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MODELLING OF INFINITY-NORMS ERRORS BETWEEN FLUCTUATING CO-EXISTENCE STEADY-STATE SOLUTIONS USING DATA FROM COWPEA AND GROUNDNUT LEGUMES.

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Abstract

The variability of the infinity-norms errors between the co-existence steady-state solutions' data in the context of a mutualistic interaction between two legumes is compared within an in-situ parameterization and with a similar competitive interaction of an earlier study. Each of the infinity-norms errors tend to change with respect to a chosen random disturbance value with a lower random disturbance value of 0.0001being associated with the infinity-norms error value of 0.363673166883669 whereas the random disturbance value of 0.002 is associated with the infinity-norms error value of 0.545619678973393 and the random disturbance value of 0.001 is associated with the infinity-norms error value of 0.448769588090975. All these three distinct infinity-norms errors between the defined data for a mutualistic interaction are smaller than the infinity-norms error value of 0.607798977901674 between the same defined data for a competitive interaction when the Poisson random disturbance value is 0.002 as observed in our previous study.

Keywords: simulation modeling, steady state solution, random disturbance, mutualistic interaction, random disturbance perturbation, dynamical system.

1.0 Introduction

One of the oldest biological interactions is the popular competition idea which is also associated with several research contributions. By definition, the highest population size of a planet specie that the surroundings can sustain in coexistence is commonly referred to as the carrying capacity of the environment. This term is not new within mathematical literatures, Coa et al. (2001), Damgard (2004), Ekeke-a and Galadima (2015), Jin, Donovan and Breffle(2016), Meyer and Ausubel (1999), Lawson et al(2019). Mayor and Ausubel (1999) for instance studied carrying capacity, a model with logistic varying limit. They extended the widely used logistic model of growth to a limit that in turn increases the carrying capacity. In contrast, the mutualistic interaction is gaining some expected research contributions but at a slower pace when compared with the competing population interactions. The focus of this present pioneering numerical simulation of the infinity-norms errors between fluctuating co-existence steady-state solutions' data for a mutualistic interaction between the cowpea and the groundnut legumes will be used to reinforce the above-mentioned knowledge-gap.

2.0 Materials and Method

The background of this study has considered the following simplifying assumptions:

1. The growth of the two-biological species over time is enhanced by their intrinsic growth rate values in the absence of the intra-competition and inter-competition.

2. The growth of the two biological species over time is inhibited by their inter-competition.

3. The growth of the two biological species over time is enhanced by their intra-competition.

The continuous dynamical system of nonlinear first order differential equation having the following mathematical structure, George (2018) was considered.

$x^{I} = \propto_1 \ x \ - \beta_1 x^2 - r_1 xy$ (1)

$y^{I} = \propto_2 \ y - \beta_2 y^2 - r_2 xy$ (2)

With the initial conditions defined as x(0)= $x_0 > 0$ and y(0)= $y_0 > 0$.

x^{I} and y^{I} are assumed continuous and partially differentiable.

The variables for these model equations are defined as:

x(t)= time-dependent variable of cowpea plant specie

y(t)= time- dependent variable of groundnut plant specie

$\propto_1$ is the intrinsic growth for the cowpea plant specie

β_1 is the intra-competition coefficient due to interaction of the population of cowpea to inhibit the growth rate of cowpea.

$\propto_2$ is the intrinsic growth rate for the groundnut specie.

β_2 is the intra-competition coefficient due to the interaction of the population of groundnut to inhibit the growth of groundnut plant specie.

r_1 is the inter competition of coefficient due to the interaction of the population of groundnut to inhibit the growth of cowpea population.

r_2 is the inter-competition coefficient due to the interaction of the population of cowpea to inhibit the growth of the groundnut population.

Determination of Steady- state solutions

From equation (1) and equation (2).

x^1= x$(\propto_1 - \beta_1 x = r, y)$

y^1 =y$(\propto_2 - \beta_2 y - r_2 x)$

At steady state solution

$$x^1 = 0, y^1 = 0$$

$$x(\propto_1 - \beta_1 x - r_1 y) = 0$$

$$y(\propto_2 - \beta_2 y - r_2 x) = 0$$

$$\propto_1 - \beta_1 x - r_1 y = 0$$

$$\propto_2 - \beta_2 y - r_2 x = 0$$

$$\beta_1 x + r_1 y = \propto_1$$

$$r_2 x + \beta_2 y = \propto_2$$

Haven established this theoretical background, we present a numerical simulation of the study. The data that we have used to conduct this simulation analysis were empirically derived, Nafo and Ekaka-a (2009), Ekpo and Nkanang (2010), for the deterministic interaction between the cowpea and the groundnut legumes under the simplifying assumption that the random disturbance perturbation does affect the intrinsic growth parameter values alone and the intra-competition coefficients outweigh the inter-competition coefficients. For the purpose of this analysis, we have considered the following parameter values: the intrinsic growth rate parameter values of 0.0225 and 0.0446; the intra-competition coefficients' values of 0.006902 and 0.0133; the inter-competition coefficients' values of 0.0018. A

MATLAB algorithm was used to study the proposed problem from which the results presented next were obtained.

