Measuring systemic risk in the US stock market using a network approach

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Abstract

In this paper, I suggest a novel systemic risk measure using a network approach. I construct stock networks using the Minimum Spanning Tree (MST) based on the Pearson and Spearman rank correlation matrices of stock returns listed in S & P 500. I use two network centralities which I suggest as another factor to explain the risk-adjusted volatilities of stocks, which is unexplained by the Fama-French five factors and the momentum factor, to quantify connectedness among stocks: the degree based on direct connections among stocks and the community's influence strength based on indirect connections in communities, detected by modularity maximization. The statistical and dynamic properties of network centralities for the Pearson and Spearman rank correlation matrices are almost identical, and stocks in similar industry sectors are in the same community for both correlation matrices. Systemic risk is defined by multiplying the risks due to direct and indirect connections among stocks without a particular assumption of the return distribution. An increase in systemic risk is observed in the US financial recessions: the recession after the dot-com bubble and the subprime mortgage crisis.

Keywords: systemic risk, minimum spanning tree, network centrality, community structure

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1 Introduction

Most stocks are strongly correlated with each other, and negative shock can propagate through a strong correlation. In particular, the negative shock from one financial institution can be amplified through shock propagation in the stock market. During the subprime mortgage crisis, the bankruptcy of Lehman Brothers propagated to the US stock market and the global stock market via strong connections among stocks and stock markets. After the subprime mortgage crisis, understanding the risk from the negative shock of a small part of the market to the whole market via strong connections in the market, called "systemic risk," has been important[1]. To identify and measure systemic risk, we need to understand the structure of connections in the market.

With the development of network theory, we can understand the interaction behaviors of agents in the system. Network theory has been applied to economics and finance (see Allen and Babus (2009); Goyal (2009); Jackson (2014); Carvalho and Tahbaz-Salehi (2019)). In particular, network theory can be useful for measuring the effect of networks on risk due to interactions (see Allen and Babus (2009)). Bardoscia et al. (2021) also suggest that the methodologies in complex networks can be useful for analyzing the risk due to complex network structures emerging by the nonlinear and complex interaction among financial institutions in the financial market.

In this paper, I suggest a novel systemic risk measure based on connections among stocks in the US stock market. The stock networks that describe the interaction structure among stocks are constructed using the Minimum Spanning Tree (MST) based on the cross-correlation matrix among daily stock returns listed in S & P 500 from 1996 to 2021. To confirm the robustness of the US stock networks, I use two kinds of cross-correlation matrices: the Pearson and Spearman rank correlation matrices. To identify the characteristics of the interaction among stocks, I use two network centrality measures that quantify

[1] There are some the review papers of systemic risk explain the progress and development of the research about systemic risk (see Bisias et al. (2012); Adrian, Covitz and Liang (2015); Engle (2018); Jackson and Pernoud (2021).

the influence of stocks in the US stock network: the degree centrality based on direct connections of stocks and the community's influence strength based on indirect connections of stocks in the same community. Communities in the US stock market are detected using modularity maximization and show the clusters of stocks strongly correlated. The statistical and dynamic properties of the network centralities in the US stock networks for the Pearson and Spearman rank correlation matrices are almost identical. Clusters of stocks in similar industry sectors are observed in the US stock networks for the Pearson and Spearman rank correlation matrices using community detection.

To measure the systemic risk due to connections among stocks in the US stock market, I use two network centralities as another factor to explain the risk-adjusted volatilities of stocks, which is unexplained volatility by the six-factor model consisting of the Fama-French five factors and the momentum factor. Then, I regress the risk-adjusted volatility on the network centrality. The systemic risk in the stock market is defined by the product of the absolute values of the regression coefficients of degree centrality and community's influence strength on the risk-adjusted volatility and measures the risk due to direct and indirect connections among stocks without a particular assumption of the return distribution. The regression coefficients of degree centrality and the community's influence strength quantify the risks due to direct and indirect connections among stocks, respectively. The regression coefficients of degree centralities of stocks each year are negative since high market-capitalized stocks have a higher degree centrality and lower volatility than low market-capitalized stocks. The dramatic increase in the absolute value is observed in the US financial recessions: the recession after the dot-com bubble and the subprime mortgage crisis. The regression coefficients of the community's influence strengths of stocks are positive due to a stronger positive correlation among stocks in the stock market as the stock market is more unstable. The dramatic increase in the absolute value is also observed in the US financial recessions. Finally, an increase in systemic risk is observed in the US financial recessions.

The remaining part of this paper is as follows. Section 2 presents the literature related to my research and the contribution of my research to the literature. Section 3 shows the data set used in my research. Section 4 describes the methodology that I use in data analysis. Section 5 represents testable hypotheses, and the results are represented in Section 6. Finally, I conclude and discuss my paper in Section 7.

2 Literature Review

The application of network theory to measuring systemic risk in the financial market has grown exponentially. Previous research on systemic risk has focused on the banking system and the stock market. The systemic risk in the banking system has been measured using the interbank network based on the credit relationship among banks by the money flows between lenders and borrowers in the interbank lending market. Thus, it is not hard to understand interactions among banks and quantify the risk in the interbank networks. In this section, I introduce the literature related to the empirical analysis of interbank networks. [2]

Fender and McGuire (2010) suggest a framework that measures system-wide funding risk and the transmission of shocks across countries via the global interbank networks constructed by the credit relationship among banks. They find that the strength of linkages from the banks in the US to the banks in other countries, measured by changes in net interbank claims (assets minus liabilities) of banks in the US on banks in the banks in other countries during the subprime mortgage crisis in the US is observed. The result shows that the negative shocks of the banks in the US during the financial crisis were transmitted to other countries via interbank networks.

Gofman (2017) shows that financial stability can be improved by restricting the interconnectedness of banks using the over-the-counter interbank lending market in the US. The

[2]Model studies about systemic risk in the banking system using interbank networks have also been conducted. Most studies have focused on the channels of amplifying negative shocks from a few banks via interbank networks, such as correlated portfolios across banks, fire sales on market prices, counterparty risk, etc. (see the survey paper of Jackson and Pernoud (2021)). The model studies can give us an understanding of the relationship between systemic risk and the interbank network structure not observed by the empirical analysis due to the need for more data.

result shows that financial stability can be worsened by strong interconnectedness among banks.

Anderson, Paddrik and Wang (2019) analyze the bank networks constructed using the data on balance sheets of banks and interbank deposits in 1862 and 1867 in Pennsylvania. They focus on the effect of the National Banking Acts (NBAs) on the interbank networks. They find that the interbank networks were denser, and interbank deposits were concentrated on a few banks due to the NBAs. They also find the robustness to mild shocks in a more concentrated bank network but the fragility to large shocks to the financial center banks. Their study shows the mechanism of how the policy of the banking industry affects the banking system from the perspective of interbank networks.

Craig and Ma (2022) analyze the interbank networks in the German interbank lending market based on the credit relationship among banks. They find a few large banks in the core and many smaller banks in the periphery of the networks. Large banks have a role in intermediate funding flows between many smaller banks. They also suggest a model that explains the link formation among banks. They estimate unobserved monitoring costs using their model and the data, and an increase in the estimated unobserved monitoring costs is observed during the subprime mortgage crisis in the US.

The studies on systemic risk using interbank networks are limited to systemic risk in the banking industry. However, during the financial crisis, the whole industry sectors in the economy are in crisis. It implies that we need to consider connections among all industry sections in the economic system to precisely measure systemic risk. If we use the data in the stock market, we can solve the limitation of the study using interbank networks.

The systemic risk in the stock market has been measured using stock networks. The stock networks can be constructed using the cross-correlation matrix of stock returns. The cross-correlation matrix of stock returns includes information about relationships among stocks. Thus, we can find hidden networks using the cross-correlation matrix of stock returns, and the networks can provide useful information about stock connections to measure systemic risk in the stock market. Then, we can understand the shock propagation among different industry sectors if we use the stock networks. In this section, I introduce literature on constructing stock networks using the cross-correlation matrix of stock returns and measuring systemic risk using the stock networks or the cross-correlation matrix (including the covariance matrix) of stock returns.

Mantegna (1999) suggests a way to construct the stock network using a cross-correlation matrix of returns in the US stock market. The stock network is constructed using the Minimum Spanning Tree (MST), in which a stock is more likely to be linked to another stock whose distance from the stock is shorter. The higher the correlation between two stocks is, the shorter the distance between two stocks. He finds that the stocks in the same industrial sectors are clustered in the network and shows that the stock network using an MST can be useful information to understand the stock market's structure. This clustering of the stocks in the same industrial sectors is also observed in other research (see MacMahon and Garlaschelli (2015)).

Kritzman et al. (2010) suggest the absorption ratio as an indicator of systemic risk measure using the principle components of the covariance matrix of stock returns in the US stock market. They find an increase in the absorption ratio during the subprime mortgage crisis in the US. It shows that the cross-correlation matrix of stock returns in the stock market can be useful for measuring systemic risk in the stock market.

