Available online www.unicrossjournals.com

UNICROSS JOURNAL OF SCIENCE AND TECHNOLOGY, UJOST

RESEARCH ARTICLE VOL. 3(1) DECEMBER, 2023 ISSN:2814-2233

Date Approved March 31, 2024 Pages 1 - 13

EVALUATION OF EQUAL INTER-COMPETITION COEFFICIENTS AND RANDOM ENVIRONMENTAL PERTURBATION ON CO-EXISTENCE AND SURVIVAL USING MATLAB ALGORITHM.

Atsu, Jeremiah Ugeh

Department of Mathematics, University of Cross River State, Calabar, Nigeria.

Email: atsujeremiah@gmail.com

Abstract

Ecological experts have experimentally tested the effect of equal inter-competition coefficients and a random environmental perturbation on the ecological theory of co-existence and survival in the instance of a single impact of a random environmental perturbation like the 2012 Nigeria flooding that was a major devastating environmental event on the livelihoods, habitats, ecosystems and agriculture to mention a few. However, on the simplifying assumption of equal inter-competition coefficients in which the intra-competition coefficients outweigh the inter-competition coefficients to an extent, how can we measure quantitatively the effect of this assumption on the ecological theory of co-existence and survival in the scenario of a relatively high Poisson random perturbation value of 4.8? We have developed a MATLAB algorithm for several variations of the inter-competition coefficients to solve and analyze this challenging mutualistic interaction between cowpea and groundnut legumes. The novel results that we have obtained have not been seen elsewhere; these are fully presented and discussed.

Keywords: Inter-competition, intra-competition, random environmental perturbation, co-existence, survival, habitat, ecosystem, mutualistic interaction.

1.0 Introduction

To the best of our knowledge most of real-life environmental perturbation is rather a sequence of distinct occurrences and not an in-situ event. It is also difficult to know exactly how the dynamics of two mutualistic interacting legumes can respond to these environmental fluctuations. It is against this background that we have assumed that the contributions of the enhancing inter-competition coefficients can be equal on a further simplifying assumption that the chosen inhibiting intra-competition coefficients in the sense of a popular Lotka-Volterra formalism can outweigh the inter-competition coefficients in the context of two mutualistic interacting legumes.

The work of Yan and Ekaka-a (2011) in stabilizing a mathematical model of plant species interaction comes in hand. Stabilizing a no linear system is very important in application. This is found in the feedback control based on the Ricatti equation of the linearized system, Barbu, Coca and Yan, (2005). The method of Barbu, Coca and Yan can be extended to stabilizing a semi parabolic equation, Yan, Coca, and Barbu (2009). A recent research, Ekaka-a (2009) and Ford, Lumb & Ekaka (2010) has shown that ecological systems behave like other real-world systems which are expected to run over a longer period of time to enable a clearer qualitative characterization. Our present study numerically estimates the depletion rates, effects of equal inter-competition coefficients and a random environmental perturbation on ecological coexistence and survival of species. Mechanisms of stabilizations for model equations of competition interaction which are most often unstable are needed to accurately model real ecosystems, Hannon (1986).

2.0 Methods and Materials

This work derives its origin from considering the steady states of the following general nonlinear system.

$$y^{\prime} = y(t)(a_1 - b_1 y(t) - c_1 z(t) \quad (1)$$

$$z^{1} = z(t)(a_2 - b_2 y(t) - c_2 z(t) \quad (2)$$

with initial conditions y(0)= $y_0 > 0, z(0) = z_0 > 0$

where $a_i, b_i, c_{i,} i = 1{,}2$ are positive constants.

The steady-state (y_e, z_e) satisfies

$$y_e(a_1 - b_1 y_e - c_1 z_e) = 0 \quad (3)$$

$$z_e(a_2 - b_2 y_e - c_2 z_e) = 0 \quad (4)$$

The implication of this is that there are for steady states

$$y_e = 0, z_e = 0,$$

$$y_e = 0, z_e = \frac{a_2}{c_2},$$

$$y_e = \frac{a_1}{b_1}, z_e = 0,$$

$$y_e = \frac{a_1 c_2 - c_1 a_2}{b_1 c_2 - c_1 b_2}, \quad z_e = \frac{b_1 a_2 - a_1 b_2}{b_1 c_2 - c_1 b_2}$$

These have been taken into consideration in our MATHLAB algorithm.

