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A MODIFICATION OF THE DESIRABILITY FUNCTION APPROACH TO MULTI-RESPONSE SURFACE OPTIMIZATION

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ABSTRACT

A recent study by Usen, Akpan, Ugbe, Ikpang, Uket and Obeten (2021) reviewed the methods developed (so far) for solving multi-response surface optimization problems, side-by-side an identification of their demerits. In particular, the review showed that these existing methods: (i) were either inflexible or non-robust – they were either too problem-specific or too situation-specific, (ii) they considered only a small set of variables – as a large set of variables make their implementation awkward, and (iii) some of the methods failed under certain conditions. The concern of this article was principally on overcoming the third demerit, particularly with regards to improving the desirability function method. Therefore, a new desirability function D^* was used to replace the conventional desirability function D - which fails when at least one of the $x_i's$ are zeros. Based on this formulation, the researchers recommended that the proposed formulation should be incorporated into newer statistical software as this would ensure solving multi-response surface optimization problems in more robust manner, especially in cases where techniques desirability function technique fails.

Keywords: Multi-Response Surface Optimization, Desirability Function

1. Introduction

Typical industrial processes often involve a combination of contributory factors at different favorable operating conditions in order to determine process responses (Ruben, Maria, Carlos & Pedro, 2018). In this regard, the interest of any process personnel may either be to optimize one response variable of interest in relation to the settings of a fixed set of contributory factors, or to optimize more than one response variable of interest in relation to the settings of a fixed set of contributory factors. The former problem is that of single response surface optimization, whereas the latter problem is that of multi-response surface optimization.

In the design of statistical experiment, as a branch of Statistics, solutions to single response surface optimization problems were formally attempted by G. E. P. Box and K. B. Wilson (in 1951), and Genichi Taguchi within the same era. Their studies gave rise to response surface methodology (RSM) and Taguchi robust method, respectively; in either case, little or no attention was given to the possibility of optimizing processes involving more than one process response (Khuri, 2017). Here, RSM was defined as a collection of mathematical and statistical techniques useful for solving problems in which a response variable of interest is influenced by a set of predictor variables, and the overall goal is to optimize the response variable.

However, to overcome this oversight by Box and Wilson, and Genichi Taguchi, Statisticians in subsequent years, have developed a number of methods comprising the: desirability function method, priority-based method, contour plots method, squared error loss function method, process capability index method, posterior preference method, distance function method, etc. Notwithstanding, while the principal idea in implementing most of these methods for solving multi-response surface optimization problems all argue that the first stage in the solution process should be to develop response surface models for each response, followed by the use of unique techniques in obtaining the optimum operating conditions, the ideas of other methods such as: constrained optimization, game theory, etc., argue otherwise.

A recent study by Usen, Akpan, Ugbe, Ikpang, Uket and Obeten (2021) reviewed the methods developed (so far) for solving multi-response surface optimization problems, side-by-side an identification of their demerits. In particular, the review showed that these existing methods: (i) were either inflexible or non-robust – they were either too problem-specific or too situation-specific, (ii) they considered only a small set of variables – as a large set of variables make their implementation awkward, and (iii) some of the methods failed under certain conditions. The concern of this research lies on overcoming the third demerit, particularly with regards to improving the desirability function method.

The desirability function technique, from when it was developed by Derringer and Suich (1980) constructs an overall desirability using a desirability function. This method has become a very popular method necessarily requiring the decision maker's preference information, in the absence of which it fails. More so, the method also fails when one or more of the factors are of zero value as the desirability function in this case will be the root of zero. In this regard, the need for a technique or modification which overcomes the latter limitation in the presence of the decision maker's preference information cannot be overemphasized. This study was an attempt to bridge the said gap through the formulation of the proposed technique.

2. Conceptual Framework

2.1. The idea of single response surface optimization

Single response surface optimization (SRSO) is a procedure which concerns itself with the optimization of one response variable that is influenced by several factors. This idea of SRSO is the basis for RSM, and was introduced by G.E.P Box and K.B. Wilson in 1951 for designing experiments and subsequent analysis of experimental data involving just one response (Zhang & Gao, 2007).

However, depending on its area and nature of usage, since invention, RSM has been defined in varying ways. For instance, it was independently defined by Dayananda, Shrikantha, Raviraj and Rajesh (2010), Amayo(2010), Arokiya many and Sivakumaar (2011), etc., as a mixture of mathematical and statistical techniques useful for modeling and analyzing problems in which a response variable is influenced by several factors, and whose objective is to optimize this response variable.