Song et al. (2011) analyze the worldwide stock markets using the global stock market networks constructed by the cross-correlation matrix among returns of worldwide stock market indices. They find an increase in the average correlation coefficient among worldwide stock market indices during the subprime mortgage crisis in the US. They also find an increase in the mutual information of connections among stock markets in the global stock market networks during the subprime mortgage crisis. The results show that the correlation and the information flow among the global stock markets significantly increase during the global financial crisis.

Billio et al. (2012) analyze the interconnections among stocks based on principle components analysis (PCA) using the covariance matrix of monthly stock returns of four sectors; hedge funds, banks, broker/dealers, and insurance companies, and the stock networks constructed using Granger-causality among stocks in the four sectors. They find an increase in the number of connections among stocks during the subprime mortgage crisis in the US. They also observe that banks and insurers more significantly impact hedge funds and broker/dealers than vice versa, and this asymmetry is enhanced during the subprime mortgage crisis in the US (2007-2009). Their results show that the asymmetry of the connections with banks can be a systemic risk measure in the financial market.

Wang, Xie and Stanley (2018) suggest another way to construct the global financial market network using a cross-correlation matrix of returns estimated by partial correlation coefficients. Using the MST based on partial correlation coefficients, they construct the networks and find that geographical characteristics cluster financial markets. Also, they show that network centrality measures (influence strength, betweenness centrality, and closeness centrality), which show the influence of stocks in the stock networks, detect the change in the global financial market network structure due to extreme events, such as the subprime mortgage crisis in the US.

Gong et al. (2019) suggest a systemic risk measure based on Granger-causality connections among financial companies' returns in the Chinese financial market, combined with a Value-at-Risk (VaR), traditionally used in measuring risk based on a particular assumption of the return distribution. They find an increase in the connectedness in the Chinese financial market during the subprime mortgage crisis in the US. They also find an increase in the systemic risk in the Chinese financial market during the subprime mortgage crisis in the US. It indicates that risk measure based on a Value-at-Risk is closely connected with connections among financial institutions in the financial market.

My paper contributes to the literature focused on measuring systemic risk in the stock market using networks based on a cross-correlation matrix of stock returns (see Mantegna

(1999); Kritzman et al. (2010); Song et al. (2011); Billio et al. (2012); Wang, Xie and Stanley (2018)). In particular, I introduce two network centralities in the stock network as another factor to explain the risk-adjusted volatilities of stocks, which is unexplained by the six factor model (the Fama-French five factors and the momentum factor), and suggest a systemic risk measure using the relationship between the risk-adjusted volatility and the network centrality. Thus, my research also contributes to the research of systemic risk using network information and Value-at-Risk (see Gong et al. (2019)). In addition, my systemic risk measure does not assume any particular assumption of the stock return distribution. Thus, it is easy to measure. As a result of data analysis, my systemic risk measure shows a consistent result with the previous research (see Song et al. (2011); Billio et al. (2012); Gong et al. (2019)). Therefore, my study develops a new systemic risk measure in the stock market.

3 Data sets

I use returns of common stocks listed in S & P 500 from 01/01/1996 to 12/31/2021. Daily returns are used to construct stock networks. To construct these networks, we need the cross-correlation matrix of stock returns. The cross-correlation matrix is measured using the daily data for a year. The daily data is enough to measure the cross-correlation matrix for a year with a high statistical significance.[3] The stock network for each year describes the interaction structure of the US stocks based on the correlation among stocks for each year. Thus, we can measure the change in the interaction structure in the US stock market from year to year. Risk-adjusted volatilities are estimated using the six-factor model, which includes the Fama-French five factors and the momentum factor in the US stock market from 01/01/1996 to 12/31/2021 (see Fama and French (2015)). I use the data from the Center for Research in Security Prices database (CRSP) to construct stock networks and the Wharton Research Data Services (WRDS) data to construct risk-adjusted volatilities.

[3] Daily stock returns have been used to measure the cross-correlation matrix for a year in the previous research, and the results have shown a high statistical significance (see Mantegna (1999); Song et al. (2011); Wang, Xie and Stanley (2018)).

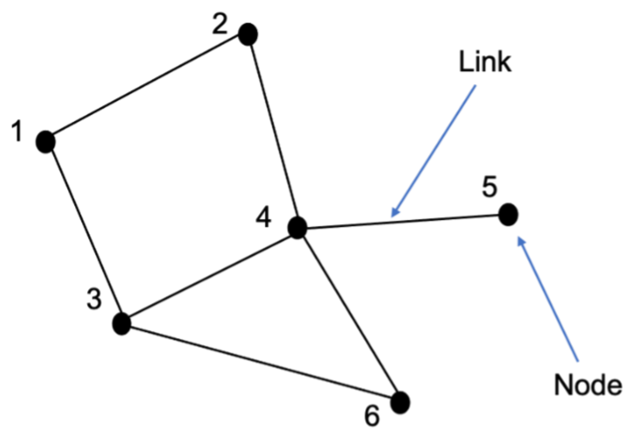

Figure 1: An example of a small network. There are six nodes and seven links in the network.

4 Methodology

4.1 What is a network?

A network is used to describe the interaction structure among agents in the system.[4] A network consists of nodes and links. A node represents an agent in the system. A link between two nodes represents the relationship between two agents. In the stock network, a node denotes a stock. A link denotes the relationship between two stocks in the stock market. In my research, a link is constructed by the distance measure based on the correlation between two stocks. If the correlation between two stocks is higher, their distance is shorter, and they are more likely to be tied in the network. Thus, two stocks can be linked if two stocks are highly correlated in the stock market.

Figure 1 depicts an example of a small network. The network consists of six nodes and seven links. The number of node 4's links is four, and the number of links of node 1 is two. The nodes connected to node i are called the *neighbors* of node i. Nodes 2, 3, 5, and 6 are connected to node 4. Thus, nodes 2, 3, 5, and 6 are the neighbors of node 4. If this

[4]Several textbooks introduce network methodology (see Easley and Kleinberg (2010); Jackson (2010); Barabási (2016); Newman (2018)). These books introduce a lot of networks in several systems, such as citation networks, friendship networks, customer-supplier networks, input-output networks, stock networks, Etc. Also, the books introduce applications of networks to several areas, such as economics, sociology, physics, biology, Etc.

network is the stock network, stock 4 has greater influence than others because the number of stocks connected to stock 4 is the highest. Also, the shortest path length between node 1 and node 4 is 2. Using the number of neighbors and the shortest path length, we can define several centralities that quantify the influence of a node in the network. I explain the network centralities used in my research in Section 4.3.

4.2 Network construction

My methodologies to construct stock networks are based on the cross-correlation matrices of stock returns (see Mantegna (1999); Wang, Xie and Stanley (2018)). I get the cross-correlation matrix from the stock returns time series using two methods to check the robustness of stock networks: Pearson correlation and Spearman rank correlation.

The return of stock i at time t ($r_i(t)$) in S & P 500 is calculated by the difference between log stock i's price at time t ($\log(P_i(t))$) and log stock i's price at time $t-1$ ($\log(P_i(t-1))$): $r_i(t) \equiv \log(P_i(t)) - \log(P_i(t-1))$. Daily stock returns are used to estimate the cross-correlation matrices of stock returns.

The cross-correlation matrix of N stocks' returns at time t ($C(t)$) is calculated by using the Pearson correlation coefficient or Spearman rank correlation between two stock returns as follows:

$$C(t) = \begin{pmatrix} \rho_{11}(t) & \cdots & \rho_{1N}(t) \\ \vdots & \ddots & \vdots \\ \rho_{N1}(t) & \cdots & \rho_{NN}(t) \end{pmatrix}, \tag{1}$$

where $\rho_{ij}(t)$ denotes the correlation coefficient between stock i's return and stock j's return using the data from $t-1-L$ and $t-1$ ($1 \leq i, j \leq N$, $-1 \leq \rho_{ij}(t) \leq 1$). L denotes the time window to calculate the cross-correlation matrix.

I define the distance measure between two stocks using the correlation between two stocks as follows: $d_{ij}(t) = \sqrt{2(1-\rho_{ij}(t))}$ ($0 \leq d_{ij}(t) \leq 2$), where $d_{ij}(t)$ denotes the distance between stocks i and j at time t. If $\rho_{ij}(t) = 1$, then $d_{ij}(t) = 0$. If $\rho_{ij}(t) = -1$, then $d_{ij}(t) = 2$. Thus, the higher the correlation between stocks i and j at time t is, the shorter $d_{ij}(t)$ is.

