighborhood of $N_F = 0$, the increase in W^d is very small. This says that the welfare benefit of introducing the first foreign acquirers is only of second order. Near $N_F = 0$ the acquisition price is still close to its equilibrium level in the closed economy, so that foreign acquirers pay close to the social value of the acquired companies. It is only as the number of foreign acquirers rises that acquisition prices rise substantially higher. This is when the injection of resources from foreign acquisitions becomes more attractive to the domestic economy. Hence $\frac{dW^d}{dN_F} > 0$ for $N_F > 0$.

The endogenous number of startups that enter every period, is thus given by

$$n_S = \frac{l^*}{\bar{l}}.$$

In the Appendix we characterize the steady-state equilibrium for the open economy model with endogenous entry. We then show the following.

Proposition 6 *The equilibrium number of startups founded in each period, n_S^*, is increasing in λ and N_F. Moreover, the domestic welfare W^d is increasing in both λ and N_F.*

The intuition behind the results from Proposition 6 is as follow. A more attractive scaleup environment (higher λ) benefits young companies. As a consequence starting a company becomes more attractive. This increases the endogenous number of startups, and therefore domestic welfare. An increase in potential foreign acquirers (higher N_F) increases the equilibrium price of acquisitions. This also benefits young companies, so the same logic applies.

6   Scaleup Externalities

So far our model considers an ecosystem with intergenerational linkages where young companies that refuse to get acquired may become future acquirers. An elegant but also limiting property of the model is that the equilibrium does not depend on the size of the ecosystem. If we doubled its size, the equilibrium price P^* and the marginal decision thresholds c^* and θ^* all remain the same. As a model extension we now consider another intergenerational linkage, namely the role of accumulated experience in the ecosystem. The seminal work of Saxenian (1994) describes how a rich set of symbiotic relationships developed in Silicon Valley as it grew in size, noting how experienced entrepreneurs and managers bolstered the next generation of ventures. Here we are particularly interested in how scaleups benefit from the accumulated experiences of prior generations of successful scaleups. We therefore extend our model to allow scaleups to learn from the cumulative experience of those who came before them.

To model intergenerational scaleup externalities we use a tractable "reduced-form" specification where the cost of scaling depends on the cumulative experience of successful scaleups from earlier generations. Specifically we assume that c_t is now drawn from a uniform distribution over the interval $[0, \overline{C}(N_{M,t})]$, where

$$\overline{C}(N_{M,t}) = \frac{1}{1 + \omega N_{M,t}} \bar{c}.$$

Recall that $N_{M,t}$ measures the stock of mature companies, this provides a useful measure of prior cumulative scaleup experience.[8] The above expression simply says that larger values of $N_{M,t}$ reduce the cost of new scaleups. For tractability we chose this functional form where higher values of $N_{M,t}$ lower the upper boundary of the uniform distribution, thereby lowering the expected cost for all new scaleups. The parameter ω is a scalar that measures the strength of the cumulative experience effect. Our base model effectively assumed $\omega = 0$, we now examine the equilibrium with $\omega > 0$.[9] For this we first focus on the efficiency of the scaling decision in a closed economy.

Proposition 7 *In the steady-state equilibrium of a closed economy with $\omega > 0$, too few young companies choose to scale instead of sell, i.e., $c^* < c^{fb}$.*

Proposition 7 is the equivalent of Proposition 2 in the base model. It shows that, in the presence of scaleup externalities, the closed economy equilibrium no longer achieves the optimal balance between scaling and selling. The reason why too few young companies scale up is that they do not internalize the externalities that they generate for future generations of scaleups.

Combining the results from Propositions 4 and 7, it comes as no surprise that in an open economy with $N_F > 0$, there are again too few young companies that scale, i.e., $c^* < c^{fb}$. The interesting question becomes how Proposition 5 changes for $\omega > 0$. For tractability reasons we focus on the case of small N_F, thereby asking what happens to the domestic equilibrium as the first foreign acquirers enter.

Proposition 8 *In the steady-state equilibrium with scaleup externalities ($\omega > 0$), the introduction of foreign acquirers has a negative effect on domestic welfare, i.e., $\left. \frac{dW^d}{dN_F} \right|_{N_F=0} < 0$.*

This result shows that, in a neighborhood of $N_F = 0$, an increase in foreign acquirers reduces domestic social welfare. The reason is that foreign acquirers further distort the balance between scaling and selling. Given that the domestic economy already has too few scaleups in the presence of scaleup externalities, foreign acquirers aggravate this problem, thereby decreasing domestic welfare. Proposition 7 stands in direct contrast to Proposition 5, as it identifies circumstances where the introduction of foreign acquirers is not desirable for the domestic

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[8]To be precise, $N_{M,t}$ measures the sum of all prior successful scaleups, minus attrition from the periodic death rate $(1 - \delta)$.

[9]While the cumulative experience affects the dynamic behaviour of the model, we can still focus on its steady-state equilibrium. Unlike in the base model, however, equilibrium properties, such as the acquisition price P^*, now depend on the size of the domestic ecosystem.

ecosystem. Foreign acquirers discourage young companies from scaling, further depriving the ecosystem of the positive externalities associated with a larger accumulation of scaleup experiences.[10]

7 Serial Entrepreneurship

7.1 Equilibrium and Efficiency in a Closed Economy

The previous section shows how intergenerational linkages at the scaleup stage can generate externalities that affect the desirability of foreign acquirers. In this section we consider an alternative extension of the base model with a different type of intergenerational linkages. In particular, we consider the role of brain drain in the context of foreign acquisitions with serial entrepreneurs. A prior literature suggests that serial entrepreneurs play a vital role in entrepreneurial ecosystems. Their accumulated experience and networks help to attract funding and build more successful ventures (Hsu, 2007, Gompers et al., 2010). In the context of foreign acquisitions, the question is not only *whether* founders become serial entrepreneurs after an acquisition, but also *where*?

An acquired team is often asked to move to the acquiring company's headquarters. In case of a foreign acquisition this means moving abroad. Founders are often contractually required to remain with the acquiring company for some time. The interesting question then is what the founders do after completing their contractual obligations? Do they start another company or not? And if so, do they start the company in their old home country or in the new foreign location?

We are interested in whether foreign acquisitions still benefit the domestic ecosystem when there is brain drain of serial entrepreneurs. Consider first the benchmark case of domestic acquisitions in a closed economy. The simplest possible specification uses an exogenous probability $\xi \in [0,1]$ that a founder becomes a serial entrepreneur after an acquisition.[11] Serial

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[10]On a technical note, it is useful to remember that Proposition 5 established that $\frac{dW^d}{dN_F} = 0$ at $N_F = 0$. This says that, in a neighborhood of $N_F = 0$, the welfare gains to having more foreign acquirers are only of second-order. However, with $\omega > 0$, any distortions to the balance between scaling and selling is a first-order effect. This explains why $\frac{dW^d}{dN_F} < 0$ in a neighborhood of $N_F = 0$.

[11]The simplest way of justifying this is to assume that after an acquisition the entrepreneur receives a random draw for the cost of starting another venture. With probability ξ this cost is zero, and with probability $(1 - \xi)$ it is very large. Note also that for simplicity, the only types of serial entrepreneurs are those who sold young ventures. Further allowing scaleup entrepreneurs (i.e., those who led their companies all the way to maturity) to also become serial entrepreneurs would complicate the model without adding significant additional insights.

entrepreneurs have valuable prior experience, so their probability of success is now given by $(1+\kappa)\rho$ where $\kappa \in [0, \frac{1-\rho}{\rho}]$.

The option to become a serial entrepreneur changes the trade-off between scaling, which requires staying with the current venture, and selling, which opens up the possibility of serial entrepreneurship. Specifically, the scaling condition now becomes

$$\lambda U_{M,t+1} - c_t \geq P_t + \xi(1+\kappa)U_{s,t+1} \quad \Leftrightarrow \quad c_t \leq c_t^* = \lambda U_{M,t+1} - P_t - \xi(1+\kappa)U_{s,t+1}.$$

The last term on the right hand side captures the expected utility of being a serial entrepreneur, which is $(1+\kappa)$ times the expected utility of a first-time entrepreneur.

In the Appendix we rederive the equilibrium with serial entrepreneur. The next proposition shows how serial entrepreneurship affects the market equilibrium and welfare in a closed economy. For this it is useful to use the amalgamated serial entrepreneurship parameter $\widetilde{\xi} \equiv (1+\kappa)\xi$, which measures both the incidence (ξ) and the performance advantage (κ) of serial entrepreneurs.

Proposition 9 *Serial entrepreneurship in a closed economy has the following effects:*

(i) *A higher serial entrepreneurship parameter $\widetilde{\xi}$ implies*

- *a lower acquisition price P^*,*
- *a lower scaleup cost ceiling c^*,*
- *a larger number of early acquisitions n_Y^{a*}, and*
- *a higher acquisition cost ceiling θ^* for mature domestic companies.*

(ii) *The competitive steady-state scaling decisions occur at the socially efficient level, i.e., $c^* = c^{fb}$.*

(iii) *A higher serial entrepreneurship parameter $\widetilde{\xi}$ implies a higher domestic welfare W^d (for $\widetilde{\xi}$ sufficiently small).*

Part (i) of Proposition 9 says that the option of becoming a serial entrepreneur makes selling a young company relatively more attractive. This lowers the scaleup cost ceiling c^*, increases the number of early acquisitions n_Y^{a*}, and consequently lowers the acquisition price P^*.[12] Part (ii) of Proposition 9 establishes that the previous result from Proposition 2, namely that the

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[12]In the Appendix we show that the effect of $\widetilde{\xi}$ on the number of scaling companies n_Y^{sc*} is ambiguous. The reason is that there are fewer young companies led by first-time entrepreneurs who want to scale up. However, there are also young companies led by serial entrepreneurs, some of which decide to scale.
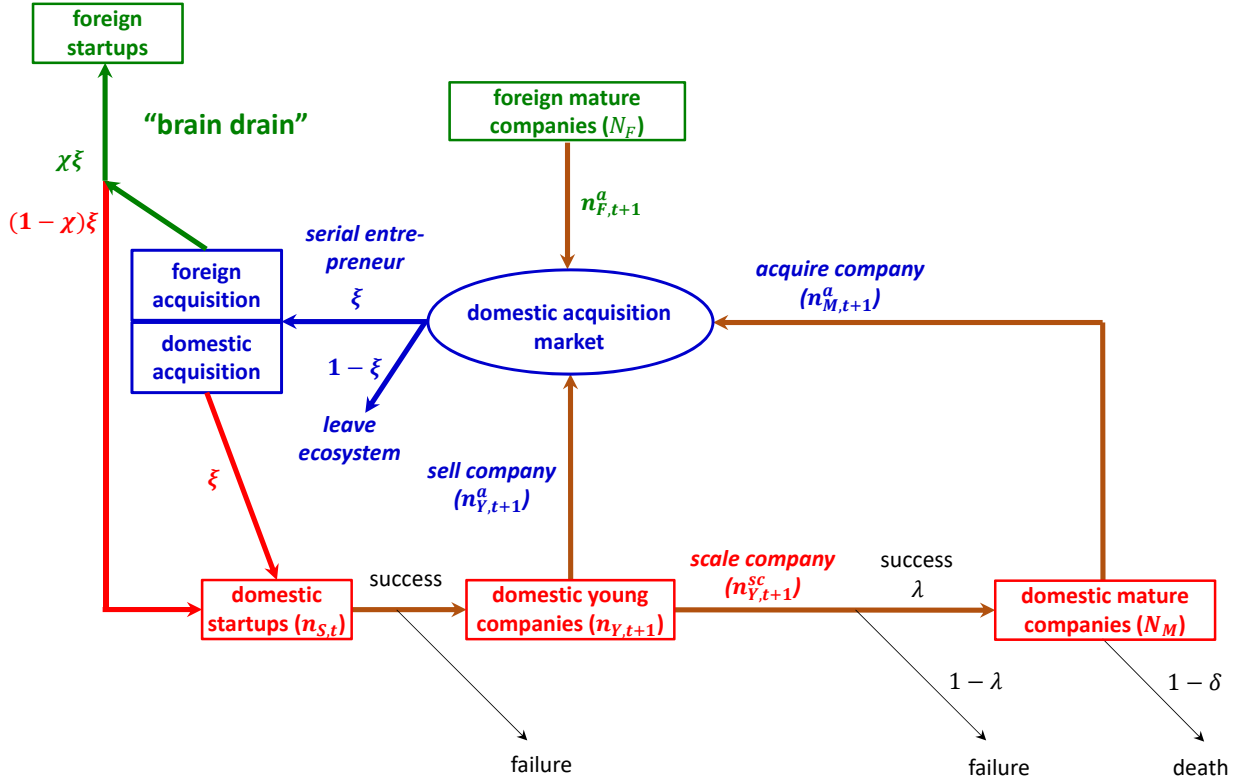
Figure 2: Serial Entrepreneurship and Brain Drain

equilibrium scaling decisions are socially efficient, remains valid. Part (iii) shows that serial entrepreneurship has a positive effect on domestic welfare, as serial entrepreneurs bring more and better quality venture into the ecosystem.

7.2 Brain Drain in an Open Economy

The analysis of serial entrepreneurs in a closed domestic ecosystem provides the foundations for identifying the effects of potential brain drain in an open economy. As a next step we examine how the location decisions of serial entrepreneurs, whose young companies were acquired by foreign buyers, affect the equilibrium outcome. A key issue is whether these serial entrepreneurs pursue their next ventures in the old domestic or new foreign country.

Figure 2 provides a graphical overview of the full model extension with serial entrepreneurs and brain drain. Consider serial entrepreneurs who sold their young companies to foreign acquirers. Let $\chi \in [0, 1]$ be an exogenous probability that they start their next ventures in the foreign location. For $\chi = 0$ there is no brain drain, and for $\chi = 1$ there is complete brain drain of foreign-acquired serial entrepreneurs. To keep our model tractable, we assume that the probabilities and the expected utilities of starting a new company are the same across the two

locations, i.e., $\xi^d = \xi^f = \xi$ and $U_S^d = U_S^f = U_S$. Moreover, we again focus on the effect of the first foreign buyers, i.e., in a neighborhood of $N_F = 0$.

