Estimating settlements of footings in sands – a probabilistic approach

Estimation des tassements de semelles dans les sables – une approche probabiliste

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ABSTRACT: This paper discuss about probabilistic settlement analysis of footings in sands, focusing on the load curve (estimated settlements). For this purpose, three methodologies that take the First and Second Order Second Moment (FOSM and SOSM), and Monte Carlo Simulation (MCS) methods for calculating mean and variance of the estimated settlements through Schmertmann's 1970 equation are discussed. The deformability modulus (E_{Si}) is considered varying according to the division of the soil into sublayers and it is analyzed as the only independent random variable. As an example of application, a hypothetical case in state of Espirito Santo, Brazil, is evaluated. Simulations indicate that there is significant similarity between SOSM and MCS methods, while the FOSM method underestimates the results due to the non-consideration of the high orders terms in Taylor's series. The contribution to the knowing of the uncertainties in settlement predictions can provides a more safety design.

RÉSUMÉ : Cet article traite de l'approche probabiliste de l'estimation des tassements de semelles dans les sables, en se concentrant sur la courbe charge-tassements estimés. Pour ce faire, trois méthodes basées sur les moments du premier et deuxième ordre (FOSM et SOSM), des moyennes et écarts-types et sur des simulations de Monte Carlo (MCS) ont été utilisées pour le calcul du tassement moyen et sa variation à l'aide de l'équation de Schmertmann (1970) et sont discutées. Le module de déformabilité (ESi) est considéré variable dû à la division du sol en sous-couches et il est considéré comme le seul paramètre indépendant et aléatoire. Comme exemple d'application, une étude de cas situé dans l'état d'Espirito Santo, au Brésil, est discutée et évaluée. Les simulations montrent qu'il y a une similitude significative entre les résultats obtenus par les simulations SOSM et MCS, alors que les estimations FOSM sous-estiment les tassements en raison de la non-prise en compte des termes d'ordre élevé des décompositions en série de Taylor. La contribution à la connaissance des incertitudes dans les prédictions de tassement peut fournir un dimensionnement plus fiable.

KEYWORDS: Sandy soils, foundations (engineering), settlement of structures, reliability (engineering), probabilities.

1 INTRODUCTION

Probabilistic or reliability studies and risk evaluation have become increasing popular in geotechnical engineering only in the last decades (Sivakugan and Johnson 2004), while geotechnical analysis are still usually made by conventional deterministic approaches, based on safety factors. Most commonly studies in probabilistic analysis reported in the literature discuss the ultimate limit state (ULS), representing the probability of a foundation to failure, p_F. According to Aoki et al. (2002), this probability is function of relative position and scatter degree of the probability density curves of solicitation, fs(S), and resistance, F_R(R), as shown in figure 1, so:

$$p_F = \int_0^\infty F_R(R).f_S(S)dx \tag{1}$$

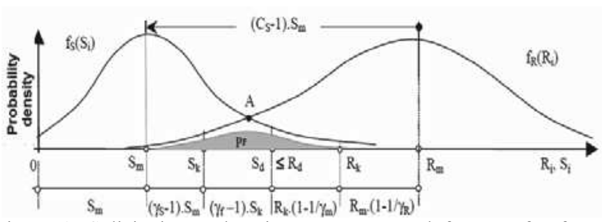

Figure 1. Solicitation and resistance curves and factors of safety – Reliability analysis of a foundation at the ULS (Aoki et al., 2002).

The same probabilistic concept can be applied to analyze the serviceability limit state (SLS) of a foundation (figure 2). In a foundation settlement analysis, the probability of failure becomes the probability of predicted or estimated settlement (calculated with service loads) to exceed limiting settlement (limiting movement affecting visual appearance, serviceability or function and stability). Here, solicitation and resistance functions assume the variability of predicted and limiting settlements, respectively, which are treated as dependent random variables. The probability of SLS to failure, p_E is then:

$$p_E = \int_0^\infty \rho(x).\rho_{\lim}(x).dx \tag{2}$$

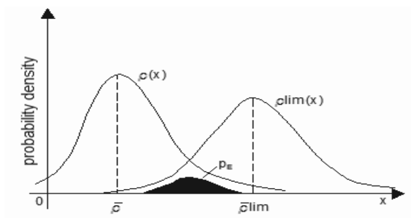

Figure 2.
Solicitation and resistance curves – Reliability analysis of a foundation at the SLS.