But aside this definition, it has been viewed by Bradley (2007) as a methodology that helps the experimenter to reach the goal of optimum response. Ebrahimjsour, Rahman, Eanchng, Basri and Salleh (2008) and Raissi and Farsani (2009) independently defined it to be a well-known up-to-date approach for constructing approximation models via physical experiments, computer simulations and experimented observations; whereas Dutta and Basu (2011), defined it to be a set of techniques used in the empirical study of relationships between one or more responses and a group of variables. Akpan, Ugbe and Usen (2013) defined it as the use of a mixture of mathematical and statistical techniques for exploring near-optimal operating conditions of a process.

The idea of RSM is based on the assumption that one has a set of data containing observations on a response variable y and the independent variables - $\pi_1, \pi_2, \pi_3, \ldots, \pi_k$ in which case what is calleda response surface model (a mathematical model)is fitted to y as a function of the independent variables π_i's in order to provide a summary representation of the behaviors of the response, as the independent variables are changed (Akpan, et al., 2013). This model is given by equation (1).

$$y = f(\pi_1, \pi_2, \pi_3, \ldots, \pi_k) + \varepsilon$$

where $\varepsilon \sim \mathrm{N}(0, \sigma^2)$ is the source of variability not accounted for (statistical error), and is independent.

In particular, regression models are often used for characterizing this relationship between the response variableand the independent variables for the subsequent analysis of the response (Montgomery, 2011). This is best suitable in systems where the nature of the relationship between y and the π_i values may be known. Thus,

$$\hat{y} = f(\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3, \ldots, \hat{\pi}_k)$$

The essence of fitting suchregression models may be to optimize the response, orto find what regions of the π -space lead to a desirable product, or to gain knowledge of the general form of the underlying process with a view to describing options (such as the first two objectives) to customers (Montgomery, 2011).

Notwithstanding, in some other systems, unlike the previous scenario, the form of the relationship between the response y and the independent variables $\pi_1, \pi_2, \pi_3, \ldots, \pi_k$ may be unknown. In such cases, the first step in RSM is to find a suitable approximation for the true functional relationship between y and $\pi_1, \pi_2, \pi_3, \ldots, \pi_k$ (Dayananda et al., 2010; Montgomery, 2011). Usually a low-degree polynomial in a relatively small region of the π -space is appropriate for use (Montgomery, 2011) based on a first-order design (FOD) or second-order design (SOD) (Alonzo & Hiroyuki, 2009; Amayo, 2010).

We note, however, that an FOD is suitable when the operating conditions are remote from the optimal setting. The use of the FOD side-by-side the method of steepest ascent/descent (MSA or MSD) leads the experimenter to the optimal region in the most efficient way. Nevertheless, if the presence of curvature is detected in the system, then a higher degree polynomial must be used by means of an SOD (Raissi & Farsani, 2009; Dayananda et al., 2010; Montgomery, 2011). Notwithstanding, polynomial models of degrees higher than two are rarely fitted in practice; and this is particularly because of the difficulty of interpreting the form of the fitted surface (which produces predictions whose standard errors are greater than those from the low-degree fit); and, more so, because the region of interest is usually chosen small enough for the first or second order to be a reasonable choice.

The SOM is widely used in RSM for several reasons: (1) SOM's are very flexible in the sense thatthey can take on a wide variety of functional forms, hence, they will often work well as an approximation to the true response surface (Montgomery, 2011), (2) parameters in SOM's are easy to estimate(Raissi & Farsani, 2009; Amayo, 2010;Dayananda et al., 2010; Buyske & Trout, 2011; Montgomery, 2011), and (3) there is considerable practical experience indicating that the SOM works well in solving real response surface problems (Alonzo & Hiroyuki, 2009).

The technique of RSM can be accomplished in a sequence of three (3) phases, starting with a phase zero, followed by phases one and two. At phase zero, some ideas are generated concerning which factors are likely to be important in the response surface study. It is usually called a "screening experiment". The objective of the factor screening is to reduce the list of candidate variables to a relatively few so that subsequent experiments will be more efficient and require fewer runs or tests. The purpose of this phase is the identification of the important independent variables.