Using the distance measure, I define the distance matrix of N stocks at time t ($D(t)$) as follows:

$$D(t) = \begin{pmatrix} d_{11}(t) & \cdots & d_{1N}(t) \\ \vdots & \ddots & \vdots \\ d_{N1}(t) & \cdots & d_{NN}(t) \end{pmatrix}. \tag{2}$$

$d_{ij}(t)$ satisfies three axioms of a metric space: (i) $d_{ii}(t) = 0$; (ii) $d_{ij}(t) = d_{ji}(t)$; (iii) $d_{ik}(t) \leq d_{ij}(t) + d_{jk}(t)$. $d_{ij}(t)$ means the farness between stocks i and j in the stock market, and we can define any measure based on $D(t)$ in the stock market because $d_{ij}(t)$ spans a metric space. Also, we can construct a network that describes the interaction structure based on a distance measure in a metric space. In this paper, I focus on the interaction structure of the US stocks based on $D(t)$. The interaction structures can be constructed using a Minimum Spanning Tree (MST). I use the MST of the US stocks using Kruskal (1956) has been widely used in building an MST. The MST of stocks is constructed by following four steps:

(i) Sort all elements of the distance matrix in ascending order.

(ii) Choose the element that has the smallest value and add the link to the network. $d_{ii}(t)$ is not chosen in this step ($\because d_{ii}(t) = 0$).

(iii) Choose the next smallest element and add the link to the network, which should be a tree [5] after adding the link.

(iv) Repeat (iii) until all nodes are connected in the network.

The constructed MST includes the hidden structure of stock connections in the correlation among stocks. By steps (ii) and (iii) in constructing the MST of stocks, strongly

[5] A *tree* is a network in which any two nodes are linked by one path without a loop. A *loop* is a path that connects a node to itself. For example, the network in Figure 1 is not a tree because there exists a loop. For example, node 1 is connected to itself:1 $\to$ 2 $\to$ 4 $\to$ 3 $\to$ 1.

correlated stocks are linked with filtering out weak correlations. Thus, the MST of stocks can have a more important correlation structure hidden in the correlation matrix of stocks. Understanding the correlation structure among stocks is necessary to measure systemic risk in the stock market since systemic risk propagates through strongly correlated stocks. It implies that the MST in the stock market delivers valuable information to measure systemic risk in the stock market.

After constructing the MST, I measure the influence of a node or a stock in the MST using the following two centralities and classify communities in the MST.

4.3 Network centralities and community detection in the stock network

A centrality quantifies the influence of a node in the network. To understand the interaction behaviors of agents in the network and network structure, we need to identify the influence of nodes in the network. Thus, a network centrality is one of the most important measures in network analysis and a useful proxy to measure the connectivity of networks (see Chapter 2 in Jackson (2010)). In this paper, I use two network centralities to measure the connectivity of the US stock networks: degree centrality and community's influence strength.

The *degree centrality* quantifies the connectivity of a node based on a direct connection. The degree centrality of node i at time t ($DC_i(t)$) is defined as follows:

$$DC_i(t) = \frac{k_i(t)}{N(t) - 1}, \tag{3}$$

where $k_i(t)$ denotes the number of node i's links in the stock network at time t. $N(t)$ denotes the total number of nodes in the stock network at time t. In the stock network, the degree centrality captures the simple connectivity of a stock or the unweighted connectivity of a stock.

The *community's influence strength* quantifies the strength of the community's members' influence. The community's influence strength of a node i at time t ($CIS_i(t)$) is defined as follows:

$$CIS_i(t) = \frac{\sum_{j \in c_i(t), j \neq i} DC_j(t)}{|c_i(t)| - 1}, \tag{4}$$

where $c_i(t)$ is the community in the stock network at time t, to which stock i belongs. $|c_i(t)|$ denotes the number of stocks in community $c_i(t)$. The community's influence strength in the stock network captures the average influence of other stocks based on connectivity in the same community or the average indirect connection of a node.

I detect communities in the stock network to measure the community's influence strength of nodes using the maximizing modularity method widely used in detecting communities in a network (see Porter et al. (2009); MacMahon and Garlaschelli (2015); Fortunato and Hric (2016)). The modularity of the stock network N_t at time t ($Q(N_t)$) is defined as follows:

$$Q(N_t) = \frac{1}{2m_t} \sum_{ij} [A_{ij}(t) - P_{ij}(t)] \delta(c_i(t), c_j(t)), \tag{5}$$

where $m_t = \frac{1}{2} \sum_{ij} A_{ij}(t)$: (1)$A_{ij}(t) = A_{ji}(t) = 1$ if stocks i and j are linked in the stock network N_t; (2)$A_{ij}(t) = A_{ji}(t) = 0$ otherwise. $P_{ij}(t)$ denotes the expected weight of a link between stocks i and j at time t: $P_{ij}(t) = \frac{k_i(t)k_j(t)}{2m_t}$, where $k_i(t) = \sum_j A_{ij}(t), k_j(t) = \sum_i A_{ji}(t)$. $\delta(c_i(t), c_j(t))$ denotes whether stocks i and j are in the same community in the stock network N_t: (1) $\delta(c_i(t), c_j(t)) = 1$ if stocks i and j are in the same community (i.e., $c_i(t) = c_j(t)$); (2) $\delta(c_i(j), c_j(t)) = 0$ otherwise (i.e., $c_i(t) \neq c_j(t)$). $Q(N_t)$ is from -1 (all links are between communities) to 1 (all links are within communities). Communities in N_t are detected by maximizing $Q(N_t)$. Communities in the stock networks are the clusters of stocks connected through strong cross-correlation.

4.4 Risk-adjusted volatilities

I construct risk-adjusted volatilities using a similar approach as Rossi et al. (2015). I regress the excess daily log return of stock i at time t, $r_i^e(t)$, on the excess daily log returns on the US stock market index return, $r_M^e(t)$, returns on a US size factor, $SMB(t)$, a US value-growth factor, $HML(t)$, returns on a US profitability factor, $RMW(t)$, returns on a US investment factor, $CMA(t)$, and a US momentum factor, $MOM(t)$.[6] The excess daily log return of a stock is calculated by the difference between the daily log return of a stock and the log of gross one daily treasury bill rate.

$$\begin{aligned} r_i^e(t) = \alpha_i(t) + \beta_{1i} r_M^e(t) + \beta_{2i} SMB(t) + \beta_{3i} HML(t) + \beta_{4i} RMW(t) \\ + \beta_{5i} CMA(t) + \beta_{6i} MOM(t) + \epsilon_i(t). \end{aligned} \tag{6}$$

The risk-adjusted return of stock i at time t ($\hat{r}_i^{adj}(t)$) is defined by the sum of estimated $\alpha_i(t)$ and $\epsilon_i(t)$: $\hat{r}_i^{adj}(t) = \hat{\alpha}_i(t) + \hat{\epsilon}_i(t)$. The risk-adjusted return captures the unexplained return by the six factors in the excess stock return. The risk-adjusted volatility is defined by the absolute value of the risk-adjusted turn: $\hat{V}_i^{adj}(t) = |\hat{r}_i^{adj}(t)| = |\hat{\alpha}_i(t) + \hat{\epsilon}_i(t)|$. Each $\beta_{ki}(k = 1, 2, ..., 6)$ denotes the risk related to each factor. Thus, risk-adjusted volatility captures the volatility unexplained by the six factors in excess stock volatility.[7] I suggest that the risk-adjusted volatility can be explained by the stock networks, and the relationship between the risk-adjusted volatility and the stock networks can be a measure of systemic risk to explain financial crisis.

[6] Wharton Research Data Services (WRDS) provides the daily and monthly Fama-French factors data, which includes five factors and the momentum factor. I use the six-factor data provided in WRDS.

[7] In general, the daily volatility of a stock is estimated by the historical volatility of a daily stock using the standard deviation of a daily stock return. The absolute value of a daily stock return can also be used for a stock's daily volatility and is more useful to detect extreme events and risks in the stock market than the historical volatility of a stock using the standard deviation (see Zheng et al. (2014)).

4.5 The regression and systemic risk measure

The research aims to measure systemic risk using the relationship between the risk-adjusted volatility of the stock ($\hat{V}_i^{adj}(t)$), which is the unexplained volatility by the six factors, and the stock networks or the stock network effect on the risk-adjusted volatility of the stock. To measure the network effect on the risk-adjusted volatility of the stock, I run the regression for all stocks listed in S & P 500 at year y_t as follows:

$$\hat{V}_i^{adj}(t) = c_{y_t} + \beta_{NET,y_t} NET_i(y_t) + \eta_{NET,i}(t), \tag{7}$$

where y_t is the year, including the date t ($t \in y_t$). $NET_i(y_t)$ denotes the network measure of stock i in the stock network constructed using the data in year y_t. The degree centrality or community's influence strength of a stock i in the stock network constructed using the data in year y_t are used for $NET_i(y_t)$. β_{NET,y_t} measures the risk associated with the network measure $NET_i(y_t)$ in the stock network at year y_t, and $NET_i(y_t)$ can be another factor to explain the volatility, which is not explained by the six-factor model consisting of the Fama-French five factors and the momentum factor.