In the specific case of settlements of footings in sands, the predicted settlements can be evaluated through traditional methods, such as: Schmertmann (1970), Schmertmann et al. (1978), Burland and Burbidge (1984), Berardi and Lancellotta (1991). The limiting settlements evaluation can be made by using observational, empirical, structural or numerical modelling methods (Negulescu and Foerster 2010), but are beyond the scope of this paper.

This paper focuses on the solicitation curve and assumes, as a simplification, that the variability of the resistance (limiting settlement) curve is zero (i.e. it has been considered constant for some specific deterministic value). Thus, the probability of occurrence of limiting settlements becomes:

$$p_E[\rho \geq \rho_{\lim}] = \int_{\rho \lim}^{\infty} \rho(x).dx \tag{3}$$

The integrals of equations (1, 2 and 3) are commonly solved using analytical approximations (or reliability methods). In the following sections three methodologies using FOSM, SOSM and MCS methods with Schmertmann's (1970) equation are shortly presented and discussed as a simple and practical way to characterize the settlement solicitation curve for a case of a single footing in a sandy soil.

2 ANALYZED METHODOLOGIES

2.1 *Main concepts adopted*

The main concepts adopted on the analyzed methodologies are:

- The total predicted settlement (ρ) is given by Schmertmann's (1970), calculated through the sum of the settlement increments (ρ_i) of each sublayer:

$$\rho = \sum_{i=1}^{N} \rho_i \tag{4}$$

where: i=1, N and N is the number of adopted sublayers.

- If the increments (ρ_i) are statistically independents and $V[\rho_i]$ are the variance increments of the sublayers then, the total variance also can be calculated as the sum of $V[\rho_i]$:

$$V[\rho] = \sum_{i=1}^{N} V[\rho_i] \tag{5}$$

The proposed simplifications consider that the predicted settlement is function of only one random variable (E_{Si}, in each sublayer), and is completely described by its first two moments (mean and variance). The evaluation must have done considering the soil stratification, first through the evaluation of each sublayer, individually, and then accounting for the entire stratum of the subsoil (sum of the increments).

2.2 *The FOSM and SOSM methods*

Consider the given form of the performance function of the random variables x_1, x_2, x_3... x_i, independent, such as: $G[X]=G(x_1, x_2, x_3... x_i)$. Developing the function G[X] about its mean and the mean of the random variables x_i, using the Taylor's expansion series, gives (Baecher and Christian 2003):

$$G[X] = G[\bar{X}] + \frac{1}{1!}\frac{\partial G}{\partial x}(X-\bar{X}) + \\ + \frac{1}{2!}\frac{\partial^2 G}{\partial x^2}(X-\bar{X})^2 + \frac{1}{3!}\frac{\partial^3 G}{\partial x^3}(X-\bar{X})^3 + ... \tag{6}$$

The mean (E[ρ]) and variance (V[ρ]) of the predicted settlement can be obtained from equation (6), considering the Schmertmann's (1970) method as the performance function and assuming the parameter E_S as the unique random variable. For the FOSM method, gives:

$$E[\rho] = \rho = C_1 C_2 \sigma^* \sum_{i=1}^{N}\left(\frac{I_{zi}\Delta_{zi}}{\bar{E}_{si}}\right) \tag{7}$$

$$V[\rho] = \left[C_1 C_2 \sigma^* \sum_{i=1}^{N} \frac{I_{zi}\Delta_{zi}}{\bar{E}_{si}^{\,2}}\right]^2 .V[E_{si}] \tag{8}$$

For the SOSM method, settlement mean and variance are:

$$E[\rho] = C_1 C_2 \sigma^* \sum_{i=1}^{N}\left(\frac{I_{Zi}\Delta_{Zi}}{\bar{E}_{Si}}\right) + \\ + C_1 C_2 \sigma^* \sum_{i=1}^{N}\left(\frac{I_{Zi}\Delta_{Zi}}{\bar{E}_{Si}^{\,3}}.V[E_{Si}]\right) \tag{9}$$

$$V[\rho] = \left[C_1 C_2 \sigma^* \sum_{i=1}^{N} \frac{I_{Zi}\Delta_{Zi}}{\bar{E}_{Si}^{\,2}}\right]^2 .V[E_{Si}] + \\ + 2\left[C_1 C_2 \sigma^* \sum_{i=1}^{N} \frac{I_{Zi}\Delta_{Zi}}{\bar{E}_{Si}^{\,3}}\right]^2 .V[E_{Si}]^2 \tag{10}$$

The first terms at the right side of equations (9 and 10) correspond exactly to the mean and variance calculated by the FOSM method, while the second terms represent the additional terms considered in the Taylor's series. This simple observation shows that the use of the FOSM method underestimates the results of mean and variance as increasing the importance of the second term of the considered performance function.