In this pioneering study, the model parameters that we have utilized were first derived by Ekaka-a et al (2013) based on the primary growth data by Ekpo et al (2010) including the several cited articles that supported their full report. For the purpose of this analysis, the intrinsic growth rate parameter values are 0.0225 grams and 0.0446 grams per area of habitat, the intra-competition coefficients are 0.006902 and 0.0133, the inter-competition coefficients are 0.0024 and 0.0024. The deterministic model formulation follows the popular Lotka-Volterra type which is not the central focus of this analysis. A MATLAB algorithm has been implemented to predict the data below under the implicit assumptions that the said environmental perturbation only affects the intrinsic growth rates provided the intra-competition coefficients outweigh the inter-competition coefficients to an extent. For the purpose of clarity, the notations represented by the model parameter K stand for the biological carrying capacity which is defined as the ratio of the intrinsic growth rate to the intra-competition coefficient while the notations represented by the model parameter alpha as the ratio of the inter-competition coefficient to the intra-competition coefficient.

3.0 Results

The results of this analysis are displayed as in Table 1, Table 2, and Table 3 under different scenarios below:

Table 1: MATLAB algorithm predicted data of co-existence and survival outcomes with a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0024 and 0.0024.

Example	C_b	G_b	α_{12}	$\frac{K_1}{K_2}$	α_{21}	$\frac{K_2}{K_1}$
1	730.5376	462.0821	0.3477	1.7255	0.1805	0.5795
2	221.2456	372.9168	0.3477	0.2750	0.1805	3.6364
3	486.9994	126.4355	0.3477	11.4907	0.1805	0.0870
4	284.5962	252.0799	0.3477	0.9812	0.1805	1.0192
5	844.3969	503.9562	0.3477	1.9033	0.1805	0.5254
6	251.6317	399.0497	0.3477	0.3192	0.1805	3.1331
7	779.9396	319.2675	0.3477	3.7469	0.1805	0.2669
8	617.5363	165.9956	0.3477	10.2605	0.1805	0.0975
9	440.2871	413.2947	0.3477	0.8884	0.1805	1.1257
10	721.0206	479.7453	0.3477	1.5851	0.1805	0.6309
11	496.0700	105.7582	0.3477	28.2786	0.1805	0.0354
12	759.8430	477.5482	0.3477	1.7442	0.1805	0.5733
13	609.8092	386.8643	0.3477	1.7169	0.1805	0.5824
14	608.6517	254.7409	0.3477	3.5890	0.1805	0.2786
15	514.0146	157.8896	0.3477	7.0486	0.1805	0.1419

Table 2: MATLAB algorithm predicted data of co-existence and survival outcomes with a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0036 and 0.0036.

Example	C_b	G_b	α_{12}	$\frac{K_1}{K_2}$	α_{21}	$\frac{K_2}{K_1}$
1	584.5491	173.0657	0.5216	33.3030	0.2707	0.0300
2	240.1983	85.0328	0.5216	9.7841	0.2707	0.1022
3	264.9791	372.2648	0.5216	0.2356	0.2707	4.2443
4	637.9924	290.4850	0.5216	4.1299	0.2707	0.2421
5	782.8498	227.6842	0.5216	42.0710	0.2707	0.0238
6	444.7500	261.4422	0.5216	2.1862	0.2707	0.4574
7	800.0276	506.8916	0.5216	1.8449	0.2707	0.5420
8	264.5077	251.7065	0.5216	0.7397	0.2707	1.3520
9	508.3213	374.1999	0.5216	1.3235	0.2707	0.7556
10	745.6771	477.5589	0.5216	1.8010	0.2707	0.5552
11	378.3333	351.0656	0.5216	0.7851	0.2707	1.2737
12	571.9577	216.8559	0.5216	7.3960	0.2707	0.1352
13	211.4291	240.4430	0.5216	0.4695	0.2707	2.1300
14	857.6185	358.3367	0.5216	5.3147	0.2707	0.1882
15	528.8251	227.2682	0.5216	4.8769	0.2707	0.2050