I suggest a systemic risk measure in the stock market at year y_t using β_{NET,y_t} as follows:

$$SR_{NET}(y_t) = |\beta_{DC,y_t}| \cdot |\beta_{CIS,y_t}|, \tag{8}$$

where $|\beta_{DC,y_t}|$ denotes the amplitude of risk-adjusted volatility change due to an increase of average direct connections among stocks in the stock network at year y_t. Thus, it measures the risk due to the direct connections among stocks. $|\beta_{CIS,y_t}|$ denotes the amplitude of risk-adjusted volatility change due to an increase of average network connections of others in the same community in the stock network at year y_t and measures the risk due to the indirect connections by other stocks. Thus, $SR_{NET}(y_t)$ measures the amplitude of risk-adjusted volatility change due to increased average connections, including direct connections and indirect connections among stocks, in the stock network at year y_t. The amplitude of volatility change per one connection will be larger during a financial crisis than during a normal period since the stock market will fluctuate more during a crisis via connections among stocks. Previous studies have shown that the information flow related to the change of stock volatility via connections in the stock network increases as the stock market is riskier (see Song et al. (2011); Billio et al. (2012); Gong et al. (2019)). It implies that the volatility change per connection increases as the stock market is riskier. A systemic risk measure should explain the financial market fluctuation due to the propagation of negative shock by the failure of financial institutions via connections among financial institutions (see the definition of systemic risk in Schwarcz (2008)). $SR_{NET}(y_t)$ measures volatility per connection without a particular assumption of return distribution and is more intuitive than previous systemic risk measures using PCA and VaR (see Billio et al. (2012); Gong et al. (2019)). Thus, $SR_{NET}(y_t)$ will be the systemic risk measure in the stock market based on connections among stocks at year y_t, which can explain the risk unexplained by the Fama-French five factors and the momentum factor.

5 Testable Hypotheses

The first hypothesis (**H1**) that I want to test is as follows:

H1. *The effect of the degree centrality of stock on the risk-adjusted volatility is negative due to the low volatility of high-capitalized stocks with high degree centrality ($\beta_{DC,y_t} < 0$).*

According to Li et al. (2019), the portfolio constructed by stocks in the network's periphery performs better than others in the network's core. In general, nodes located in the core have a higher degree centrality than the periphery in the network. Thus, the degree centrality of the stock would be negatively correlated with the risk-adjusted return. The low risk-adjusted return implies low risk-adjusted volatility. It indicates that the degree centrality of the stock is negatively correlated with the risk-adjusted volatility.

The second hypothesis (**H2**) that I want to test is as follows:

H2. *The effect of the community's influence strength on the risk-adjusted volatility is positive due to the positive correlation among stocks in the same community* ($\beta_{CIS,y_t} > 0$)*.*

As I have already explained in the literature review, previous research has shown that the stocks in the same industry group are strongly connected and in the same community detected in the stock network (see Mantegna (1999); MacMahon and Garlaschelli (2015)). In addition, Song et al. (2011) show a correlation among stocks becomes higher as the stock market is more unstable. In particular, they observe a significant increase in correlation among stocks in the financial crisis. It implies that the number of stocks positively correlated in the stock market increases as the stock market is unstable. In general, the stock market volatility increases as the stock market is unstable. Thus, the effect of the community's influence strength on the risk-adjusted volatility is positive.

The third hypothesis (**H3**) that I want to test is as follows:

H3. $SR_{NET}(y_t)$ *is higher as the stock market is more unstable.*

As I have already explained in the literature review, it has been observed that the information flow among stocks via stock networks, measured by connectedness in the stock market, increases as the stock market is more unstable (see Song et al. (2011); Billio et al. (2012); Gong et al. (2019)). It implies that the relationship between the volatility and connections among stocks is stronger as the stock market is riskier. Thus, $SR_{NET}(y_t)$, which measures the amplitude of the relationship between the risk-adjusted volatility and average stock connections in the stock market, including direct and indirect connections among stocks, is higher as the stock market is more unstable.

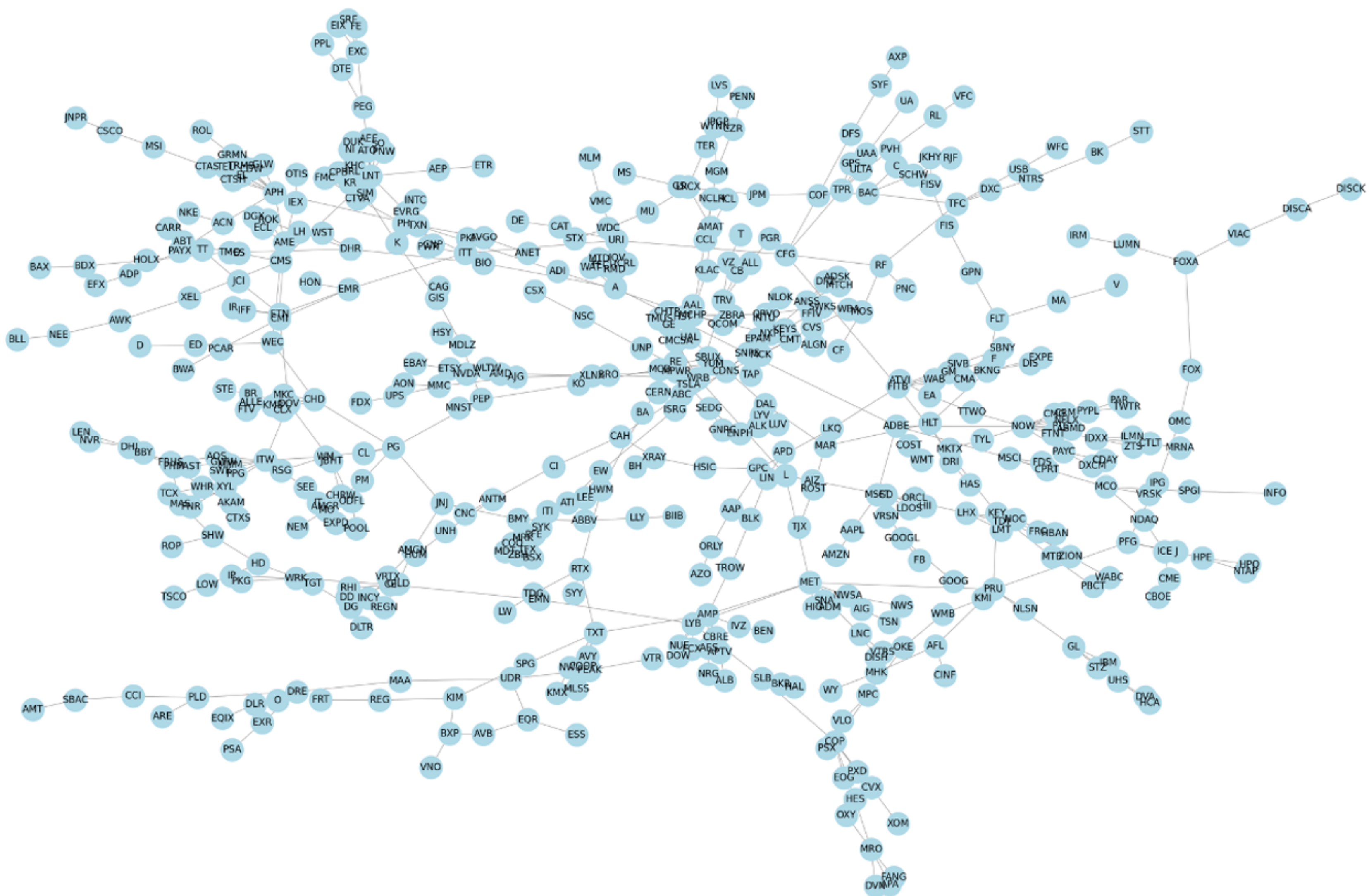

Figure 2: The US stock network using the Pearson correlation matrix in 2021. Nodes indicate stocks. Labels in nodes indicate stock symbols or ticker symbols.