The next proposition shows how brain drain affects the domestic economy.[13]

Proposition 10 *Stronger brain drain (i.e., a higher χ) implies*

- *fewer early acquisitions n_Y^{a*},*

- *fewer scaleups n_Y^{sc*}, and*

- *a lower domestic welfare W^d.*

Brain drain effectively shrinks the domestic ecosystem. Proposition 10 finds that stronger brain drain leads to both fewer acquisitions and fewer scaleups, so that there are fewer young companies overall. Put differently, brain drain results in a loss of entrepreneurial activity due to the loss of serial entrepreneurs.

The next proposition examines how serial entrepreneurs and brain drain affect the key insight from Proposition 5. For this we assume that the domestic welfare function only accounts for the expected utilities of entrepreneurs who remain in the domestic ecosystem. This means that once a serial entrepreneur starts a new company in the foreign country, the utility going forward is no longer included in the domestic welfare function.

Proposition 11 *Consider an open economy with serial entrepreneurs.*

(i) *If brain drain is relatively weak, then the introduction of foreign buyers always increases domestic welfare. Formally $\frac{dW^d}{dN_F} > 0$ in the neighborhood of $\chi = 0$ and $N_F = 0$.*

(ii) *If brain drain is relatively strong, then the introduction of foreign buyers always decreases domestic welfare. Formally $\frac{dW^d}{dN_F} < 0$ in the neighborhood of $\chi = 1$ and $N_F = 0$.*

Similar to Proposition 8, Proposition 11 identifies another set of circumstances under which the introduction of foreign acquirers can be detrimental to the domestic ecosystem. It is not the presence of serial entrepreneurs that generates this result, the real issue is brain drain. Proposition 11 shows that if brain drain is weak (χ near 0), the model with serial entrepreneurs behaves just like our main model. In that case the central finding from Proposition 5, namely that foreign acquirers increase domestic welfare, continues to hold. However, with strong brain drain (χ near 1) the presence of foreign acquirers reduces domestic welfare. The key intuition is that

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[13]Proposition 10 focuses on the most interesting comparative statics results, but the Appendix includes a more comprehensive analysis with additional comparative statics results.

brain drain permanently lowers the level of entrepreneurial activity in the domestic ecosystem (see Proposition 10).[14] Put differently, while the acquisition price may compensate the domestic ecosystem for the loss of a young promising company, it does not compensate for the loss of serial entrepreneurs who start their next companies abroad.[15]

8 Concluding Discussion

In this paper we examine the impact of foreign acquirers on a domestic entrepreneurial ecosystem. The challenge is to maintain a balance between having enough ventures scaling up to become large established companies, and having an lively acquisition market to facilitate exits for those ventures not suited for scaling. In our base model we find that even though foreign acquirers distort the efficient balance between scaling and selling, their presence remains beneficial to the ecosystem as a whole. They raise acquisition prices, which in turn increases the returns to entrepreneurship and encourages more entrepreneurial entry. We also consider two model extensions where the net impact of foreign acquirers can turn negative. If there is a cumulative learning externality at the scaleup stage, or if there is strong brain drain of serial entrepreneurs, then the introduction of foreign acquirers can actually harm the domestic ecosystem. The key novelty of this paper is that it adopts an ecosystem perspective to examine the welfare properties of foreign acquisitions. Naturally our model requires some simplifying assumptions, leaving room for future research.

Another topic worthy of future research concerns the inner mechanics of brain drain, and also brain regain. For analytical tractability we use a highly stylized model of brain drain, but in reality this involves complex decisions with many grey areas. Consider the example of the three founders of Skype, sold to eBay in 2005. They started several new companies that combined activities in Silicon Valley with activities in their home countries (e.g., Estonia) and even third countries (e.g., the UK). Another interesting example is Deep Mind where the founders agreed to sell to Google on the condition of *not* relocating the core team to Silicon Valley. All this suggests a future research agenda around the movements of serial entrepreneurs. More broadly,

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[14]For tractability the model with serial entrepreneurs does not include any scaleup externalities as discussed in Section 6. Adding scaleup externalities into the model with serial entrepreneurs would further reinforce this effect, as the loss of entrepreneurial activity from brain drain would further depress the cumulative scaleup experience.

[15]Note that foreign acquirers do not capture the rents from serial entrepreneurship either. With brain drain, the ultimate winner is the foreign ecosystem which gets an inflow of entrepreneurs, some of whom start their next companies there. These serial entrepreneurs are lost to the domestic ecosystem and become a net gain to the foreign ecosystem.

the work of Saxenian (2006) describes the efforts of policy makers in different countries to engage with their expat communities in Silicon Valley, with a view to facilitate brain regain.

Our paper establishes welfare properties of foreign acquisitions. An interesting topic for future research would be to examine what public policies are suitable to remedy these inefficiencies. We already noted that governments sometimes evoke national security concerns when blocking foreign acquisitions of technology companies. This argument may also be used as an excuse to block other foreign acquisitions. In addition, there are many policy instruments that also affect the likelihood of foreign acquisitions, including taxes and subsidies. Governments can also help to improve the domestic scaling environment by offering more training and support to their domestic scaleups (Coutu, 2014, Hellmann and Kavadias, 2016). The current model provides a useful starting point to further explore such policy questions.

Appendix

Proof of Proposition 1.

Using (2) and (3) with (1), we can write the market clearing condition (5) as follows:

$$\left[1 - \frac{1}{\bar{c}}\left(\lambda U_M - P\right)\right] \rho n_S = \frac{1}{\bar{\theta}}\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\left(\lambda U_M - P\right)\rho n_S.$$

Using (4) and re-arranging yields the following market clearing condition which defines the equilibrium acquisition price P^*:

$$\frac{1}{\bar{c}}\left[1 + \frac{1}{\bar{\theta}}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right]\underbrace{\left[\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\bar{\theta}}\left(\pi^d - P\right)^2\right] - P\right]}_{=c^*>0} = 1. \tag{9}$$

Using (9) we can implicitly differentiate P^* w.r.t. λ:

$$\frac{dP^*}{d\lambda} = \frac{\frac{1}{\bar{\theta}}\frac{1}{1-\delta}\left(\pi^d - P\right)c^* + \frac{1}{1-\delta}\left[1 + \frac{1}{\bar{\theta}}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right]\left[\Psi + \frac{1}{2}\frac{1}{\bar{\theta}}\left(\pi^d - P\right)^2\right]}{\frac{1}{\bar{\theta}}\frac{\lambda}{1-\delta}c^* + \left[1 + \frac{1}{\bar{\theta}}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right]\left[\frac{1}{\bar{\theta}}\frac{\lambda}{1-\delta}\left(\pi^d - P\right) + 1\right]} > 0.$$

Next, we have

$$\frac{dc^*}{d\lambda} = \underbrace{\frac{\partial c^*}{\partial P}}_{<0}\underbrace{\frac{dP^*}{d\lambda}}_{>0} + \underbrace{\frac{\partial c^*}{\partial \lambda}}_{>0}.$$

Note that in equilibrium the direct effect $\frac{\partial c^*}{\partial \lambda}$ (positive) must be greater than the indirect price effect $\frac{\partial c^*}{\partial P}\frac{dP^*}{d\lambda}$ (negative). Thus, $\frac{dc^*}{d\lambda} > 0$. Furthermore,

$$\frac{dn_Y^{sc*}}{d\lambda} = \frac{1}{\bar{c}}\underbrace{\frac{dc^*}{d\lambda}}_{>0}\rho n_S > 0 \qquad \frac{dn_Y^{a*}}{d\lambda} = -\frac{1}{\bar{c}}\underbrace{\frac{dc^*}{d\lambda}}_{>0}\rho n_S < 0 \qquad \frac{d\theta^*}{d\lambda} = -\underbrace{\frac{dP^*}{d\lambda}}_{>0} < 0.$$

Finally, recall that $N_M^* = \frac{1}{1-\delta}\lambda n_Y^{sc*}$. Because $\frac{dn_Y^{sc*}}{d\lambda} > 0$, we can immediately see that $\frac{dN_M^*}{d\lambda} > 0$. $\qquad\square$

Proof of Proposition 2.

Let c_t^{fb} denote the scaling cost ceiling that maximizes domestic welfare at time t. The domestic welfare at time t is then given by

$$W_t^d(c_t^{fb}) = n_{Y,t}^{sc}\left(-E[c_t|c_t \le c_t^{fb}]\right) + n_{Y,t}^a P_t + N_{M,t}\Psi + n_{M,t}^a\left[\pi^d - P_t - E[\theta_t|\theta_t \le \theta_t^*]\right], \quad (10)$$

where the first term is the total cost for all young companies that choose to scale at time t, and the second term is the total payoff for the young companies that sell at price P_t. The third term is the total core profit across all mature companies. And the last term is the total expected payoff for all mature companies when acquiring a young company at time t. Because c_t and θ_t are uniformly distributed, we have $E[c_t|c_t \le c_t^{fb}] = \frac{1}{2}c_t^{fb}$ and $E[\theta_t|\theta_t \le \theta_t^*] = \frac{1}{2}\theta_t^*$. Using $n_{Y,t}^{sc} = \frac{c_t^{fb}}{\bar{c}}\rho n_{S,t-1}$, $n_{Y,t}^a = \left(1 - \frac{c_t^{fb}}{\bar{c}}\right)\rho n_{S,t-1}$, $n_{M,t}^a = \frac{1}{\bar{\theta}}\left(\pi^d - P_t\right)N_{M,t}$, and $\theta_t^* = \pi^d - P_t$ we can write the domestic welfare function as

$$W_t^d(c_t^{fb}) = \rho n_{S,t-1}\left[\left(1 - \frac{c_t^{fb}}{\bar{c}}\right)P_t^* - \frac{1}{2}\frac{1}{\bar{c}}\left(c_t^{fb}\right)^2\right] + N_{M,t}\left[\Psi + \frac{1}{2}\frac{1}{\bar{\theta}}\left(\pi^d - P_t^*\right)^2\right].$$

Now consider the steady state where $N_M = \frac{1}{1-\delta}\lambda n_Y^{sc}$. Using (4), the steady-state domestic welfare function can be written as

$$W^d(c^{fb}) = \rho n_S\left[P^* + \left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\bar{\theta}}\left(\pi^d - P^*\right)^2\right] - P^*\right)\frac{c^{fb}}{\bar{c}} - \frac{1}{2}\frac{1}{\bar{c}}\left(c^{fb}\right)^2\right].$$

Next, we can infer from (6) that

$$U_S(c^{fb}) = \rho\left[P^* + \frac{1}{\bar{c}}\left(\lambda U_M - P^*\right)c^{fb} - \frac{1}{2}\frac{1}{\bar{c}}\left(c^{fb}\right)^2\right]. \quad (11)$$

Using (4) we get

$$U_S\left(c^{fb}\right) = \rho\left[P^* + \left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\bar{\theta}}\left(\pi^d - P^*\right)^2\right] - P^*\right)\frac{c^{fb}}{\bar{c}} - \frac{1}{2}\frac{1}{\bar{c}}\left(c^{fb}\right)^2\right]. \quad (12)$$

Consequently, $W^d(c^{fb}) = n_S U_S\left(c^{fb}\right)$.

The welfare-maximizing scaling cost ceiling c^{fb} satisfies

$$\frac{dW^d(c^{fb})}{dc^{fb}} = \rho n_S \frac{dP^*}{dc^{fb}} \left[1 - \left(\frac{\lambda}{1-\delta} \frac{1}{\theta} \left(\pi^d - P^* \right) + 1 \right) \frac{c^{fb}}{\bar{c}} \right]$$
$$+ \rho n_S \left[\left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} \left(\pi^d - P^* \right)^2 \right] - P^* \right) \frac{1}{\bar{c}} - \frac{1}{\bar{c}} c^{fb} \right] = 0. \quad (13)$$

From the market-clearing condition (9) we know that $\frac{c^{fb}}{\bar{c}} \left[1 + \frac{1}{\theta} \frac{\lambda}{1-\delta} \left[\pi^d - P^* \right] \right] = 1$. Thus,

$$\frac{dW^d(c^{fb})}{dc^{fb}} = \rho n_S \left[\left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} \left(\pi^d - P^* \right)^2 \right] - P^* \right) \frac{1}{\bar{c}} - \frac{1}{\bar{c}} \left(c^{fb} \right) \right] = 0.$$

This implies that $c^{fb} = c^* = \frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} \left(\pi^d - P^* \right)^2 \right] - P^*$. Thus, the equilibrium threshold c^* maximizes domestic welfare, and is therefore efficient.

Finally we show that $\frac{dW^d}{d\lambda} > 0$. Recall that $W^d = n_S U_S$. Thus,

$$\frac{dW^d}{d\lambda} = n_S \left[\frac{\partial U_S}{\partial P} \underbrace{\frac{dP^*}{d\lambda}}_{>0} + \frac{\partial U_S}{\partial \lambda} \right].$$

Using $c^* = \lambda U_M - P^*$ (instead of c^{fb}) and (4) in (11) we get the following expression for U_S:

$$U_S = \rho \left[P^* + \frac{1}{2} \frac{1}{\bar{c}} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} \left(\pi^d - P^* \right)^2 \right] - P^* \right)^2 \right]. \quad (14)$$

Hence,

$$\frac{\partial U_S}{\partial P} = \rho \left[1 - \frac{1}{\bar{c}} \underbrace{\left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} \left(\pi^d - P^* \right)^2 \right] - P^* \right)}_{=c^*} \left(1 + \frac{\lambda}{1-\delta} \frac{1}{\theta} \left(\pi^d - P^* \right) \right) \right].$$

According to the market clearing condition (9), $\frac{c^*}{\bar{c}} \left[1 + \frac{1}{\theta} \frac{\lambda}{1-\delta} \left[\pi^d - P^* \right] \right] = 1$. Consequently, $\frac{\partial U_S}{\partial P} = 0$. Moreover,

$$\frac{\partial U_S}{\partial \lambda} = \rho \frac{1}{\bar{c}} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} \left(\pi^d - P^* \right)^2 \right] - P^* \right) \frac{1}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} \left(\pi^d - P^* \right)^2 \right] > 0.$$

Thus, $\frac{dW^d}{d\lambda} > 0$. □

Open Economy – Comparative Statics for λ.