Table 3: MATLAB algorithm predicted data of co-existence and survival outcomes with a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0048 and 0.0048: Scenario One

Example	C_b	G_b	α_{12}	$\frac{K_1}{K_2}$	α_{21}	$\frac{K_2}{K_1}$
1	95.9745	57.4615	0.6955	2.4541	0.3609	0.4075
2	761.4027	559.3485	0.6955	1.3087	0.3609	0.7641
3	918.2189	381.6241	0.6955	12.9948	0.3609	0.0770
4	692.9050	422.8285	0.6955	2.3087	0.3609	0.4331
5	131.4850	172.4750	0.6955	0.0923	0.3609	10.8364
6	424.2118	443.1115	0.6955	0.4002	0.3609	2.4990
7	473.5357	365.0023	0.6955	1.1319	0.3609	0.8835
8	362.9907	351.6142	0.6955	0.5370	0.3609	1.8623
9	470.8115	409.3289	0.6955	0.7775	0.3609	1.2862
10	898.0985	597.4887	0.6955	1.7653	0.3609	0.5665
11	453.8786	197.4105	0.6955	9.4210	0.3609	0.1061
12	526.1242	522.8583	0.6955	0.4880	0.3609	2.0491
13	425.6750	455.0197	0.6955	0.3624	0.3609	2.7592
14	841.1128	666.4201	0.6955	1.0408	0.3609	0.9608
15	228.3906	245.5437	0.6955	0.3533	0.3609	2.8306

Table 4: MATLAB algorithm predicted data of co-existence and survival outcomes with a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0048 and 0.0048: Scenario Two

Example	C_b	G_b	α_{12}	$\frac{K_1}{K_2}$	α_{21}	$\frac{K_2}{K_1}$
1	293.3581	239.0200	0.6955	0.9548	0.3609	1.0473
2	234.9971	276.5141	0.6955	0.2227	0.3609	4.4900
3	378.1650	203.2334	0.6955	3.5478	0.3609	0.2819
4	504.7839	512.2030	0.6955	0.4502	0.3609	2.2213
5	791.5431	458.0943	0.6955	2.7430	0.3609	0.3646
6	889.2280	361.8158	0.6955	15.5926	0.3609	0.0641
7	946.4196	610.6389	0.6955	1.9391	0.3609	0.5157
8	590.8727	283.0767	0.6955	5.6424	0.3609	0.1772
9	662.4797	350.6917	0.6955	3.7508	0.3609	0.2666
10	203.2398	153.4315	0.6955	1.2055	0.3609	0.8296
11	862.3562	340.3672	0.6955	21.4698	0.3609	0.0466
12	250.6257	113.2049	0.6955	7.5548	0.3609	0.1324
13	422.0527	160.4671	0.6955	38.1053	0.3609	0.0262
14	906.4007	401.4498	0.6955	8.4385	0.3609	0.1185
15	197.1569	185.4372	0.6955	0.5967	0.3609	1.6758

Table 5: MATLAB algorithm predicted data of co-existence and survival outcomes with a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0048 and 0.0048: Scenario Three

Example	C_b	G_b	α_{12}	$\frac{K_1}{K_2}$	α_{21}	$\frac{K_2}{K_1}$
1	981.8670	419.2207	0.6955	10.6428	0.3609	0.0940
2	379.7896	283.6261	0.6955	1.2455	0.3609	0.8029
3	305.4067	360.4672	0.6955	0.2187	0.3609	4.5732
4	710.4238	614.4544	0.6955	0.7907	0.3609	1.2648
5	588.8749	439.8816	0.6955	1.2446	0.3609	0.8035
6	278.5840	241.5233	0.6955	0.7846	0.3609	1.2745
7	411.1174	425.3311	0.6955	0.4164	0.3609	2.4016
8	933.8298	466.9708	0.6955	4.6870	0.3609	0.2134
9	742.5486	377.4997	0.6955	4.3832	0.3609	0.2281
10	779.0923	584.9530	0.6955	1.2255	0.3609	0.8160
11	674.5924	367.8299	0.6955	3.3673	0.3609	0.2970
12	436.9557	324.3933	0.6955	1.2679	0.3609	0.7887
13	520.3916	320.9466	0.6955	2.2322	0.3609	0.4480
14	774.6845	550.9251	0.6955	1.4430	0.3609	0.6930
15	545.3327	355.1206	0.6955	1.8847	0.3609	0.5306