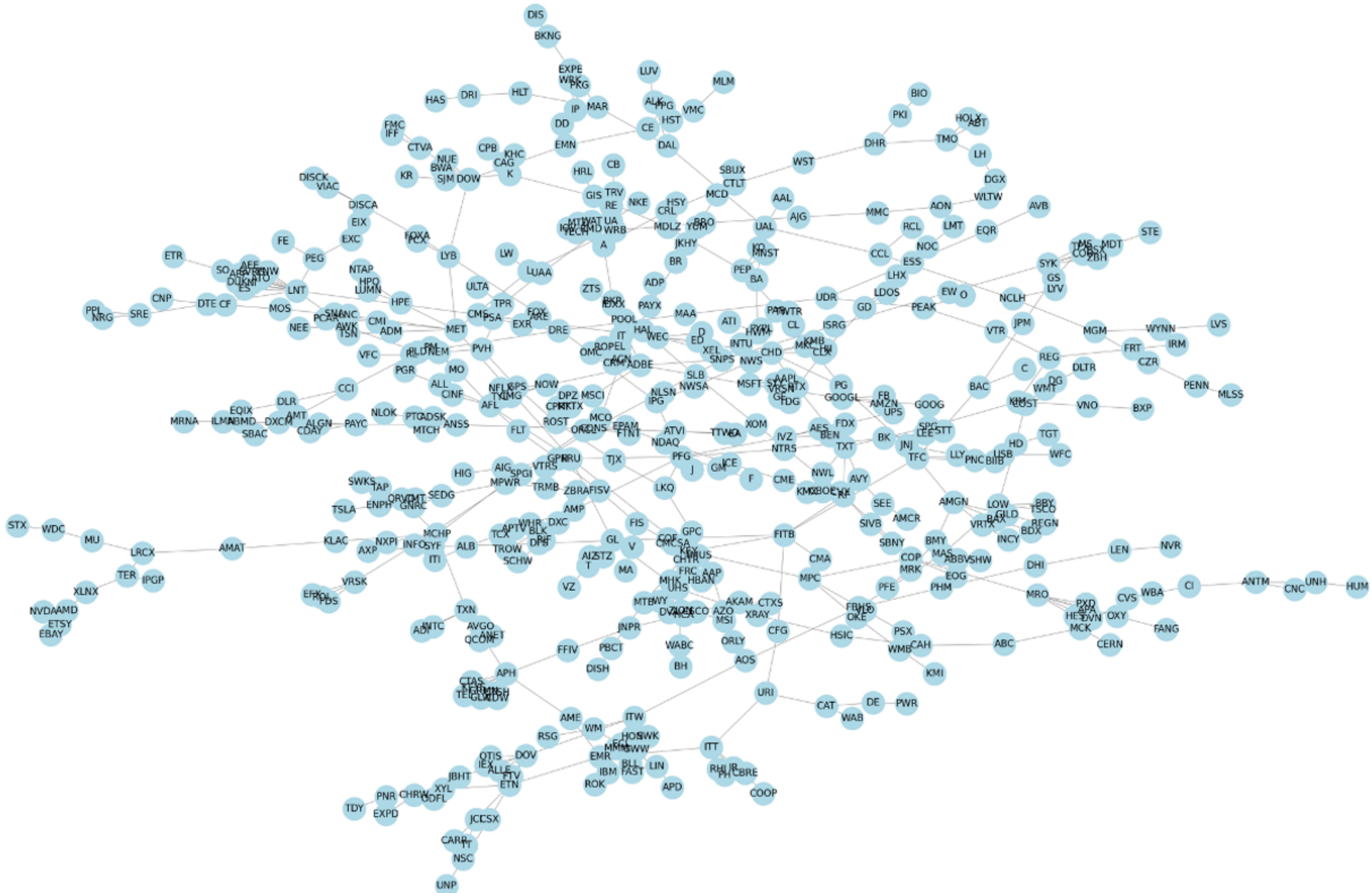

Figure 3: The US stock network using the Spearman rank correlation matrix in 2021. Nodes indicate stocks. Labels in nodes indicate stock symbols or ticker symbols.

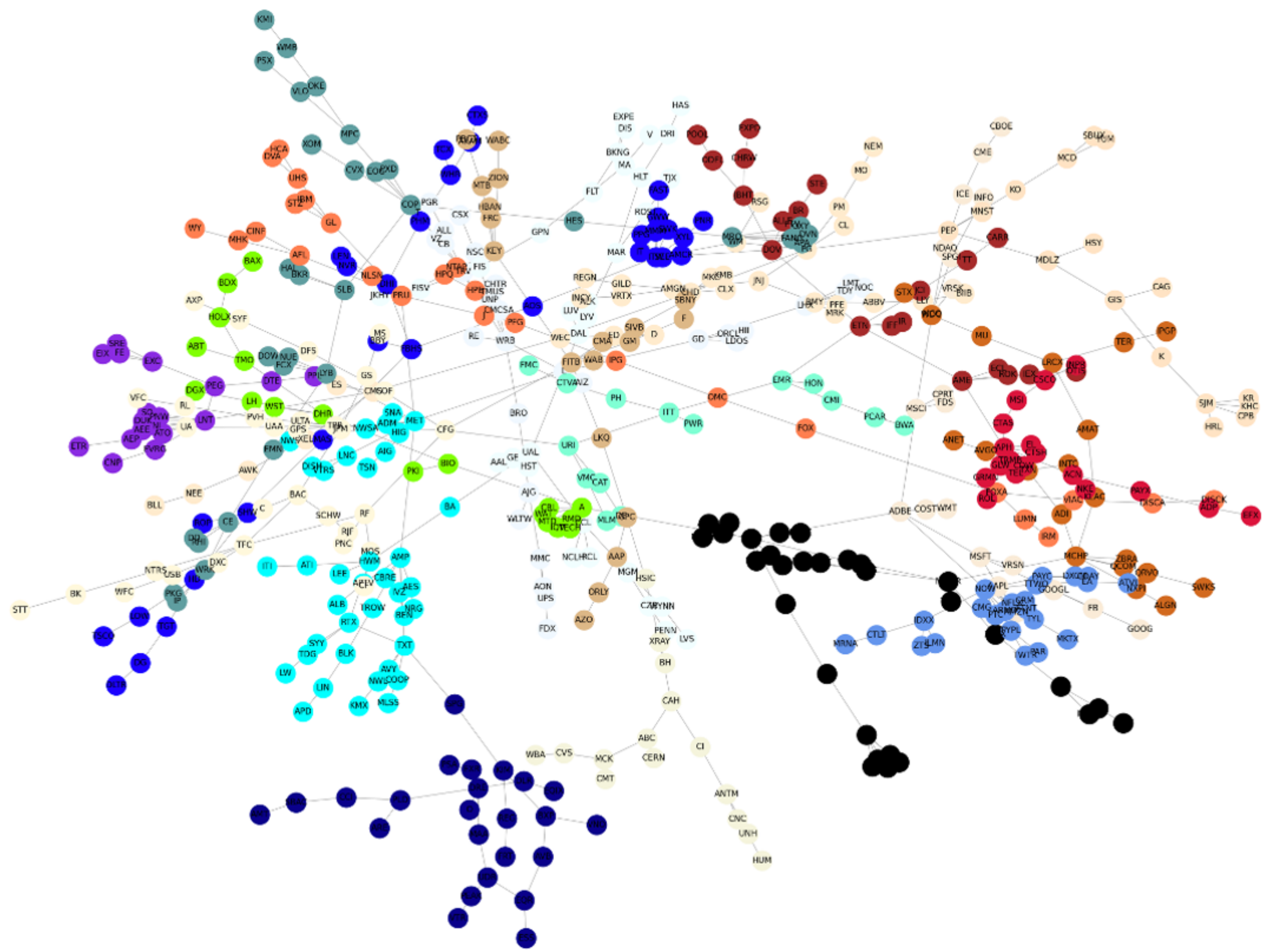

Figure 4: Communities detected in the US stock network using the Pearson correlation matrix in 2021. Stocks with the same color are in the same community. 22 communities are detected in the US stock network.

