# New Product Design under Channel Acceptance: Brick-and-Mortar, Online-Exclusive, or Brick-and-Click

Lan Luo and Jiong Sun

University of Southern California and Purdue University

2016

## Appendix A: Empirical Study on Consumers' Willingness-to-Pay for New Product Purchases from Offline vs. Online

We conducted an empirical study to examine consumers' willingness-to-pay for a newly introduced product from a traditional vs. a web store. Our study comprises a variety of product categories including apparel, electronics, furniture, home appliances, jewelry, and books. Within each product category, we present each consumer with two scenarios that vary by the intrinsic value (low vs. high) of the new product. In each scenario, the consumer is asked to indicate his/her willingness-to-pay if he/she had to purchase the product from a web store without physical inspection. Using apparel as an example, the two scenarios are presented as follows:

"Scenario 1: You are considering purchasing a piece of new apparel (e.g., sweater; coat; suit; pants). You can get a piece of new apparel with average quality from a traditional store at \$50. How much would you be willing to pay for this item if you had to purchase it from a web store without being able to physically touch and feel the product?

Scenario 2: You are considering purchasing a piece of new apparel (e.g., sweater; coat; suit; pants). You can get a piece of new apparel with excellent quality from a traditional store at $500. How much would you be willing to pay for this item if you had to purchase it from a web store without being able to physically touch and feel the product?"

Table A1. Consumers' Willingness-to-Pay for New Product Purchases from Offline vs. Online

| **Product Category** | **Apparel** | | **Electronics** | | **Furniture** | |
|---|---|---|---|---|---|---|
| *Offline Base Price* | $50 | $500 | $100 | $1,000 | $200 | $2,000 |
| *Online Willingness-to-Pay* | $37.13 | $333.33 | $84.34 | $873.24 | $144.55 | $1,473.45 |
| *% of Offline Equivalent* | 74.26% | 66.67% | 84.34% | 87.32% | 72.27% | 73.67% |
| *Ave. Category %* | 70.46% | | 85.33% | | 72.97% | |
| **Product Category** | **Home Appliance** | | **Jewelry** | | **Book** | |
| *Offline Base Price* | $400 | $2,000 | $200 | $2,000 | $10 | $100 |
| *Online Willingness-to-Pay* | $328.61 | $1,608.83 | $147.78 | $1,473.37 | $8.85 | $85.81 |
| *% of Offline Equivalent* | 82.15% | 80.44% | 73.89% | 73.67% | 88.52% | 85.81% |
| *Ave. Category %* | 81.30% | | 73.78% | | 87.17% | |

One-hundred and fifteen participants recruited from Amazon's Mechanical Turk Online Consumer Panel completed our study. The results are presented in Table A1. In this table, we report the offline base price, average online willingness-to-pay, and the ratio of the two (i.e., % of offline equivalent) by scenario and by product category. Our key findings can be summarized as follows. First, due to the lack of physical inspection, consumers' willingness-to-pay for purchasing a newly launched product online is less than its offline equivalent for all the product categories in our study across both scenarios. Second, by comparing scenario 1 with scenario 2 in each product category, we find that, consistent with our model setup, consumers' willingness-to-pay for a newly introduced product online is proportional to the intrinsic value of the product (rather than equivalent to the product's intrinsic value minus a fixed disutility term as proposed in some prior research). Lastly, in line with Chiang et al. (2003), we discover that consumers' willingness-to-pay for new products online varies by product category. In general, consumers' willingness-to-pay is closer to its offline equivalent when it comes to books, electronics, and home appliances. In contrast, their willingness-to-pay associated with buying apparel, furniture, and jewelry online is considerably less than the offline equivalent, due to the importance of touch-and-feel in these categories.

## Appendix B: Proofs

**Proof.** of Lemma 1: The retailer sets his retailer price at $p = q$ if carrying the product, i.e., when $\pi^r(q) = (q - w)\alpha \geq R$ or $w \leq q - \frac{R}{\alpha}$. The manufacturer sets her wholesale price at the maximum possible level at $w = q - \frac{R}{\alpha} \doteq w_F$ such that $\pi^r \geq R$. By maximizing her profit $\pi^m(q, q - \frac{R}{\alpha}) = (q - \frac{R}{\alpha} - cq^2)\alpha$, the manufacturer sets $q_0 = \frac{1}{2c}$, $w_0 = \frac{1}{2c} - \frac{R}{\alpha}$. And $Q_0 = \alpha$, $p_0 = \frac{1}{2c}$, $\pi_0^r = R$, and $\pi_0^m = \frac{\alpha}{4c} - R$ at the equilibrium. ■

**Proof.** of Propositions 1 and 2: 1) We first derive the retailer's best response (BR). If Online-Exclusive, the retailer gets $\pi_2^r(q, w) = (\delta_l q + s - w) + R$ if $w < \frac{\delta_l - \beta\delta_h}{1-\beta} q + s \doteq w'$; $\pi_2^r(q, w) = (\delta_h q + s - w)\beta + R$ o/w. If Offline-Exclusive, $\pi_1^r(q, w) = (q - w)\alpha + r$. Assuming $\alpha > \beta$, we have the following cases. (i) For $q > \hat{q}$, $\pi_2^r(q, w) = (\delta_l q + s - w) + R$ if $w < \frac{(\delta_l - \alpha)q + s + R - r}{(1-\alpha)} \doteq \underline{w}$; $(q - w)\alpha + r$ if $w \in [\underline{w}, \frac{(\alpha - \beta\delta_h)q - \beta s - R + r}{(\alpha - \beta)} \doteq \overline{w}]$; $(\delta_h q + s - w)\beta + R$ if $w \in [\overline{w}, \delta_h q + s - \frac{r}{\beta} \doteq w_O]$. Note that from $\frac{(\delta_l - \alpha)q + s + R - r}{(1-\alpha)} < \frac{(\alpha - \beta\delta_h)q - \beta s - (R - r)}{(\alpha - \beta)}$, we have $q > \frac{(\alpha - \beta)(s + R - r) + (1-\alpha)(\beta s + R - r)}{(1-\alpha)(\alpha - \beta\delta_h) + (\alpha - \beta)(\alpha - \delta_l)} \doteq \hat{q}$. (ii) For $q \leq \hat{q}$, $\pi^r(q, w) = (\delta_l q + s - w) + R$ if $w < \frac{\delta_l - \beta\delta_h}{1-\beta} q + s = w'$; $\pi^r(q, w) = (\delta_h q + s - w)\beta + R$ if $w \in [w', w_O]$. Note that from $\overline{w} = w_O$, we get $q = \frac{\beta R - \alpha r + \alpha\beta s}{\alpha\beta(1-\delta_h)} \doteq \hat{q}'$, and $w = \frac{\beta\delta_h R + \alpha\beta s - r\alpha}{\alpha\beta(1-\delta_h)} \doteq \hat{w}'$. We summarize his BR below

| | | | | |
|---|---|---|---|---|
| If $q \leq \hat{q}$, | $w \leq w'$ : | Online | $p = \delta_l q + s$ | All purchase |
| | $w \in (w', w_O]$ : | Online | $p = \delta_h q + s$ | More-online-skilled segments ($\beta$) purchase |
| If $q > \hat{q}$, | $w \leq \underline{w}$ : | Online | $p = \delta_l q + s$ | All |
| | $w \in (\underline{w}, \overline{w}]$ : | Offline | $p = q$ | Offline-accessible segments ($\alpha$) purchase |
| | $w \in (\overline{w}, \max[w_O, w_F]]$ : | Online | $p = \delta_h q + s$ | More-online-skilled segments ($\beta$) purchase |

Outside these regions, the retailer chooses not to carry.

2) We now derive the equilibrium. Because $\pi^m(q,w) = (w - cq^2) \cdot Q^*(q,w)$, we know that, in any case, the manufacturer will set his wholesale price at the upper bound. There are four possible cases. (1) Carry online and serve both segments for $w < \underline{w} = \frac{(\delta_l - \alpha)q + s + R - r}{(1-\alpha)}$ & $q > \hat{q}$, and $w \leq w' = \frac{\delta_l - \beta\delta_h}{1-\beta} q + s$ & $q \leq \hat{q}$. The manufacturer gets $\pi^m_{2b}(q,w) = (w - cq^2)$ subject to $(\delta_l q + s - w) + R \geq R + r$. Note that both $\underline{w}$ and $w'$ are decreasing in $q$, and $w = \delta_l q + s - r$ intersects with $w' = \frac{\delta_l - \beta\delta_h}{1-\beta} q + s$ at $q = \frac{(1-\beta)r}{\beta(\delta_l + \delta_h)}$; with $\underline{w} = \frac{(\delta_l - \alpha)q + s + R - r}{(1-\alpha)}$ at $q = \frac{\alpha(s-r)+R}{(1-\delta_l)\alpha}$. Hence, $(q^*, w^*) = (\min[\frac{\delta_l}{2c}, \frac{(1-\beta)r}{\beta(\delta_l+\delta_h)}, \frac{\alpha(s-r)+R}{(1-\delta_l)\alpha}], \delta_l q + s - r)$. (2) Offline for $w \in (\underline{w}, \overline{w} = \frac{(\alpha - \beta\delta_h)q - \beta s - R + r}{(\alpha-\beta)}]$ and $q \in [\hat{q}, \hat{q}' = \frac{\beta R - \alpha r + \alpha\beta s}{\alpha\beta(1-\delta_h)}]$. The manufacturer gets $\pi^m_1(q,w) = (w - cq^2)\alpha$. Hence, $(q^*, w^*) = (\frac{\alpha - \beta\delta_h}{2c(\alpha-\beta)}, \frac{(\alpha-\beta\delta_h)q^* - \beta s - R + r}{(\alpha-\beta)})$ if $\frac{\alpha-\beta\delta_h}{2c(\alpha-\beta)} < \hat{q}'$; $= (\hat{q}', \hat{w}')$ o/w. (3) Offline for $w \in \left(\underline{w}, w_F = q - \frac{R}{\alpha}\right]$ and $q > \hat{q}'$. The manufacturer gets $\pi^m_1(q,w) = (w - cq^2)\alpha$. Hence, $(q^*, w^*) = (\hat{q}', \hat{w}')$ if $\frac{1}{2c} < \hat{q}'$; $= \left(\frac{1}{2c}, q^* - \frac{R}{\alpha}\right)$ o/w. (4) Online and serve $\delta_h$ segment only for $w \in (\overline{w}', w_O]$ & $q \in [\hat{q}, \hat{q}']$, and $w \in (w', w_O]$ & $q \leq \hat{q}$. The manufacturer gets $\pi^m_{2a}(q,w) = (w - cq^2)\beta$. Hence, $(q^*, w^*) = (\min[\frac{\delta_h}{2c}, \hat{q}'], \delta_h q^* + s - \frac{r}{\beta})$.