Table 6: MATLAB algorithm predicted data of co-existence and survival outcomes with a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0048 and 0.0048: Scenario Four

Example	C_b	G_b	α_{12}	$\frac{K_1}{K_2}$	α_{21}	$\frac{K_2}{K_1}$
1	95.9745	57.4615	0.6955	2.4541	0.3609	0.4075
2	761.4027	559.3485	0.6955	1.3087	0.3609	0.7641
3	918.2189	381.6241	0.6955	12.9948	0.3609	0.0770
4	692.9050	422.8285	0.6955	2.3087	0.3609	0.4331
5	131.4850	172.4750	0.6955	0.0923	0.3609	10.8364
6	424.2118	443.1115	0.6955	0.4002	0.3609	2.4990
7	473.5357	365.0023	0.6955	1.1319	0.3609	0.8835
8	362.9907	351.6142	0.6955	0.5370	0.3609	1.8623
9	470.8115	409.3289	0.6955	0.7775	0.3609	1.2862
10	898.0985	597.4887	0.6955	1.7653	0.3609	0.5665
11	453.8786	197.4105	0.6955	9.4210	0.3609	0.1061
12	526.1242	522.8583	0.6955	0.4880	0.3609	2.0491
13	425.6750	455.0197	0.6955	0.3624	0.3609	2.7592
14	841.1128	666.4201	0.6955	1.0408	0.3609	0.9608
15	228.3906	245.5437	0.6955	0.3533	0.3609	2.8306

Table 7: MATLAB algorithm predicted data of co-existence and survival outcomes with a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0048 and 0.0048: Scenario Five

Example	C_b	G_b	α_{12}	$\frac{K_1}{K_2}$	α_{21}	$\frac{K_2}{K_1}$
1	872.4751	471.0567	0.6955	3.4888	0.3609	0.2866
2	181.0879	164.8786	0.6955	0.6674	0.3609	1.4983
3	244.2985	192.9367	0.6955	1.0511	0.3609	0.9514
4	592.7247	407.5161	0.6955	1.5977	0.3609	0.6259
5	725.5138	581.1165	0.6955	1.0066	0.3609	0.9935
6	804.6768	634.3186	0.6955	1.0571	0.3609	0.9460
7	920.4927	681.1952	0.6955	1.2801	0.3609	0.7812
8	457.5259	412.4896	0.6955	0.6899	0.3609	1.4495
9	500.9799	426.6152	0.6955	0.8311	0.3609	1.2032
10	675.6828	271.7476	0.6955	17.4492	0.3609	0.0573
11	319.1114	199.3780	0.6955	2.1429	0.3609	0.4667
12	910.4954	636.6963	0.6955	1.5180	0.3609	0.6587
13	588.8450	497.5602	0.6955	0.8519	0.3609	1.1739
14	636.7571	235.5840	0.6955	81.8632	0.3609	0.0122
15	696.1819	394.1936	0.6955	2.9526	0.3609	0.3387

Table 8: MATLAB algorithm predicted data of co-existence and survival outcomes with a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0048 and 0.0048: Scenario Six

