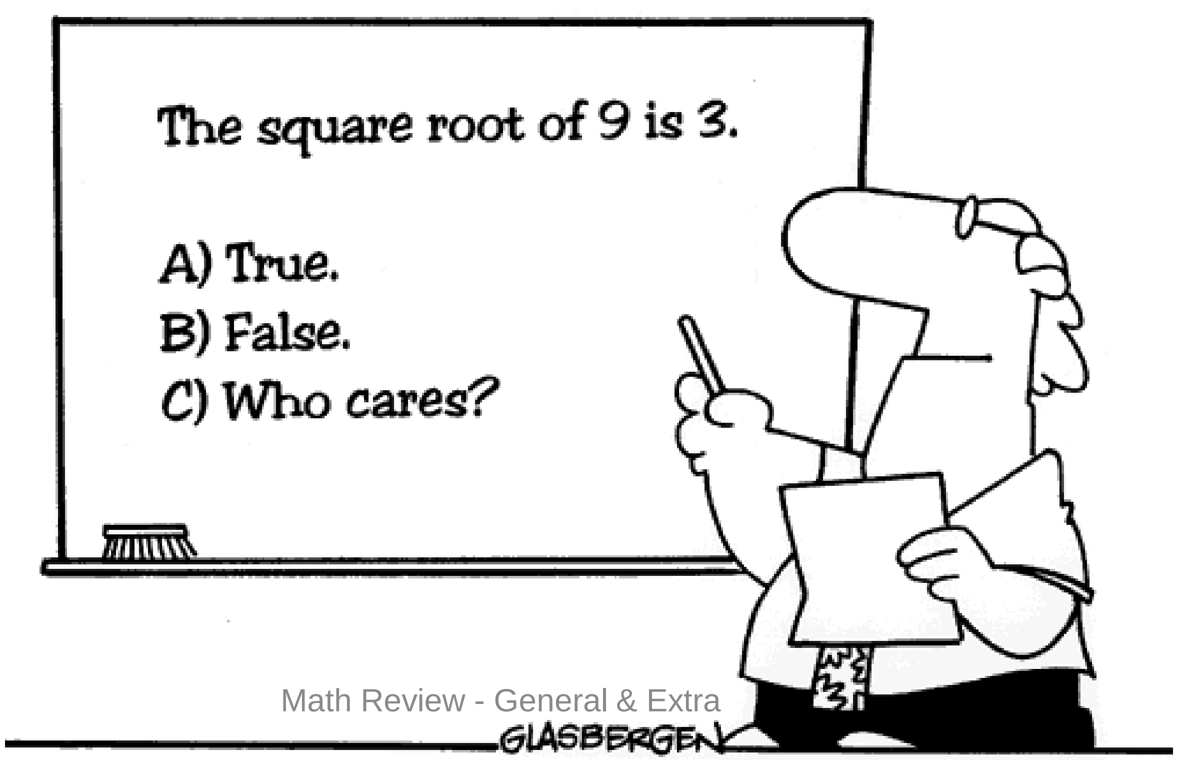
Basic Electronics Series

Mathematics for the Ham Radio Operator

Mathematics is the way we objectively describe the physical world. Without it, we would still be living in caves!



Math Vocabulary

- What is an equation?
- What is an operator?
- What is a variable?
- What is a formula?

Equation

- What does solving an equation mean?
  - Getting the equation to its most simple form so it can be of use to answer a practical question.

Equation

$$C = A + B$$

- The value on the left side of the equal sign in the same as the value on the right side.
- The "answer" or what you're trying to solve for is, by convention, always on the left side of the equal sign.
- All equation manipulation seeks to get the unknown by itself on the left side, in its simplest form.
- Equation manipulation to get the left term in its simplest form requires one to do the same operation on each side of the equal sign, e.g., subtract both sides by 3, divide both sides by ½, etc.
- Implies a single answer. Compare this to an inequality, whose answer is a range of values, e.g., $C < A + B$

Operators

- Operators tell us to do something to the numbers or variables on each side of the operator
  - Addition $+$
  - Subtraction $-$
  - Multiplication x or $\bullet$
  - Division $\div$ or $/$ or $\overline{)\quad}$
  - Exponent Y^X
  - Root $\sqrt{}$ or $\sqrt[n]{}$
  - Reciprocal $1/n$

Operators

- Opposite math operations:

Addition ⇔ Subtraction

Multiplication ⇔ Division

Root ⇔ Exponent

- A number divided by itself is 1… $X / X = 1$

- A number multiplied by 1 is itself… $Y \cdot 1 = Y$

Whatever you do to one side of the equation, you must do to the other side of the equation. There are very defined rules that dictate the order operations are executed… PEMDAS!