From $\frac{1}{2c} < \hat{q}'$, we have $R > \underline{R} = \frac{\alpha(1-\delta_h)}{2c} + \frac{\alpha}{\beta} r - \alpha s$. Hence, when $R > \underline{R}$, the optimum is achieved at $q = \hat{q}'$; otherwise, at $q = \frac{1}{2c}$. From $\frac{\alpha-\beta\delta_h}{2c(\alpha-\beta)} < \hat{q}'$, we have $R > \overline{R} = \frac{\alpha(1-\delta_h)(\alpha-\beta\delta_h)}{2c(\alpha-\beta)} + \frac{\alpha}{\beta} r - \alpha s$. Hence, when $R < \overline{R}$, the optimum is achieved $q = \hat{q}'$; otherwise at $q = \frac{\alpha-\beta\delta_h}{2c(\alpha-\beta)}$. In addition, $\underline{R} < \overline{R}$

holds. The equilibrium quality, prices and profits are summarized below:

| | | |
|---|---|---|
| If $R \le \underline{R}$, | Offline | $q^* = \frac{1}{2c}, w^* = \frac{1}{2c} - \frac{R}{\alpha}, p^* = q^*, Q^* = \alpha, \pi_{1a}^{m*} = \frac{\alpha}{4c} - R, \pi_{1a}^{r*} = R + r$ |
| If $R \in \left(\underline{R}, \overline{R}\right]$, | Offline | $q^* = \frac{\beta R - \alpha r + \alpha\beta s}{\alpha\beta(1-\delta_h)} > \frac{1}{2c}, w^* = \frac{\beta\delta_h R - \alpha r + \alpha\beta s}{\alpha\beta(1-\delta_h)}, p^* = q^*, Q^* = \alpha,$ <br> $\pi_{1b}^{m*} = \frac{\alpha\beta(1-\delta_h)(\beta\delta_h R + \alpha\beta s - \alpha r) - c(\beta R + \alpha\beta s - \alpha r)^2}{\alpha\beta^2(1-\delta_h)^2}, \pi_{1b}^{r*} = R + r$ |
| If $R \in \left(\overline{R}, \hat{R}\right]$, | Offline | $q^* = \frac{\alpha - \beta\delta_h}{2c(\alpha-\beta)} > \frac{1}{2c}, w^* = \frac{(\alpha-\beta\delta_h)^2}{2c(\alpha-\beta)^2} - \frac{\beta s + R - r}{(\alpha-\beta)}, p^* = q^*, Q^* = \alpha,$ <br> $\pi_{1c}^{m*} = \frac{\alpha(\alpha-\beta\delta_h)^2}{4c(\alpha-\beta)^2} - \frac{\alpha(\beta s + R - r)}{(\alpha-\beta)}, \pi_{1c}^{r*} = \frac{\alpha(\beta s + R - r)}{(\alpha-\beta)} - \frac{\alpha\beta(1-\delta_h)(\alpha-\beta\delta_h)}{2c(\alpha-\beta)^2}$ |
| If $R > \hat{R}$, | Online | $\begin{cases} s \le \hat{s}: q^* = \frac{\delta_h}{2c}, w^* = \delta_h q^* + s - r/\beta, p^* = \delta_h q^* + s, Q^* = \beta, \pi_{2a}^{m*} = \frac{\beta\delta_h^2}{4c} + s\beta - r \\ s > \hat{s}: q^* = \frac{\delta_l}{2c}, w^* = \delta_l q^* + s - r, p^* = \delta_l q^* + s, Q^* = 1, \pi_{2b}^{m*} = \frac{\delta_l^2}{4c} + s - r \end{cases}$ <br> $\pi_2^{r*} = R + r$ |

where $\underline{R} = \frac{\alpha(1-\delta_h)}{2c} - \alpha s + \alpha r/\beta$, $\overline{R} = \frac{\alpha(1-\delta_h)(\alpha-\beta\delta_h)}{2c(\alpha-\beta)} - \alpha s + \alpha r/\beta$, $\hat{R}$ is such that $\pi_{1c}^{m*} = \max\left(\pi_{2a}^{m*}, \pi_{2b}^{m*}\right)$, and $\hat{s} = \frac{\beta\delta_h^2 - \delta_l^2}{4c(1-\beta)}$. ■

**Proof.** of Propositions 3 and 4: 1) We first derive the retailer's BR. The following tables list consumers' channel choice in different price regions when the product is carried as a brick-and-click.

| For $q \le \delta_l q + s$ | $U_1$ | $U_2$ | if $p \le q$ | if $p \in (q, \delta_l q + s]$ | if $p \in (\delta_l q + s, \delta_h q + s]$ | if $p > \delta_h q + s$ |
|---|---|---|---|---|---|---|
| $\alpha\beta$ | $q - p$ | $\delta_h q + s - p$ | online | online | online | no buy |
| $\alpha(1-\beta)$ | $q - p$ | $\delta_l q + s - p$ | online | online | no buy | no buy |
| $(1-\alpha)\beta$ | / | $\delta_h q + s - p$ | online | online | online | no buy |
| $(1-\alpha)(1-\beta)$ | / | $\delta_l q + s - p$ | online | online | online | no buy |

| For $q \in (\delta_l q + s, \delta_h q + s]$ | $U_1$ | $U_2$ | if $p \le \delta_l q + s$ | if $p \in (\delta_l q + s, q]$ | if $p \in (q, \delta_h q + s]$ | if $p > \delta_h q + s$ |
|---|---|---|---|---|---|---|
| $\alpha\beta$ | $q - p$ | $\delta_h q + s - p$ | online | online | online | no buy |
| $\alpha(1-\beta)$ | $q - p$ | $\delta_l q + s - p$ | offline | offline | no buy | no buy |
| $(1-\alpha)\beta$ | / | $\delta_h q + s - p$ | online | online | online | no buy |
| $(1-\alpha)(1-\beta)$ | / | $\delta_l q + s - p$ | online | no buy | no buy | no buy |

| For $q > \delta_h q + s$ | $U_1$ | $U_2$ | if $p \leq \delta_l q + s$ | if $p \in (\delta_l q + s, \delta_h q + s]$ | if $p \in (\delta_h q + s, q]$ | if $p > q$ |
|---|---|---|---|---|---|---|
| $\alpha\beta$ | $q - p$ | $\delta_h q + s - p$ | offline | offline | offline | no buy |
| $\alpha(1-\beta)$ | $q - p$ | $\delta_l q + s - p$ | offline | offline | offline | no buy |
| $(1-\alpha)\beta$ | / | $\delta_h q + s - p$ | online | online | no buy | no buy |
| $(1-\alpha)(1-\beta)$ | / | $\delta_l q + s - p$ | online | no buy | no buy | no buy |

Hence, $Q_3(p)$ is given by

| | |
|---|---|
| When $q \leq \delta_l q + s$ | $Q_3(p) = \begin{cases} 0+1 & \text{if } p \leq q \\ 0+1 & \text{if } p \in (q, \delta_l q + s] \\ \beta & \text{if } p \in (\delta_l q + s, \delta_h q + s] \\ 0 & \text{otherwise} \end{cases}$ |
| When $q \in (\delta_l q + s, \delta_h q + s]$ | $Q_3(p) = \begin{cases} 1 & \text{if } p \leq \delta_l q + s \\ \alpha + (1-\alpha)\beta & \text{if } p \in (\delta_l q + s, q] \\ \beta & \text{if } p \in (q, \delta_h q + s] \\ 0 & \text{otherwise} \end{cases}$ |
| When $q > \delta_h q + s$ | $Q_3(p) = \begin{cases} 1 & \text{if } p \leq \delta_l q + s \\ \alpha + (1-\alpha)\beta & \text{if } p \in (\delta_l q + s, \delta_h q + s] \\ \alpha & \text{if } p \in (\delta_h q + s, q] \\ 0 & \text{otherwise} \end{cases}$ |

As discussed in the paper, only two cases are not dominated by the other strategies: (i) $\pi_3^r = (q-w)\left[\alpha(1-\beta)+\beta\right] \doteq \pi_a^r$ for $q < \frac{s}{1-\delta_h}$; and (ii) $\pi_3^r = (\delta_h q + s - w)\left[\alpha + (1-\alpha)\beta\right] \doteq \pi_b^r$ o/w, where $p = q$ if $q < \frac{s}{1-\delta_h}$; $\delta_h q + s$ o/w. Note that the retailer can price $\delta_l q + s$ when carrying in both channels, but all consumers will purchase from the online channel, which is dominated by carrying online only (because of the offline-participation criterion $R$). We know that carrying the product in both channels gives $\pi_3^r(q,w) = \min\left[\pi_b^r, \pi_a^r\right]$, carrying offline only gives $\pi_1^r(q,w) = (q - w)\alpha + r$, and carrying it online only gives $\pi_2^r(q,w) = \max\left[(\delta_l q + s - w) + R, (\delta_h q + s - w)\beta + R\right] \doteq$

$\max\left[\pi^r_{2b}, \pi^r_{2a}\right]$.