Using (2), (3) with (1), and (7) with $\phi^* = \pi^f - P$, we can write the market-clearing condition (8) for the open economy as

$$\left(1 - \frac{c^*}{\overline{c}}\right) \rho n_S = \frac{1}{\overline{\theta}} \left(\pi^d - P\right) \frac{\lambda}{1-\delta} \frac{c^*}{\overline{c}} \rho n_S + \frac{1}{\overline{\phi}} \left(\pi^f - P\right) N_F. \tag{15}$$

Rearranging and using $c^* = \frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2}\frac{1}{\overline{\theta}} \left(\pi^d - P\right)^2\right] - P$, the market-clearing condition can be written as

$$
\begin{aligned}
H &\equiv \frac{1}{\overline{c}} \left[1 + \frac{1}{\overline{\theta}} \frac{\lambda}{1-\delta} \left(\pi^d - P\right)\right] \left[\frac{\lambda}{1-\delta} \left(\Psi + \frac{1}{2}\frac{1}{\overline{\theta}} \left(\pi^d - P\right)^2\right) - P\right] \\
&\quad + \frac{1}{\overline{\phi}} \left(\pi^f - P\right) \frac{N_F}{\rho n_S} - 1 = 0.
\end{aligned}
\tag{16}
$$

Using H we can implicitly differentiate P^* w.r.t. λ:

$$
\begin{aligned}
\frac{dP^*}{d\lambda} &= \frac{\frac{1}{\overline{c}}\frac{1}{\overline{\theta}}\frac{1}{1-\delta} \left(\pi^d - P\right) \overbrace{\left[\frac{\lambda}{1-\delta} \left(\Psi + \frac{1}{2}\frac{1}{\overline{\theta}} \left(\pi^d - P\right)^2\right) - P\right]}^{=c^*>0}}{-\frac{\partial H}{\partial P}} \\
&\quad + \frac{\frac{1}{\overline{c}} \left[1 + \frac{1}{\overline{\theta}}\frac{\lambda}{1-\delta} \left(\pi^d - P\right)\right] \frac{1}{1-\delta} \left(\Psi + \frac{1}{2}\frac{1}{\overline{\theta}} \left(\pi^d - P\right)^2\right)}{-\frac{\partial H}{\partial P}}.
\end{aligned}
$$

Note that the numerators are positive. Moreover, we can immediately see that $\frac{\partial H}{\partial P} < 0$. Thus, $\frac{dP^*}{d\lambda} > 0$.

Next, we have

$$\frac{dc^*}{d\lambda} = \underbrace{\frac{\partial c^*}{\partial P}}_{<0} \underbrace{\frac{dP^*}{d\lambda}}_{>0} + \underbrace{\frac{\partial c^*}{\partial \lambda}}_{>0}.$$

Note that in equilibrium the direct effect $\frac{\partial c^*}{\partial \lambda}$ (positive) must be greater than the indirect price effect $\frac{\partial c^*}{\partial P}\frac{dP^*}{d\lambda}$ (negative). Thus, $\frac{dc^*}{d\lambda} > 0$. This also implies that

$$\frac{dn_Y^{sc*}}{d\lambda} = \frac{1}{\bar{c}}\underbrace{\frac{dc^*}{d\lambda}}_{>0}\rho n_S > 0 \qquad \frac{dn_Y^{a*}}{d\lambda} = -\frac{1}{\bar{c}}\underbrace{\frac{dc^*}{d\lambda}}_{>0}\rho n_S < 0 \qquad \frac{d\theta^*}{d\lambda} = -\underbrace{\frac{dP^*}{d\lambda}}_{>0} < 0.$$

Finally, recall that $N_M^* = \frac{\lambda}{1-\delta}n_Y^{sc*}$. Because $\frac{dn_Y^{sc*}}{d\lambda} > 0$, we can immediately see that $\frac{dN_M^*}{d\lambda} > 0$.

Proof of Proposition 3.

Using the market-clearing condition (16) we can implicitly differentiate P^* w.r.t. N_F:

$$\frac{dP^*}{dN_F} = -\frac{\overbrace{\frac{1}{\bar{\phi}}\left(\pi^f - P\right)\frac{1}{\rho n_S}}^{>0}}{\frac{\partial H}{\partial P}}.$$

Again we can see that $\frac{\partial H}{\partial P} < 0$. Thus, $\frac{dP^*}{dN_F} > 0$. Moreover, recall that $c^* = \frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^*$, which is decreasing in P^*. And because $\frac{dP^*}{dN_F} > 0$, we have $\frac{dc^*}{dN_F} < 0$. Note that $\frac{dc^*}{dN_F} < 0$ and $\frac{dP^*}{dN_F} > 0$ also imply that $\frac{dn_Y^{sc*}}{dN_F} < 0$, $\frac{dn_Y^{a*}}{dN_F} > 0$, and $\frac{d\theta^*}{dN_F} < 0$. Finally, because $\frac{dn_Y^{sc*}}{dN_F} < 0$, we have $\frac{dN_M^*}{dN_F} < 0$. $\qquad\square$

Proof of Proposition 4.

Using c^{fb} we can write the market-clearing condition for the open economy, (16), as

$$1 - \left[1 + \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right]\frac{c^{fb}}{\bar{c}} = \frac{1}{\bar{\phi}}\left(\pi^f - P\right)\frac{N_F}{\rho n_S}. \tag{17}$$

We know from Proof of Proposition 2 that the steady-state domestic welfare, as a function of c^{fb}, is given by $W^d(c^{fb}) = n_S U_S\left(c^{fb}\right)$. The welfare-maximizing scaling cost ceiling c^{fb} satisfies $\frac{dW^d(c^{fb})}{dc^{fb}} = 0$. Using (17) in (13) we get

$$\frac{dW^d(c^{fb})}{dc^{fb}} = \rho n_S\left[\frac{dP^*}{dc^{fb}}\frac{1}{\bar{\phi}}\left(\pi^f - P^*\right)\frac{N_F}{\rho n_S} + \left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^*\right)\frac{1}{\bar{c}} - \frac{1}{\bar{c}}\left(c^{fb}\right)\right].$$

Evaluating $\frac{dW^d(c^{fb})}{dc^{fb}}$ at $c^{fb} = c^* = \frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^*$ we get

$$\left.\frac{dW^d(c^{fb})}{dc^{fb}}\right|_{c^{fb}=c^*} = \frac{dP^*}{dc^{fb}}\frac{1}{\phi}\left(\pi^f - P^*\right)N_F.$$

Using (17) we can implicitly differentiate P^* w.r.t. c^{fb}:

$$\frac{dP^*}{dc^{fb}} = \frac{\left[1 + \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right]\frac{1}{\bar{c}}}{\frac{1}{\theta}\frac{\lambda}{1-\delta}\frac{c^{fb}}{\bar{c}} + \frac{1}{\phi}\frac{N_F}{\rho n_S}} > 0.$$

Thus, $\left.\frac{dW^d(c^{fb})}{dc^{fb}}\right|_{c^{fb}=c^*} > 0$. This implies that $c^*(N_F > 0) < c^{fb}$.

Next, the global welfare function at time t is given by

$$\overline{W}_t(c_t^{fb}) = n_{Y,t}^{sc}\left(-E[c_t|c_t \leq c_t^{fb}]\right) + n_{Y,t}^a P_t + N_{M,t}\Psi + n_{M,t}^a\left[\pi^d - P_t - E[\theta_t|\theta_t \leq \theta_t^*]\right]$$
$$+ n_{F,t}^a\left[\pi^f - P_t - E[\phi_t|\phi_t \leq \phi_t^*]\right].$$

Because c_t, θ_t, and ϕ_t, are uniformly distributed, we have $E[c_t|c_t \leq c_t^{fb}] = \frac{1}{2}c_t^{fb}$, $E[\theta_t|\theta_t \leq \theta_t^*] = \frac{1}{2}\theta_t^*$, and $E[\phi_t|\phi_t \leq \phi_t^*] = \frac{1}{2}\phi_t^*$.

Using (1) and (2) with c^{fb}, (3) with $\theta_t^* = \pi^d - P_t$, and (7) with $\phi_t^* = \pi^f - P_t$ we can write $\overline{W}_t(c_t^{fb})$ as

$$\overline{W}_t(c_t^{fb}) = \rho n_{S,t-1}\left[\left(1 - \frac{c_t^{fb}}{\bar{c}}\right)P_t^* - \frac{1}{2}\frac{1}{\bar{c}}\left(c_t^{fb}\right)^2\right] + N_{M,t}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P_t^*\right)^2\right]$$
$$+ \frac{1}{\bar{\phi}}\frac{1}{2}\left(\pi^f - P_t^*\right)^2 N_F.$$

Now consider the steady state where $N_M = \frac{1}{1-\delta}\lambda n_Y^{sc}$. The steady-state global welfare function can be written as

$$\overline{W}(c^{fb}) = \rho n_S\left[P^* + \left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^*\right)\frac{c^{fb}}{\bar{c}} - \frac{1}{2}\frac{1}{\bar{c}}\left(c^{fb}\right)^2\right]$$
$$+ \frac{1}{\bar{\phi}}\frac{1}{2}\left(\pi^f - P^*\right)^2 N_F$$

The global welfare-maximizing cost ceiling c^{fb} satisfies $\frac{d\overline{W}(c^{fb})}{dc^{fb}} = 0$, where

$$
\begin{aligned}
\frac{d\overline{W}(c^{fb})}{dc^{fb}} &= \rho n_S \frac{dP^*}{dc^{fb}} \left[1 - \left(\frac{\lambda}{1-\delta} \frac{1}{\theta} \left(\pi^d - P^* \right) + 1 \right) \frac{c^{fb}}{\overline{c}} \right] \\
&\quad + \rho n_S \left[\left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} \left(\pi^d - P^* \right)^2 \right] - P^* \right) \frac{1}{\overline{c}} - \frac{1}{\overline{c}} c^{fb} \right] \\
&\quad - \frac{1}{\overline{\phi}} \frac{dP^*}{dc^{fb}} \left(\pi^f - P^* \right) N_F.
\end{aligned}
$$

Using the market-clearing condition (15) with c^{fb} we find that

$$
\left[1 + \frac{1}{\theta} \frac{\lambda}{1-\delta} \left(\pi^d - P^* \right) \right] \frac{c^{fb}}{\overline{c}} = 1 - \frac{1}{\overline{\phi}} \left(\pi^f - P^* \right) \frac{N_F}{\rho n_S}.
$$

Using this we can write $\frac{d\overline{W}(c^{fb})}{dc^{fb}} = 0$, after some simplifications, as follows:

$$
\rho n_S \left[\left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} \left(\pi^d - P^* \right)^2 \right] - P^* \right) \frac{1}{\overline{c}} - \frac{1}{\overline{c}} c^{fb} \right] = 0.
$$

Solving for c^{fb} we get

$$
c^{fb} = \frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} \left[\pi^d - P^* \right]^2 \right] - P^*.
$$

Thus, $c^{fb} = c^*$, i.e., the equilibrium scaling cost ceiling c^* maximizes global welfare. $\qquad\square$

Proof of Proposition 5.

Recall that $W^d = n_S U_S$, where U_S is given by (14). Thus,

$$
\frac{dW^d}{dN_F} = n_S \left[\frac{\partial U_S}{\partial P} \frac{dP^*}{dN_F} + \frac{\partial U_S}{\partial N_F} \right].
$$

We can immediately see from (14) that $\frac{\partial U_S}{\partial N_F} = 0$. And we know from Proposition 3 that $\frac{dP^*}{dN_F} > 0$. Moreover, recall from Proof of Proposition 2 that

$$
\frac{\partial U_S}{\partial P} = \rho \left[1 - \frac{1}{\overline{c}} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} \left(\pi^d - P^* \right)^2 \right] - P^* \right) \left(1 + \frac{\lambda}{1-\delta} \frac{1}{\theta} \left(\pi^d - P^* \right) \right) \right].
$$

Note that the market-clearing condition for the open economy, (16), can be written as

$$1 - \frac{1}{\bar{c}} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2}\frac{1}{\bar{\theta}} \left(\pi^d - P \right)^2 \right] - P \right) \left(1 + \frac{1}{\bar{\theta}} \frac{\lambda}{1-\delta} \left(\pi^d - P \right) \right) = \frac{1}{\bar{\phi}} \left(\pi^f - P \right) \frac{N_F}{\rho n_S}.$$

Using this we get

$$\frac{\partial U_S}{\partial P} = \frac{1}{\bar{\phi}} \left(\pi^f - P^* \right) \frac{N_F}{n_S}.$$

Consequently, $\frac{dW^d}{dN_F}|_{N_F=0} = 0$ and $\frac{dW^d}{dN_F}|_{N_F>0} > 0$. $\qquad\square$

Proof of Proposition 6.