3.0 Results

The full results of this pioneering study are presented as shown in Table 1-Table 6.

Table 1: Constructing the difference between fluctuating co-existence steady-state solutions' data using a MATLAB algorithm with a Poisson random disturbance value of 0.002

The cowpea co-ordinate of the co-existence data (C)	The groundnut co-ordinate of the co-existence data (G)	D = C-G

1-norm to 10-norm error calculation of D data	11-norm to infinity-norm error calculation of D data
4.679893319892285	0.605171997155550
1.488450347256439	0.596826549778781
1.019724093262310	0.590061949699854
0.846200675745810	0.584496775908588
0.758055471721123	0.579860092285293
0.705497521407870	0.575954575634935
0.670984539857489	0.572633427660256
0.646815974635833	0.569785418013869
0.629096853583523	0.567324907889008
0.615651487837968	**0.545619678973393**

Table 3: Constructing the difference between fluctuating co-existence steady-state solutions' data using a MATLAB algorithm with a Poisson random disturbance value of 0.001

The cowpea co-ordinate of the co-existence data (C)	The groundnut co-ordinate of the co-existence data (G)	D = C-G
4.309393419979431	4.003163621365824	0.306229798613607
4.439080156420360	4.014024673318937	0.425055483101423
4.305881678351319	3.955823925180684	0.350057753170636
4.349915831050248	3.993201236636676	0.356714594413572
4.320901730918684	3.992394783025217	0.328506947893467
4.315056088275518	3.986530699816879	0.328525388458639
4.375786857886578	4.004170531385498	0.371616326501081
4.411491522390546	4.018376338410474	0.393115183980072
4.426330310812824	3.977560722721849	0.448769588090975
4.394696468225829	3.963027328533265	0.431669139692564

Table 4: Calculating the p-norms error between the fluctuating co-existence steady-state solutions' data denoted by the notation D using a MATLAB algorithm with a Poisson random disturbance value of 0.001

1-norm to 10-norm error calculation of D data	11-norm to infinity-norm error calculation of D data
3.740260203916035	0.492577983402226
1.191826733518054	0.486408992751976
0.818125711896567	0.481451528747280
0.680286224023490	0.477403939920525
0.610661834593595	0.474053376799578
0.569453547914751	0.471246037636763
0.542630897312752	0.468868510042817
0.524030640799938	0.466835677369115
0.510533812674460	0.465082657593424
0.500398311826898	**0.448769588090975**

Table 5: Constructing the difference between fluctuating co-existence steady-state solutions' data using a MATLAB algorithm with a Poisson random disturbance value of 0.0001

The cowpea co-ordinate of the co-existence data (C)	The groundnut co-ordinate of the co-existence data (G)	D = C-G
4.287704731498852	3.939268867874788	0.348435863624064
4.294218791425385	3.938164363810549	0.356054427614836
4.300560958453075	3.939999728575774	0.360561229877301
4.296757056088072	3.941361423422793	0.355395632665278
4.299003245466008	3.939539698751901	0.359463546714107
4.288968576551315	3.935649371342048	0.353319205209266
4.299106133460801	3.935432966577132	0.363673166883669
4.293432626511677	3.935712138517989	0.357720487993689
4.301880721131173	3.940951484601805	0.360929236529368
4.294207582128757	3.938096427465110	0.356111154663647

Table 6: Calculating the p-norms error between the fluctuating co-existence steady-state solutions' data denoted by the notation D using a MATLAB algorithm with a Poisson random disturbance value of 0.0001

1-norm to 10-norm error calculation of D data	11-norm to infinity-norm error calculation of D data
3.571663951775224	0.440619651394623
1.129534617874522	0.433027886616641
0.769594074461168	0.426710421730387
0.635268174656686	0.421372574744492
0.566220640207528	0.416803889704856
0.524421926205313	0.412850086041442
0.496477619855022	0.409395507520377
0.476508632082276	0.406351758648594
0.461540698428075	0.403650126791870
0.449911290135396	0.363673166883669

4.0 Discussion of Results

We have observed that the infinity-norms error is 0.363673166883669 when the random disturbance value is 0.0001 whereas the infinity-norms error is 0.448769588090975 when the random disturbance value is 0.001 and the infinity-norms error is 0.545619678973393 when the random disturbance value is 0.002. These empirical results are only unique for the mutualistic interaction between the cowpea and the groundnut legumes.

5.0 Conclusion

For the mutualistic interaction between the cowpea and the groundnut legumes, we have used a MATLAB algorithm to find out that the infinity-norms errors between fluctuating co-existence steady-state solutions' data with a low differential random disturbance scenario dominantly changes from a random disturbance value of 0.0001 to a random disturbance value of 0.002. These observed infinity-norms errors for a mutualistic interaction are unique and generally smaller than the infinity-norms errors for a competitive interaction which was 0.607798977901674 as clearly shown in our most recent study. In a future publication, the same numerical idea will be extended to consider other types of interactions such as the commensalism and predation.

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