At phase one, the process engineerattempts to determine if the current operating conditions of the factors result in a value of the response that is near the optimum. This is done at a point on the response surface remote from the optimum (where there is little curvature in the system). However, ifthe current operating conditions of the factors do not result in a value of the response that is near the optimum, the objective of the process engineer becomes to lead his process rapidly and efficiently to the general vicinity of the optimum. At this phase, the FOM is appropriate (Montgomery, 2011). Orthogonal first-order (OFO) designs are used with the MSA or MSD to achieve this (Bradley, 2007; Montgomery, 2011). The most commonly used OFO designs are: 2^k factorial, Plackett-Burman, and Simplex designs.Experiments are then conducted along the PSA until no further increase (or decrease) in response is determined. Then a new FOM may be fit, a new PSA (or PSD) determined, and the procedure continued. Eventually, the experimenter will arrive at the vicinity of the optimum. This is usually indicated by lack of fit of the FOM. At this time, additional experiments are conducted to obtain the precise estimate of the optimum.

Phase two begins when the process is near the optimum. At this phase, the desire of the process engineeris to obtain a model that will accurately approximate the true response function within a relatively small region around the optimum. And since the true response surface usually exhibits curvature near the optimum, an SOM (or perhaps some higher order polynomials) is often used. Once an appropriate model has been obtained, this model may be analyzed to determine the optimum conditions for the process. The preferred class of experimental designs used for this purpose is the rotatable designs. Some of the frequently used rotatable designs for fitting the SOMs are: 3^k factorial, central composite designs, and the Box-Benhken designs(Montgomery, 2011). However, on fitting the SOM with any of these designs, a canonical analysis is used to:(1) locate the optimum,(2) characterize the stationary point,(3) explain the ridge systems, and (4) explain the relationship between the canonical variables $\{w_i\}$ and the independent (coded) variables $\{x_i\}$ (Montgomery, 2011).

2.2. Multi-response surface optimization (MRSO)

Unlike SRSO problems in which the concern is to optimize one process response variable, a common problem experienced in process design is the selection of optimal parameter levels involving a simultaneous consideration of multiple response variables. MRSO has emerged, in recent years, as the use of available mathematical and statistical optimization techniques for tackling problems of this nature. Regardless of its importance in practice, the development of an optimization scheme for multi-response surface problems has received little attention, even as the few available techniques are being applied in a variety of industrial processes.

2.3. Existing techniques for multi-response surface optimization

From its initial development till date, a variety of methods have been introduced for multiple-response optimization, all of which have been categorized at different times by several authors. For instance, Pignatiello (2004) categorized the existing methods into three; Tajbakhsh and Norossana (2006) categorized the existing methods into four. This dissertation considers the most recent classifications being that of Amineh and Kazem (2011), and Taha, Mirmehdi, and Majid (2014). These categories are the approaches of: overlapping contour plots, constrained optimization problem, desirability function, loss function, process capability, distance function, and game theory.

2.3.1. Overlapping contour plots

Myers and Montgomery (2002) suggested that an approach for optimizing several responses is to overlay the contour plots for each response. Here, the experimenter can visually examine the contour plot to discover the appropriate operating conditions. Amineh and Kazem (2011) emphasized that this technique is mainly suitable when there are few design variables, since it becomes awkward for more than three design variables. More so, in this approach Amineh and Kazem (2011) explains that there is no need for the decision maker's information especially as contour plots play the main role.

2.3.2. Constrained optimization problem

The formulation and solving of the multi-response problem like a constrained optimization problem was described by Flavio (2010) as a popular approach. Kim, Min and Jeong (2001) classified it as a priority based approach. The priority-based approach which is similar to a method-bounded objective in the multi-objective decision-making problem chooses the response with the highest importance as the objective function and the rest of the functions are considered as constraints, although it is not always much straightforward. This idea was first suggested by Myers and Carter (1973). In their study, the responses were assumed and referred to as a "primary response" and a "constraint response". The objective was to find conditions on a set of designed variables which maximizes the primary response function subject to the constraint response function. Subsequently, Biles (1975) considered multiple process responses by extending the study and formulation of Myers and Carter (1973).

2.3.3. Loss function approach

The squared error loss function was first suggested by Pignatiello (1993) as:

$$L(y(x)) = (y(x) - \Phi)' C (y(x) - \Phi)$$

where $y(x)$ is the response vector, $\Phi(x)$ is the target vector, and C is the cost matrix and is used to determine the relative importance of the response variables.