Using $l^* = U_S$ we get $n_S^* = \frac{1}{\bar{l}} U_S$. Thus, $\frac{dn_S^*}{d\lambda} = \frac{1}{\bar{l}}\frac{dU_S}{d\lambda}$. With $n_S^* = \frac{1}{\bar{l}} U_S$, the equilibrium with endogenous entry in the open economy is now defined by

$$
\begin{aligned}
F \equiv & \left[\frac{1}{\bar{c}} \left(1 + \frac{1}{\bar{\theta}} \frac{\lambda}{1-\delta} \left(\pi^d - P \right) \right) \left(\frac{\lambda}{1-\delta} \left(\Psi + \frac{1}{2}\frac{1}{\bar{\theta}} \left(\pi^d - P \right)^2 \right) - P \right) - 1 \right] \frac{\rho}{\bar{l}} U_S \\
& + \frac{1}{\bar{\phi}} \left(\pi^f - P \right) N_F = 0
\end{aligned}
\tag{18}
$$

$$
G \equiv U_S - \rho \left[P + \frac{1}{2}\frac{1}{\bar{c}} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2}\frac{1}{\bar{\theta}} \left(\pi^d - P \right)^2 \right] - P \right)^2 \right] = 0,
\tag{19}
$$

where (18) is the market-clearing condition, and (19) defines U_S; see (14). Using Cramer's rule we get

$$
\frac{dU_S}{d\lambda} = \frac{\overbrace{\begin{vmatrix} \frac{\partial F}{\partial P} & -\frac{\partial F}{\partial \lambda} \\ \frac{\partial G}{\partial P} & -\frac{\partial G}{\partial \lambda} \end{vmatrix}}^{\equiv T_1}}{\underbrace{\begin{vmatrix} \frac{\partial F}{\partial P} & \frac{\partial F}{\partial U_S} \\ \frac{\partial G}{\partial P} & \frac{\partial G}{\partial U_S} \end{vmatrix}}_{\equiv T_2}}.
$$

We can immediately see that $\frac{\partial F}{\partial \lambda} > 0$, $\frac{\partial G}{\partial \lambda} < 0$, $\frac{\partial G}{\partial U_S} = 1 > 0$, and $\frac{\partial F}{\partial P} < 0$. Moreover,

$$\frac{\partial F}{\partial U_S} = \left[\frac{1}{\bar{c}} \left(1 + \frac{1}{\bar{\theta}} \frac{\lambda}{1-\delta} \left[\pi^d - P \right] \right) \left(\frac{\lambda}{1-\delta} \left(\Psi + \frac{1}{2}\frac{1}{\bar{\theta}} \left(\pi^d - P \right)^2 \right) - P \right) - 1 \right] \frac{\rho}{\bar{l}}.$$

Using F we can write this as

$$\frac{\partial F}{\partial U_S} = -\frac{1}{U_S}\frac{1}{\bar{\phi}}\left(\pi^f - P\right)N_F < 0.$$

We already know that $\frac{\partial U_S}{\partial P} > 0$, which implies that $\frac{\partial G}{\partial P} < 0$. Thus,

$$T_1 = -\overbrace{\frac{\partial F}{\partial P}}^{<0}\overbrace{\frac{\partial G}{\partial \lambda}}^{<0} + \overbrace{\frac{\partial G}{\partial P}}^{<0}\overbrace{\frac{\partial F}{\partial \lambda}}^{>0} < 0$$

$$T_2 = \underbrace{\frac{\partial F}{\partial P}}_{<0}\underbrace{\frac{\partial G}{\partial U_S}}_{>0} - \underbrace{\frac{\partial G}{\partial P}}_{<0}\underbrace{\frac{\partial F}{\partial U_S}}_{<0} < 0.$$

Thus, $\frac{dU_S}{d\lambda} > 0$, and therefore, $\frac{dn_S^*}{d\lambda} > 0$.

Likewise,

$$\frac{dU_S}{dN_F} = \frac{\overbrace{\begin{vmatrix} \frac{\partial F}{\partial P} & -\frac{\partial F}{\partial N_F} \\ \frac{\partial G}{\partial P} & -\frac{\partial G}{\partial N_F} \end{vmatrix}}^{\equiv T_3}}{\underbrace{\begin{vmatrix} \frac{\partial F}{\partial P} & \frac{\partial F}{\partial U_S} \\ \frac{\partial G}{\partial P} & \frac{\partial G}{\partial U_S} \end{vmatrix}}_{=T_2}}.$$

We already know that $T_2 < 0$. Moreover, we can see that $\frac{\partial F}{\partial N_F} > 0$ and $\frac{\partial G}{\partial N_F} = 0$. Thus,

$$T_3 = -\frac{\partial F}{\partial P}\overbrace{\frac{\partial G}{\partial N_F}}^{=0} + \overbrace{\frac{\partial G}{\partial P}}^{<0}\overbrace{\frac{\partial F}{\partial N_F}}^{>0} < 0.$$

This implies that $\frac{dU_S}{dN_F} > 0$, and therefore, $\frac{dn_S^*}{dN_F} > 0$.

Finally, because $W^d = n_S^* U_S = \frac{1}{l}\left[U_S\right]^2$ with $\frac{dU_S}{d\lambda} > 0$ and $\frac{dU_S}{dN_F} > 0$, we have $\frac{dW^d}{d\lambda} > 0$ and $\frac{dW^d}{dN_F} > 0$. $\qquad\square$

Model with Scaling Externality.

Consider first a closed economy. Using $N_M = \frac{1}{1-\delta}\lambda n_Y^{sc}$ we get

$$n_Y^{sc} = \frac{c^{fb}}{\bar{C}(N_M)}\rho n_S = \frac{1}{\bar{c}}\left(1 + \omega\frac{\lambda}{1-\delta}n_Y^{sc}\right)c^{fb}\rho n_S,$$

where c^{fb} is the scaling cost ceiling that maximizes domestic welfare. Solving for n_Y^{sc} we get

$$n_Y^{sc}(c^{fb}) = \frac{\frac{1}{\bar{c}}c^{fb}\rho n_S}{1 - \omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^{fb}\rho n_S}. \tag{20}$$

Furthermore, using (20) and $\theta^* = \pi^d - P$, we find that

$$n_Y^a(c^{fb}) = \left(1 - \frac{c^{fb}}{\bar{C}(N_M)}\right)\rho n_S = \left(1 - \frac{\frac{1}{\bar{c}}c^{fb}}{1 - \omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^{fb}\rho n_S}\right)\rho n_S \tag{21}$$

$$n_M^a(c^{fb}) = \frac{1}{1-\delta}\frac{\theta^*}{\bar{\theta}}\lambda n_Y^{sc} = \frac{1}{\bar{\theta}}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\frac{\frac{1}{\bar{c}}c^{fb}\rho n_S}{1 - \omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^{fb}\rho n_S}. \tag{22}$$

Using (21) and (22) we can write the market-clearing condition, $n_M^a = n_Y^a$, for the closed economy as a function of c^{fb}:

$$\frac{1}{\bar{\theta}}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\frac{\frac{1}{\bar{c}}c^{fb}\rho n_S}{1 - \omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^{fb}\rho n_S} = \left(1 - \frac{\frac{1}{\bar{c}}c^{fb}}{1 - \omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^{fb}\rho n_S}\right)\rho n_S$$

$$\Leftrightarrow \quad \frac{1}{\bar{c}}\left[1 + \frac{1}{\bar{\theta}}\frac{\lambda}{1-\delta}\left(\pi^d - P\right) + \omega\frac{\lambda}{1-\delta}\rho n_S\right]c^{fb} = 1. \tag{23}$$

Next, recall from Proof of Proposition 2 that the domestic welfare is given by $W^d(c^{fb}) = n_S U_S\left(c^{fb}\right)$. From (12) we can infer that

$$U_S\left(c^{fb}\right) = \rho\left[P^* + \left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\bar{\theta}}\left(\pi^d - P^*\right)^2\right] - P^* - \frac{1}{2}c^{fb}\right)\frac{c^{fb}}{\bar{c}}\left(1 + \omega N_M\right)\right].$$

Using $N_M = \frac{1}{1-\delta}\lambda n_Y^{sc}$ with (20) we get

$$U_S\left(c^{fb}\right) = \rho\left[P^* + \left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^* - \frac{1}{2}c^{fb}\right)\frac{\frac{1}{\bar{c}}c^{fb}}{1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^{fb}\rho n_S}\right].$$
(24)

Thus,

$$W^d(c^{fb}) = \rho n_S\left[P^* + \left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^* - \frac{1}{2}c^{fb}\right)\frac{\frac{1}{\bar{c}}c^{fb}}{1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^{fb}\rho n_S}\right].$$
(25)

The welfare-maximizing scaling cost ceiling, c^{fb}, is defined by

$$\frac{dW^d(c^{fb})}{dc^{fb}} = \frac{\partial W^d(c^{fb})}{\partial P}\frac{dP^*}{dc^{fb}} + \frac{\partial W^d(c^{fb})}{\partial c^{fb}}.$$

We get

$$\frac{\partial W^d(c^{fb})}{\partial P} = \rho n_S\left[1 - \left(1 + \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P^*\right)\right)\frac{\frac{1}{\bar{c}}c^{fb}}{1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^{fb}\rho n_S}\right].$$

We can infer from the market-clearing condition (23) that

$$1 + \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P^*\right) = \frac{1}{\frac{1}{\bar{c}}c^{fb}}\left[1 - \frac{1}{\bar{c}}c^{fb}\omega\frac{\lambda}{1-\delta}\rho n_S\right].$$

Using this we can immediately see that $\frac{\partial W^d(c^{fb})}{\partial P} = 0$.

For parsimony define $Y \equiv \frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^*$. We then get

$$\frac{\partial W^d(c^{fb})}{\partial c^{fb}} = \rho n_S\frac{1}{\bar{c}}\frac{\left[Y - c^{fb}\right]\left[1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^{fb}\rho n_S\right] + \left[Yc^{fb} - \frac{1}{2}\left(c^{fb}\right)^2\right]\frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}\rho n_S}{\left[1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^{fb}\rho n_S\right]^2}.$$
(26)

Next, we evaluate $\frac{dW^d(c^{fb})}{dc^{fb}}$ at $c^{fb} = c^* = \frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^* = Y$:

$$
\left.\frac{dW^d(c^{fb})}{dc^{fb}}\right|_{c^{fb}=c^*} = \underbrace{\left.\frac{\partial W^d(c^{fb})}{\partial P}\right|_{c^{fb}=c^*}\left.\frac{dP^*}{dc^{fb}}\right|_{c^{fb}=c^*}}_{=0} + \left.\frac{\partial W^d(c^{fb})}{\partial c^{fb}}\right|_{c^{fb}=c^*}
$$

$$
= \rho n_S \frac{1}{\bar{c}}\frac{\frac{1}{2}\left(c^*\right)^2\frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}\rho n_S}{\left[1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^*\rho n_S\right]^2} > 0.
$$

This implies that $c^* < c^{fb}$, i.e., there is not enough scaling in the presence of scaling externalities.

Next consider an open economy. Using $n_F^a = \frac{1}{\phi}\left[\pi^f - P\right]N_F$ we can write the market-clearing condition, $n_M^a + n_F^a = n_Y^a$, as

$$
\frac{1}{\bar{c}}\left[1 + \frac{1}{\bar{\theta}}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right]\frac{c^{fb}}{1 - \omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^{fb}\rho n_S} + \frac{1}{\bar{\phi}}\left[\pi^f - P\right]\frac{N_F}{\rho n_S} = 1. \tag{27}
$$

Recall that $W^d(c^{fb}) = n_S U_S\left(c^{fb}\right)$, where c^{fb} satisfies $\frac{dW^d(c^{fb})}{dc^{fb}} = 0$. Note that

$$
\frac{dW^d(c^{fb})}{dc^{fb}} = \frac{\partial W^d(c^{fb})}{\partial P}\frac{dP^*}{dc^{fb}} + \frac{\partial W^d(c^{fb})}{\partial c^{fb}}.
$$

Using (25) we get

$$
\frac{\partial W^d(c^{fb})}{\partial P} = \rho n_S\left[1 - \left(1 + \frac{1}{\bar{\theta}}\frac{\lambda}{1-\delta}\left(\pi^d - P^*\right)\right)\frac{\frac{1}{\bar{c}}c^{fb}}{1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^{fb}\rho n_S}\right].
$$

Note that (27) implies that

$$
1 - \left[1 + \frac{1}{\bar{\theta}}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right]\frac{\frac{1}{\bar{c}}c^{fb}}{1 - \omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^{fb}\rho n_S} = \frac{1}{\bar{\phi}}\left[\pi^f - P\right]\frac{N_F}{\rho n_S}.
$$

Thus,

$$
\frac{\partial W^d(c^{fb})}{\partial P} = \frac{1}{\bar{\phi}}\left[\pi^f - P\right]N_F > 0.
$$

Using (27) we can implicitly differentiate P^* w.r.t. c^{fb}:

$$\frac{dP^*}{dc^{fb}} = \frac{\frac{1}{\bar{c}}\left[1 + \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right]\frac{1}{\left[1-\omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^{fb}\rho n_S\right]^2}}{\frac{1}{\bar{c}}\frac{1}{\theta}\frac{\lambda}{1-\delta}\frac{c^{fb}}{1-\omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^{fb}\rho n_S} + \frac{1}{\phi}\frac{N_F}{\rho n_S}} > 0.$$

Moreover, $\frac{\partial W^d(c^{fb})}{\partial c^{fb}}$ is given by (26), with $Y = \frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^*$.

Evaluating $\frac{dW^d(c^{fb})}{dc^{fb}}$ at $c^{fb} = c^* = Y$ we get

$$\left.\frac{dW^d(c^{fb})}{dc^{fb}}\right|_{c^{fb}=c^*} = \left.\frac{\partial W^d(c^{fb})}{\partial P}\right|_{c^{fb}=c^*}\left.\frac{dP^*}{dc^{fb}}\right|_{c^{fb}=c^*} + \left.\frac{\partial W^d(c^{fb})}{\partial c^{fb}}\right|_{c^{fb}=c^*},$$

where

$$\left.\frac{\partial W^d(c^{fb})}{\partial P}\right|_{c^{fb}=c^*} = \frac{1}{\bar{\phi}}\left[\pi^f - P\right]N_F > 0$$

$$\left.\frac{dP^*}{dc^{fb}}\right|_{c^{fb}=c^*} = \frac{\frac{1}{\bar{c}}\left[1 + \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right]\frac{1}{\left[1-\omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^*\rho n_S\right]^2}}{\frac{1}{\bar{c}}\left[\frac{1}{\theta}\frac{\lambda}{1-\delta}\right]\frac{c^*}{1-\omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^*\rho n_S} + \frac{1}{\phi}\frac{N_F}{\rho n_S}} > 0$$

$$\left.\frac{\partial W^d(c^{fb})}{\partial c^{fb}}\right|_{c^{fb}=c^*} = \rho n_S\frac{1}{\bar{c}}\frac{\frac{1}{2}(c^*)^2\frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}\rho n_S}{\left[1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^{fb}\rho n_S\right]^2} > 0.$$

Thus, $\left.\frac{dW^d(c^{fb})}{dc^{fb}}\right|_{c^{fb}=c^*} > 0$, which implies that $c^* < c^{fb}$ in the open economy.