With the calculated values of settlement mean, variance and standard deviation (root square of variance) in hands, the probabilistic analysis can be made by setting the lognormal distribution to represent the predict settlement and specifying deterministic values to limiting settlement.

The lognormal distribution was used here for being a strictly positive distribution, while having a simple relationship with the normal distribution (Bredja et al. 2000, Fenton and Griffiths 2002, Goldsworthy 2006).

The methodologies assume the analysis of an isolated footing. Nevertheless, if two non-correlated footings are being evaluated, differential settlement can be obtained by:

$$E[\Delta\rho] = E[\rho_1] - E[\rho_2] \tag{11}$$

$$V[\Delta\rho] = V[\rho_1] + V[\rho_2] \tag{12}$$

where: $E[\rho_1]$ and $E[\rho_2]$ are mean predicted settlements of the footings and $V[\rho_1]$ and $V[\rho_2]$ are its variances.

2.3 *The MCS method*

The Monte Carlo Simulation method consists basically in the simulation of all random variables and the resolution of the performance function for all those generated values. Here again, deformability modulus is the only random variable.

As a simplification, is proposed a number of 1.000 simulations of modulus for each sublayer, using lognormal distribution. The simulation can be done by using random number generator algorithms for Microsoft Excel. The main steps are summarized below:

- Analysis of mean and variance of q_{ci} results, for each sublayer.
- Estimation of mean and variance of E_{Si}.
- Simulation of E_{Si} (using mean, variance and lognormal distribution).
- Calculus of settlement mean and variance increment for each sublayer.
- Calculus of total settlement mean and variance.
- Probabilistic settlement analysis using lognormal distribution and an adopted limiting settlement value(s).

2.4 *Evaluation of deformability modulus in the sublayers*

In reliability analysis, independent random variables are influenced by uncertainties and it must be appropriate quantified. In the proposed methodologies, only one random variable is adopted (E_{Si}) for each sublayer.

The uncertainties in E_{Si} can be analyzed by attributing values to E_{Si} variance ($V[E_{Si}]$), or by analyzing the sources of uncertainties in the E_{Si} estimations. Considering that the moduli E_{Si} are estimated from CPT, three sources of uncertainty are suggested to be accounted for:

(i) The uncertainties due to field measurements (q_{ci}, in this case) – in other words, the sum of inherent soil variability and equipments/measurement procedures errors of CPT. This variance is named $V_1[E_{Si}]$.

(ii) The uncertainties due to transformation models – in other words, the empirical correlations used to transform the field measurement results (q_{ci}) into required design parameters (E_{Si}). This variance is named $V_2[E_{Si}]$;

(iii) Statistical uncertainties – due to limited sampling or insufficient representative sampling data in the field. This variance is named $V_3[E_{Si}]$.

The sources of uncertainties represented by $V_1[E_{Si}]$ and $V_2[E_{Si}]$ are explicit in the E_{Si} x q_{ci} correlations. The typical form of those correlations is:

$$E_{Si} = \alpha . q_{Ci} \tag{13}$$

Observe that in equation (13) two variables can contribute for the uncertainties in E_{Si} estimations, which are: q_{ci} and α. It represents the uncertainties $V_1[E_S]$ and $V_2[E_S]$, as assumed before. The FOSM method is applied to equation (13) to give those sources of uncertainties. Then, $V_1[E_S]$ e $V_2[E_S]$ are:

$$V_1[E_{Si}] = \alpha_{average}{}^2 . V[q_{Ci}] \tag{14}$$

in which: $V[q_{ci}]$ is the sampling variance, calculated using q_{ci} results, of the ith sublayer, and $\alpha_{average}$ is the average or mean α-value, from the choosed correlations.

$$V_2[E_{Si}] = q_{Ci\,average}{}^2 . V[\alpha] \tag{15}$$

in which: $V[\alpha]$ is the variance of α –values, supposed to be equally likely. To evaluate $V_2[E_{Si}]$, two or more empirical correlations are needed or, in other case, it results zero.

The third source of uncertainties evaluated is due to the representative of sampling data. Assuming that this source of uncertainties is function only of the amount of sampling (size of sample), it can be calculated using the following equation proposed by DeGroot (1986; apud Goldsworthy 2006):

$$V_3[E_S] = \frac{V_1[E_S]}{n} \tag{16}$$

in which: $V_1[E_S]$ is the sampling variance from E_S results; n is the number of data obtained from CPT.