The above results immediately follow from the fact that $\frac{\partial \pi^r_{2b}}{\partial w} = -1 < \frac{\partial \pi^r}{\partial w} = -\left[\alpha + (1-\alpha)\beta\right] < \frac{\partial \pi^r_1}{\partial w} = -\alpha < \frac{\partial \pi^r_{2a}}{\partial w} = -\beta$. The boundaries between $\pi^r_{2b}$ and $\pi^r_b$ and between $\pi^r_{2b}$ and $\pi^r_a$ are given by $w'_{OB} = \frac{q(\delta_l - \delta_h[\alpha+(1-\alpha)\beta]) + R}{(1-\alpha)(1-\beta)} + s$ and $w_{OB} = \frac{q[\delta_l - \alpha - (1-\alpha)\beta] + s + R}{(1-\alpha)(1-\beta)}$. The boundaries between $\pi^r_b$ and $\pi^r_1$ and between $\pi^r_a$ and $\pi^r_1$ are given by $w'_{BF} = \frac{q(\delta_h[\alpha+(1-\alpha)\beta]-\alpha) + s[\alpha+(1-\alpha)\beta] - r}{(1-\alpha)\beta}$ and $w_{BF} = q - \frac{r}{(1-\alpha)\beta}$. The boundaries between $\pi^r_1$ and $\pi^r_{2a}$ and between $\pi^r_{2b}$ and $\pi^r_1$ are given by $w_{FO} = \frac{(\alpha - \delta_h \beta)q - s\beta + r - R}{\alpha - \beta}$ and $w_{OF} = \frac{(\delta_l - \alpha)q + s + R}{(1-\alpha)}$. The boundaries between $\pi^r_1$ and $R + r$ and between $\pi^r_{2a}$ and $R + r$ are given by $w_F = q - \frac{R}{\alpha}$ and $w_O = \delta_h q + s - \frac{r}{\beta}$. The boundary between $\pi^r_{2b}$ and $\pi^r_{2a}$ is given by $w' = \frac{\delta_l - \beta\delta_h}{1-\beta} q + s$. Denote the intersection of $w_F$ and $w_O$ by $\hat{q}' = \frac{\alpha\beta s + \beta R - \alpha r}{\alpha\beta(1-\delta_h)}$. Denote the intersection of $w_{OB}$ and $w_{BO}$ by $\hat{q} = \frac{[\alpha+(1-\alpha)\beta]s + R}{(1-\alpha)\beta + \alpha - (1-\alpha)\beta\delta_h - \alpha\delta_l}$. If $(1-\alpha)\beta R > (1-\beta)\alpha r$, $w'_{BF}$ and $w_{FO}$ intersect at $\hat{q}'' = \frac{(1-\alpha)\beta R - (1-\beta)\alpha r}{(1-\delta_h)(1-\beta)\alpha^2} + \frac{s}{(1-\delta_h)} > \frac{s}{1-\delta_h}$. The boundaries between $\pi^r_b$ and $\pi^r_{2a}$ and between $\pi^r_a$ and $\pi^r_{2a}$ are given by $w'_{BO} = \delta_h q + s - \frac{R}{\alpha(1-\beta)}$ and $w_{BO} = \frac{q[\alpha + (1-\alpha)\beta - \delta_h\beta] - s\beta - R}{\alpha(1-\beta)}$. If $(1-\alpha)\beta R < (1-\beta)\alpha r$, $w_{BF}$ and $w_{FO}$ intersect at $\hat{q}'' < \frac{s}{1-\delta_h}$.

| | | | | |
|---|---|---|---|---|
| If $q \le \hat{q}$, | $w \le w'$ | Online | $p = \delta_l q + s$ | All |
| | $w \in (w', w_O]$ | Online | $p = \delta_h q + s$ | $\beta$ |

| | | | | |
|---|---|---|---|---|
| If $q \in (\hat{q}, \hat{q}'']$, | $w \le w_{OB}$ | Online | $p = \delta_l q + s$ | All |
| | $w \in \left(\max\left(w_{OB}, w'_{OB}\right), \min\left(w_{BO}, w'_{BO}\right)\right]$ | Both | $q < \frac{s}{1-\delta_h} : p = q$ | $\alpha(1-\beta)$ offline, $\alpha\beta$ and $(1-\alpha)\beta$ online |
| | | | $q \ge \frac{s}{1-\delta_h} : p = \delta_h q + s$ | $\alpha\beta$ and $\alpha(1-\beta)$ offline, $(1-\alpha)\beta$ online |
| | $w \in (w', w_O]$ | Online | $p = \delta_h q + s$ | $\beta$ |

| | | | | |
|---|---|---|---|---|
| If $q \in (\hat{q}'', \hat{q}']$, | $w \le \max\left(w_{OB}, w'_{OB}\right)$ | Online | $p = \delta_l q + s$ | All |
| | $w \in \left(\max\left(w_{OB}, w'_{OB}\right), \min\left(w_{BF}, w'_{BF}\right)\right]$ | Both | same above | same above |
| | $w \in \left(\min\left(w_{BF}, w'_{BF}\right), w_{FO}\right]$ | Offline | $p = q$ | $\alpha$ |
| | $w \in (w_{FO}, w_O]$ | Online | $p = \delta_h q + s$ | $\beta$ |

| | | | | |
|---|---|---|---|---|
| If $q > \hat{q}'$ | $w \leq w'_{OB}$ | Online | $p = \delta_l q + s$ | All |
| | $w \in (w'_{OB}, w'_{BO}]$ | Both | $p = \delta_h q + s$ | $\alpha\beta$ and $\alpha(1-\beta)$ offline, $(1-\alpha)\beta$ online |
| | $w \in (w'_{BO}, w_F]$ | Offline | $p = q$ | $\alpha$ |

where $\hat{q}'' < \frac{s}{1-\delta_h}$ if $(1-\alpha)\beta R < (1-\beta)\alpha r$; $\geq \frac{s}{1-\delta_h}$ o/w. Outside these regions, the retailer chooses not to carry the product.

2) We now derive the equilibrium. (1) In the region of $\pi_1^r(q,w)$ (offline), (i) at the upper boundary, $w_F = q - \frac{R}{\alpha}$, by maximizing $\pi^m(q,w) = (w - cq^2)\alpha$, the manufacturer sets $q^* = \frac{1}{2c}$, and gets $\pi_{1a}^{m*} = \frac{\alpha}{4c} - R$, (ii) at $(\hat{q}', \hat{w}')$, by maximizing $\pi^m(q,w)$, the manufacturer sets $q^* = \frac{\beta R - \alpha r + \alpha\beta s}{\alpha\beta(1-\delta_h)} > \frac{1}{2c}$, $w^* = \frac{\beta\delta_h R - \alpha r + \alpha\beta s}{\alpha\beta(1-\delta_h)}$, and gets $\pi_{1b}^{m*} = \frac{\alpha\beta(1-\delta_h)(\beta\delta_h R + \alpha\beta s - \alpha r) - c(\beta R + \alpha\beta s - \alpha r)^2}{\alpha\beta^2(1-\delta_h)^2}$, and (iii) at the upper boundary, $w_{FO} = \frac{(\alpha - \delta_h\beta)q - s\beta + r - R}{\alpha - \beta}$, by maximizing $\pi^m(q,w)$, the manufacturer sets $q^* = \frac{(\alpha - \delta_h\beta)}{2c(\alpha-\beta)} > \frac{1}{2c}$, and gets $\pi_{1c}^{m*} = \frac{\alpha(\alpha - \beta\delta_h)^2}{4c(\alpha-\beta)^2} - \frac{\alpha(\beta s + R - r)}{(\alpha-\beta)}$.

(2a) In the region of $\pi_3^r(q,w) = \pi_a^r$ (both), which happens when $\hat{q}'' \leq \frac{s}{1-\delta_h}$, (i) at the upper boundary, $w_{BF} = q - \frac{r}{(1-\alpha)\beta}$, by maximizing $\pi^m(q,w) = (w - cq^2)[\alpha + (1-\alpha)\beta]$, the manufacturer sets $q^* = \frac{1}{2c}$, and gets $\pi_a^{m*} = \left[\frac{1}{4c} - \frac{r}{(1-\alpha)\beta}\right][\alpha + (1-\alpha)\beta]$, (ii) at $(\hat{q}'', \hat{w}'')$, the manufacturer sets $q^* = \frac{(1-\alpha)\beta R - (1-\beta)\alpha r}{(1-\delta_h)(1-\alpha)\beta^2} + \frac{s}{(1-\delta_h)} > \frac{1}{2c}$ (because $(1-\alpha)\beta R < (1-\beta)\alpha r$), and gets $\pi_b^{m*} = \left[\frac{(1-\alpha)\beta R - (1-\beta)\alpha r}{(1-\delta_h)(1-\alpha)\beta^2} + \frac{s}{(1-\delta_h)} - \frac{r}{(1-\alpha)\beta} - c\left[\frac{(1-\alpha)\beta R - (1-\beta)\alpha r}{(1-\delta_h)(1-\alpha)\beta^2} + \frac{s}{(1-\delta_h)}\right]^2\right][\alpha + (1-\alpha)\beta]$, and (iii) at the upper boundary, $w_{BO} = \frac{q[\alpha + (1-\alpha)\beta - \delta_h\beta] - s\beta - R}{\alpha(1-\beta)}$, the manufacturer sets $q^* = \frac{1}{2c}\left[1 + \frac{(1-\delta_h)\beta}{\alpha(1-\beta)}\right] > \frac{1}{2c}$, and gets $\pi_c^{m*} = \left[\frac{1}{4c}\left[1 + \frac{(1-\delta_h)\beta}{\alpha(1-\beta)}\right]^2 - \frac{s\beta + R}{\alpha(1-\beta)}\right][\alpha + (1-\alpha)\beta]$.

(2b) In the region of $\pi_3^r(q,w) = \pi_b^r$ (both), which happens when $\hat{q}'' > \frac{s}{1-\delta_h}$, (i) at the upper boundary, $w'_{BF} = \frac{q(\delta_h[\alpha + (1-\alpha)\beta] - \alpha) + s[\alpha + (1-\alpha)\beta] - r}{(1-\alpha)\beta}$, by maximizing $\pi^m(q,w) = (w - cq^2)[\alpha + (1-\alpha)\beta]$, the manufacturer sets $q^* = \frac{\delta_h[\alpha + (1-\alpha)\beta] - \alpha}{2c(1-\alpha)\beta} < \frac{1}{2c}$, and gets $\pi_a^{m*} = [\frac{1}{4c}\left[\frac{\delta_h[\alpha + (1-\alpha)\beta] - \alpha}{(1-\alpha)\beta}\right]^2$ $+ \frac{s[\alpha + (1-\alpha)\beta] - r}{(1-\alpha)\beta}][\alpha + (1-\alpha)\beta]$, (ii) at $(\hat{q}'', \hat{w}'')$, the manufacturer sets $q^* = \frac{(1-\alpha)\beta R - (1-\beta)\alpha r}{(1-\delta_h)(1-\beta)\alpha^2} + \frac{s}{(1-\delta_h)} < \frac{1}{2c}$ (because $(1-\alpha)\beta R > (1-\beta)\alpha r$), and gets $\pi_b^{m*} = [\frac{\delta_h[\alpha + (1-\alpha)\beta] - \alpha}{(1-\delta_h)(1-\beta)\alpha^2} R - c(\frac{(1-\alpha)\beta R - (1-\beta)\alpha r}{(1-\delta_h)(1-\beta)\alpha^2} + \frac{s}{(1-\delta_h)})^2 - \frac{\delta_h r}{(1-\delta_h)\alpha} + \frac{s}{1-\delta_h}][\alpha + (1-\alpha)\beta]$, and (iii) at the upper boundary, $w'_{BO} = \delta_h q + s - \frac{R}{\alpha(1-\beta)}$, the manufacturer sets $q^* = \frac{\delta_h}{2c} < \frac{1}{2c}$, and gets $\pi_c^{m*} = \left[\frac{\delta_h^2}{4c} + s - \frac{R}{\alpha(1-\beta)}\right][\alpha + (1-\alpha)\beta]$.