Next, because $W^d = n_S U_S$, we have

$$\frac{dW^d}{dN_F} = n_S\left[\frac{\partial U_S}{\partial P}\frac{dP^*}{dN_F} + \frac{\partial U_S}{\partial N_F}\right].$$

Using the market-clearing condition (27) we define

$$H \equiv \frac{1}{\bar{c}}\left[1 + \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right]\frac{c^*}{1 - \omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^*\rho n_S} + \frac{1}{\bar{\phi}}\left[\pi^f - P\right]\frac{N_F}{\rho n_S} - 1 = 0.$$

Note that $\frac{\partial H}{\partial c^*} > 0$. Moreover, $\frac{\partial c^*}{\partial P} = -\left[1 + \frac{\lambda}{1-\delta}\frac{1}{\theta}\left(\pi^d - P\right)\right] < 0$. We can then see that $\frac{\partial H}{\partial P} < 0$.
Using H, we can implicitly differentiate P^* w.r.t. N_F:

$$\frac{dP^*}{dN_F} = -\frac{\frac{1}{\phi}\left[\pi^f - P\right]\frac{1}{\rho n_S}}{\underbrace{\frac{\partial H}{\partial P}}_{<0}} > 0.$$

Next, using $c^* = \frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^*$ we can write (24) as

$$U_S = \rho\left[P^* + \frac{\frac{1}{2}\frac{1}{\bar{c}}\left[c^*\right]^2}{1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^*\rho n_S}\right].$$

Note that $\frac{\partial U_S}{\partial N_F} = 0$. Moreover,

$$\frac{\partial U_S}{\partial P} = \rho\left[1 + \frac{\frac{1}{\bar{c}}c^*\frac{\partial c^*}{\partial P}\left[1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^*\rho n_S\right] + \frac{1}{2}\frac{1}{\bar{c}}\left[c^*\right]^2\frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}\frac{\partial c^*}{\partial P}\rho n_S}{\left[1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^*\rho n_S\right]^2}\right].$$

Note that $\frac{\partial c^*}{\partial P} = -\left[1 + \frac{\lambda}{1-\delta}\frac{1}{\theta}\left(\pi^d - P^*\right)\right]$. Thus,

$$\frac{\partial U_S}{\partial P} = \rho\left[1 - \frac{\frac{1}{\bar{c}}c^*\left[1 + \frac{\lambda}{1-\delta}\frac{1}{\theta}\left(\pi^d - P^*\right)\right]}{1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^*\rho n_S} - \frac{\frac{1}{2}\frac{1}{\bar{c}}\left[c^*\right]^2\frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}\left[1 + \frac{\lambda}{1-\delta}\frac{1}{\theta}\left(\pi^d - P^*\right)\right]\rho n_S}{\left[1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^*\rho n_S\right]^2}\right].$$

The market-clearing condition (27) implies that

$$1 - \frac{1}{\bar{c}}\left[1 + \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right]\frac{c^*}{1 - \omega\frac{1}{\bar{c}}\frac{\lambda}{1-\delta}c^*\rho n_S} = \frac{1}{\phi}\left[\pi^f - P\right]\frac{N_F}{\rho n_S}.$$

Consequently,

$$\frac{\partial U_S}{\partial P} = \rho\left[\frac{1}{\phi}\left[\pi^f - P\right]\frac{N_F}{\rho n_S} - \frac{\frac{1}{2}\frac{1}{\bar{c}}\left[c^*\right]^2\frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}\left[1 + \frac{\lambda}{1-\delta}\frac{1}{\theta}\left(\pi^d - P^*\right)\right]\rho n_S}{\left[1 - \frac{1}{\bar{c}}\omega\frac{\lambda}{1-\delta}c^*\rho n_S\right]^2}\right].$$

We can then see that for $\omega > 0$ we have $\frac{dU_S}{dN_F}\Big|_{N_F=0} < 0$, and therefore $\frac{dW^d}{dN_F}\Big|_{N_F=0} < 0$.

Proof of Proposition 9.

With serial entrepreneurs, the number of young companies choosing to sell in t is defined by

$$n_{Y,t}^{a*} = \left(1 - \frac{c_t^*(\widetilde{\xi})}{\overline{c}}\right) \rho \left(n_{S,t-1} + \widetilde{\xi} n_{Y,t-1}^{a*}\right), \tag{28}$$

and the number of scaleups is given by

$$n_{Y,t}^{sc*} = \frac{c_t^*(\widetilde{\xi})}{\overline{c}} \rho \left(n_{S,t-1} + \widetilde{\xi} n_{Y,t-1}^{a*}\right). \tag{29}$$

Moreover, recall that $\theta_t^* = \pi^d - P_t$ and $n_{M,t}^{a*} = \frac{\theta_t^*}{\theta} N_{M,t}$.

The expected utility of an entrepreneur at time t is given by

$$
\begin{aligned}
U_{S,t} &= \rho \left[\int_0^{c_t^*(\widetilde{\xi})} (\lambda U_{M,t+2} - c_t) \frac{1}{\overline{c}} dc_t + \int_{c_t^*(\widetilde{\xi})}^{\overline{c}} \left(P_{t+1} + \widetilde{\xi} U_{S,t+2}\right) \frac{1}{\overline{c}} dc_t \right] \\
&= \rho \left[P_{t+1} + \widetilde{\xi} U_{S,t+2} + \frac{1}{\overline{c}} \left(\lambda U_{M,t+2} - P_{t+1} - \widetilde{\xi} U_{S,t+2}\right) c_t^*(\widetilde{\xi}) - \frac{1}{\overline{c}} \frac{1}{2} \left(c_t^*(\widetilde{\xi})\right)^2 \right] .
\end{aligned}
\tag{30}
$$

Now consider the steady state. Using $c^*(\widetilde{\xi}) = \lambda U_M - P - \widetilde{\xi} U_S$ and (4) we get the following equation which implicitly defines U_S in the steady state:

$$U_S = \rho \left[P + \widetilde{\xi} U_S + \frac{1}{2} \frac{1}{\overline{c}} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\theta} (\pi^d - P)^2 \right] - P - \widetilde{\xi} U_S \right)^2 \right].$$

In the steady state we have $N_M = \frac{\lambda}{1-\delta} n_Y^{sc}$. And the equilibrium acquisition price P^* is defined by the market clearing condition $n_Y^a = n_M^a$. Solving (28) for n_Y^a we get

$$n_Y^{a*} = \frac{\left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right) \rho n_S}{1 - \widetilde{\xi} \left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right) \rho}. \tag{31}$$

This implies that

$$n_Y^{sc*} = \frac{c^*(\widetilde{\xi})}{\overline{c}}\rho\left(n_S + \widetilde{\xi}\frac{\left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right)\rho n_S}{1 - \widetilde{\xi}\left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right)\rho}\right) = \frac{c^*(\widetilde{\xi})}{\overline{c}}\rho n_S\frac{1}{1 - \widetilde{\xi}\left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right)\rho},$$

$$n_M^{a*} = \frac{1}{\overline{\theta}}\left(\pi^d - P\right)\frac{\lambda}{1 - \delta}n_Y^{sc*} = \frac{1}{\overline{\theta}}\left(\pi^d - P\right)\frac{\lambda}{1 - \delta}\frac{c^*(\widetilde{\xi})}{\overline{c}}\rho n_S\frac{1}{1 - \widetilde{\xi}\left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right)\rho}. \quad (32)$$

Using (31) and (32) with $c^*(\widetilde{\xi}) = \lambda U_M - P - \widetilde{\xi}U_S$ and (4), we can write the market clearing condition $n_Y^a = n_M^a$ as

$$\frac{1}{\overline{c}}\left[1 + \frac{1}{\overline{\theta}}\frac{\lambda}{1 - \delta}\left(\pi^d - P\right)\right]\underbrace{\left[\frac{\lambda}{1 - \delta}\left[\Psi + \frac{1}{2}\frac{1}{\overline{\theta}}\left(\pi^d - P\right)^2\right] - P - \widetilde{\xi}U_S\right]}_{=c^*(\widetilde{\xi})} = 1.$$

Thus, the steady-state equilibrium in the closed economy with serial entrepreneurs is defined by

$$F \equiv \frac{1}{\overline{c}}\left[1 + \frac{1}{\overline{\theta}}\frac{\lambda}{1 - \delta}\left(\pi^d - P\right)\right]\left[\frac{\lambda}{1 - \delta}\left[\Psi + \frac{1}{2}\frac{1}{\overline{\theta}}\left(\pi^d - P\right)^2\right] - P - \widetilde{\xi}U_S\right] - 1$$
$$= 0 \quad (33)$$

$$G \equiv U_S - \rho\left[P + \widetilde{\xi}U_S + \frac{1}{2}\frac{1}{\overline{c}}\left(\frac{\lambda}{1 - \delta}\left[\Psi + \frac{1}{2}\frac{1}{\overline{\theta}}\left(\pi^d - P\right)^2\right] - P - \widetilde{\xi}U_S\right)^2\right] = 0, \quad (34)$$

where (33) is the market-clearing condition, and (34) defines U_S.

Applying Cramer's rule we get

$$\frac{dP^*}{d\widetilde{\xi}} = \frac{\overbrace{\begin{vmatrix} -\frac{\partial F}{\partial\widetilde{\xi}} & \frac{\partial F}{\partial U_S} \\ -\frac{\partial G}{\partial\widetilde{\xi}} & \frac{\partial G}{\partial U_S} \end{vmatrix}}^{\equiv T_1}}{\underbrace{\begin{vmatrix} \frac{\partial F}{\partial P} & \frac{\partial F}{\partial U_S} \\ \frac{\partial G}{\partial P} & \frac{\partial G}{\partial U_S} \end{vmatrix}}_{\equiv T_2}}.$$

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We can immediately see that $\frac{\partial F}{\partial P} < 0$, $\frac{\partial F}{\partial U_S} < 0$, and $\frac{\partial F}{\partial \widetilde{\xi}} < 0$. Moreover,

$$\frac{\partial G}{\partial P} = -\rho \left[1 - \underbrace{\frac{1}{c} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2}\frac{1}{\theta} \left(\pi^d - P \right)^2 \right] - P - \widetilde{\xi} U_S \right) \left(1 + \frac{\lambda}{1-\delta}\frac{1}{\theta} \left(\pi^d - P \right) \right)}_{\equiv X} \right].$$

According to F we have $X = 1$. Thus, $\frac{\partial G}{\partial P} = 0$. Furthermore,

$$\frac{\partial G}{\partial U_S} = \underbrace{1 - \rho\widetilde{\xi}}_{>0} + \rho\widetilde{\xi}\frac{1}{c} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2}\frac{1}{\theta} \left(\pi^d - P \right)^2 \right] - P - \widetilde{\xi} U_S \right) > 0.$$

We also get

$$\frac{\partial G}{\partial \widetilde{\xi}} = -\rho U_S \left[1 - \frac{1}{c} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2}\frac{1}{\theta} \left(\pi^d - P \right)^2 \right] - P - \widetilde{\xi} U_S \right) \right].$$

Note that we can write F as

$$\frac{1}{c} \left[\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2}\frac{1}{\theta} \left(\pi^d - P \right)^2 \right] - P - \widetilde{\xi} U_S \right] = \frac{1}{1 + \frac{1}{\theta}\frac{\lambda}{1-\delta} \left(\pi^d - P \right)}.$$

Thus,

$$\frac{\partial G}{\partial \widetilde{\xi}} = -\rho U_S \left[1 - \frac{1}{1 + \frac{1}{\theta}\frac{\lambda}{1-\delta} \left(\pi^d - P \right)} \right] = -\rho U_S \left[\frac{\frac{1}{\theta}\frac{\lambda}{1-\delta} \left(\pi^d - P \right)}{1 + \frac{1}{\theta}\frac{\lambda}{1-\delta} \left(\pi^d - P \right)} \right] < 0.$$

Consequently,

$$\begin{aligned}
T_1 &= -\overbrace{\frac{\partial F}{\partial \widetilde{\xi}}}^{<0}\overbrace{\frac{\partial G}{\partial U_S}}^{>0} + \overbrace{\frac{\partial G}{\partial \widetilde{\xi}}}^{<0}\overbrace{\frac{\partial F}{\partial U_S}}^{<0} > 0 \\
T_2 &= \underbrace{\frac{\partial F}{\partial P}}_{<0}\underbrace{\frac{\partial G}{\partial U_S}}_{>0} - \underbrace{\frac{\partial G}{\partial P}}_{=0}\frac{\partial F}{\partial U_S} < 0.
\end{aligned}$$

Thus, $\frac{dP^*}{d\widetilde{\xi}} < 0$. This also implies that $\frac{\partial \theta^*}{\partial \widetilde{\xi}} = -\frac{dP^*}{d\widetilde{\xi}} > 0$.

Likewise,

$$\frac{dU_S}{d\widetilde{\xi}} = \frac{\begin{vmatrix} \frac{\partial F}{\partial P} & -\frac{\partial F}{\partial \widetilde{\xi}} \\ \frac{\partial G}{\partial P} & -\frac{\partial G}{\partial \widetilde{\xi}} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F}{\partial P} & \frac{\partial F}{\partial U_S} \\ \frac{\partial G}{\partial P} & \frac{\partial G}{\partial U_S} \end{vmatrix}} = \frac{-\overbrace{\frac{\partial F}{\partial P}}^{<0} \overbrace{\frac{\partial G}{\partial \widetilde{\xi}}}^{<0} + \overbrace{\frac{\partial G}{\partial P}}^{=0} \frac{\partial F}{\partial \widetilde{\xi}}}{\underbrace{\frac{\partial F}{\partial P}}_{<0} \underbrace{\frac{\partial G}{\partial U_S}}_{>0} - \underbrace{\frac{\partial G}{\partial P}}_{=0} \frac{\partial F}{\partial U_S}} > 0.$$

Next, recall that $c^*(\widetilde{\xi}) = \frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^* - \widetilde{\xi}U_S$. Thus,

$$\frac{dc^*(\widetilde{\xi})}{d\widetilde{\xi}} = \underbrace{\frac{\partial c^*(\widetilde{\xi})}{\partial P}}_{<0}\underbrace{\frac{dP^*}{d\widetilde{\xi}}}_{<0} + \underbrace{\frac{\partial c^*(\widetilde{\xi})}{\partial U_S}}_{<0}\underbrace{\frac{dU_S}{d\widetilde{\xi}}}_{>0} + \underbrace{\frac{\partial c^*(\widetilde{\xi})}{\partial \widetilde{\xi}}}_{<0}.$$

Note that in equilibrium the direct effect $\frac{\partial c^*(\widetilde{\xi})}{\partial \widetilde{\xi}}$ (negative) must be greater than the indirect price effect $\frac{\partial c^*(\widetilde{\xi})}{\partial P}\frac{dP^*}{d\widetilde{\xi}}$ (positive). Thus, $\frac{dc^*(\widetilde{\xi})}{d\widetilde{\xi}} < 0$.