Thus, the equation to account for all sources of uncertainties on the variance of E_{Si}, of the ith sublayer is:

$$V[E_{Si}] = V_1[E_{Si}] + V_2[E_{Si}] + V_3[E_{Si}] \tag{17}$$

2.5 *Further discussions*

Comparative analysis has showed that the use of the FOSM method underestimates the results for $COV[E_S]$>30%, reaching up to 50% error when $COV[E_S]$=100%, due to the non-consideration of the higher orders terms in Taylor's series, while SOSM and MCS methods seems to converge, approximately, to same results for all $COV[E_S]$ values.

It has been also observed that the depth where the major variance contribution occurs is highly dependent of the E_{Si} values, with strong influence of the I_Z distribution factor, from Schmertmann's (1970). So, the significance of settlement variance contribution ($V[\rho_i]$), of the ith sublayer, in total settlement variance ($V[\rho]$) increases as the lower the mean value of E_{Si} and the closer the sublayer is to I_{Zmax} depth.

As being simplified methods, is important to summarize the advantages and limitations of its use. Some advantages are:

- Easy application, trough electronic spreadsheets, without having finite element or advanced calculation software's.
- It's very helpful for giving guidance on the sensivity of design results (Griffiths et al. 2002), outcome from Schmertmann's (1970) equation, to variations of deformability modulus.
- Is possible to verify the distribution and the contribution of settlement variances in the sublayers.
- Despite the non-account for spatial correlations or scale of fluctuation of deformability modulus, the use of Taylor's methods is not against safety, as observed previously by Gimenes and Hachich (1992).

Some limitations are:

- It's assumed a single and isolated footing (i.e. there are no interaction among strain bulbs of adjacent footings and no soil-structure interaction effects).
- In a foundation SLS analysis is necessary to account for the variability of other important parameters as: geometry and load of footings, which were considered constants for the present study.

On the use of the proposed methodologies, is recommended that the sublayer thickness be considered as small as possible, so the influence of tendencies in vertical variability is minimal (Campanella et al, 1987). For example, in mechanical CPT with 20cm interval data, is indicated to set 20cm for sublayer thickness, so the vertical variability is already considered in the subsoil stratification and is not necessary to detrend the data (since the sublayers are treated as independent from each other). In this case, the evaluated uncertainties in moduli are only from horizontal variability of the sublayers.

3 EXAMPLE OF APPLICATION

This section presents an example of application of the SOSM methodology. The case considers one footing with 1600 kN centrally applied load, size of 2,0m x 2,0m, embedded 1,0m below ground surface. The subsoil stratum is showed in figure 3. This situation with shallow stratum composed by sand with varied relative density is a typical soil formation from the coastal of Vitoria/ES, influenced by the transgression/regression marine phenomena, occurred in Quaternaries' period.

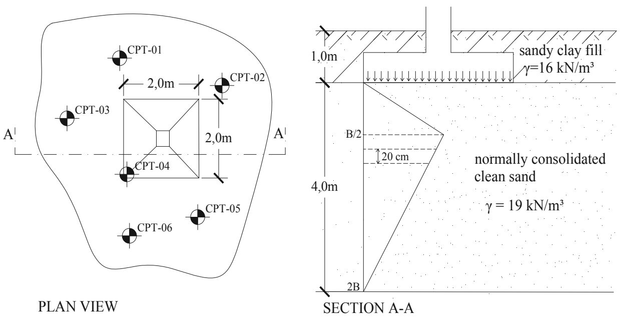

Figure 3. Subsoil stratum adopted for the example of application.

The results of 06 mechanical cone penetration tests (CPT), with 20 cm limit interval data, are hypothetically assumed to be available in a region around the footing, which is represented by the shown subsoil stratum.

For Schmertmann's (1970) equation, sublayer thickness was set at 20 cm. To account for soil variability in this region is firstly necessary to analyze statistically the CPT data. For each sublayer, q_{ci} mean and variance values must be calculated.

After that, deformability modulus has to be estimated for each sublayer, through the adopted(s) empirical correlation(s). Here, it's assumed the use of only one correlation, which is given by Schmertmann's (1970):

$$E_{Si} = 2.q_{Ci} \tag{18}$$

The transformation must be done using mean q_{ci} values.