Example	C_b	G_b	α_{12}	$\frac{K_1}{K_2}$	α_{21}	$\frac{K_2}{K_1}$
1	95.9745	57.4615	0.6955	2.4541	0.3609	0.4075
2	761.4027	559.3485	0.6955	1.3087	0.3609	0.7641
3	918.2189	381.6241	0.6955	12.9948	0.3609	0.0770
4	692.9050	422.8285	0.6955	2.3087	0.3609	0.4331
5	131.4850	172.4750	0.6955	0.0923	0.3609	10.8364
6	424.2118	443.1115	0.6955	0.4002	0.3609	2.4990
7	473.5357	365.0023	0.6955	1.1319	0.3609	0.8835
8	362.9907	351.6142	0.6955	0.5370	0.3609	1.8623
9	470.8115	409.3289	0.6955	0.7775	0.3609	1.2862
10	898.0985	597.4887	0.6955	1.7653	0.3609	0.5665
11	453.8786	197.4105	0.6955	9.4210	0.3609	0.1061
12	526.1242	522.8583	0.6955	0.4880	0.3609	2.0491
13	425.6750	455.0197	0.6955	0.3624	0.3609	2.7592
14	841.1128	666.4201	0.6955	1.0408	0.3609	0.9608
15	228.3906	245.5437	0.6955	0.3533	0.3609	2.8306

Table 9: MATLAB algorithm predicted data of co-existence and survival outcomes with a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0048 and 0.0048: Scenario Seven

Example	C_b	G_b	α_{12}	$\frac{K_1}{K_2}$	α_{21}	$\frac{K_2}{K_1}$
1	872.4751	471.0567	0.6955	3.4888	0.3609	0.2866
2	181.0879	164.8786	0.6955	0.6674	0.3609	1.4983
3	244.2985	192.9367	0.6955	1.0511	0.3609	0.9514
4	592.7247	407.5161	0.6955	1.5977	0.3609	0.6259
5	725.5138	581.1165	0.6955	1.0066	0.3609	0.9935
6	804.6768	634.3186	0.6955	1.0571	0.3609	0.9460
7	920.4927	681.1952	0.6955	1.2801	0.3609	0.7812
8	457.5259	412.4896	0.6955	0.6899	0.3609	1.4495
9	500.9799	426.6152	0.6955	0.8311	0.3609	1.2032
10	675.6828	271.7476	0.6955	17.4492	0.3609	0.0573
11	319.1114	199.3780	0.6955	2.1429	0.3609	0.4667
12	910.4954	636.6963	0.6955	1.5180	0.3609	0.6587
13	588.8450	497.5602	0.6955	0.8519	0.3609	1.1739
14	636.7571	235.5840	0.6955	81.8632	0.3609	0.0122
15	696.1819	394.1936	0.6955	2.9526	0.3609	0.3387

Table 10: MATLAB algorithm predicted data of co-existence and survival outcomes with a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0048 and 0.0048: Scenario Eight

Example	C_b	G_b	α_{12}	$\frac{K_1}{K_2}$	α_{21}	$\frac{K_2}{K_1}$
1	858.3428	313.5466	0.6955	169.8912	0.3609	0.0059
2	579.0438	365.4801	0.6955	2.0758	0.3609	0.4817
3	693.5001	531.5915	0.6955	1.1511	0.3609	0.8688
4	569.8409	492.2244	0.6955	0.7940	0.3609	1.2595
5	457.1015	181.2292	0.6955	20.3604	0.3609	0.0491
6	412.6216	412.7535	0.6955	0.4759	0.3609	2.1011
7	498.2703	238.2977	0.6955	5.6874	0.3609	0.1758
8	527.7308	413.0207	0.6955	1.0806	0.3609	0.9254
9	432.9432	426.1035	0.6955	0.5062	0.3609	1.9754
10	540.3744	529.4762	0.6955	0.5147	0.3609	1.9428
11	513.8029	465.0567	0.6955	0.6808	0.3609	1.4688
12	278.9764	207.7953	0.6955	1.2554	0.3609	0.7966
13	285.1490	314.2196	0.6955	0.3153	0.3609	3.1716
14	825.1243	498.4093	0.6955	2.3851	0.3609	0.4193
15	618.7016	459.2251	0.6955	1.2687	0.3609	0.7882

4.0 Discussion of results

In Table 1, we have used a MATLAB algorithm to predict two (2) instances in which the cowpea legume may not survive and only thirteen (13) instances of valid co-existence and survival. In contrast, we have observed only five (5) instances in which the groundnut legume may not survive and only ten (10) instances of valid co-existence and survival. Therefore, the cowpea is more likely to co-exist and survive than the groundnut when a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0024 and 0.0024 is considered.