(3a) In the region of $\pi_2^r(q,w) = \pi_{2b}^r$ (online), the upper boundaries $w'_{OB}$ between $\pi_{2b}^r$ and $\pi_b^r$, $w_{OB}$ between $\pi_{2b}^r$ and $\pi_a^r$, $w_{OF}$ between $\pi_{2b}^r$ and $\pi_1^r$, and $w'$ between $\pi_{2b}^r$ and $\pi_{2a}^r$ are all decreasing in $q$. Hence, the manufacturer sets $q^* = \frac{\delta_l}{2c}, w^* = \delta_l q^* + s - r, \pi_{2b}^{m*} = \frac{\delta_l^2}{4c} + s - r$.

(3b) In the region of $\pi_2^r(q,w) = \pi_{2b}^r$ (online), at the upper boundary, $w_O = \delta_h q + s$, by maximizing $\pi^m(q,w) = (w - cq^2)\beta$, the manufacturer sets $q^* = \frac{\delta_h}{2c}, w^* = \delta_h q^* + s - r/\beta, \pi_{2a}^{m*} = \frac{\beta\delta_h^2}{4c} + s\beta - r$.

The decreasing rates of the manufacturer's profits in $R$ determine the sequence of emergence of each of the above equilibria: (i) When $\frac{s}{1-\delta_h}$ is high enough, we have offline (case 1) appear first, then both (case 2a), and finally online (case 3a or 3b), as $R$ increases; (ii) Otherwise, we have offline (1), then both (2b), and finally online (3a or 3b). The equilibrium quality, prices and profits are summarized below:

(i) When $\frac{s}{1-\delta_h}$ is high,

| | | |
|---|---|---|
| If $R \leq \underline{R}_F$, | Offline | $q^* = \frac{1}{2c}, w^* = q^* - \frac{R}{\alpha}, p^* = q^*, Q^* = \alpha, \pi_{1a}^{m*} = \frac{\alpha}{4c} - R, \pi_{1a}^{r*} = R + r$ |
| If $R \in \left(\underline{R}_F, \overline{R}_F\right]$, | Offline | $q^* = \frac{\beta R - \alpha r + \alpha\beta s}{\alpha\beta(1-\delta_h)} > \frac{1}{2c}, w^* = \frac{\beta\delta_h R - \alpha r + \alpha\beta s}{\alpha\beta(1-\delta_h)}, p^* = q^*, Q^* = \alpha,$ <br> $\pi_{1b}^{m*} = \frac{\alpha\beta(1-\delta_h)(\beta\delta_h R + \alpha\beta s - \alpha r) - c(\beta R + \alpha\beta s - \alpha r)^2}{\alpha\beta^2(1-\delta_h)^2}, \pi_{1b}^{r*} = R + r$ |
| If $R \in \left(\overline{R}_F, \hat{R}_1\right]$, | Offline | $q^* = \frac{\alpha - \beta\delta_h}{2c(\alpha-\beta)} > \frac{1}{2c}, w^* = \frac{(\alpha-\beta\delta_h)^2}{2c(\alpha-\beta)^2} - \frac{\beta s + R - r}{(\alpha-\beta)}, p^* = q^*, Q^* = \alpha,$ <br> $\pi_{1c}^{m*} = \frac{\alpha(\alpha-\beta\delta_h)^2}{4c(\alpha-\beta)^2} - \frac{\alpha(\beta s + R - r)}{(\alpha-\beta)}, \pi_{1c}^{r*} = \frac{\alpha(\beta s + R - r)}{(\alpha-\beta)} - \frac{\alpha\beta(1-\delta_h)(\alpha-\beta\delta_h)}{2c(\alpha-\beta)^2} \geq R + r$ |

| | | |
|---|---|---|
| If $R \in \left(\hat{R}_1, \underline{R}_B\right]$, | Both | $q^* = \frac{1}{2c}, w^* = q^* - \frac{r}{(1-\alpha)\beta}, p^* = q^*, Q^* = \alpha + (1-\alpha)\beta,$ <br> $\pi_a^{m*} = \left[\frac{1}{4c} - \frac{r}{(1-\alpha)\beta}\right][\alpha + (1-\alpha)\beta], \pi_a^{r*} = \frac{r}{(1-\alpha)\beta}[\alpha + (1-\alpha)\beta] \geq R + r$ |

| | | |
|---|---|---|
| If $R \in \left(\underline{R}_B, \overline{R}_B\right]$, | Both | $q^* = \frac{(1-\alpha)\beta R - (1-\beta)\alpha r}{(1-\delta_h)(1-\alpha)\beta^2} + \frac{s}{(1-\delta_h)} > \frac{1}{2c}, w^* = q^* - \frac{r}{(1-\alpha)\beta}, p^* = q^*, Q^* = \alpha + (1-\alpha)\beta,$ <br> $\pi_b^{m*} = \left[q^* - \frac{r}{(1-\alpha)\beta} - cq^{*2}\right][\alpha + (1-\alpha)\beta], \pi_b^{r*} = \frac{r}{(1-\alpha)\beta}[\alpha + (1-\alpha)\beta] \geq R + r$ |

| | | |
|---|---|---|
| If $R \in \left(\overline{R}_B, \hat{R}_2\right]$, | Both | $q^* = \frac{1}{2c}\left[1 + \frac{(1-\delta_h)\beta}{\alpha(1-\beta)}\right] > \frac{1}{2c}, w^* = \frac{1}{2c}\left[1 + \frac{(1-\delta_h)\beta}{\alpha(1-\beta)}\right]^2 - \frac{s\beta + R}{\alpha(1-\beta)},$ <br> $p^* = q^*, Q^* = \alpha + (1-\alpha)\beta,$ <br> $\pi_c^{m*} = \left[\frac{1}{4c}\left[1 + \frac{(1-\delta_h)\beta}{\alpha(1-\beta)}\right]^2 - \frac{s\beta + R}{\alpha(1-\beta)}\right][\alpha + (1-\alpha)\beta],$ <br> $\pi_c^{r*} = \left[-\frac{1}{2c}\left[1 + \frac{(1-\delta_h)\beta}{\alpha(1-\beta)}\right]\frac{(1-\delta_h)\beta}{\alpha(1-\beta)} + \frac{s\beta + R}{\alpha(1-\beta)}\right][\alpha + (1-\alpha)\beta] \geq R + r$ |

$$
\text{If } R > \hat{R}_2, \quad \text{Online} \quad \begin{cases} s \le \hat{s}: & q^* = \frac{\delta_h}{2c}, w^* = \delta_h q^* + s - r/\beta, \\ & p^* = \delta_h q^* + s, Q^* = \beta, \pi_{2a}^{m*} = \frac{\beta \delta_h^2}{4c} + s\beta - r \\ s > \hat{s}: & q^* = \frac{\delta_l}{2c}, w^* = \delta_l q^* + s - r, \\ & p^* = \delta_l q^* + s, Q^* = 1, \pi_{2b}^{m*} = \frac{\delta_l^2}{4c} + s - r \end{cases} , \pi_2^{r*} = R + r
$$

where, $\widehat{R}_1$ is such that $\pi_{1c}^{m*} = \pi_a^{m*}$, and $\widehat{R}_2$ is such that $\pi_c^{m*} = \pi_{2a}^{m*}$ or $\pi_{2b}^{m*}$.

(ii) When $\frac{s}{1-\delta_h}$ is low, for $R \le \hat{R}_1$, and $R > \hat{R}_2$, the equilibrium is the same as in (i).

$$
\text{If } R \in \left(\hat{R}_1, \underline{R}_B\right], \quad \text{Both} \quad
\begin{gathered}
q^* = \frac{\delta_h[\alpha+(1-\alpha)\beta]-\alpha}{2c(1-\alpha)\beta} < \frac{\delta_h}{2c}, w^* = \frac{1}{2c}\frac{(\delta_h[\alpha+(1-\alpha)\beta]-\alpha)^2}{(1-\alpha)^2\beta^2} + \frac{s[\alpha+(1-\alpha)\beta]-r}{(1-\alpha)\beta}, \\
p^* = \delta_h q^* + s, Q^* = \alpha + (1-\alpha)\beta, \\
\pi_a^{m*} = \left[\frac{1}{4c}\left[\frac{\delta_h[\alpha+(1-\alpha)\beta]-\alpha}{(1-\alpha)\beta}\right]^2 + \frac{s[\alpha+(1-\alpha)\beta]-r}{(1-\alpha)\beta}\right][\alpha + (1-\alpha)\beta], \\
\pi_a^{r*} = \left[\frac{(1-\delta_h)\alpha(\delta_h[\alpha+(1-\alpha)\beta]-\alpha)}{2c(1-\alpha)^2\beta^2} + s - \frac{s[\alpha+(1-\alpha)\beta]-r}{(1-\alpha)\beta}\right][\alpha + (1-\alpha)\beta] \ge R + r
\end{gathered}
$$