Next, using (28) we get

$$\frac{dn_Y^{a*}}{d\widetilde{\xi}} = \frac{\partial n_Y^{a*}}{\partial c^*}\underbrace{\frac{dc^*(\widetilde{\xi})}{d\widetilde{\xi}}}_{<0} + \underbrace{\frac{\partial n_Y^{a*}}{\partial \widetilde{\xi}}}_{>0},$$

with

$$\frac{\partial n_Y^{a*}}{\partial c^*} = \frac{-\frac{1}{\overline{c}}\rho n_S\left[1 - \widetilde{\xi}\left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right)\rho\right] - \left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right)\rho n_S\widetilde{\xi}\frac{1}{\overline{c}}\rho}{\left[1 - \widetilde{\xi}\left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right)\rho\right]^2} = -\frac{\frac{1}{\overline{c}}\rho n_S}{\left[1 - \widetilde{\xi}\left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right)\rho\right]^2} < 0.$$

Thus, $\frac{dn_Y^{a*}}{d\widetilde{\xi}} > 0$. Finally recall that $n_Y^{sc*} = \frac{c^*(\widetilde{\xi})}{\overline{c}}\left(\rho n_S + \rho\widetilde{\xi}n_Y^{a*}\right)$. Thus,

$$\frac{dn_Y^{sc*}}{d\widetilde{\xi}} = \frac{1}{\overline{c}}\rho\left[\frac{dc^*(\widetilde{\xi})}{d\widetilde{\xi}}\left(n_S + \widetilde{\xi}n_Y^{a*}\right) + c^*(\widetilde{\xi})\left(n_Y^{a*} + \widetilde{\xi}\frac{dn_Y^{a*}}{d\widetilde{\xi}}\right)\right].$$

Because $\frac{dc^*(\widetilde{\xi})}{d\widetilde{\xi}} < 0$ and $\frac{dn_Y^{a*}}{d\widetilde{\xi}} > 0$, the sign of $\frac{dn_Y^{sc*}}{d\widetilde{\xi}}$ is ambiguous.

Next, we derive the domestic welfare function. Again, let c_t^{fb} denote the scaleup cost ceiling that maximizes domestic welfare at time t. For our derivations we need

$$U_S(\widetilde{\xi} = 0, c_t^{fb}) = \rho\left[P_{t+1} + \frac{1}{\overline{c}}\left(\lambda U_{M,t+2} - P_{t+1}\right)c_t^{fb} - \frac{1}{\overline{c}}\frac{1}{2}\left(c_t^{fb}\right)^2\right], \tag{35}$$

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which follows from (30). Using $E[c_t | c_t \leq c_t^{fb}] = \frac{1}{2} c_t^{fb}$, $E[\theta_t | \theta_t \leq \theta_t^*] = \frac{1}{2} \theta_t^*$, $n_{Y,t}^a = \left(1 - \frac{c_t^{fb}}{\bar{c}}\right) \rho \left(n_{S,t-1} + \tilde{\xi} n_{Y,t-2}^a\right)$, $n_{Y,t}^{sc} = \frac{c_t^{fb}}{\bar{c}} \rho \left(n_{S,t-1} + \tilde{\xi} n_{Y,t-2}^a\right)$, $n_{M,t}^a = \frac{1}{\bar{\theta}} \left(\pi^d - P_t\right) N_{M,t}$, and $\theta_t^* = \pi^d - P_t$, we can write the domestic welfare function (10) as

$$W_t^d(c_t^{fb}) = \rho \left(n_{S,t-1} + \tilde{\xi} n_{Y,t-2}^a\right) \left[P_t^* - P_t^* \frac{c_t^{fb}}{\bar{c}} - \frac{1}{2} \frac{1}{\bar{c}} \left(c_t^{fb}\right)^2\right] + N_{M,t} \left[\Psi + \frac{1}{2} \frac{1}{\bar{\theta}} \left(\pi^d - P_t^*\right)^2\right].$$

In the steady state we have $N_M = \frac{\lambda}{1-\delta} n_Y^s$. Using (4), the steady-state domestic welfare function can then be written as

$$
\begin{aligned}
W^d(c^{fb}) &= \rho \left(n_S + \tilde{\xi} n_Y^a \left(c^{fb}\right)\right) \left[P^* + \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\bar{\theta}} \left(\pi^d - P^*\right)^2\right] - P^*\right) \frac{c^{fb}}{\bar{c}} - \frac{1}{2} \frac{1}{\bar{c}} \left(c^{fb}\right)^2\right] \\
&= \left(n_S + \tilde{\xi} n_Y^a \left(c^{fb}\right)\right) U_S(\tilde{\xi} = 0, c^{fb}),
\end{aligned}
$$

where the equilibrium price P^* is defined by

$$\left[1 + \frac{1}{\bar{\theta}} \frac{\lambda}{1-\delta} \left(\pi^d - P\right)\right] \frac{c^{fb}}{\bar{c}} = 1. \tag{36}$$

Note that c^{fb} satisfies

$$\frac{dW^d(c^{fb})}{dc^{fb}} = \frac{dn_Y^a \left(c^{fb}\right)}{dc^{fb}} \tilde{\xi} U_S(\tilde{\xi} = 0, c^{fb}) + \left[n_S + \tilde{\xi} n_Y^a \left(c^{fb}\right)\right] \frac{dU_S(\tilde{\xi} = 0, c^{fb})}{dc^{fb}} = 0.$$

Recall that $n_Y^a \left(c^{fb}\right)$ is defined by $n_Y^a = \left(1 - \frac{c^{fb}}{\bar{c}}\right) \rho \left(n_S + \tilde{\xi} n_Y^a\right)$. Implicitly differentiating n_Y^a w.r.t. c^{fb} yields

$$\frac{dn_Y^a}{dc^{fb}} = -\frac{\frac{1}{\bar{c}} \rho \left(n_S + \tilde{\xi} n_Y^a\right)}{1 - \left(1 - \frac{c^{fb}}{\bar{c}}\right) \rho \tilde{\xi}}.$$

Moreover, using (35) with (4), and (36), we get

$$\frac{dU_S(\tilde{\xi} = 0, c^{fb})}{dc^{fb}} = \rho \left[\frac{1}{\bar{c}} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\bar{\theta}} \left(\pi^d - P^*\right)^2\right] - P^*\right) - \frac{1}{\bar{c}} c^{fb}\right].$$

Consequently,

$$
\begin{aligned}
\frac{dW^d(c^{fb})}{dc^{fb}} &= -\frac{\frac{1}{\bar{c}}\rho\left(n_S + \widetilde{\xi}n_Y^a\right)}{1 - \left(1 - \frac{c^{fb}}{\bar{c}}\right)\rho\widetilde{\xi}}\widetilde{\xi}U_S(\widetilde{\xi} = 0, c^{fb}) \\
&\quad + \frac{1}{\bar{c}}\rho\left(n_S + \widetilde{\xi}n_Y^a\right)\left[\left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^*\right) - c^{fb}\right].
\end{aligned}
$$

Next, we evaluate $\frac{dW^d(c^{fb})}{dc^{fb}}$ at $c^{fb} = c^*(\widetilde{\xi}) = \lambda U_M - P^* - \widetilde{\xi}U_S$. We have $\frac{dW^d(c^{fb})}{dc^{fb}}\bigg|_{c^{fb}=c^*(\widetilde{\xi})} = 0$ if $\left(1 - \left(1 - \frac{c^*(\widetilde{\xi})}{\bar{c}}\right)\rho\widetilde{\xi}\right)U_S = U_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi}))$. Using (30) with $c^*(\widetilde{\xi})$ and (35), we can write the condition as

$$
\begin{aligned}
&\left(1 - \left(1 - \frac{c^*(\widetilde{\xi})}{\bar{c}}\right)\rho\widetilde{\xi}\right)\rho\left[P^* + \widetilde{\xi}U_S + \frac{1}{\bar{c}}\left(\lambda U_M - P^* - \widetilde{\xi}U_S\right)c^*(\widetilde{\xi}) - \frac{1}{\bar{c}}\frac{1}{2}\left(c^*(\widetilde{\xi})\right)^2\right] \\
&= \rho\left[P^* + \frac{1}{\bar{c}}\left(\lambda U_M - P^*\right)c^*(\widetilde{\xi}) - \frac{1}{\bar{c}}\frac{1}{2}\left(c^*(\widetilde{\xi})\right)^2\right],
\end{aligned}
$$

which can be simplified to

$$
U_S = \rho\left[P^* + \widetilde{\xi}U_S + \frac{1}{\bar{c}}\left(\lambda U_M - P^* - \widetilde{\xi}U_S\right)c^*(\widetilde{\xi}) - \frac{1}{\bar{c}}\frac{1}{2}\left(c^*(\widetilde{\xi})\right)^2\right].
$$

Note that the expression on the RHS is U_S. Thus, $\frac{dW^d(c^{fb})}{dc^{fb}}\bigg|_{c^{fb}=c^*(\widetilde{\xi})} = 0$, i.e., the equilibrium cost ceiling $c^*(\widetilde{\xi})$ maximizes domestic welfare, and is therefore efficient (as $c^*(\widetilde{\xi}) = c^{fb}$).

Finally, we have

$$
\frac{dW^d}{d\widetilde{\xi}} = \left(n_Y^{a*} + \widetilde{\xi}\frac{dn_Y^{a*}}{d\widetilde{\xi}}\right)U_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi})) + \left(n_S + \widetilde{\xi}n_Y^{a*}\right)\frac{dU_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi}))}{d\widetilde{\xi}}.
$$

We can see from (30) that $\frac{\partial U_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{\partial\widetilde{\xi}} = 0$. Thus, $\frac{dU_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{d\widetilde{\xi}} = \frac{\partial U_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{\partial c^*}\frac{dc^*(\widetilde{\xi})}{d\widetilde{\xi}}$. Using (35) with (4) and $c^*(\widetilde{\xi})$, and (36), we get $\frac{\partial U_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{\partial c^*} = \rho\frac{1}{\bar{c}}\left[\left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^*\right) - c^*(\widetilde{\xi})\right]$. Using $c^*(\widetilde{\xi}) = \frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^* - \widetilde{\xi}U_S$, we find that $\frac{\partial U_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{\partial c^*} = \rho\frac{1}{\bar{c}}\widetilde{\xi}U_S > 0$. Evaluating $\frac{dW^d}{d\widetilde{\xi}}$ at $\widetilde{\xi} \to 0$ we then get $\frac{dW^d}{d\widetilde{\xi}}\bigg|_{\widetilde{\xi}\to 0} = n_Y^a U_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi})) > 0$. $\qquad\square$

Proof of Proposition 10.

For parsimony define $\overline{\chi} \equiv 1 - \chi$. With serial entrepreneurs and brain drain in the open economy, the number of early acquisitions and the number of scaleups in t are given by

$$
n_{Y,t}^a = \left(1 - \frac{c_t^*(\widetilde{\xi})}{\overline{c}}\right) \rho \left(n_{S,t-1} + \widetilde{\xi}\left(n_{M,t-1}^a + \overline{\chi} n_{F,t-1}^a\right)\right), \tag{37}
$$

$$
n_{Y,t}^{sc} = \frac{c_t^*(\widetilde{\xi})}{\overline{c}} \rho \left(n_{S,t-1} + \widetilde{\xi}\left(n_{M,t-1}^a + \overline{\chi} n_{F,t-1}^a\right)\right). \tag{38}
$$

Moreover, recall that $n_{M,t-1}^a = \frac{\theta_{t-1}^*}{\theta} N_{M,t-1}$ with $\theta_{t-1}^* = \pi^d - P_{t-1}$, and $n_{F,t-1}^a = \frac{\phi_{t-1}^*}{\widetilde{\phi}} N_F$ with $\phi_{t-1} = \pi^f - P_{t-1}$.

Now consider the steady state. Using $N_M = \frac{\lambda}{1-\delta} n_Y^{sc}$ we get

$$
n_M^a = \frac{1}{\theta} \frac{\lambda}{1-\delta} \left(\pi^d - P^*\right) \frac{c^*(\widetilde{\xi})}{\overline{c}} \rho \left(n_S + \widetilde{\xi}\left(n_M^a + \overline{\chi} n_F^a\right)\right). \tag{39}
$$

For parsimony define $\Gamma \equiv \frac{1}{\theta} \frac{\lambda}{1-\delta} \left(\pi^d - P^*\right)$. Solving (39) for n_M^a we get

$$
n_M^{a*} = \frac{\Gamma \frac{c^*(\widetilde{\xi})}{\overline{c}} \rho \left(n_S + \widetilde{\xi} \overline{\chi} n_F^a\right)}{1 - \Gamma \frac{c^*(\widetilde{\xi})}{\overline{c}} \rho \widetilde{\xi}}. \tag{40}
$$

To ensure that $n_M^{a*} > 0$, we assume that the parameter values are such that $1 - \Gamma \frac{c^*(\widetilde{\xi})}{\overline{c}} \rho \widetilde{\xi} > 0$.