2.3.4. Process capability approach

Process capability index is used to evaluate whether a process is able to meet current specification limits. Hsiang and Taguchi (1985) presented the index C_{pm} as:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

where USL and LSL are specification limits, and μ, σ^2 and T respectively denote the mean, variance, and target in the above equation. Subsequently, in an independent work, Chan, Cheng and Spiring (1988) further extended this index; hence, this index could now be applied in multi-response optimization. The maximization of process capability as a criterion for multi-response optimization was further considered by Plante (2001).

2.3.5. Distance function approach

The distance function approach was proposed by Khuri and Conlon (1981). The distance function is

$$\text{Distance}\{\hat{y}(x), T\} = \left\{ \hat{y}(x - T)' \sum\nolimits_{\hat{y}(x)}^{-1} \{\hat{y}(x) - T\} \right\} \quad (5$$

where T represents the target value, $\hat{y}(x)$ is the predicted response, and $\sum_{\hat{y}(x)}$ is the variance-covariance matrix of the predicted responses. The optimal operating condition is achieved if the distance function gets minimized.

2.3.6. Game theory approach

Navidi, Amiri and Kamranrad (2014) proposed a game theoretic-based approach for multi-response optimization by viewing each response as a player and each factor as strategies of each player. Their approach could determine the best predictor factor sets in order to obtain the best joint desirability of responses. This was achieved this, the signal to noise ratio (SN) index for each response will be calculated by considering the joint values of strategies, following which the obtained SN ratios for each strategy are modeled in the game theory table. To end the procedure, via Nash Equilibrium, the best strategy which is the best values of predictor factors is then determined.

3. Empirical Review

A growing number of scholarly research articles on the theory and applications of RSM are endlessly flooding the internet in reputable international journals.

For instance, Olusola and Akindele (2019) used an eco-friendly natural coagulant, *Moringa oleifera* seed for the treatment of surface water. In order to obtain lowest turbidity of surface water, optimization of process variable affecting the coagulation of water was carried out using response surface methodology (RSM). Four parameters were varied viz. settling time, agitation time, agitation speed and *Moringa oleifera* seed extract (MOSE) concentration, and their effects on the turbidity of the surface water were established. The data obtained was fitted to a quadratic model which was also validated. The model predicted lowest turbidity of 5.49 NTU with optimal condition of 120min of settling time, agitation speed of 100rpm, 10 min of agitation time and 3g/l of MOSE concentration. The condition was verified in replicates and turbidity of 5.51 NTU was obtained.

Ming-dong, Chao, Lei, Li and Cheng (2018) selected and A-pillar as an example to investigate the effect of stamping parameters on the parts forming quality of AA5754 sheet. A finite element model was established using commercial stamping software PAMSTAMP2G. Barlat2000 yield function was used to describe the yield behavior of the material. Stamping experiment was conducted to validate the reliability of the model. The studied parameters were blank-holder force (100 – 700KN) and draw-bead's geometrical variables, including two fillet radii R1 (8 – 12MM), R2 (4 – 8mm) and the height of draw-bead D (2 – 6mm). The central composite experiment design method was used to design the simulation matrix. In order to obtain stamped parts with optimal forming quality, response surface methodology was used to establish relationship between stamping parameters and forming quality (rupture and spring back). The non-domination sorting genetic algorithm II (NSGA-II) was adopted to conduct an optimal calculation of the models. A Pareto-optimal solution set in the solution space was obtained. A reasonable optimized scheme was selected. The optimum blank-holder force was found to be 700KN with the draw-bead's geometrical parameters R1, R2 and D of 12mm, 6.6mm and 3.8mm, respectively.

Murdani, Jakfar, Ekawati, Nadira and Darmadi (2018) conducted and experiment by an electrochemical method and electrolysis using aluminium electrodes with the independent variables being voltage, contact time and concentration of electrolytes. The response optimization via RSM showed that optimum conditions of contact time 34.26 min, voltage 12V, concentration of electrolyte 0.38M could decrease COD to 65.039 percent. The model recommended by RSM for the three (3) variables was a quadratic regression model.