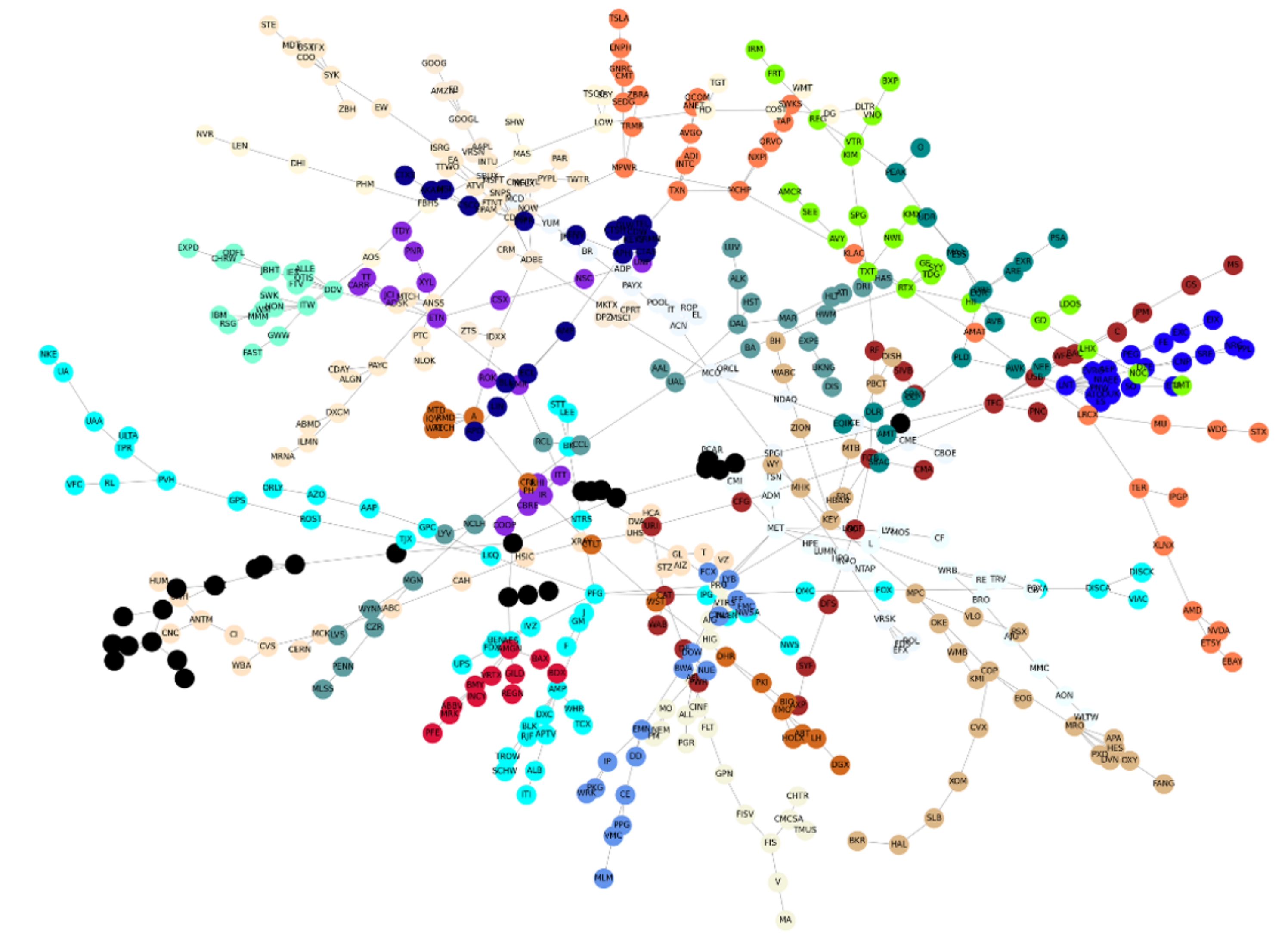

Figure 5: Communities detected in the US stock network using the Spearman rank correlation matrix in 2021. Stocks with the same color are in the same community. 23 communities are detected in the US stock network.

Ticker symbol	Name	Industry title	Ticker symbol	Name	Industry title
VNO	Vornado Realty Trust	REAL ESTATE INVESTMENT TRUSTS	MAA	Mid-America Apartment Communities, Inc.	REAL ESTATE INVESTMENT TRUSTS
FRT	Federal Reality Investment Trust	REAL ESTATE INVESTMENT TRUSTS	AVB	AvalonBay Communities, Inc.	REAL ESTATE INVESTMENT TRUSTS
UDR	UDR, Inc	REAL ESTATE INVESTMENT TRUSTS	ESS	Essex Property Trust, Inc.	REAL ESTATE INVESTMENTS TRUSTS
PSA	Public Storage	INVESTORS, NEC	O	Realty Income Corporation	REAL ESTATE INVESTMENT TRUSTS
PEAK	Healthpeak Properties	REAL ESTATE INVESTMENT TRUSTS - Healthcare facilities	ARE	Alexandria Real Estate Equities, Inc	REAL ESTATE AGENTS & MANAGERS (FOR OTHERS)
DRE	Duke Reality Corporation	REAL ESTATE INVESTMENT TRUSTS	BXP	Boston Properties, Inc.	REAL ESTATE INVESTMENT TRUSTS
VTR	Ventas Inc.	REAL ESTATE INVESTMENT TRUSTS	PLD	Prologis, Inc.	REAL ESTATE INVESTMENT TRUSTS
KIM	Kimco Realty Corporation	REAL ESTATE INVESTMENT TRUSTS	AMT	American Tower Corporation	TELEPHONE COMMUNICATIONS (NO RADIOTELEPHONE)
EQR	Equity Residental	REAL ESTATE INVESTMENT TRUSTS	CCI	Crown Castle Inc.	RADIOTELEPHONE COMMUNICATIONS
REG	Regency Centers Corporation	REAL ESTATE INVESTMENT TRUSTS	SBAC	SBA Communications Corporation	COMMUNICATIONS SERVICES, NEC
SPG	Simon Property Group, Inc.	REAL ESTATE INVESTMENT TRUSTS	EQUIX	Equinix, Inc.	TELEPHONE COMMUNICATIONS (NO RADIOTELEPHONE)
DLR	Digital Realty Trust, Inc.	REAL ESTATE INVESTMENT TRUSTS			

Table 1: The stocks in the community detected in the US stock network using the Pearson correlation matrix among stocks in 2021.

Ticker symbol	Name	Industry title	Ticker symbol	Name	Industry title
CDW	CDW Corporation	PRODUCTS AND SERVICES	CTXS	Citrix Systems, Inc.	SERVICES-COMPUTER PROGRAMMING, DATA PROCESSING, ETC.
FRT	Keysight Technologies, Inc.	WHOLESALE-MOTOR VEHICLES & MOTOR VEHICLE PARTS & SUPPLIES	APH	Amphenol Corporation	ELECTRONIC CONNECTORS
GLW	Corning Incorporated	DRAWING & INSULATING OF NONFERROUS WIRE	AME	AMETEK, Inc.	INDUSTRIAL INSTRUMENTS FOR MEASUREMENT, DISPLAY, AND CONTROL
MSI	Motorola Solutions, Inc.	TELEPHONE COMMUNICATIONS EQUIPMENTS	CTSH	Cognizant Technology Solutions Corporation	SERVICES-COMPUTER PROGRAMMING, DATA PROCESSING, ETC.
CTAS	Cintas Corporation	MEN'S & BOYS' FURNISHGS, WORK CLOTHG, & ALLIED GARMENTS	FFIV	F5, Inc.	SERVICES-PREPACKAGED SOFTWARE
APD	Air Products and Chemicals, Inc.	SPECIALITY CLEARNING, POLISHING AND SANITATION PREPARATIONS	JNPR	Juniper Networks, Inc	COMPUTER PERIPHERAL EQUIPMENT, NETC
BLL	Ball Corporation	METAL CANS	AKAM	Akamai Technologies, Inc.	SERVICES-PREPACKAGED SOFTWARE
ECL	Ecolab Inc.	AUTO CONTROLS FOR REGULATING RESIDENTIAL & COMML ENVIRONMENTS	TEL	TE Connectivity Ltd.	WHOLESALE-ELECTRONIC PARTS & EQUIPMENT, NEC
CSCO	Cisco Systems, Inc	SEMICONDUCTORS & RELEATED DEVICES			

Table 2: The stocks in the community detected in the US stock network using the Spearman rank correlation matrix among stocks in 2021.

Descriptive statistics				
Panel A: Degree centrality				
Cross-correlation matrix type	Average	Standard deviation	Minimum	Maximum
Pearson	0.004	0.004	0.002	0.100
Spearman	0.004	0.004	0.002	0.065
Panel B: Community's influence strength				
Cross-correlation matrix type	Average	Standard deviation	Minimum	Maximum
Pearson	0.004	0.001	0.002	0.005
Spearman	0.004	0.004	0.002	0.005

Table 3: The descriptive statistics of degree centralities and community's influence strengths of nodes in the US stock networks.

6 Results

Figures 2 and 3 show the stock networks constructed using MSTs based on the Pearson correlation matrix among stocks and Spearman rank correlation matrix among stocks, respectively. We can see the interaction structures among stocks using the stock networks. The connections among stocks are heterogeneous, and clusters are observed in the stock networks.

I apply community detection to the US stock networks to identify the clusters in the US stock networks. Figures 4 and 5 show the communities detected in the US stock networks constructed using MSTs based on the Pearson correlation matrix among stocks and Spearman rank correlation matrix among stocks, respectively. Stocks with the same color are in the same community. 22 communities are detected in the US stock network based on the Pearson correlation matrix, and 23 communities are detected in the US stock network based on the Spearman rank correlation matrix.

Table 3 shows the descriptive statistics of network centralities of nodes in the US stock networks using the Pearson correlation matrix among stocks and Spearman rank correlation matrix among stocks. The descriptive statistics for the Pearson and Spearman correlation matrices are almost identical. It implies that the characteristics of the US stock networks

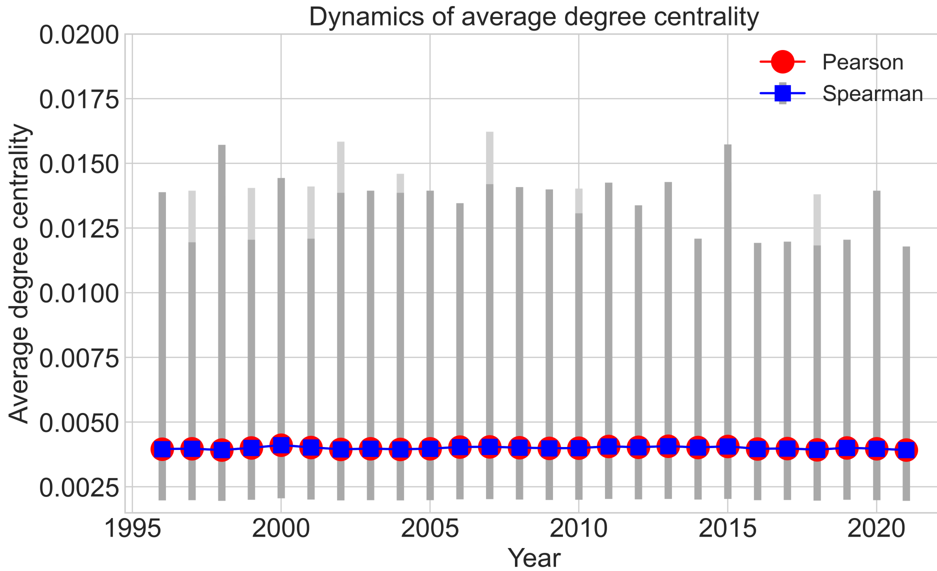

Figure 6: The dynamics of the average degree centralities in the US stock networks using the Pearson and Spearman rank correlation matrices. Red-filled circles (Blue-filled squares) indicate the average degree centrality in the US stock networks using the Pearson correlation (Spearman rank correlation). Light gray (Dark gray) bars indicate 95 percent confidence intervals of the average degree centrality in the US stock network using the Pearson correlation (Spearman rank correlation).