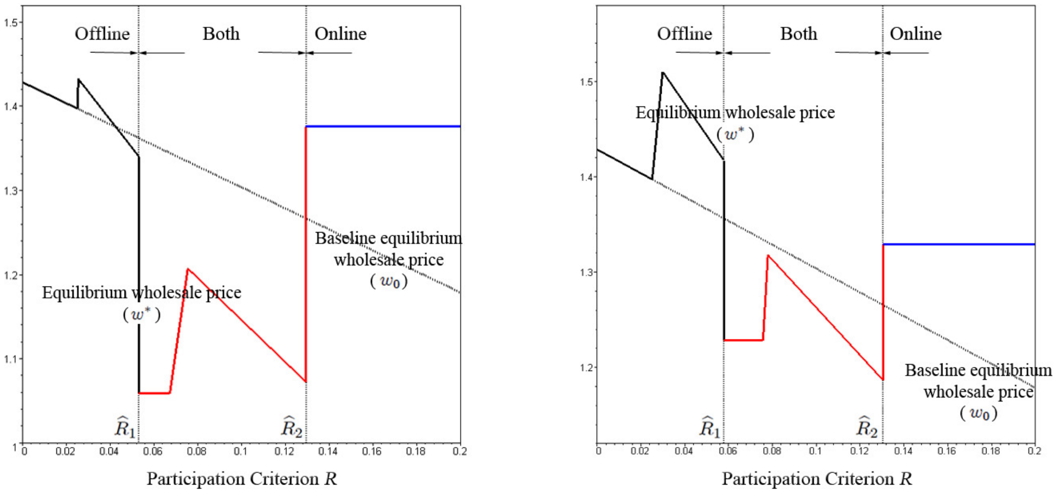
$$
\text{If } R \in \left(\underline{R}_B, \overline{R}_B\right], \quad \text{Both} \quad
\begin{gathered}
q^* = \frac{(1-\alpha)\beta R-(1-\beta)\alpha r}{(1-\delta_h)(1-\beta)\alpha^2} + \frac{s}{(1-\delta_h)} \le \frac{\delta_h}{2c}, w^* = \frac{(\delta_h[\alpha+(1-\alpha)\beta]-\alpha)}{(1-\delta_h)(1-\beta)\alpha^2}R + \frac{\alpha s-\delta_h r}{(1-\delta_h)\alpha}, \\
p^* = \delta_h q^* + s, Q^* = \alpha + (1-\alpha)\beta, \\
\pi_b^{m*} = \left[\begin{array}{c} \frac{\delta_h[\alpha+(1-\alpha)\beta]-\alpha}{(1-\delta_h)(1-\beta)\alpha^2}R - c\left(\frac{(1-\alpha)\beta R-(1-\beta)\alpha r}{(1-\delta_h)(1-\beta)\alpha^2} + \frac{s}{(1-\delta_h)}\right)^2 \\ -\frac{\delta_h r}{(1-\delta_h)\alpha} + \frac{s}{1-\delta_h} \end{array}\right] \cdot [\alpha + (1-\alpha)\beta], \\
\pi_b^{r*} = \frac{R}{\alpha(1-\beta)}[\alpha + (1-\alpha)\beta] \ge R + r
\end{gathered}
$$

$$
\text{If } R \in \left(\overline{R}_B, \hat{R}_2\right], \quad \text{Both} \quad
\begin{gathered}
q^* = \frac{\delta_h}{2c} < \frac{1}{2c}, w^* = \frac{\delta_h^2}{2c} + s - \frac{R}{\alpha(1-\beta)}, p^* = \delta_h q^* + s, Q^* = \alpha + (1-\alpha)\beta, \\
\pi_c^{m*} = \left[\frac{\delta_h^2}{4c} + s - \frac{R}{\alpha(1-\beta)}\right][\alpha + (1-\alpha)\beta], \pi_c^{r*} = \frac{R}{\alpha(1-\beta)}[\alpha + (1-\alpha)\beta] \ge R + r
\end{gathered}
$$

Note that in the region, $R \in (\hat{R}_1, \hat{R}_2]$, the manufacturer's profit $\pi^m$ is higher than that in Section 3.2. ■

Figure A1 depicts the equilibrium wholesale prices in the model of Section 3.3.

**Proof.** of results in Section 4.1 (Two Wholesale Prices) and Lemma 2: When carrying the product in both channels, only two cases are not dominated by the other strategies: (i) $\pi_3^r = (q - w_1)\,\alpha(1-\beta) + (q - w_2)\,\beta \doteq \pi_a^r$ for $q < \frac{s}{1-\delta_h}$; and (ii) $\pi_3^r = (\delta_h q + s - w_1)\,\alpha + (\delta_h q + s - w_2)\,(1-\alpha)\beta \doteq \pi_b^r$ o/w, where $p = q$ if $q < \frac{s}{1-\delta_h}$; $\delta_h q + s$ o/w. Carrying the product in both channels gives $\pi_3^r = \min[\pi_b^r, \pi_a^r]$, carrying it offline only gives $\pi_1^r = (q - w_1)\alpha + r$, and carrying it online




(a) When $s$ and/or $\delta_h$ is Small.

(b) When $s$ and/or $\delta_h$ is High.

Figure 1: Equilibrium Wholesale Price as a Function of $R$ in the Model of Section 3.3. (The parameter values are the same as in Figure 3.)

only gives $\pi_2^r = \max\left[(\delta_l q + s - w_2) + R, (\delta_h q + s - w_2)\beta + R\right] \doteq \max\left[\pi_{2b}^r, \pi_{2a}^r\right]$. The retailer's best response remains the same, except that the boundary conditions are obtained using the above profit functions. Below, we derive the equilibrium in the region of carrying the product in both channels, with two subcases. In the case of carrying the product offline- or online-exclusive, the equilibrium remains the same because of the single wholesale price.

(a) In the region of $\pi_3^r(q, w) = \pi_a^r$, which happens when $\hat{q}'' \leq \frac{s}{1-\delta_h}$,

Subregion (i) at the upper boundary, $w_{BF2} = \alpha w_{BF1} + (1-\alpha)q - r/\beta$, by maximizing $\pi^m(q, w) = (w_1 - cq^2)\alpha(1-\beta) + (w_2 - cq^2)\beta$, the manufacturer sets $w_1^* = q^*$, $w_2^* = q^* - r/\beta$, $q^* = \frac{1}{2c}$, which is the same as above, and gets $\pi_a^{m*} = \frac{1}{4c}[\alpha(1-\beta) + \beta] - r$, which is higher than above. Subregion (iii) at the upper boundary, $w_{BO1} = \frac{q[\alpha(1-\beta)+\beta(1-\delta_h)]-s\beta-R}{\alpha(1-\beta)}$ (same as above), the manufacturer sets $w_1^* = \frac{q^*[\alpha(1-\beta)+\beta(1-\delta_h)]-s\beta-R}{\alpha(1-\beta)} = q^* - \frac{[s-(1-\delta_h)q^*]\beta+R}{\alpha(1-\beta)}$, $w_2^* = q^*$, $q^* = \frac{1}{2c}\left[1 + \frac{(1-\delta_h)\beta}{\alpha(1-\beta)+\beta}\right] > \frac{1}{2c}$ which is lower than above ($q^* = \frac{1}{2c}\left[1 + \frac{(1-\delta_h)\beta}{\alpha(1-\beta)}\right] > \frac{1}{2c}$), and gets $\pi^m = (q^* - cq^{*2})[\alpha(1-\beta)+\beta] - [s\beta + R - \beta(1-\delta_h)q^*]$ which is higher than above. According to continuity theory, in the intermediate region (ii), the optimal $q^*$ and $w^*$ are linear functions between the above values. it

(b) In the region of $\pi_3^r(q,w) = \pi_b^r$, which happens when $\hat{q}'' > \frac{s}{1-\delta_h}$,

Subregion (i) at the upper boundary, $w'_{BF2} = (\delta_h q + s) - \frac{[q-(\delta_h q+s)]\alpha+r}{(1-\alpha)\beta}$ (same as above), by maximizing $\pi^m(q,w) = (w_1 - cq^2)\alpha + (w_2 - cq^2)(1-\alpha)\beta$, the manufacturer sets $w_1^* = \delta_h q^* + s$, $w_2^* = (\delta_h q^* + s) - \frac{[q^*-(\delta_h q^*+s)]\alpha+r}{(1-\alpha)\beta}$, $q^* = \frac{\delta_h}{2c} - \frac{\alpha}{2c[\alpha+(1-\alpha)\beta]} < \frac{1}{2c}$ which is higher than above ($q^* = \frac{\delta_h}{2c} - \frac{\alpha}{2c(1-\alpha)\beta} < \frac{1}{2c}$), and gets $\pi^m = (\delta_h q^* + s - cq^{*2})[\alpha + (1-\alpha)\beta] - ([q^* - (\delta_h q^* + s)]\alpha + r)$ which is higher than above. Subregion (iii) at the upper boundary, $w'_{BO1} = (\delta_h q + s)(1-\beta) + w_2\beta - \frac{R}{\alpha}$, the manufacturer sets $w_1^* = (\delta_h q^* + s) - \frac{R}{\alpha}$, $w_2^* = \delta_h q^* + s$, $q^* = \frac{\delta_h}{2c} < \frac{1}{2c}$ which is the same as above, and gets $\pi^m = \left[\frac{\delta_h^2}{4c} + s - \frac{R}{\alpha+(1-\alpha)\beta}\right][\alpha + (1-\alpha)\beta]$ which is higher than above. According to continuity theory, in the intermediate region (ii), the optimal $q^*$ and $w^*$ are linear functions between the above values.

Hence,

(i) when $\frac{s}{1-\delta_h}$ is high (switch segment $\alpha\beta$ buy online)

(a) The manufacturer offers a product of high quality $q^* = \frac{1}{2c} + \frac{(1-\delta_h)\beta}{2c[\alpha(1-\beta)+\beta]}$, which happens in the upper region of $R$.

(b) The retailer still gets $\pi_3^r = \left(s - (1-\delta_h)\frac{1}{2c}\left[1 + \frac{(1-\delta_h)\beta}{\alpha(1-\beta)+\beta}\right]\right)\beta + R$.

(c) The manufacturer's wholesale prices $w_1^* = q^* >$ ($w_2^* = q^* - r/\beta$ in main model), and $w_1^* = q^* - \frac{[s-(1-\delta_h)q^*]\beta+R}{\alpha(1-\beta)}$ ($< w_2^* = q^*$ in the main model).

(ii) when $\frac{s}{1-\delta_h}$ is low (switch segment $\alpha\beta$ buy offline)

(a) The manufacturer offers a product of low quality $q^* = \frac{\delta_h}{2c} - \frac{\alpha}{2c[\alpha+(1-\alpha)\beta]}$, which happens in the lower region of $R$;

(b) The retailer still gets $\pi_3^r = \left[(1-\delta_h)\left(\frac{\delta_h}{2c} - \frac{\alpha}{2c[\alpha+(1-\alpha)\beta]}\right) - s\right]\alpha + r$.

