**Product Line Design for Consumer Durables:**
**An Integrated Marketing and Engineering Approach**

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## Web Appendix A: Mapping from Design Variables to Consumer Attributes

In this appendix, we discuss the general steps that guide the mapping from design variables to consumer attributes. Let us denote the sets of design variables as $\mathbf{y} = (y_1, y_2, \ldots, y_m)$ and consumer attributes as $\mathbf{x} = (x_1, x_2, \ldots, x_n)$. First, we examine how each element of the consumer attribute vector corresponds to the respective elements in the design variable vector. For example, we may find that $x_1=f(y_1,y_2)$, $x_2=f(y_1,y_3,y_5)$, $x_3= f(y_4)$, and etc.

Next, we proceed to investigate the specifics of these mapping functions. There are two basic types of mapping relationships. In the *first* type, the relationship between the design variable and the consumer attribute can be specified in a simple look-up table and such specification does not require the construction of an engineering simulation. For example, in a power tool design, given the selection of the design variable housing type (A or B), the product designer can easily determine the value of the consumer attribute product girth (small or large).

In the *second* type, an engineering simulation is needed to calculate the value of the consumer attribute as a function of the corresponding design variables. Using the same power tool example, the product designer needs to use an engineering simulation to determine the value of the consumer attribute product life as a joint function of the values of the design variables motor type, gearbox type, and gear ratio. The general guidelines of how to build such simulations have been well established in the literature (e.g. Doebelin 1998; Papalambros and Wilde 2000; Ulrich and Eppinger 2004). In the following, we briefly explain the basic steps involved in the development of such a simulation.

Step one: Determine the inputs and the outputs of the simulation. The inputs are a set of design variables (e.g. motor types, gear types, and gear ratio). The outputs include: 1) a set of consumer attributes (e.g. power amp, product life) and 2) some measures of the product's engineering performance metrics (e.g. motor output speed, armature temperature). The former directly influence consumers' purchase decisions. The latter are typically used to assess whether the product satisfies the engineering constraints imposed by the product designer.

Step two: Identify the set of engineering parameters (**pa**) that characterize the uncontrollable variations in the product's usage environment (e.g. power voltage). For each engineering parameter, proceed to specify its nominal value $\mathbf{pa_0}$ (i.e. the most likely value) and the typical range of its value (i.e. the lower bound $\mathbf{pa_U}$ and the upper bound $\mathbf{pa_L}$). For example, the nominal source voltage is 110V and the actual source voltage typically varies between 95V and 125V, depending on the product's usage environment.

Step three: Simulate product performance under various usage situations. A finite element method is typically used to perform this task. This method discretizes all the continuous variables and parameters into sets of discrete sub-domains and simulates how different product configurations will perform under different environmental conditions (see Schenk and Schueller (2005) for a detailed discussion on finite element methods). Finally, the product designer documents the nominal, the upper, and the lower bound values of each output variable, as a function of the design variable configuration. The mapping relationships between the design variables (i.e. the $y_s$) and the consumer attributes (i.e. the $x_s$) are established accordingly.

## Web Appendix B: Implementation Details of Proposed Model

### *CONSUMER PREFERENCE MODEL*

The probability of consumer $i$'s sequence of choices $\boldsymbol{\delta}_i$ can be written as follows:

(A1) $$l(\boldsymbol{\delta}_i|\boldsymbol{\xi}_i) = \prod_{d=1}^{D}\prod_{k=1}^{K}\Pr(\boldsymbol{\delta}_{idk} = 1|\boldsymbol{\xi}_i)^{\delta_{idk}}$$

where $\delta_{idk}$ is the 0/1 choice indicator variable, and

(A2) $$\Pr(\boldsymbol{\delta}_{idk} = 1|\boldsymbol{\xi}_i) = \frac{\exp\left(\mathbf{x}_{dk}\boldsymbol{\beta}_{ix} + p_{dk}\beta_{ip}\right)}{\sum_{d'=1}^{D}\exp(\mathbf{x}_{d'k}\boldsymbol{\beta}_{ix} + p_{d'k}\beta_{ip}) + \exp(\alpha_i)}$$

where $\boldsymbol{\xi}_i \sim \sum_{s=1}^{S}\theta_{is}MVN(\overline{\boldsymbol{\xi}}_s, \boldsymbol{\Omega}_s)$ with $\sum_{s=1}^{S}\theta_{is} = 1$.