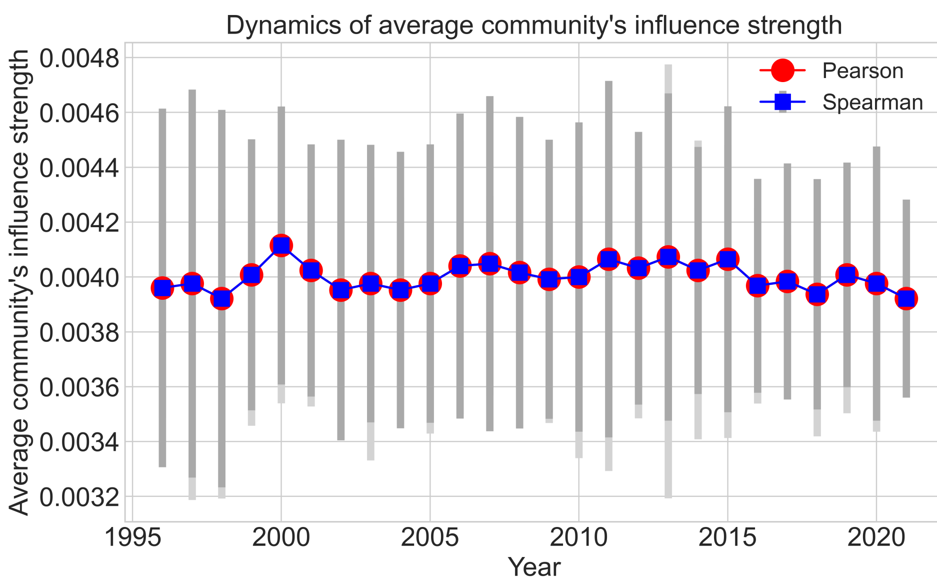

Figure 7: The dynamics of the average community's influence strengths in the US stock networks using the Pearson and Spearman rank correlation matrices. Red-filled circles (Blue-filled squares) indicate the average community's influence strength in the US stock networks using the Pearson correlation (Spearman rank correlation). Light gray (Dark gray) bars indicate 95 percent confidence intervals of the average community's influence strength in the US stock network using the Pearson correlation (Spearman rank correlation).

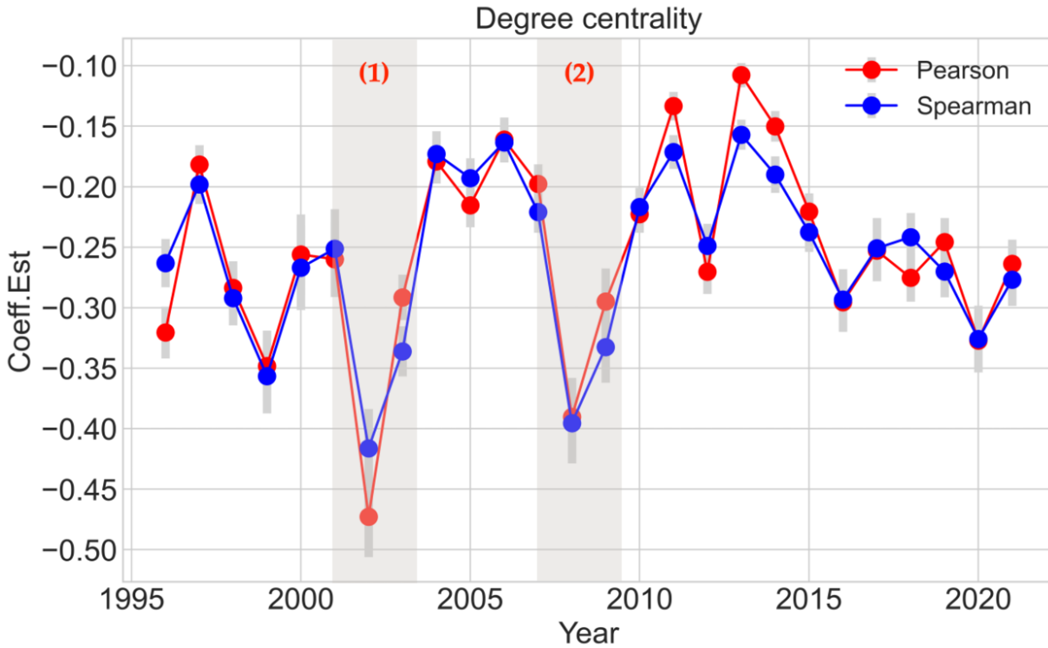

Figure 8: The dynamics of β_{DC,y_t} in the US stock networks using the Pearson and Spearman rank correlation matrices. Red-filled (Blue-filled) circles indicate the β_{DC,y_t} in the US stock networks using the Pearson correlation (Spearman rank correlation). Light gray bars indicate 95 percent confidence intervals of the β_{DC,y_t} in the US stock network using the Pearson correlation or Spearman rank correlation. (1) indicates the period of recession after the dot-com bubble in the US. (2) indicates the period of the subprime mortgage crisis in the US.

constructed using the Pearson correlation matrix among stocks and Spearman correlation matrix among stocks are similar.

Figure 6 shows the dynamics of average degree centralities in the US stock networks using the Pearson and Spearman rank correlation matrices. The dynamical properties of degree centralities in the US stock networks for the Pearson and Spearman rank correlation matrices are almost the same. The dynamic properties of the average community's influence strengths in the US stock networks for the Pearson and Spearman rank correlation matrices are almost identical (see Figure 7). Those results also support the similarity of structural properties of the US stock networks for the Pearson and Spearman rank correlation matrices.

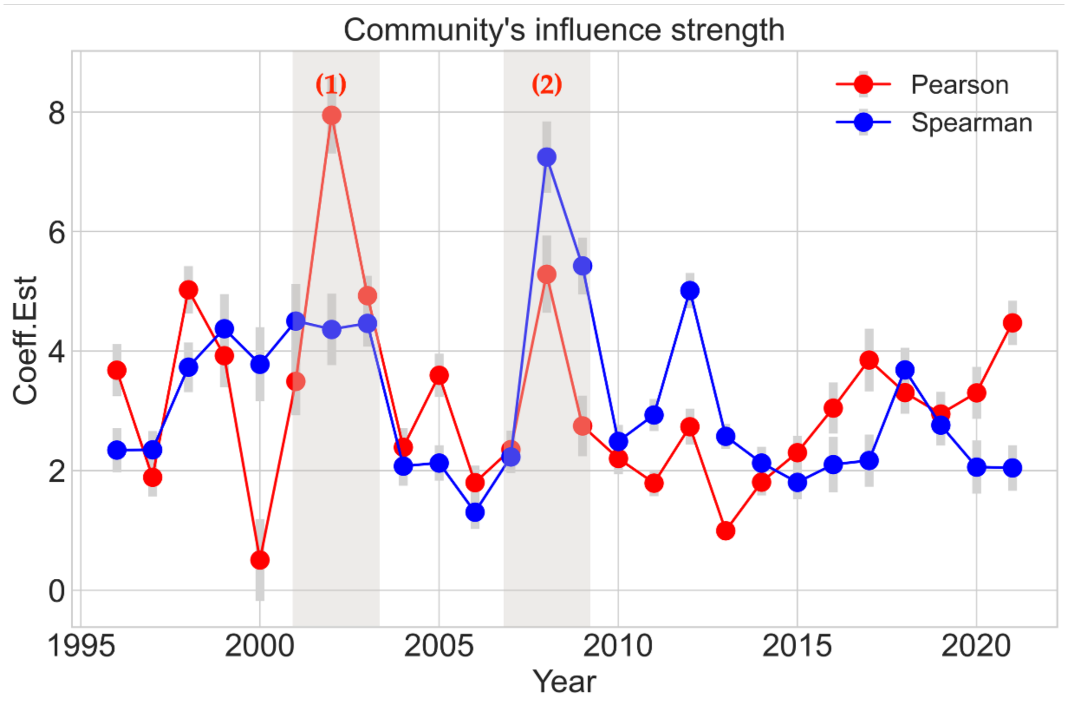

Figure 9: The dynamics of β_{CIS,y_t} in the US stock networks using the Pearson and Spearman rank correlation matrices. Red-filled (Blue-filled) circles indicate the β_{CIS,y_t} in the US stock networks using the Pearson correlation (Spearman rank correlation). Light gray bars indicate 95 percent confidence intervals of the β_{CIS,y_t} in the US stock network using the Pearson correlation or Spearman rank correlation. (1) indicates the period of recession after the dot-com bubble in the US. (2) indicates the period of the subprime mortgage crisis in the US.