Ruben, Maria, Carlos and Pedro (2018) sought to determine regression models by the use of a response surface method that relate the zinc coating parameters to the input parameters in steel screws. When considering the coating requirements of cost, coating process speed, corrosion resistance, and coating thickness, the optimal input parameters were found by using a multi-response surface. Input parameters of 0.3 amps/dm^2, 20.0 ^{0}C, 13.9 g/L, 45 min, 28.5 ML/L, and 2.8 ML/L, respectively were obtained when considering the cost. Considering minimization of the deposition time, the input parameters obtained were 0.5 amps/dm^2, 24.6 ^{0}C, 13.9 g/L, 45 min, and 26.9 ML/L, respectively. The optimal inputs to maximize the corrosion resistance were 0.6

amps/dm^2, 32.4 ^{0}C, 14.0 g/L, 45 min, 28.7 mL/L, and 2.5 ML/L, respectively. Finally, when maximizing the coating thickness, the inputs were 0.7 amps/dm^2, 38.4 ^{0}C, 12.2 g/L, 45 min, 26.5 mL/L, and 1.5 ML/L respectively.

Edy and Kariyam (2017) discussed M-estimation as a parameter estimator in multivariate response surface models containing outliers. The authors used a case-study on the experimental results to the enhancement of the surface layer of aluminium alloy air by shot peening as a case study.

Mahdi, et al. (2017) discussed the use of multi-response surface optimization in the selection of preferred solutions from among various non-dominated solutions. Their study proposed a three-stage method for solving multi-response surface optimization problem. In the first stage, a robust approach was used to construct a regression model. In the second stage, non-dominated solutions were generated by the $\in -$ constraint approach. The robust solutions obtained in the third phase were non-dominated solutions that were more likely to be Pareto solutions during consecutive iterations. A simulation study was then presented in order to show the effective performance of the proposed approach. Finally, a numerical example from the literature was brought in to demonstrate the efficiency and applicability of the proposed methodology.

Anthony (2015) focused on the use of the methods of optimal experimental design to solve number of nonstandard problems arising particularly in experiments in process industries. Examples included response-surface designs for an arbitrary number of observations. And because some combinations of factors may produce unstable results, designs were found over irregular design regions. The method of dividing such designs into blocks was also shown. Mixture designs, again over constrained regions, were mentioned. The theory underlying the construction of the designs was integrated using general equivalence theorem for determinant optimality, which led to algorithms for the construction of designs as well as the assessment of their efficiency. These designs were compared with those that minimized the variance of prediction over a specified region, for which an equivalence theorem was also described, together with the algorithm for construction of these I-optimal designs. References were given to recent works.

Edy, Suryo and Sri (2014) focused on the steps of model building using multivariate regression procedure. They proposed an M-estimation for estimating parameters in multi-response surface models. The proposed method was illustrated using the well-known problem “tire treads compound problem’, which was originally presented by Derringer and Suich (1980). Based on this example, the performance of the OLS and M-estimation was compared, by comparing the SSE of OLS and M-estimation. The comparison showed that M-estimation approach produced smaller SSE. These results indicated that for the parameter estimates of multivariate response surface models with the data outliers, the M-estimation had a better performance than the OLS.

Navidi, et al. (2014) proposed a new game theoretic-based approach for multi-response optimization problem acknowledging that game theory is a useful tool for decision-making in the conflict of interests between intelligent players in order to select the best joint strategy for them through selecting the best joint desirability. Their study used the game theory approach via definition of each response as each player and factors as strategies of each player. This approach could determine the best predictor factor sets in order to obtain the best joint desirability of responses. For this aim, the signal to noise ration index for each response was calculated with consideration made of the joint values of strategies, following which the signal to noise ratio for each strategy was modeled in the game theory table. Finally, the authors used Nash equilibrium to determine the best strategy (the best values of predictor factors). A real case and a numerical example were given to show the efficiency of the proposed method. In addition, the performance of the proposed method was compared with the VIKOR method.

Taha, Mirmehdi and Majid (2014) presented a new approach which took the benefits of principal component analysis and multivariate regression. Global criterion method of multi-objective optimization was also used to reach a compromise solution which improved all response variables simultaneously. And at the end of their study, the proposed approach was analytically described.