Therefore, the total likelihood function can be specified as:

(A3) $$\begin{aligned} L = \prod_{i=1}^{N} l(\boldsymbol{\delta}_i|\boldsymbol{\xi}_i) f(\boldsymbol{\xi}_i|\overline{\boldsymbol{\xi}}_1, \overline{\boldsymbol{\xi}}_2, \ldots, \overline{\boldsymbol{\xi}}_S, \boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2, \ldots, \boldsymbol{\Omega}_S, \theta_{i1}, \theta_{i2}, \ldots, \theta_{iS}) \\ \prod_{i=1}^{N}\prod_{s=1}^{S} f(\theta_{is}|\boldsymbol{\gamma}_s) f(\overline{\boldsymbol{\xi}}_1, \overline{\boldsymbol{\xi}}_2, \ldots, \overline{\boldsymbol{\xi}}_S, \boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2, \ldots, \boldsymbol{\Omega}_S, \boldsymbol{\gamma}_s) \end{aligned}$$

The priors are specified as follows: $\overline{\boldsymbol{\xi}}_s \sim MVN(\boldsymbol{\psi}_s, \boldsymbol{\Xi}_s)$ with $\boldsymbol{\psi}_s = 0$ and $\boldsymbol{\Xi}_s^{-1} \sim W(\rho, R)$ with $\rho$ = 10 and $R$ = 10**I**; and $\boldsymbol{\gamma}_s \sim MVN(\varpi, \boldsymbol{\Phi})$ with $\varpi = 0$ and $\boldsymbol{\Phi}^{-1} \sim W(\varsigma, \boldsymbol{\Theta})$ with $\varsigma$ = 10 and $\boldsymbol{\Theta}$ = 10**I**.

For identification purpose, we set the hyper-parameter of the "no-choice" option in segment 1 to be zero. Additionally, we would like to point out that, although Bayesian estimation of a finite mixture model is subject to the well-known label switching problem, the introduction of the covariates $\mathbf{z}_i$ provides additional information that improves identification and alleviates this issue (Congdon 2003).

Our MCMC procedure is carried out by sequentially generating draws from the following distributions:

1. Generate $\boldsymbol{\xi}_i$

(A4) $$f(\boldsymbol{\xi}_i|*) \propto l(\boldsymbol{\delta}_i|\boldsymbol{\xi}_i) f(\boldsymbol{\xi}_i)$$

where $l(\boldsymbol{\delta}_i|\boldsymbol{\xi}_i)$ is provided in Equation (A3) and $\boldsymbol{\xi}_i \sim \sum_{s=1}^{S} \theta_{is} MVN(\overline{\boldsymbol{\xi}}_s, \boldsymbol{\Omega}_s)$.

2. Generate $\overline{\boldsymbol{\xi}}_s$

(A5) $$f(\overline{\boldsymbol{\xi}}_s|*) \sim MVN\left(\boldsymbol{\Delta}_s\left(\left(\frac{\boldsymbol{\Omega}_s}{N_s}\right)^{-1}\frac{\sum_{i=1}^{N_s}\boldsymbol{\xi}_i I(q_i = s)}{N_s} + \boldsymbol{\Xi}_s^{-1}\boldsymbol{\psi}_s\right), \boldsymbol{\Delta}_s\right)$$

where $\boldsymbol{\Delta}_s = \left(\left(\frac{\boldsymbol{\Omega}_s}{N_s}\right)^{-1} + \boldsymbol{\Xi}^{-1}\right)^{-1}$, $I(q_i = s)$ is an indicator function which takes the value of one if consumer *i* belongs to segment *s.* Following Diebolt and Robert (1994), the segment membership follows a multinomial distribution with $q_i \sim Multin(s; \theta_{i1}, \theta_{i2}, ..., \theta_{iS})$. And $N_s = \sum_{i=1}^{N} d(q_i = s)$ represents the number of consumers in segment *s*.

3. Generate $\boldsymbol{\Omega}_s$

(A6) $$f\left(\boldsymbol{\Omega}_s^{-1}|*\right) \sim W\left(\rho + N_s, \mathbf{R} + \sum_{i=1}^{N} I(q_i = s)(\boldsymbol{\xi}_i - \overline{\boldsymbol{\xi}}_s)(\boldsymbol{\xi}_i - \overline{\boldsymbol{\xi}}_s)'\right)$$

4. Generate $\theta_{is}$

(A7) $$f\left(\theta_{is}|*\right) \propto f\left(\boldsymbol{\xi}_i|\theta_{is}\right) f\left(\theta_{is}|\boldsymbol{\gamma}_s\right) f\left(\boldsymbol{\gamma}_s\right)$$

where $\boldsymbol{\xi}_i \sim \sum_{s=1}^{S} \theta_{is} MVN(\overline{\boldsymbol{\xi}}_s, \boldsymbol{\Omega}_s)$, $\theta_{is} = \frac{\exp(\boldsymbol{\gamma}_s \mathbf{z}_i)}{\sum_{s'=1}^{S} \exp(\boldsymbol{\gamma}_{s'} \mathbf{z}_i)}$, and $\boldsymbol{\gamma}_s \sim MVN(\varpi, \boldsymbol{\Phi})$.