(c) The manufacturer's wholesale prices $w_1^* = \delta_h q^* + s >$ ($> w_2^* = (\delta_h q^* + s) - \frac{[q^*-(\delta_h q^*+s)]\alpha+r}{(1-\alpha)\beta}$ in main model), and $w_1^* = (\delta_h q^* + s) - \frac{R}{\alpha}$ ($< w_2^* = \delta_h q^* + s$ in main model). ■

**Proof.** of results in Section 4.2 (Two Retail Prices): The firm has two possible pricing strategies:

1) $p_1 = q$, and $p_2 = \delta_h q + s$; and 2) $p_1 = q$, and $p_2 = \delta_l q + s$, and each consumer type purchases from either channel, as follows:

Table A3. Consumers' Channel Outlet Choice with Differential Pricing

| Consumer Type | When $p_2 = \delta_h q + s$ | When $p_2 = \delta_l q + s$ |
|---|---|---|
| More-online-skilled, offline-accessible | buy in either channel | buy online |
| Less-online-skilled, offline-accessible | buy offline | buy in either channel |
| More-online-skilled, offline-inaccessible | buy online | buy online |
| Less-online-skilled, offline-inaccessible | no purchase | buy online |

We analyze below the conditions under which the retailer will choose strategy (1) over (2). Under strategy (1), $\pi^r_{(1)} = [\max(q, \delta_h q + s) - w]\,\alpha\beta + (q - w)\,\alpha(1-\beta) + (\delta_h q + s - w)\,(1-\alpha)\beta$ and $\pi^m_{(1)} = (w - cq^2)[1-(1-\alpha)\,(1-\beta)]$. The manufacturer will price the product such that the retailer will gain more than her profits under online-exclusive, i.e., $w \leq \frac{\max(q,\delta_h q+s)\alpha\beta+(\delta_h q+s)(1-\alpha)\beta+q\alpha(1-\beta)-(\delta_h q+s)\beta}{\alpha+(1-\alpha)\beta+\beta} \doteq w_{(1)}$. Similarly, under strategy (2), where $\pi^r_{(2)} = (\delta_l q + s - w)\,\alpha\beta + [\max(q, \delta_l q + s) - w]\alpha(1-\beta) + (\delta_l q + s - w)[(1-\alpha)\beta + (1-\alpha)(1-\beta)]$ and $\pi^m_{(2)} = \left(w - cq^2\right)$, the manufacturer will price the product such that $w \leq \frac{\max(q,\delta_l q+s)\alpha(1-\beta)+(\delta_l q+s)[(1-\alpha)+\alpha\beta]-(\delta_h q+s)\beta}{1+\beta} \doteq w_{(2)}$. The manufacturer then chooses $q$ by maximizing $\pi^m_{(1)}$ or $\pi^m_{(2)}$. Because $\frac{\partial w_{(1)}}{\partial q} > \frac{\partial w_{(2)}}{\partial q}$, it must hold that the equilibrium quality $q^*_{(1)} > q^*_{(2)}$. In addition, the higher $\delta_h$, the higher $q^*_{(1)}$. Therefore, when $\delta_h$ is sufficiently high, the manufacturer will prefer (1) to (2) because of a sufficiently high profit margin, even though the demand under (1) is lower than (2); he will prefer (2) otherwise. ■

**Proof.** of results in Section 4.3 (Retailer Dictating Wholesale Price): We know that the retailer sets his retail price at $q$ in the case of offline-exclusive, at $\delta_h q + s$ in the case of online-exclusive, and at $q$ (or $\delta_h q + s$) when $\frac{s}{1-\delta_h}$ is high (or low) in the case of brick-and-click. It follows that the retailer will set the optimal quality $q^*$ at $\frac{1}{2c}$ in the case of offline-exclusive, at $\frac{\delta_h}{2c}$ in the case of online-exclusive, and at $\frac{1}{2c}$ (or $\frac{\delta_h}{2c}$) when $\frac{s}{1-\delta_h}$ is high (or low) in the case of brick-and-click. ■

**Proof.** of results in Section 4.4 (Two Qualities): Suppose that the retailer accepts both products. Given consumers' utilities below

| Consumer Type | Proportion | $U_1$ | $U_2$ |
|---|---|---|---|
| More-online-skilled, offline-accessible | $\alpha\beta$ | $q_1 - p_1$ | $\delta_h q_2 + s - p_2$ |
| Less-online-skilled, offline-accessible | $\alpha(1-\beta)$ | $q_1 - p_1$ | $\delta_l q_2 + s - p_2$ |
| More-online-skilled, offline-inaccessible | $(1-\alpha)\beta$ | / | $\delta_h q_2 + s - p_2$ |
| Less-online-skilled, offline-inaccessible | $(1-\alpha)(1-\beta)$ | / | $\delta_l q_2 + s - p_2$ |

there are three scenarios in the retailer's pricing strategies:

(a) The retailer sets prices $p_1$ and $p_2$, such that $q_1 - p_1 \geq \delta_h q_2 + s - p_2$ and $\delta_h q_2 + s - p_2 \geq 0$. Consequently, $p_1 = q_1$ and $p_2 = \delta_h q_2 + s$. And the market demand and firms' profits are given by

$$
\begin{aligned}
(D_1, D_2) &= (\alpha\beta + \alpha(1-\beta), (1-\alpha)\beta) \\
\pi^r_{3(a)} &= (q_1 - w_1)\alpha + (\delta_h q_2 + s - w_2)(1-\alpha)\beta \\
\pi^m_{3(a)} &= (w_1 - cq_1^2)\alpha + (w_2 - cq_2^2)(1-\alpha)\beta
\end{aligned}
$$

(b) The retailer sets prices $p_1$ and $p_2$, such that $q_1 - p_1 \geq \delta_l q_2 + s - p_2$ and $\delta_l q_2 + s - p_2 \geq 0$. Consequently, $p_1 = q_1$ and $p_2 = \delta_l q_2 + s$. And the market demand and firms' profits are given by

$$
\begin{aligned}
(D_1, D_2) &= (\alpha(1-\beta), \alpha\beta + (1-\alpha)\beta + (1-\alpha)(1-\beta) \\
\pi^r_{3(b)} &= (q_1 - w_1)\alpha(1-\beta) + (\delta_l q_2 + s - w_2)[\alpha\beta + (1-\alpha)] \\
\pi^m_{3(b)} &= (w_1 - cq_1^2)\alpha(1-\beta) + (w_2 - cq_2^2)[\alpha\beta + (1-\alpha)]
\end{aligned}
$$

(c) The retailer sets prices $p_1$ and $p_2$, such that $q_1 - p_1 \geq \delta_h q_2 + s - p_2$ and $\delta_l q_2 + s - p_2 \geq 0$. Consequently, $p_1 = q_1 - q_2(\delta_h - \delta_l)$ and $p_2 = \delta_l q_2 + s$. And the market demand and firms' profits are given by

$$\begin{aligned}
(D_1, D_2) &= (\alpha\beta + \alpha(1-\beta), (1-\alpha)\beta + (1-\alpha)(1-\beta) \\
\pi^r_{3(c)} &= [q_1 - q_2(\delta_h - \delta_l) - w_1]\,\alpha + (\delta_l q_2 + s - w_2)(1-\alpha) \\
\pi^m_{3(c)} &= (w_1 - cq_1^2)\alpha + (w_2 - cq_2^2)(1-\alpha)
\end{aligned}$$

By comparing $\pi^r_{3(a)}$, $\pi^r_{3(b)}$, and $\pi^r_{3(c)}$, it is easy to show that the retailer chooses pricing strategy (a) when $\delta_h$ is large, (b) when $\delta_h$ is intermediate, and (c) when $\delta_h$ is small because

$$\begin{aligned}
\pi^r_{3(a)} &\leq \pi^r_{3(b)} \Rightarrow \\
\delta_h &\leq \delta_l \frac{\alpha\beta + (1-\alpha)}{(1-\alpha)\beta} + \frac{(s-w_2)[\alpha\beta + (1-\alpha)] - (q_1 - w_1)\alpha\beta}{(1-\alpha)\beta q_2} - \frac{(s-w_2)}{q_2}
\end{aligned}$$

$$\pi^r_{3(a)} \leq \pi^r_{3(c)} \Rightarrow \delta_h \leq \delta_l \frac{1}{[(1-\alpha)\beta + \alpha]} + \frac{(s-w_2)(1-\alpha)(1-\beta)}{q_2\,[(1-\alpha)\beta + \alpha]}$$

$$\pi^r_{3(b)} \leq \pi^r_{3(c)} \Rightarrow \delta_h \leq \delta_l(1-\beta) + \frac{(q_1 - w_1)\beta}{q_2}$$

Suppose that the retailer accepts only the low end product, and carries it online only. We have that $p_2 = \delta_l q_2 + s$. Firms' profits are given by

$$\begin{aligned}
\pi^r_2 + R &= \max\left[(\delta_l q_2 + s - w_2), (\delta_h q_2 + s - w_2)\beta\right] + R \\
\pi^m_2 &= (w_2 - cq_2^2) \text{ or } (w_2 - cq_2^2)\beta
\end{aligned}$$

Suppose that the retailer accepts only the high end product, and carries it online only. We have that $p_1 = q_1$. Firms' profits are given by

$$\pi^r_1 + r = (q_1 - w_1)\alpha + r, \pi^m_1 = (w_1 - cq_1^2)\alpha$$

The retailer will accept only the low end product if

$$(\delta_l q_2 + s - w_2) + R \geq \max \begin{bmatrix} (q_1 - w_1)\alpha + (\delta_h q_2 + s - w_2)(1-\alpha)\beta, \\ (q_1 - w_1)\alpha(1-\beta) + (\delta_l q_2 + s - w_2)[\alpha\beta + (1-\alpha)], \\ [q_1 - q_2(\delta_h - \delta_l) - w_1]\,\alpha + (\delta_l q_2 + s - w_2)(1-\alpha) \end{bmatrix}$$

The retailer will accept only the high end product if

$$(q_1 - w_1)\alpha + r \geq \max \begin{bmatrix} (q_1 - w_1)\alpha + (\delta_h q_2 + s - w_2)(1-\alpha)\beta, \\ (q_1 - w_1)\alpha(1-\beta) + (\delta_l q_2 + s - w_2)[\alpha\beta + (1-\alpha)], \\ [q_1 - q_2(\delta_h - \delta_l) - w_1]\,\alpha + (\delta_l q_2 + s - w_2)(1-\alpha) \end{bmatrix}$$

We limit our attention to the optimal quality levels at the tipping point from brick-and-click to online-only and the tipping point from offline-only to brick-and-click. Following a similar arguement above, the manufactuer determines her wholesale prices and product quality levels by maximizing her profits in each of the following scenarios.