The next step is to calculate $V[E_{Si}]$. Observe that, as only one empirical correlation was adopted, $V[\alpha] = 0$ and then, $V_2[E_{Si}]$ becomes automatically null.

Following, the settlement mean and variance contribution of each sublayer has to be evaluated. Table 1 shows the main calculation steps and results for the given example, where: q_{ci} and E_{Si} are given in MPa and predicted settlements results (ρ, $\sigma[\rho]$) are given in mm. Variances are given in square units.

Table 1. Evaluation of CPT results, uncertainties in E_{Si}, and application of the SOSM method.

Sublayer	q_{ci}	$V[q_{ci}]$	E_{Si}	$V[E_{Si}]$	ρ_i	$V[\rho_i]$	% in $V[\rho]$
1	10,0	10,3	20,1	48,2	0,252	0,007	0,2
2	9,6	9,9	19,2	46,0	0,791	0,077	2,0
3	9,7	9,9	19,4	46,0	1,308	0,207	5,4
4	9,2	9,4	18,4	44,1	1,937	0,481	12,5
5	8,9	9,3	17,9	43,3	2,576	0,884	22,9
6	9,4	9,6	18,8	44,7	2,607	0,845	21,9
7	9,7	9,8	19,3	45,7	2,358	0,672	17,4
8	11,9	12,1	23,8	56,4	1,737	0,297	7,7
9	13,3	13,5	26,6	63,0	1,414	0,176	4,6
10	15,4	15,5	30,8	72,2	1,104	0,092	2,4
11	18,1	18,0	36,2	83,8	0,839	0,045	1,2
12	21,5	21,6	42,9	100,8	0,628	0,022	0,6
13	24,2	24,7	48,4	115,3	0,489	0,012	0,3
14	24,8	25,8	49,7	120,5	0,413	0,008	0,2
15	21,8	22,6	43,7	105,5	0,400	0,009	0,2
16	20,4	21,0	40,7	97,9	0,352	0,007	0,2
17	19,2	19,8	38,5	92,3	0,291	0,005	0,1
18	16,1	16,3	32,3	76,1	0,250	0,005	0,1
19	15,9	16,0	31,7	74,5	0,153	0,002	0,0
20	15,9	16,0	31,8	74,5	0,051	0,000	0,0
Sum	-	-	-	-	19,95	3,86	100,0
$\sigma[\rho]$	-	-	-	-	-	1,96	-
COV (%)	-	-	-	-	-	9,84	-

The mean and variance of the predicted settlement are then the sum of the increments of each sub-layer, as suggested by the sum at the bottom of the table 1. So, the predicted settlement can now be represented by the form:

$$\rho(mm) = 20 \pm 2 \tag{19}$$

For the complete characterization of the solicitation curve (predicted settlement) lognormal distribution was used. Figure 4 shows the results for the probability of the predicted settlement to exceed different values of limiting settlements in a range between 10 to 50 mm. For example, the probability of the predicted settlement to exceed 25 mm is about 1,1%. For exceeding values of over 40 mm, P [$\rho \geq 40$mm] ≈ 0.

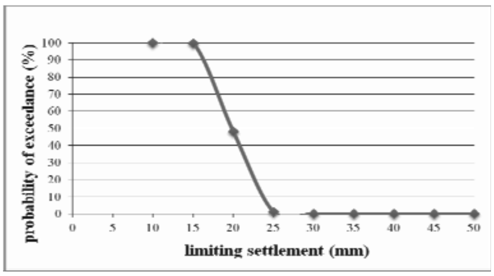

Figure 4. Probability of the predicted settlement to exceed different values of limiting settlement.

The analysis of the sources of uncertainties indicates that about 80% of the settlement variance is influenced by the uncertainties due to inherent soil variability and measurement test errors. It is important to emphasize that the uncertainties due to transformation model was not evaluated in the example.

4 CONCLUSIONS

It has been proposed and briefly discussed three simplified methodologies for probabilistic analysis of settlements of footings in sands, which adopts the soil stratification to compute the only considered random variable (deformability modulus).

Despite the presented limitations adopted on methodologies proposal, it can be assumed as a first approximation for evaluating the uncertainties (especially in deformability modulus) at the SLS analysis of a foundation. The association between probabilistic analysis and settlement predictions can become an interesting tool for geotechnical engineering in the knowing of soil variability and related uncertainties.

Therefore, any attempt to quantify the sources of uncertainties and its effects in geotechnical analysis, through probabilistic models, may become an important tool for helping engineers to make better and consistent design decisions.

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