6.1 The regression and systemic risk measure results

Figure 8 shows the dynamics of β_{DC,y_t} in the US stock networks using the Pearson and Spearman rank correlation matrices. All β_{DC,y_t}s are estimated with a significance level at 1 percent level. All estimated coefficients β_{DC,y_t}s are negative for the Pearson and Spearman rank correlation matrices. The result strongly supports **H1**. In addition, $|\beta_{DC,y_t}|$ increases during the US financial recessions: (1) the recession after the dot-com bubble; (2) the subprime mortgage crisis. It implies that the risk due to direct connections among stocks in the US stock market increases during the US financial recessions.

Figure 9 depicts the dynamics of β_{CIS,y_t} in the US stock networks using the Pearson and Spearman rank correlation matrices. All β_{CIS,y_t}s are estimated with a significance

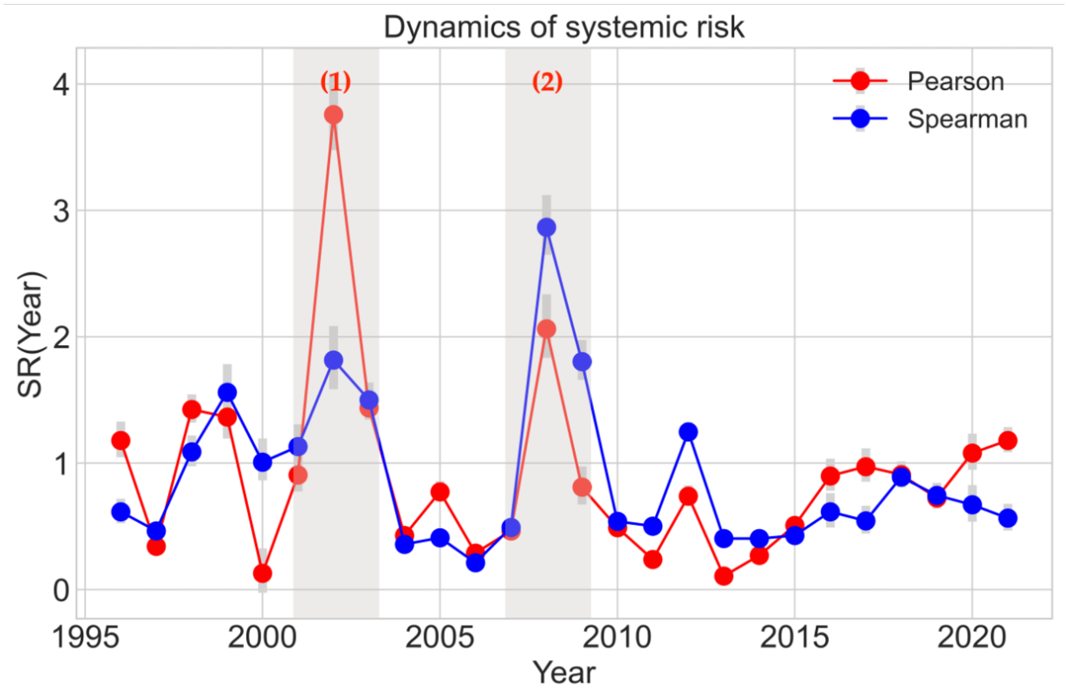

Figure 10: The dynamics of $SR_{NET}(y_t)$ in the US stock networks using the Pearson and Spearman rank correlation matrices. Red-filled (Blue-filled) circles indicate the $SR_{NET}(y_t)$ in the US stock networks using the Pearson correlation (Spearman rank correlation). Light gray bars indicate 95 percent confidence intervals of the $SR_{NET}(y_t)$ in the US stock network using the Pearson correlation or Spearman rank correlation. (1) indicates the period of recession after the dot-com bubble in the US. (2) indicates the period of the subprime mortgage crisis in the US.

level at 1 percent level. All estimated coefficients β_{CIS,y_t}s are positive for the Pearson and Spearman rank correlation matrices. The result strongly supports **H2**. In addition, $|\beta_{CIS,y_t}|$ increases during the US financial recessions: (1) the recession after the dot-com bubble; (2) the subprime mortgage crisis. It implies that the risk due to indirect connections among stocks in the US stock market increases during the US financial recessions.

Figure 10 shows the dynamics of $SR_{NET}(y_t)$ in the US stock networks using the Pearson and Spearman rank correlation matrices. $SR_{NET}(y_t)$ increases during the US financial recessions: (1) the recession after the dot-com bubble; (2) the subprime mortgage crisis. It implies that systemic risk in the US stock market due to direct and indirect connections among stocks increases during the US financial recessions. The result strongly supports **H3**.

All results using the Pearson and Spearman correlation matrices among stocks are almost identical. It shows that the results using the MST based on the cross-correlation matrix among stocks are robust regardless of the type of cross-correlation matrix. My results are consistent with previous studies about measuring systemic risk in the stock market based on network connectivity (see Billio et al. (2012); Gong et al. (2019)). Thus, my systemic risk measure ($SR_{NET}(y_t)$) can be a novel systemic risk measure.

7 Conclusion and discussion

In this paper, I suggest a novel systemic risk measure based on connections among stocks in the US stock market. I construct the US stock networks using a Minimum Spanning Tree (MST) based on a cross-correlation matrix among daily stock returns listed in S & P 500 from 1996 to 2021. To confirm the robustness of the stock networks, I use two types of correlation matrices: the Pearson and Spearman rank correlation matrices. The stock networks describe the interaction structure among stocks connected by a strong positive correlation in the US stock market. To identify the characteristics of the stock networks, I use two network centralities: the degree centrality and the community's influence strength. The degree centrality quantifies the direct connections of stocks in the US stock network. The community's influence strength quantifies the indirect connections of stocks by other stocks in the same community. The communities are detected using the maximum modularity method. The stocks in similar industry sectors are observed in the same community, and the statistical and dynamic properties of network centralities in the US stock networks using the Pearson and Spearman rank correlation matrices are almost identical.

To quantify systemic risk in the US stock market, I use the network centrality as another factor to explain the risk-adjusted volatilities of stocks, unexplained by the risks due to the Fama-French five factors and the momentum factor, and regress the risk-adjusted volatility on the network centrality of stocks. The effect of degree centrality on the risk-adjusted volatility is negative, and the effect of the community's influence of strength on the risk-adjusted volatility is positive. The systemic risk measure, defined by the multiplication between the absolute values of the regression coefficients of degree centrality and the community's influence strength on the risk-adjusted volatility, explains the effect of stock connections, including direct and indirect connections in the stock market. An increase in systemic risk is observed during the US financial recession: the recession after the dot-com bubble and the subprime mortgage crisis.

My study contributes to the studies on measuring systemic risk. First, my research provides risk factors related to network structures constructed by a cross-correlation among stocks to measure systemic risk, such as the degree centrality and community's influence strength, unexplained by the Fama-French five factors and the momentum factor. In addition, my systemic risk measure does not assume a particular assumption of return distribution and is easy to understand.

Future research using my research could be as follows. My study cannot identify the source to make a positive correlation among stocks since I use network centrality as an independent variable using the stock network based on the cross-correlation matrix of stock returns. However, in the banking sectors, the credit relationship among banks can make a positive correlation among banks in the stock market and can be the channel of systemic risk. In other sectors, the customer-supplier relationship among stocks can be the source of positive correlation among stocks. Cohen and Frazzini (2008) analyze the relationship between the network constructed by the customer-supplier relationship of firms and stock returns. They find that customer-supplier networks predict stock returns. Thus, if the network analyses of banking sectors using the credit relationship and other sectors using the customer-supplier relationship among stocks are added to my analysis, we can further understand systemic risk in the stock market.

In addition, if we analyze the connections among the global stock markets, we can measure systemic risk more precisely. All stock markets are globally connected, and the stocks in the US stock markets can be affected by the stocks in the stock markets in other countries, such as UK, China, and Japan stock markets. Song et al. (2011) find that the information flow among the global stock markets increased during the subprime mortgage crisis in the US. Wang, Xie and Stanley (2018) show that the correlation among the global stock markets becomes enhanced, and transmission of information among the global stock market is done more quickly during the subprime mortgage crisis in the US than in other normal periods. It implies that systemic risk in the stock market will propagate globally via connections with the stock markets in other countries. Thus, if the analysis of the global stock networks is added to my analysis, we can measure systemic risk in the stock market more precisely.

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