(a-1) From both to online only,

$$\begin{aligned} \max\ \pi^m_{3(a)} &= (w_1 - cq_1^2)\alpha + (w_2 - cq_2^2)(1-\alpha)\beta \\ \text{s.t., } \pi^r_{3(a)} &= (q_1 - w_1)\alpha + (\delta_h q_2 + s - w_2)(1-\alpha)\beta \geq (\delta_l q_2 + s - w_2) + R \end{aligned}$$

We then have $q_1^* = \frac{1}{2c}$ and $q_2^* = \frac{\delta_h}{2c}$.

(a-2) From both to offline only

$$\begin{aligned} \max\ \pi^m_{3(a)} &= (w_1 - cq_1^2)\alpha + (w_2 - cq_2^2)(1-\alpha)\beta \\ \text{s.t., } \pi^r_{3(a)} &= (q_1 - w_1)\alpha + (\delta_h q_2 + s - w_2)(1-\alpha)\beta \geq (q_1 - w_1)\alpha + r \end{aligned}$$

We then have $q_1^* = \frac{1}{2c}$ and $q_2^* = \frac{\delta_h}{2c}$.

(b-1) From both to online only,

$$
\begin{aligned}
\max\ \pi_{3(b)}^m &= (w_1 - cq_1^2)\alpha(1-\beta) + (w_2 - cq_2^2)[\alpha\beta + (1-\alpha)] \\
\text{s.t., } \pi_{3(b)}^r &= (q_1 - w_1)\alpha(1-\beta) + (\delta_l q_2 + s - w_2)[\alpha\beta + (1-\alpha)] \geq (\delta_l q_2 + s - w_2) + R
\end{aligned}
$$

We then have $q_1^* = \frac{1}{2c}$ and $q_2^* = \frac{\delta_l}{2c}$.

(b-2) From both to offline only

$$
\begin{aligned}
\max\ \pi_{3(b)}^m &= (w_1 - cq_1^2)\alpha(1-\beta) + (w_2 - cq_2^2)[\alpha\beta + (1-\alpha)] \\
\text{s.t., } \pi_{3(b)}^r &= (q_1 - w_1)\alpha(1-\beta) + (\delta_l q_2 + s - w_2)[\alpha\beta + (1-\alpha)] \geq (q_1 - w_1)\alpha + r
\end{aligned}
$$

We then have $q_1^* = \frac{1}{2c}$ and $q_2^* = \frac{\delta_l}{2c}$.

(c-1) From both to online only,

$$
\begin{aligned}
\max\ \pi_{3(c)}^m &= (w_1 - cq_1^2)\alpha + (w_2 - cq_2^2)(1-\alpha) \\
\text{s.t., } \pi_{3(c)}^r &= [q_1 - q_2(\delta_h - \delta_l) - w_1]\,\alpha + (\delta_l q_2 + s - w_2)(1-\alpha) \geq (\delta_l q_2 + s - w_2) + R
\end{aligned}
$$

We then have $q_1^* = \frac{1}{2c}$ and $q_2^* = \frac{\delta_l - (\delta_h - \delta_l)\frac{2\alpha}{1-\alpha}}{2c}$.

(c-2) From both to offline only

$$
\begin{aligned}
\max\ \pi_{3(c)}^m &= (w_1 - cq_1^2)\alpha + (w_2 - cq_2^2)(1-\alpha) \\
\text{s.t., } \pi_{3(c)}^r &= [q_1 - q_2(\delta_h - \delta_l) - w_1]\,\alpha + (\delta_l q_2 + s - w_2)(1-\alpha) \geq (q_1 - w_1)\alpha + r
\end{aligned}
$$

We then have $q_1^* = \frac{1}{2c}$ and $q_2^* = \frac{\delta_l - (\delta_h - \delta_l)\frac{2\alpha}{1-\alpha}}{2c}$. $\blacksquare$

**Proof.** of results in Section 4.5 (Two-Part Tariff Supply Contract): Results are obvious because the fixed fee does not interact with product quality in the retailer's best response function or the manufacturer's profit function. ∎

**Proof.** of results in Section 4.6 (Retailer's Endogeneous Participation Criterion): Results immediately follow by inspecting the retailer's profit in Figure 4(b), which is weakly increasing for $R < \hat{R}_2$, followed by a drop at $\hat{R}_2$, and then is increasing for $R > \hat{R}_2$. The equilibrium quality level remains the same as in the main model, because the retailer's endogenous participation criterion does not affect the basic premises of the manufacturer's profit maximization. ∎

**Proof.** of results in Section 4.7 (Partial Online Access): When the product is carried as brick-and-click, $\pi_3^r(q,w) = \min\left[\pi_b^r, \pi_a^r\right]$, where $\pi_3^r = (q-w)\left[\alpha(1-\beta)\gamma + \beta\gamma\right] \doteq \pi_a^r$ for $q < \frac{s}{1-\delta_h}$, and $\pi_3^r = (\delta_h q + s - w)\left[\alpha\gamma + (1-\alpha)\beta\gamma\right] \doteq \pi_b^r$ o/w. When the product is carried as offline-exclusive $\pi_1^r(q,w) = (q-w)\alpha + r$, and online-exclusive $\pi_2^r(q,w) = \max\left[\pi_{2b}^r, \pi_{2a}^r\right]$, where $\pi_{2b}^r = (\delta_l q + s - w)\gamma + R$ and $\pi_{2a}^r = (\delta_h q + s - w)\beta\gamma + R$. Following a similar argument. We obtain the equilibrium quality as follows. When offline-exclusive, for $R \in \left(\underline{R}_F, \overline{R}_F\right]$, $q^* = \frac{\beta\gamma R - \alpha r + \alpha\beta\gamma s}{\alpha\beta\gamma(1-\delta_h)} > \frac{1}{2c}$; $R \in \left(\overline{R}_F, \hat{R}_1\right]$, $q^* = \frac{\alpha - \beta\gamma\delta_h}{2c(\alpha - \beta\gamma)} > \frac{1}{2c}$. When brick-and-click, (i) if $\frac{s}{1-\delta_h}$ is high, the equilibrium quality $q^* = \frac{1}{2c}\left[1 + \frac{(1-\delta_h)\beta\gamma}{\alpha(1-\beta\gamma)}\right] \left(< q^* = \frac{1}{2c}\left[1 + \frac{(1-\delta_h)\beta}{\alpha(1-\beta)}\right] \text{ in main model}\right)$ for $R \in \left(\overline{R}_B, \hat{R}_2\right]$; (ii) if $\frac{s}{1-\delta_h}$ is low, $q^* = \frac{\delta_h[\alpha + (1-\alpha)\beta\gamma] - \alpha}{2c(1-\alpha)\beta\gamma} < \frac{\delta_h}{2c} \left(> q^* = \frac{\delta_h[\alpha + (1-\alpha)\beta] - \alpha}{2c(1-\alpha)\beta} \text{ in the main model}\right)$ for $R \in \left(\hat{R}_1, \underline{R}_B\right]$. When online-exclusive, the equilibrium quality remains the same. ∎

**Proof.** of results in Section 4.8 (Continuous Consumer Heterogeneities) and Lemma 3: The retailer's profit from the new product is given by $\pi^r(p,q,w) = Q_I(p) \cdot (p-w)$ and the manufacturer's profit is given by $\pi^m(p,q,w) = Q_I(p) \cdot \left(w - cq^2\right)$, with $I = \{0,1,2,3\}$. In the offline-exclusive case, the offline demand is given by $Q_1(p) = \frac{q-p}{t}$. In the online-exclusive case, the online demand is given by $Q_2(p) = \begin{cases} 1 & \text{if } p \leq \delta_l q + s \\ \beta & \text{if } p \in (\delta_l q + s, \delta_h q + s] \\ 0 & \text{otherwise} \end{cases}$. In the case of brick-and-click, the demand is given by $Q_3(p) = \begin{cases} 1 & \text{if } p \leq \delta_l q + s \\ \beta + (1-\beta)\frac{q-p}{t} & \text{if } p \in (\delta_l q + s, \delta_h q + s] \\ \frac{q-p}{t} & \text{otherwise} \end{cases}$. The manufacturer decides $(q, w)$, and then the retailer decides whether to carry the product and, if carrying it, his retail price $p$.

In the case of offline-exclusive (the same as in the baseline case), the equilibrium results are below. From FOC, the retailer's best response retail price is $p^*(q,w) = \frac{1}{2}(q+w)$, and he carries the manufacturer's product when $\pi^r(q,w) = \frac{1}{4t}(q-w)^2 \geq R$. Substituting the best response retail price into the manufacturer's profit and from FOC, we obtain the equilibrium $q^* = \frac{1}{2c}$, $w^* = \frac{3}{8c}$, $p^* = \frac{7}{16c}$, $\pi^{r*} = \frac{1}{256c^2t}$, and $\pi^{m*} = \frac{1}{128c^2t}$. To satisfy the retailer's participation criterion, we need $\pi^{r*} = \frac{1}{256c^2t} > R$. When it does not hold, i.e., $c \geq \frac{1}{16\sqrt{Rt}}$, we have the manufacturer's maximum possible wholesale price given by $q - 2\sqrt{Rt}$. Substituting this maximum wholesale price into the manufacturer's profit and from FOC of quality $q$, we get $q^* = \frac{1}{2c}$, $w^* = \frac{1}{2c} - 2\sqrt{Rt}$, $p^* = \frac{1}{2c} - \sqrt{Rt}$, $\pi^{r*} = R$, and $\pi^{m*} = \frac{\sqrt{Rt}}{4ct} - 2R$. That is, when the margin cost is low, i.e., $c \leq \frac{1}{16\sqrt{Rt}}$ or $R < \frac{1}{16^2c^2t}$, the retailer's participation constraint is not binding, i.e., $\pi^{r*} = \frac{1}{256c^2t} > R$. That the retailer gains more than $R$ is due to his added ability to fine-tune the retail price in the linear model. In the case of online-exclusive, the equilibrium remains the same.