Using (37) and (40), the market clearing-condition, $n_Y^a = n_M^a + n_F^a$, can be written as

$$
\left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right) \rho \left(n_S + \widetilde{\xi}\left(\frac{\Gamma \frac{c^*(\widetilde{\xi})}{\overline{c}} \rho \left(n_S + \widetilde{\xi} \overline{\chi} n_F^a\right)}{1 - \Gamma \frac{c^*(\widetilde{\xi})}{\overline{c}} \rho \widetilde{\xi}} + \overline{\chi} n_F^a\right)\right) = \frac{\Gamma \frac{c^*(\widetilde{\xi})}{\overline{c}} \rho \left(n_S + \widetilde{\xi} \overline{\chi} n_F^a\right)}{1 - \Gamma \frac{c^*(\widetilde{\xi})}{\overline{c}} \rho \widetilde{\xi}} + n_F^a.
$$

Rearranging yields

$$
\left[(1 + \Gamma) \frac{c^*(\widetilde{\xi})}{\overline{c}} - 1\right] \rho \left(n_S + \widetilde{\xi} \overline{\chi} n_F^a\right) + \left[1 - \Gamma \frac{c^*(\widetilde{\xi})}{\overline{c}} \rho \widetilde{\xi}\right] n_F^a = 0. \tag{41}
$$

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Using $c^*(\widetilde{\xi}) = \frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P\right)^2\right] - P - \widetilde{\xi}U_S$, $n_F^a = \frac{1}{\phi}\left(\pi^f - P\right)N_F$, and $\Gamma = \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)$, we can write the market-clearing condition (41) as

$$
\begin{aligned}
F \;\equiv\; & \left[\left(1 + \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right)\frac{1}{\bar{c}}\left[\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P\right)^2\right] - P - \widetilde{\xi}U_S\right] - 1\right]\rho\left(n_S + \widetilde{\xi}\overline{\chi}\frac{1}{\phi}\left(\pi^f - P\right)N_F\right) \\
& + \frac{1}{\phi}\left(\pi^f - P\right)N_F\left[1 - \rho\widetilde{\xi}\frac{1}{\theta}\frac{\lambda}{1-\delta}\frac{1}{\bar{c}}\left(\pi^d - P\right)\left[\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P\right)^2\right] - P - \widetilde{\xi}U_S\right]\right].
\end{aligned}
\tag{42}
$$

Moreover, we know from (34) that U_S is defined by

$$
G \equiv U_S - \rho\left[P + \widetilde{\xi}U_S + \frac{1}{2}\frac{1}{\bar{c}}\left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P\right)^2\right] - P - \widetilde{\xi}U_S\right)^2\right] = 0.
\tag{43}
$$

The steady-state equilibrium is then defined by (42) and (43).

For the comparative statics we focus on the limit case with $N_F \to 0$. We first note that $\frac{\partial F}{\partial \overline{\chi}}\big|_{N_F \to 0} = 0$ and $\frac{\partial G}{\partial \overline{\chi}}\big|_{N_F \to 0} = 0$. This immediately implies that $\frac{dP^*}{d\overline{\chi}}\big|_{N_F \to 0} = 0$ and $\frac{dU_S}{d\overline{\chi}}\big|_{N_F \to 0} = 0$. Thus, $\frac{dP^*}{d\chi}\big|_{N_F \to 0} = 0$ and $\frac{dU_S}{d\chi}\big|_{N_F \to 0} = 0$. This also implies that $\frac{dc^*(\widetilde{\xi})}{d\chi}\big|_{N_F \to 0} = 0$ and $\frac{d\theta^*}{d\chi}\big|_{N_F \to 0} = 0$. Furthermore, note that

$$
\frac{dn_F^{a*}}{d\overline{\chi}}\big|_{N_F \to 0} \;=\; 0
$$

$$
\frac{dn_M^{a*}}{d\overline{\chi}}\big|_{N_F \to 0} \;=\; \frac{\partial n_M^{a*}}{\partial U_S}\underbrace{\frac{dU_S}{d\overline{\chi}}\big|_{N_F \to 0}}_{=0} + \frac{\partial n_M^{a*}}{\partial P}\underbrace{\frac{dP^*}{d\overline{\chi}}\big|_{N_F \to 0}}_{=0} + \underbrace{\frac{\partial n_M^{a*}}{\partial \overline{\chi}}}_{>0} > 0.
$$

Consequently, $\frac{dn_F^{a*}}{d\chi}\big|_{N_F \to 0} = 0$ and $\frac{dn_M^{a*}}{d\chi}\big|_{N_F \to 0} < 0$. Moreover,

$$
\frac{dn_Y^{sc*}}{d\overline{\chi}}\big|_{N_F \to 0} \;=\; \frac{\partial n_Y^{sc*}}{\partial c^*}\underbrace{\frac{dc^*(\widetilde{\xi})}{d\overline{\chi}}}_{=0} + \frac{\partial n_Y^{sc*}}{\partial n_M^a}\underbrace{\frac{dn_M^{a*}}{d\overline{\chi}}}_{>0} + \frac{\partial n_Y^{sc*}}{\partial n_F^a}\underbrace{\frac{dn_F^{a*}}{d\overline{\chi}}}_{=0} + \underbrace{\frac{\partial n_Y^{sc*}}{\partial \overline{\chi}}}_{>0} > 0
$$

$$
\frac{dn_Y^{a*}}{d\overline{\chi}}\big|_{N_F \to 0} \;=\; \frac{\partial n_Y^{a*}}{\partial c^*}\underbrace{\frac{dc^*(\widetilde{\xi})}{d\overline{\chi}}}_{=0} + \frac{\partial n_Y^{a*}}{\partial n_M^a}\underbrace{\frac{dn_M^{a*}}{d\overline{\chi}}}_{>0} + \frac{\partial n_Y^{a*}}{\partial n_F^a}\underbrace{\frac{dn_F^{a*}}{d\overline{\chi}}}_{=0} + \underbrace{\frac{\partial n_Y^{a*}}{\partial \overline{\chi}}}_{>0} > 0.
$$

Thus, $\frac{dn_Y^{sc*}}{d\chi}\big|_{N_F \to 0} < 0$ and $\frac{dn_Y^{a*}}{d\chi}\big|_{N_F \to 0} < 0$. Finally, recall that $N_M^* = \frac{\lambda}{1-\delta}n_Y^{sc*}$. Because $\frac{dn_Y^{sc*}}{d\chi}\big|_{N_F \to 0} < 0$, we can immediately see that $\frac{dN_M^*}{d\chi}\big|_{N_F \to 0} < 0$.

Next, following along the lines of the derivation of the domestic welfare function W^d in Proof of Proposition 9, we find that the domestic welfare function in the presence of serial entrepreneurs with brain drain is given by

$$W^d = \left[n_S + \widetilde{\xi} \left(n_M^{a*} + \overline{\chi} n_F^{a*} \right) \right] U_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi})),$$

where

$$U_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi})) = \rho \left[P^* + \frac{1}{2} \frac{1}{\overline{c}} \left[\left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\overline{\theta}} \left(\pi^d - P^* \right)^2 \right] - P^* \right)^2 - \left(\widetilde{\xi} U_S \right)^2 \right] \right].$$
(44)

We have already shown that $\frac{dP^*}{d\overline{\chi}} \big|_{N_F \to 0} = 0$ and $\frac{dU_S}{d\overline{\chi}} \big|_{N_F \to 0} = 0$. Thus, $\frac{dU_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{d\overline{\chi}} = 0$. Moreover, $\frac{dn_M^{a*}}{d\overline{\chi}} \big|_{N_F \to 0} > 0$ and $\frac{dn_F^{a*}}{d\overline{\chi}} \big|_{N_F \to 0} = 0$. Thus, $\frac{dW^d}{d\overline{\chi}} \big|_{N_F \to 0} > 0$, which implies that $\frac{dW^d}{d\chi} \big|_{N_F \to 0} < 0$. $\qquad \square$

Proof of Proposition 11.

Recall from Proof of Proposition 10 that $W^d = \left[n_S + \widetilde{\xi} \left(n_M^{a*} + \overline{\chi} n_F^{a*} \right) \right] U_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi}))$, where $\overline{\chi} \equiv 1 - \chi$, $U_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi}))$ is given by (44), n_M^{a*} is given by (40), and $n_F^{a*} = \frac{1}{\phi} \left(\pi^f - P^* \right) N_F$. Moreover, the market-clearing condition is given by (42), and U_S is defined by (43).

First suppose that $\chi = 1$. The market equilibrium is then defined by

$$
\begin{aligned}
F \equiv & \; \frac{1}{\overline{c}} \left(1 + \frac{1}{\overline{\theta}} \frac{\lambda}{1-\delta} \left(\pi^d - P \right) \right) \left[\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\overline{\theta}} \left(\pi^d - P \right)^2 \right] - P - \widetilde{\xi} U_S \right] - 1 \\
& + \frac{1}{\phi} \left(\pi^f - P \right) \frac{N_F}{\rho n_S} \left[1 - \rho \widetilde{\xi} \frac{1}{\overline{\theta}} \frac{\lambda}{1-\delta} \frac{1}{\overline{c}} \left(\pi^d - P \right) \left[\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\overline{\theta}} \left(\pi^d - P \right)^2 \right] - P - \widetilde{\xi} U_S \right] \right] \\
= & \; 0
\end{aligned}
$$
(45)

$$G \equiv U_S - \rho \left[P + \widetilde{\xi} U_S + \frac{1}{2} \frac{1}{\overline{c}} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\overline{\theta}} \left(\pi^d - P \right)^2 \right] - P - \widetilde{\xi} U_S \right)^2 \right] = 0,$$
(46)

where (45) is the market-clearing condition for $\chi = 1$, and (46) the condition defining U_S. With $\chi = 1$ we get $W^d(\chi = 1) = \left[n_S + \widetilde{\xi} n_M^{a*} \right] U_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi}))$. Thus,

$$\frac{dW^d(\chi = 1)}{dN_F} \bigg|_{N_F=0} = \widetilde{\xi} \frac{dn_M^{a*}}{dN_F} \big|_{N_F=0} U_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi})) + \left[n_S + \widetilde{\xi} n_M^{a*} \right] \frac{dU_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi}))}{dN_F} \big|_{N_F=0}.$$

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To find the signs of $\frac{dn_M^{a*}}{dN_F}|_{N_F=0}$ and $\frac{dU_S(\tilde{\xi}=0,c^*(\tilde{\xi}))}{dN_F}|_{N_F=0}$ we need to do the following basic comparative statics. First, applying Cramer's rule we get

$$
\frac{dP^*}{dN_F}\bigg|_{N_F=0} = \frac{\overbrace{\begin{vmatrix} -\frac{\partial F}{\partial N_F}\big|_{N_F=0} & \frac{\partial F}{\partial U_S}\big|_{N_F=0} \\ -\frac{\partial G}{\partial N_F}\big|_{N_F=0} & \frac{\partial G}{\partial U_S}\big|_{N_F=0} \end{vmatrix}}^{\equiv T_1}}{\underbrace{\begin{vmatrix} \frac{\partial F}{\partial P}\big|_{N_F=0} & \frac{\partial F}{\partial U_S}\big|_{N_F=0} \\ \frac{\partial G}{\partial P}\big|_{N_F=0} & \frac{\partial G}{\partial U_S}\big|_{N_F=0} \end{vmatrix}}_{\equiv T_2}}.
$$

Note that $\frac{\partial G}{\partial N_F}\big|_{N_F=0} = 0$ and

$$
\frac{\partial F}{\partial N_F}\bigg|_{N_F=0} = \frac{1}{\phi}\left(\pi^f - P\right)\frac{1}{\rho n_S}\underbrace{\left[1 - \rho\tilde{\xi}\frac{1}{\theta}\frac{\lambda}{1-\delta}\frac{1}{\bar{c}}\left(\pi^d - P\right)\left[\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P\right)^2\right] - P - \tilde{\xi}U_S\right]\right]}_{=1-\Gamma\frac{c^*(\tilde{\xi})}{\bar{c}}\rho\tilde{\xi}>0} > 0.
$$

We can also immediately see that $\frac{\partial F}{\partial P}\big|_{N_F=0} < 0$. Moreover,

$$
\frac{\partial G}{\partial P}\bigg|_{N_F=0} = -\rho\left[1 - \frac{1}{\bar{c}}\left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P\right)^2\right] - P - \tilde{\xi}U_S\right)\left(1 + \frac{\lambda}{1-\delta}\frac{1}{\theta}\left(\pi^d - P\right)\right)\right].
$$

Using the market-clearing condition (45) with $N_F \to 0$ we get

$$
1 = \frac{1}{\bar{c}}\left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P\right)^2\right] - P - \tilde{\xi}U_S\right)\left(1 + \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right). \tag{47}
$$

Thus, $\frac{\partial G}{\partial P}\big|_{N_F=0} = 0$. Furthermore, we can see that $\frac{\partial F}{\partial U_S}\big|_{N_F=0} < 0$. Moreover,

$$
\frac{\partial G}{\partial U_S}\bigg|_{N_F=0} = 1 - \rho\tilde{\xi}\left[1 - \frac{1}{\bar{c}}\left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P\right)^2\right] - P - \tilde{\xi}U_S\right)\right].
$$

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Using (47),

$$
\begin{aligned}
\left.\frac{\partial G}{\partial U_S}\right|_{N_F=0} &= 1 - \rho\widetilde{\xi}\left[\frac{1}{\bar{c}}\left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P\right)^2\right] - P - \widetilde{\xi}U_S\right)\frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P\right)\right] \\
&= 1 - \Gamma\frac{c^*(\widetilde{\xi})}{\bar{c}}\rho\widetilde{\xi} > 0.
\end{aligned}
$$

Consequently,

$$
T_1 = -\overbrace{\left.\frac{\partial F}{\partial N_F}\right|_{N_F=0}}^{>0}\overbrace{\left.\frac{\partial G}{\partial U_S}\right|_{N_F=0}}^{>0} + \overbrace{\left.\frac{\partial G}{\partial N_F}\right|_{N_F=0}}^{=0}\left.\frac{\partial F}{\partial U_S}\right|_{N_F=0} < 0
$$

$$
T_2 = \underbrace{\left.\frac{\partial F}{\partial P}\right|_{N_F=0}}_{<0}\underbrace{\left.\frac{\partial G}{\partial U_S}\right|_{N_F=0}}_{>0} - \underbrace{\left.\frac{\partial G}{\partial P}\right|_{N_F=0}}_{=0}\left.\frac{\partial F}{\partial U_S}\right|_{N_F=0} < 0.
$$

Thus, $\left.\frac{dP^*}{dN_F}\right|_{N_F=0} > 0$. Likewise,

$$
\left.\frac{dU_S}{dN_F}\right|_{N_F=0} = \frac{\overbrace{\left|\begin{matrix} \left.\frac{\partial F}{\partial P}\right|_{N_F=0} & -\left.\frac{\partial F}{\partial N_F}\right|_{N_F=0} \\ \left.\frac{\partial G}{\partial P}\right|_{N_F=0} & -\left.\frac{\partial G}{\partial N_F}\right|_{N_F=0} \end{matrix}\right|}^{\equiv T_3}}{\underbrace{\left|\begin{matrix} \left.\frac{\partial F}{\partial P}\right|_{N_F=0} & \left.\frac{\partial F}{\partial U_S}\right|_{N_F=0} \\ \left.\frac{\partial G}{\partial P}\right|_{N_F=0} & \left.\frac{\partial G}{\partial U_S}\right|_{N_F=0} \end{matrix}\right|}_{=T_2}}.
$$

We already know that $T_2 < 0$. Moreover,

$$
T_3 = -\left.\frac{\partial F}{\partial P}\right|_{N_F=0}\overbrace{\left.\frac{\partial G}{\partial N_F}\right|_{N_F=0}}^{=0} + \overbrace{\left.\frac{\partial G}{\partial P}\right|_{N_F=0}}^{=0}\left.\frac{\partial F}{\partial N_F}\right|_{N_F=0} = 0.
$$

Thus, $\left.\frac{dU_S}{dN_F}\right|_{N_F=0} = 0$. Moreover, recall that $c^*(\widetilde{\xi}) = \frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^* - \widetilde{\xi}U_S$.
Because $\left.\frac{dP^*}{dN_F}\right|_{N_F=0} > 0$ and $\left.\frac{dU_S}{dN_F}\right|_{N_F=0} = 0$, we have $\left.\frac{dc^*(\widetilde{\xi})}{dN_F}\right|_{N_F=0} < 0$.