In Table 2, we have used a MATLAB algorithm to similarly predict two (2) instances in which the cowpea legume may not survive and only thirteen (13) instances of valid co-existence and survival. In contrast, we have observed only seven (7) instances in which the groundnut legume may not survive and only eight (8) instances of valid co-existence and survival. In this scenario, we can deduce that the cowpea is more likely to co-exist and survive than the groundnut when a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0036 and 0.0036 is considered.

However, in Table 3, we have used a MATLAB algorithm to predict six (6) instances in which the cowpea legume may not survive and only nine (9) instances of valid co-existence and survival whereas we have observed only two (2) instances in which the groundnut legume may not survive and only thirteen (13) instances of valid co-existence and survival. In this scenario, we can deduce that the groundnut is more likely to co-exist and survive than the cowpea when a random noise intensity value of 4.8 for a mutualistic interaction between cowpea and groundnut for equal inter-competition coefficients of 0.0048 and 0.0048 is considered. An exceptional repeated observation has been made for the Table 6 and Table 8 data for the same random noise and equal inter-competition coefficients for a mutualistic interaction between cowpea and groundnut.

Without loss of generality, we have observed as shown in Table 4, Table 5, and Table 7 that the cowpea legume is more likely to co-exist and survive than the groundnut legume. In particular, the groundnut legume is more likely to co-exist and survive as shown in Table 5 and Table 9 than the groundnut legume as shown in Table 4. It is also very clear as shown in Table 10 that the cowpea legume relatively tends to co-exist and survive better than the groundnut legume.

5.0 Conclusion

The variability of co-existence and survival for two mutualistic interacting legumes with the equal-effect of the inter-competition coefficients simplifying assumption and a defined random noise intensity has been observed. In some of the predicted data, we have found that the cowpea legume tends to co-exist and survive better than the groundnut legume whereas in a few instances the groundnut legume tends to co-exist and survive better than the cowpea legume irrespective of the random environmental disturbance inclusion.

References

Ekaka-a, E.N., Nafo N.M. Ugwu, C. Agwu, I.A (2013). Concergence Characterization of the Border Steady-State Solution of Two Interacting Legumes, Scientia Africana, Vol. 12(1), Pp.89-92.

Ekpo, M.A & Nkanang A.J. (2010). Nitrogen fixing capacity of legumes and their rhizosphereal microflora in diesel oil polluted oil in the tropics, Journal of Petroleum and Gas Engineering, 1(4), Pp. 76-83.

Yan, Y. & Ekaka-a, E.N. (2011). Stabilizing a Mathematical Model of Population System. Journal of the Franklin Institute, 348, Pp. 2744-2758.

Barbu, V & Coca, D. & Yan, Y. (2005). Stabilizing Semi linear Parabolic Equations, Numerical Functional Analysis and Optimization 26 Pp. 449-480

Ekaka-a, E.N. (2009) Computational and Mathematical Modelling of Plant Species Interactions in a Harsh Climate, PhD Thesis, Department of Mathematics, The University of Liverpool and The University of Chester, United Kingdom.

Ford, N. J., Lumb, P.M., Ekaka-a, E.N. (2010). Mathematical modeling of plant species interactions in a harsh climate, Journal of Computational and Applied Mathematics **234**, Pp. 2732-2744

Hannon, B. (1986) Ecosystem Control Theory, Journal of Theoretical Biology **121**, Pp. 417-437

Yan, Y. (2005). Galerkin finite element methods for Stochastic Parabolic partial differential equations, SIAM J. Numer, Anal. **43**. Pp. 1363-1384

Yan, Y., Coca, D. & Barbu, V. (2009). Finite dimensional controller design for semi linear parabolic systems, Nonlinear Analysis: Theory, methods and Applications **70**, Pp. 4451-4475

Yan, Y., Coca, D. & Barbu, V. (2008). Feedback control for Navier-Stokes equation, Nonlinear Functional Analysis and Optimization, **29**. Pp.225-242