Order of Operations

- ***P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally*
  - ***P*** – parentheses
  - ***E*** – exponents
  - ***M*** – multiplication
  - ***D*** – division
  - ***A*** – addition
  - ***S*** subtraction
- This mnemonic will help clarify the order in which a complex equation is worked

Solving an Equation

- Pythagorean Theorem

$$C^2 = A^2 + B^2$$

Solving for C (the length of the hypotenuse in this case)

$$\sqrt{C^2} = \sqrt{(A^2 + B^2)}$$

Apply same square root operation to both sides

$$\cancel{\sqrt{C^2}} = \sqrt{(A^2 + B^2)}$$

Square and square root ops cancel each other

$$C = \sqrt{(A^2 + B^2)}$$

The answer in simplest from for what we're given.

Solving an Equation

- Ohm's Law:

$$E = I \cdot R$$

- The variables:
  - **E** represents voltage
  - **I** represents current
  - **R** represents resistance
- The · operator is multiplication
- The order of the equation implies we know I and R, and need to solve for E

Solving an Equation

- $E = I \cdot R$
  - Current is 10 amps
  - Resistance is 50 ohms
- Therefore, E = 10 amps * 50 ohms
- E = 500 volts

- It's important that the dimensions of the variables (voltage, resistance, current in this case) are carried through the problem to ensure the answer represents the answer you were after. You will see this in antenna length calculations in this course (inches vs. feet). This is vital when problems become more complicated.

Solving an Equation

- What if we know the voltage and the current and want to find the resistance?

$$E = I \cdot R \Rightarrow R = E / I$$

Divide both sides by I

Swap the right & left side of the equal sign to get the unknown value (R) on the left.

Example – Series Resistance

$$R_1 + R_2 + R_3 + R_N = R_T$$

- Calculating resistance of resistors in series, where R_N is the last resistor and R_T is the total resistance

Example – Parallel Resistance

$$R_T = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

- R_1 = 50 ohms
- R_2 = 200 ohms
- R_T = Total resistance

- Calculating total resistance for two resistors in parallel
- Multiply R_1 and R_2
  - Write the number down
- Add R_1 and R_2
  - Write the number down
- Divide the first number by the second to find R_T.

Example – Parallel Resistance

$$R_T = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

- R_1 = 50 ohms
- R_2 = 200 ohms
- R_T = ?

- $R_1 \cdot R_2 = ?$
  - $50 * 200 = 10{,}000$
- $R_1 + R_2 = ?$
  - $50 + 200 = 250$
- $R_T = 10{,}000/250 = 40 \text{ ohms}$

Example – Parallel Resistance

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_N}}$$

- Multiple (n) resistors in parallel
- Do each fraction in the denominator in turn $1/R_n$
  - Write the number down
- Add all fraction results together.
  - Write the number down
- Divide 1 by the sum of the fractions.

Example – Parallel Resistance

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_N}}$$

- R_1 = 50 ohms
- R_2 = 100 ohms
- R_3 = 200 ohms

- $1/R_1 = ?$
  - $1/50 = 0.02$
- $1/R_2 = ?$
  - $1/100 = 0.01$
- $1/R_3 = ?$
  - $1/200 = 0.005$
- Sum of fractions $= ?$
  - $0.02 + 0.01 + 0\ .005 = 0.035$
- $1/$Sum of fractions $= ?$
  - $R_T = 1/0.035 = 28.6$ ohms
- Alternate method: Calculate all values using fractions, versus converting fractions to decimals

Example - Power

$$P = \frac{E^2}{R}$$

- E = 300
- R = 450

- Power formula
- Square the numerator E
  - E * E
  - Write the number down
- Divide the squared voltage E by R.

Example - Power

$$P = \frac{E^2}{R}$$

- E = 300 volts
- R = 450 ohms

- $E^2 = ?$ (square E)
  - $300^2 = 300 * 300 = 90{,}000$
- $90{,}000/R = ?$
  - $P = 90000 / 450 =$ 200 watts

Solving an Equation

$$V_{PEAK} = \sqrt{2} \cdot V_{RMS}$$

$$V_{PEAK} = 1.414 \cdot V_{RMS}$$

$$V_{RMS} = V_{PEAK} \,/\, 1.414$$

$$V_{RMS} = 100 \,/\, 1.414$$

$$V_{RMS} = 70.721V$$

- $V_{Peak} = 100$
- $V_{RMS} = ?$

- Solving for RMS voltage
- Move V_{RMS} to left side
  - $V_{RMS} = V_{PEAK} / 1.414$
- Substitute value for V_{PEAK}
  - $V_{RMS} = 100/1.414 = 70.7\ V_{RMS}$

Example - PEP

$$V_{PEAK} = \sqrt{2} \cdot V_{RMS}$$

$$V_{PEAK} = 1.414 \cdot V_{RMS}$$

$$300 = 1.414 \cdot V_{RMS}$$

$$\mathbf{V_{RMS} = 300 / 1.414}$$

$$V_{RMS} = 212.164V$$

$$PEP = (V_{RMS})^2 / R$$

$$PEP = 212.164V^2 / 50\Omega$$

$$PEP = 900.272W$$

- Answer found from one formula inserted into a second formula to solve the problem.
- Solve first equation for V_{RMS}
- Substitute the V_{RMS} value into the second equation to find the PEP.
    - $V_{PEAK} = 300V$
    - $R = 50\Omega$
    - $PEP = ?\,W$