Response surface methodology and Box-Behnken design were utilized by Diyar, Kasolang, Basim and Abudullah (2013) to predict and evaluate the effects of the three-level-three-independent variables on oil-film friction. The design independent variables including speed, load and oil-feed pressure were used to develop an empirical model for oil-film friction in hydrodynamic plain journal bearing. The salient feature of their study was to investigate the interaction between the main three (3) factors and provide a profound understanding on the significance of each factor. The analysis of variance (ANOVA) of the regression model as well as a comparison of predicted and observed response values showed good correspondence, implying that the empirical model derived from response surface approach could predict the response adequately.

Hakan and Semet (2013) carried out the optimization of a manufacturing problem with two responses by the application of response surface methodology and desirability function.

Taha, et al. (2012) modeled and optimized multi-response surfaces and their related stochastic nature using Goal Programming (GP) in which the weights of response variables have been obtained through a Group Decision Making (GDM) process. Because of existing uncertainty in the stochastic model, some stochastic optimization methods have been applied to find robust optimum results. At the end the proposed method was described mathematically and analytically.

Amineh and Kazem (2011) gave a detailed look at prominent approaches that have been suggested so far in MRS; it also reviews and discusses the classifications with a special focus on the decision maker's preference information. Results of the case-study analysis show that applying a meta-heuristic algorithm with existing MRS approaches lead to better findings.

Arokiyamany and Sivakumaar (2011) used RSM in the optimization process for Bacteriocin. Dutta and Basu (2011) used RSM for the removal of Methylene Blue by *Acaecia Auricultiformis* scrap wood char.

In the study of Molani (2011) a systematic methodology based on the response surface methodology was coupled with an effective simulated annealing algorithm to find the optimum process parameter values.

Amayo, (2010) applied RSM to automotive suspension designs. Dayananda et al. (2010) applied RSM in the study of surface roughness in grinding of aerospace materials (6061AI-15Vol%SiC_{25p}). Lenth (2010) reviewed and advanced RSMs in R using RSM updated to version 1.40.

Alonzo and Hiroyuki (2009) applied RSM in the formulation of nutrient broth systems with predetermined P.H and water activities.

Raissi and Farsani (2009) proposed a procedure which can resolve a complex parameter design problem with more than two responses. Their method can be applied to those areas where there are large data sets and a number of responses are to be optimized simultaneously. In addition, the proposed procedure is relatively simple and can be implemented easily by using ready-made standard statistical package.

Ebrahimjsour et al. (2008) experimented on a modeling study by RSM and Artificial Neural Network or Culture Parameters Optimization for Thermostable Lipase production from a newly *Thermophilic Geobacillus Sp.* Strain ARM.

Sadjadi, Habibian and Khaledi (2008) studied different types of multi-objective decision making methods and examined some of them on some real world RSM problems. The results were compared to show that even some trivial method can lead to some promising result.

Zhang and Gao (2007) applied RSM in medium optimization for Pyruvic acid production of *Torulopsis Glabrata* TP19 in Batch Fermentation.

4. Materials and Method

In this section of the article, we put forward materials needed for the formulation of the proposed modification, alongside an outline of the proposed procedure of the said formulation. These material are that of the geometric mean and desirability function.

4.1. The geometric mean formulae

In mathematics, the geometric mean (GM) is the average value or mean which signifies the central tendency of the set of numbers by finding the product of their values. Basically, we multiply the numbers altogether and take the k^{th} root of the multiplied numbers, where k is the total number of data values. In other words, the geometric mean is defined as the k^{th} root of the product of k numbers. It is noted that the geometric mean is different from the arithmetic mean because the in the latter we add values and then divide their result by the total number of values; whereas in the geometric mean, we multiply the given data values and then take the root with the radical index for the total number of data values.

The formula to calculate the geometric mean may be traditionally expressed as:

$$GM = \sqrt[k]{x_1 \times x_2 \times x_3 \times \cdots \times x_k} \quad (6)$$

However, it may alternatively be defined as:

$$GM = \text{Anti}\log\left(\frac{\sum_{i=1}^{k} \log x_i}{k}\right) \quad (7)$$

4.2. The desirability function approach

The desirability function approach is one of the most widely used methods in industry for the optimization of multiple response processes. It is based on the idea that the "quality" of a product or process that has multiple quality characteristics, with one of them outside of some "desired limits", is completely unacceptable. The method finds operating conditions $\mathbf{x}$ that provide the "most desirable" response values.