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Next, using $\Gamma = \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P^*\right)$ and $\chi = 1$ in (40) we get

$$n_M^{a*} = \frac{\frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P^*\right)\frac{c^*(\widetilde{\xi})}{\bar{c}}\rho n_S}{1 - \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P^*\right)\frac{c^*(\widetilde{\xi})}{\bar{c}}\rho\widetilde{\xi}}.$$

We can immediately see that $\frac{\partial n_M^{a*}}{\partial P} < 0$ and $\frac{\partial n_M^{a*}}{\partial c^*} > 0$. And because $\left.\frac{dP^*}{dN_F}\right|_{N_F=0} > 0$ and $\left.\frac{dc^*(\widetilde{\xi})}{dN_F}\right|_{N_F=0} < 0$, we have $\left.\frac{dn_M^{a*}}{dN_F}\right|_{N_F=0} < 0$.

Furthermore, using (44) we get

$$\left.\frac{dU_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{dN_F}\right|_{N_F=0} = \left.\frac{\partial U_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{\partial P}\right|_{N_F=0}\underbrace{\left.\frac{dP^*}{dN_F}\right|_{N_F=0}}_{>0} + \left.\frac{\partial U_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{\partial U_S}\right|_{N_F=0}\underbrace{\left.\frac{dU_S}{dN_F}\right|_{N_F=0}}_{=0}$$

$$+ \underbrace{\left.\frac{\partial U_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{\partial N_F}\right|_{N_F=0}}_{=0}. \tag{48}$$

We have

$$\left.\frac{\partial U_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{\partial P}\right|_{N_F=0} = \rho\left[1 - \frac{1}{\bar{c}}\left(\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P^*\right)^2\right] - P^*\right)\left(1 + \frac{\lambda}{1-\delta}\frac{1}{\theta}\left(\pi^d - P^*\right)\right)\right].$$

Using (47) we get

$$\left.\frac{\partial U_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{\partial P}\right|_{N_F=0} = -\rho\frac{1}{\bar{c}}\left(1 + \frac{1}{\theta}\frac{\lambda}{1-\delta}\left(\pi^d - P^*\right)\right)\widetilde{\xi}U_S < 0.$$

This implies that $\left.\frac{dU_S(\widetilde{\xi}=0,c^*(\widetilde{\xi}))}{dN_F}\right|_{N_F=0} < 0$. Consequently, $\left.\frac{dW^d(\chi=1)}{dN_F}\right|_{N_F=0} < 0$.

Now suppose that $\chi = 0$. The number of young companies in the steady state is then given by $n_Y^* = \rho\left(n_S + \widetilde{\xi}\left(n_M^{a*} + n_F^{a*}\right)\right)$. Because in equilibrium $n_Y^a = n_M^a + n_F^a$, we have $n_Y^* = \rho\left(n_S + \widetilde{\xi}n_Y^{a*}\right)$. Thus, $n_Y^{sc*} = \frac{c^*(\widetilde{\xi})}{\bar{c}}\rho\left(n_S + \widetilde{\xi}n_Y^{a*}\right)$ and

$$n_Y^a = \left(1 - \frac{c^*(\widetilde{\xi})}{\bar{c}}\right)\rho\left(n_S + \widetilde{\xi}n_Y^a\right) \quad \Leftrightarrow \quad n_Y^{a*} = \frac{\left(1 - \frac{c^*(\widetilde{\xi})}{\bar{c}}\right)\rho n_S}{1 - \left(1 - \frac{c^*(\widetilde{\xi})}{\bar{c}}\right)\rho\widetilde{\xi}}. \tag{49}$$

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Furthermore, using $N_F = \frac{\lambda}{1-\delta} n_Y^s$ and (49), we get

$$n_M^{a*} = \frac{1}{\overline{\theta}} \frac{\lambda}{1-\delta} \left(\pi^d - P^*\right) \frac{c^*(\widetilde{\xi})}{\overline{c}} \rho \left(n_S + \widetilde{\xi} n_Y^{a*}\right) = \frac{1}{\overline{\theta}} \frac{\lambda}{1-\delta} \left(\pi^d - P^*\right) \frac{\frac{c^*(\widetilde{\xi})}{\overline{c}}}{1 - \left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right) \rho \widetilde{\xi}} \rho n_S.$$

The market clearing-condition, $n_Y^a = n_M^a + n_F^a$, then becomes

$$\frac{\left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right) \rho n_S}{1 - \left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right) \rho \widetilde{\xi}} = \frac{1}{\overline{\theta}} \frac{\lambda}{1-\delta} \left(\pi^d - P\right) \frac{\frac{c^*(\widetilde{\xi})}{\overline{c}}}{1 - \left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right) \rho \widetilde{\xi}} \rho n_S + \frac{1}{\overline{\phi}} \left(\pi^f - P\right) N_F,$$

which can be written as

$$0 = \left[1 + \frac{1}{\overline{\theta}} \frac{\lambda}{1-\delta} \left(\pi^d - P\right)\right] \frac{c^*(\widetilde{\xi})}{\overline{c}} - 1 + \frac{1}{\overline{\phi}} \left(\pi^f - P\right) \frac{N_F}{\rho n_S} \left[1 - \left(1 - \frac{c^*(\widetilde{\xi})}{\overline{c}}\right) \rho \widetilde{\xi}\right].$$

Using $c^*(\widetilde{\xi}) = \frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\overline{\theta}} \left(\pi^d - P\right)^2\right] - P - \widetilde{\xi} U_S$, we find that the market equilibrium is defined by the following two equations for $\chi = 0$:

$$
\begin{aligned}
H &\equiv \frac{1}{\overline{c}} \left[1 + \frac{1}{\overline{\theta}} \frac{\lambda}{1-\delta} \left(\pi^d - P\right)\right] \left[\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\overline{\theta}} \left(\pi^d - P\right)^2\right] - P - \widetilde{\xi} U_S\right] - 1 \\
&\quad + \frac{1}{\overline{\phi}} \left(\pi^f - P\right) \frac{N_F}{\rho n_S} \left[1 - \rho \widetilde{\xi} + \frac{1}{\overline{c}} \rho \widetilde{\xi} \left[\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\overline{\theta}} \left(\pi^d - P\right)^2\right] - P - \widetilde{\xi} U_S\right]\right] = 0
\end{aligned}
$$

$$G = U_S - \rho \left[P + \widetilde{\xi} U_S + \frac{1}{2} \frac{1}{\overline{c}} \left(\frac{\lambda}{1-\delta} \left[\Psi + \frac{1}{2} \frac{1}{\overline{\theta}} \left(\pi^d - P\right)^2\right] - P - \widetilde{\xi} U_S\right)^2\right] = 0.$$

With $\chi = 0$ and $n_M^a + n_F^a = n_Y^a$ we get $W^d(\chi = 0) = \left[n_S + \widetilde{\xi} n_Y^{a*}\right] U_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi}))$. Thus,

$$\left. \frac{dW^d(\chi = 0)}{dN_F} \right|_{N_F=0} = \widetilde{\xi} \frac{dn_Y^{a*}}{dN_F} \Big|_{N_F=0} U_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi})) + \left[n_S + \widetilde{\xi} n_Y^{a*}\right] \frac{dU_S(\widetilde{\xi} = 0, c^*(\widetilde{\xi}))}{dN_F} \Big|_{N_F=0}.$$

Again, to find the signs we first need to do some basic comparative statics. Using Cramer's rule,

$$
\frac{dP^*}{dN_F}\bigg|_{N_F=0} = \frac{\overbrace{\begin{vmatrix} -\left.\frac{\partial H}{\partial N_F}\right|_{N_F=0} & \left.\frac{\partial H}{\partial U_S}\right|_{N_F=0} \\ -\left.\frac{\partial G}{\partial N_F}\right|_{N_F=0} & \left.\frac{\partial G}{\partial U_S}\right|_{N_F=0} \end{vmatrix}}^{\equiv Z_1}}{\underbrace{\begin{vmatrix} \left.\frac{\partial H}{\partial P}\right|_{N_F=0} & \left.\frac{\partial H}{\partial U_S}\right|_{N_F=0} \\ \left.\frac{\partial G}{\partial P}\right|_{N_F=0} & \left.\frac{\partial G}{\partial U_S}\right|_{N_F=0} \end{vmatrix}}_{\equiv Z_2}}.
$$

Recall that $\left.\frac{\partial G}{\partial N_F}\right|_{N_F=0} = \left.\frac{\partial G}{\partial P}\right|_{N_F=0} = 0$ and $\left.\frac{\partial G}{\partial U_S}\right|_{N_F=0} > 0$. And we can also see that $\left.\frac{\partial H}{\partial P}\right|_{N_F=0} < 0$ and $\left.\frac{\partial H}{\partial U_S}\right|_{N_F=0} < 0$. Moreover,

$$
\left.\frac{\partial H}{\partial N_F}\right|_{N_F=0} = \frac{1}{\phi}\left(\pi^f - P\right)\frac{1}{\rho n_S}\left[\underbrace{1 - \rho\widetilde{\xi}}_{>0} + \frac{1}{c}\rho\widetilde{\xi}\underbrace{\left[\frac{\lambda}{1-\delta}\left[\Psi + \frac{1}{2}\frac{1}{\theta}\left(\pi^d - P\right)^2\right] - P - \widetilde{\xi}U_S\right]}_{=c^*(\widetilde{\xi})>0}\right] > 0.
$$

Thus,

$$
Z_1 = -\overbrace{\left.\frac{\partial H}{\partial N_F}\right|_{N_F=0}}^{>0}\overbrace{\left.\frac{\partial G}{\partial U_S}\right|_{N_F=0}}^{>0} + \overbrace{\left.\frac{\partial G}{\partial N_F}\right|_{N_F=0}}^{=0}\left.\frac{\partial H}{\partial U_S}\right|_{N_F=0} < 0
$$

$$
Z_2 = \underbrace{\left.\frac{\partial H}{\partial P}\right|_{N_F=0}}_{<0}\underbrace{\left.\frac{\partial G}{\partial U_S}\right|_{N_F=0}}_{>0} - \underbrace{\left.\frac{\partial G}{\partial P}\right|_{N_F=0}}_{=0}\left.\frac{\partial H}{\partial U_S}\right|_{N_F=0} < 0.
$$

Hence, $\left.\frac{dP^*}{dN_F}\right|_{N_F=0} > 0$. Likewise,

$$
\left.\frac{dU_S}{dN_F}\right|_{N_F=0} = \frac{\overbrace{\begin{vmatrix} \left.\frac{\partial H}{\partial P}\right|_{N_F=0} & -\left.\frac{\partial H}{\partial N_F}\right|_{N_F=0} \\ \left.\frac{\partial G}{\partial P}\right|_{N_F=0} & -\left.\frac{\partial G}{\partial N_F}\right|_{N_F=0} \end{vmatrix}}^{\equiv Z_3}}{\underbrace{\begin{vmatrix} \left.\frac{\partial H}{\partial P}\right|_{N_F=0} & \left.\frac{\partial H}{\partial U_S}\right|_{N_F=0} \\ \left.\frac{\partial G}{\partial P}\right|_{N_F=0} & \left.\frac{\partial G}{\partial U_S}\right|_{N_F=0} \end{vmatrix}}_{=Z_2}}.
$$

We already know that $Z_2 < 0$. Moreover,

$$
Z_3 = -\left.\frac{\partial H}{\partial P}\right|_{N_F=0} \overbrace{\left.\frac{\partial G}{\partial N_F}\right|_{N_F=0}}^{=0} + \overbrace{\left.\frac{\partial G}{\partial P}\right|_{N_F=0}}^{=0} \left.\frac{\partial H}{\partial N_F}\right|_{N_F=0} = 0.
$$

Thus, $\left.\frac{dU_S}{dN_F}\right|_{N_F=0} = 0$. This also implies that $\left.\frac{dc^*(\widetilde{\xi})}{dN_F}\right|_{N_F=0} < 0$. Moreover, we can see that $\frac{\partial n_Y^{a*}}{\partial c^*} < 0$; hence, $\left.\frac{dn_Y^{a*}}{dN_F}\right|_{N_F=0} > 0$. Consequently, $\left.\frac{dW^d(\chi=0)}{dN_F}\right|_{N_F=0} > 0$. $\qquad\square$

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