Solving an Equation

$E_S / E_P = N_S / N_P$

$(E_S / E_P) \cdot E_P = (N_S / N_P) \cdot E_P$

$E_S = (N_S \cdot E_P) / N_P$

$E_S = (300 \cdot 115) / 2{,}100$

$E_S – 34{,}500 / 2{,}100$

$E_S = 16.429V$

- Transformer turns vs. voltage
- Solve for E_S
- Multiply both sides by E_P
- The E_P values on the left cancel
- Formula becomes:
  - $E_S = (N_S \cdot E_P) / N_P$
    - $N_S = 300$
    - $N_P = 2{,}100$
    - $E_P = 115$
    - $E_S = ?$

Example – Turns Ratio

$N_P / N_S = \sqrt{}(ZP / ZS)$

$N_P / N_S = \sqrt{}(1600 / 8$

$N_P / N_S = \sqrt{200}$

$N_P / N_S = 14.142$

$N_P / N_S = 14.142$

$N_P / N_S \approx 14{:}1$

- $Z_P = 1600$
- $Z_S = 8$
- Ratio of N_P to N_S = ?

- $Z_P / Z_S = ?$
  - $1600/8 = 200$
  - Write the number down
- √200 = ?
  - √200 = 14.142
- Ratio of N_P to N_S = 14.142 / 1
  - Ratio is 14.142 to 1
  - Also written as 14.142 : 1
- There will be ~ 14 primary turns for every secondary turn

Solving an Equation

Special Case: $10^L = N$ Base 10 logarithm

Exponents: $Y^X =$ Y multiplied by itself X times

$10^3 = 1000$

$10^2 = 100$

$10^1 = 10$

$10^0 = 1$

$10^{-1} = 1/10^1 = 1/10 = 0.1$

$10^{-2} = 1/10^2 = 1/100 = 0.01$

$X^{1/2} = \sqrt{(X)}$ $Y^{1/3} = \sqrt[3]{(Y)}$ $Z^{1/X} = \sqrt[X]{(Z)}$

Solving an Equation

$$N = 10^L$$

← Anti-log: Reverse or opposite of the logarithm

$$L = \log_{10} N$$

← Logarithms

- "the log of N is L."
- or "What power of 10 will give you N?"

How I remember: $\text{Log}_3 9 = 2$ => $3^2 = 9$

Do not confuse "log" with natural logarithm "ln" (area under a curve)

$\text{ln}_e\, y = x$ => $e^x = y$, where $e = 2.71\ldots$

Solving an Equation

$$N = 10^{L} \qquad L = \log_{10} N$$

$$1000 = 10^{3} \qquad 3 = \log_{10} 1000$$

$$100 = 10^{2} \qquad 2 = \log_{10} 100$$

$$10 = 10^{1} \qquad 1 = \log_{10} 10$$

$$1 = 10^{0} \qquad 0 = \log_{10} 1$$

$$0.1 = 10^{-1} \qquad -1 = \log_{10} 0.1$$

$$0.01 = 10^{-2} \qquad -2 = \log_{10} 0.01$$

Decibels

Ratio of the Power Out to the Power In

$$dB = 10 * \log_{10}\left(\frac{P2}{P1}\right)$$

Examples of Power Ratios commonly expressed in dB:

- Gain of an amplifier stage
- Pattern of an antenna
- Loss of a transmission line
- Voltage ratio uses 20 in the formula vice 10

Common Decibel Table

1dB	$= 10 \times \log_{10} 1.26$	-1dB	$= 10 \times \log_{10} 1/1.26$
3dB	$= 10 \times \log_{10} 2$	-3dB	$= 10 \times \log_{10} 1/2$
6dB	$= 10 \times \log_{10} 4$	-6dB	$= 10 \times \log_{10} 1/4$
7dB	$= 10 \times \log_{10} 5$	-7dB	$= 10 \times \log_{10} 1/5$
9dB	$= 10 \times \log_{10} 8$	-9dB	$= 10 \times \log_{10} 1/8$
10dB	$= 10 \times \log_{10} 10$	-10dB	$= 10 \times \log_{10} 1/10$
13dB	$= 10 \times \log_{10} 20$	-13dB	$= 10 \times \log_{10} 1/20$
17dB	$= 10 \times \log_{10} 50$	-17dB	$= 10 \times \log_{10} 1/50$
20dB	$= 10 \times \log_{10} 100$	-20dB	$= 10 \times \log_{10} 1/100$

Example - Decibels

$$dB = 10 \log_{10} (P_2 / P_1)$$
$$dB = 10 \log_{10} (200 / 50)$$
$$dB = 10 \log_{10} (4)$$
$$dB = 10 \cdot 0.602$$
$$dB = 6.02$$

- $P_2/P_1 = ?$
  - $200/50 = 4$
  - Write the number down.
- log 4 = ?
  - log 4 = 0.602
  - Write the number down.
- $10 * 0.602 = ?$
  - $0.602 * 10 = 6.02$ dB

- P_2 = 200 watts
- P_1 = 50 watts
- dB = ?

Math is not to be feared, but revered!