For each response y_i, a desirability function $d_i(y_i)$ assigns numbers between 0 and 1 to the possible values of y_i with $d_i(y_i)=0$ representing a completely undesirable value of y_i and $d_i(y_i)=1$ representing a completely desirable or ideal response value. The individual desirabilities are then combined using the geometric mean, which gives the overall desirability $D=\{d_1(y_1)\times d_2(y_2)\times d_3(y_3)\times\cdots\times d_k(y_k)\}^{\frac{1}{k}}$ with k denoting the number of responses. Notice that if any response is completely undesirable (that is, if $d_i(y_i)=0$), then the overall desirability is zero. In practice, fitted response values $\hat{y}_i$ are used in place of the y_i.

Depending on whether a particular response y_i is to be maximized, minimized, or assigned a target value, different desirability functions $d_i(y_i)$ can be used. A useful class of desirability functions was proposed by Derringer and Suich (1980). Let L_i, U_i and T_i be the lower, upper and target values, respectively, that are desired for response y_i, with $L_i \le T_i \le U_i$.

If a response is of the "target is best" kind, then its individual desirability function is:

$$d_i(\hat{y}_i)=\begin{cases} 0 & if\ \hat{y}_i < L_i \\ \left\{\dfrac{\hat{y}_i-L_i}{T_i-L_i}\right\}^s & if\ L_i \le \hat{y}_i \le T_i \\ \left\{\dfrac{\hat{y}_i-U_i}{T_i-U_i}\right\}^t & if\ T_i \le \hat{y}_i \le U_i \\ 0 & if\ \hat{y}_i > U_i \end{cases}$$

with the exponent s and t determining how important it is to hit the target value. For $s=t=1$, the desirability function increases linearly towards T_i; for $s<1$, $t<1$, the function is convex; for $s>1$, $t>1$, the function is concave (see the example below for an illustration).

If a response is to be maximized instead, the individual desirability is defined as:

$$d_i(\hat{y}_i)=\begin{cases} 0 & if\ \hat{y}_i < L_i \\ \left\{\dfrac{\hat{y}_i-L_i}{T_i-L_i}\right\}^s & if\ L_i \le \hat{y}_i \le T_i \\ 1.0 & if\ \hat{y}_i > T_i \end{cases}$$

with T_i in this case interpreted as a large enough value for the response.

Finally, if we want to minimize a response, we could use:

$$d_i(\hat{y}_i)=\begin{cases} 1.0 & if\ \hat{y}_i < T_i \\ \left\{\dfrac{\hat{y}_i-U_i}{T_i-U_i}\right\}^s & if\ T_i \le \hat{y}_i \le U_i \\ 0 & if\ \hat{y}_i > U_i \end{cases}$$

with T_i denoting a small enough value for the response.

The desirability approach consists of the following steps:

1. Conduct experiments and fit response models for all k responses;

2. Define individual desirability functions for each response;
3. Maximize the overall desirability D with respect to the controllable factors.

However, the most important advantage of this approach is that the Decision Maker's preference information can be easily applied in the model. In addition, it is easy to use, and is popular among available methods.

5. The proposed formulation

In order to overcome the demerit of the already existing method for multi-response surface optimization (usingthe desirability function approach), we have suggested a new desirability function, denoted by D^* to be used. The new steps are as follows:

Step 1: Conduct experiments and fit response models for all k responses;
Step 2: Define individual desirability functions for each response using D^*;

Where we define D^* as in (7) with a slight modification which requires that we introduce a replacement value $x_i^* = x_i + 1$ for any $x_i \neq 0$ ($i = 1, 2, 3, \ldots, k$). That is,

$$D^* = \text{Anti}\log\left(\frac{\sum_{i=1}^{k} \log x_i}{k}\right) \qquad (8)$$

Step 3: Maximize the overall desirability D^* with respect to the controllable factors.

6. Conclusion

In conclusion, the proposed modification holds tendencies of overcoming the lapses encountered in using the conventional desirability function D - which fails when at least one of the $x_i's$ are zeros. This is a plus to any process engineer who aims to optimize or her process with such variables regardless. Based on this formulation, the researchers have recommended that the proposed formulation should be incorporated into existing statistical software as this would ensure solving multi-response surface optimization problems in more robust manner, especially in cases where techniques desirability function technique fails.

As suggestions for further study, the researchers have suggested that: (i) attempts should be made at a coding the proposed technique as this will allow for its implementation in robust and flexible manner whilst saving time, and (ii) other techniques aside averaging should be explored as this may result in outstanding theoretical results.

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