In the case of brick-and-click, the retailer's best response retail price is given by $p^*(q,w) = \begin{cases} \delta_l q + s & \text{if } \frac{t\beta}{2(1-\beta)} \leq s + \delta_l q - \frac{q+w}{2} \\ \frac{q+w}{2} + \frac{t\beta}{2(1-\beta)} & \text{if } \frac{t\beta}{2(1-\beta)} \in \left(s + \delta_l q - \frac{q+w}{2}, s + \delta_h q - \frac{q+w}{2}\right] \\ \delta_h q + s & \text{if } \frac{t\beta}{2(1-\beta)} > s + \delta_h q - \frac{q+w}{2} \end{cases}$. Note that the retailer will not set his retail price higher than $\delta_h q + s$ because, otherwise, this case reduces to the offline-online case.

The equilibrium quality and wholesale price are given by

$$
q^* = \begin{cases} \frac{\delta_l}{2c} & \text{if } \frac{t\beta}{(1-\beta)} \leq \frac{4}{3}\left(s + \frac{\delta_l}{2c} - \frac{7}{16c}\right) \\ \frac{1}{2c} & \text{if } \frac{t\beta}{(1-\beta)} \in \left(\frac{4}{3}\left(s + \frac{\delta_l}{2c} - \frac{7}{16c}\right), \frac{4}{3}\left(s + \frac{\delta_h}{2c} - \frac{7}{16c}\right)\right] \\ q" & \text{if } \frac{t\beta}{(1-\beta)} > \frac{4}{3}\left(s + \frac{\delta_h}{2c} - \frac{7}{16c}\right) \end{cases}
$$

$$
w^* = \begin{cases} \frac{\delta_l^2}{2c} + s - R & \text{if } \frac{t\beta}{(1-\beta)} \leq \frac{4}{3}\left(s + \frac{\delta_l}{2c} - \frac{7}{16c}\right) \\ \frac{3}{8c} + \frac{t\beta}{2(1-\beta)} & \text{if } \frac{t\beta}{(1-\beta)} \in \left(\frac{4}{3}\left(s + \frac{\delta_l}{2c} - \frac{7}{16c}\right), \frac{4}{3}\left(s + \frac{\delta_h}{2c} - \frac{7}{16c}\right)\right] \\ w" & \text{if } \frac{t\beta}{(1-\beta)} > \frac{4}{3}\left(s + \frac{\delta_h}{2c} - \frac{7}{16c}\right) \end{cases}
$$

where

$$
\begin{aligned}
q" &= \frac{cs(1-\beta) - c\beta t + \delta_h(1-\delta_h)(1-\beta)}{3(1-\delta_h)(1-\beta)c} + \frac{\sqrt{\begin{array}{c} c^2\left[\beta t - s\left(1-\beta\right)\right]^2 \\ +\delta_h c\left[\beta t - s\left(1-\beta\right)\right]\left(1-\delta_h\right)\left(1-\beta\right) \\ +\left(\delta_h^2 + 3cs\right)\left(1-\delta_h\right)^2\left(1-\beta\right)^2 \end{array}}}{3(1-\delta_h)(1-\beta)c} \\
w" &= \delta_h q" + s - \frac{(R+r)t}{(1-\delta_h)(1-\beta)q" - (1-\beta)s + \beta t}
\end{aligned}
$$

One can observe that the impact of $s$ and $\delta_h$ on $q"$ is mainly dictated by the first term, $\frac{cs(1-\beta)-c\beta t+\delta_h(1-\delta_h)(1-\beta)}{3(1-\delta_h)(1-\beta)c}$, which is increasing in both $s$ and $\delta_h$. Hence, consistent with our main model, the manufacturer designs a product of higher (lower) quality than the baseline case when $s$ and/or $\delta_h$ is high (low), i.e., $q_{Con} > \frac{1}{2c}$ ($q_{Con} < \frac{1}{2c}$). ■

**Proof.** of results in Section 4.9 (Inspect Offline and Buy Online): As discussed above, consumers $x \leq \frac{1}{t}[(1-\delta)q - s]$ will first inspect the product in-store and then purchase it online. Meanwhile, we need to satisfy $p_2 < p_1$. Hence, consumers either purchase the product online directly or inspect it offline and then buy online. The firm's profit in brick-and-click is given by

$\pi_3^r = Q_3(p_2) \cdot (p_2 - w)$, where $Q_3(p_2) = \begin{cases} 1 & \text{if } p_2 \leq \delta_l q + s \\ \beta + (1-\beta)\frac{q-p_2}{t} & \text{if } p_2 \in (\delta_l q + s, \delta_h q + s] \\ \frac{q-p_2}{t} & \text{otherwise} \end{cases}$. Hence, brick-and-click in this case is identical to that without such strategic behavior except that we require $p_2 < p_1$. ■

**Proof.** of results in Section 4.10 (Uncertain Demand): We illustrate the extension using Section 3.2. We show below the derivation of the equilbrium qualities by using the same notations as above.

$$\begin{aligned} \pi^m(q,w) &= \left\{[\rho(1-\lambda)+(1-\rho)]\,w - [\rho\lambda h + (1-\rho)\,\lambda l] - cq^2\right\} \cdot Q(q) \\ \pi^r(q,w) &= [\rho(1-\lambda)+(1-\rho)] \cdot (p-w) \cdot Q(q) \end{aligned}$$

(1) Equilibrium quality in region $R \in \left(\overline{R}, \hat{R}\right]$. From $\pi_1^r(q,w)$ $(= [\rho(1-\lambda)+(1-\rho)]\,(q-w)\alpha + r) = \pi_2^r(q,w)$ $(= [\rho(1-\lambda)+(1-\rho)]\,(\delta_h q + s - w)\beta + R)$, we get

$$\overline{w} = \frac{[(\alpha-\beta\delta_h)q - \beta s]\,[\rho(1-\lambda)+(1-\rho)] - R + r}{(\alpha-\beta)\,[\rho(1-\lambda)+(1-\rho)]}$$

Substituting it into $\pi^m(q,w) = \left\{[\rho(1-\lambda)+(1-\rho)]\,w - [\rho\lambda h + (1-\rho)\,\lambda l] - cq^2\right\}\alpha$ and optimizing $q$ get

$$\overline{q} = \frac{(\alpha-\beta\delta_h)}{2c(\alpha-\beta)}$$

(2) Equilibrium quality in region $R \in \left(\underline{R}, \overline{R}\right]$. From $\overline{w} = w_O$, where $w_O = \delta_h q + s - \frac{r}{\lambda\beta[\rho(1-\lambda)+(1-\rho)]}$, we get

$$q' = \frac{\beta R - \alpha r + \alpha\beta s}{\alpha\beta(1-\delta_h)\,[\rho(1-\lambda)+(1-\rho)]}$$

(3) Equilibrium quality in regions $R \leq \underline{R}$ and $R > \hat{R}$, the equilibrium quality is $\frac{1}{[\rho(1-\lambda)+(1-\rho)]}$ of

that in Section 3.2. As a summary,

| | | |
|---|---|---|
| If $R \le \underline{R}$, | Offline | $q^* = \frac{1}{2c[\rho(1-\lambda)+(1-\rho)]}, w^* = \frac{1}{[\rho(1-\lambda)+(1-\rho)]}\left(\frac{1}{2c} - \frac{R}{\alpha}\right), \pi_1^{r*} = R + r$ |
| If $R \in \left(\underline{R}, \overline{R}\right]$, | Offline | $q^* = \frac{\beta R - \alpha r + \alpha\beta s}{\alpha\beta(1-\delta_h)[\rho(1-\lambda)+(1-\rho)]}, w^* = \frac{\beta\delta_h R - \alpha r + \alpha\beta s}{\alpha\beta(1-\delta_h)[\rho(1-\lambda)+(1-\rho)]}, \pi_1^{r*} = R + r$ |
| If $R \in \left(\overline{R}, \hat{R}\right]$, | Offline | $q^* = \frac{\alpha-\beta\delta_h}{2c(\alpha-\beta)}, w^* = \frac{(\alpha-\beta\delta_h)q-\beta s}{(\alpha-\beta)} - \frac{R-r}{(\alpha-\beta)[\rho(1-\lambda)+(1-\rho)]},$<br>$\pi_1^{r*} = \left[\frac{\alpha-\beta\delta_h}{2c(\alpha-\beta)} - \frac{(\alpha-\beta\delta_h)q-\beta s}{2c(\alpha-\beta)}\right][\rho(1-\lambda)+(1-\rho)]\,\alpha + \frac{R-r}{\alpha-\beta}\alpha + r$ |
| If $R > \hat{R}$, | Online | $\begin{cases} s \le \hat{s} : q^* = \frac{\delta_h}{2c[\rho(1-\lambda)+(1-\rho)]}, w^* = \delta_h q^* + s - \frac{r}{\beta[\rho(1-\lambda)+(1-\rho)]}, \pi_2^{r*} = R + r \\ s > \hat{s} : q^* = \frac{\delta_l}{2c[\rho(1-\lambda)+(1-\rho)]}, w^* = \delta_l q^* + s - \frac{r}{[\rho(1-\lambda)+(1-\rho)]}, \pi_2^{r*} = R + r \end{cases}$ |

where $\underline{R} = \frac{\alpha(1-\delta_h)}{2c} - \alpha s\left[\rho(1-\lambda)+(1-\rho)\right] + \frac{\alpha r}{\beta}$, $\overline{R} = \frac{\alpha(1-\delta_h)(\alpha-\beta\delta_h)}{2c(\alpha-\beta)} - \alpha s\left[\rho(1-\lambda)+(1-\rho)\right] + \frac{\alpha r}{\beta}$. It is easy to see that the equilibrium quality $q^*$ in regions $R \le \underline{R}$, $R \in \left(\underline{R}, \overline{R}\right]$ and $R > \hat{R}$ increases and $w^*$ also increases. In the region $R \in \left(\overline{R}, \hat{R}\right]$, $q^*$ stays the highest as in Section 3.2, while $w^*$ reduces. The retailer's profit or surplus in $R \in \left(\overline{R}, \hat{R}\right]$ increases with $R$ as in Section 3.2. It is easy to see that $\frac{\partial \overline{R}}{\partial \lambda} > 0$ and $\frac{\partial \hat{R}}{\partial \lambda} < 0$. Similar results can be obtained for an extension similar to that in Section 3.3. ■