5. Generate $\boldsymbol{\gamma}_s$

(A8) $$f\left(\boldsymbol{\gamma}_s|*\right) \propto \prod_{i=1}^{N} f(\mathbf{z}_i, \theta_{is}|\boldsymbol{\gamma}_s) f\left(\boldsymbol{\gamma}_s\right)$$

MARKET RESPONSES FROM INCUMBENT MANUFACTURERS AND RETAILER

*First-Order-Conditions of Retailer's Profit Maximization*

Given the retailer's profit maximization function in Equation (6), we obtain the first-order-conditions (Equation (A9)). As discussed in Luo et al. (2007), the optimal retail prices can be estimated after we express $m_{kl}$ and $\frac{\partial m_{k'l'}}{\partial p_{kl}}$ as functions of the retail prices

$\left(p_{11},..., p_{1L_1},..., p_{K1},..., p_{KL_K}\right)$.

(A9) $$\frac{\partial \pi^r}{\partial p_{kl}} = m_{kl} + \sum_{k'=1}^{K}\sum_{l'=1}^{L_{k'}}[(p_{k'l'} - w_{k'l'})\frac{\partial m_{k'l'}}{\partial p_{kl}}] = 0 \qquad k=1,\ldots, K;\ l=1,\ldots,L_k$$

*First-Order-Conditions of Each Manufacturer's Profit Maximization*

The first-order-conditions of Equation (8) are given in Equation (A10). Note that, different from Luo et al. (2007), the manufacturer will choose a set of wholesale prices rather than a single price to maximize its product line profit.

(A10)

$$\frac{\partial \pi_k^m}{\partial w_{k1}} = m_{k1} + (w_{k1} - c_{k1})\sum_{k'=1}^{K}\sum_{l'=1}^{L_{k'}}\frac{\partial m_{k1}}{\partial p_{k'l'}}\frac{\partial p_{k'l'}}{\partial w_{k1}} + (w_{k2} - c_{k2})\sum_{k'=1}^{K}\sum_{l'=1}^{L_{k'}}\frac{\partial m_{k2}}{\partial p_{k'l'}}\frac{\partial p_{k'l'}}{\partial w_{k1}} + ... + (w_{kL_k} - c_{kL_k})\sum_{k'=1}^{K}\sum_{l'=1}^{L_{k'}}\frac{\partial m_{kL_k}}{\partial p_{k'l'}}\frac{\partial p_{k'l'}}{\partial w_{k1}} = 0$$

$$\frac{\partial \pi_k^m}{\partial w_{k2}} = (w_{k1} - c_{k1})\sum_{k'=1}^{K}\sum_{l'=1}^{L_{k'}}\frac{\partial m_{k1}}{\partial p_{k'l'}}\frac{\partial p_{k'l'}}{\partial w_{k2}} + m_{k2} + (w_{k2} - c_{k2})\sum_{k'=1}^{K}\sum_{l'=1}^{L_{k'}}\frac{\partial m_{k2}}{\partial p_{k'l'}}\frac{\partial p_{k'l'}}{\partial w_{k2}} + ... + (w_{kL_k} - c_{kL_k})\sum_{k'=1}^{K}\sum_{l'=1}^{L_{k'}}\frac{\partial m_{kL_k}}{\partial p_{k'l'}}\frac{\partial p_{k'l'}}{\partial w_{k2}} = 0$$

...

$$\frac{\partial \pi_k^m}{\partial w_{kL_k}} = (w_{k1} - c_{k1})\sum_{k'=1}^{K}\sum_{l'=1}^{L_{k'}}\frac{\partial m_{k1}}{\partial p_{k'l'}}\frac{\partial p_{k'l'}}{\partial w_{kL_k}} + (w_{k2} - c_{k2})\sum_{k'=1}^{K}\sum_{l'=1}^{L_{k'}}\frac{\partial m_{k2}}{\partial p_{k'l'}}\frac{\partial p_{k'l'}}{\partial w_{kL_k}} + ... + m_{kL_k} + (w_{kL_k} - c_{kL_k})\sum_{k'=1}^{K}\sum_{l'=1}^{L_{k'}}\frac{\partial m_{kL_k}}{\partial p_{k'l'}}\frac{\partial p_{k'l'}}{\partial w_{kL_k}} = 0$$

where $\left(\frac{\partial p_{11}}{\partial w_{kl}},...,\frac{\partial p_{1L_1}}{\partial w_{kl}},...,\frac{\partial p_{K1}}{\partial w_{kl}},...\frac{\partial p_{KL_K}}{\partial w_{kl}}\right)' = \left(\frac{\partial m_{kl}}{\partial p_{11}},...,\frac{\partial m_{kl}}{\partial p_{1L_1}},...,\frac{\partial m_{kl}}{\partial p_{K1}},...\frac{\partial m_{kl}}{\partial p_{KL_K}}\right)'\left(G^{-1}\right)'$, and $G$ is a $\widehat{L}\times\widehat{L}$

($\widehat{L} = \sum_{k=1}^{K} L_k$) matrix.

Let $j$ denote the $kl^{th}$ product and $g$ denote the $\widetilde{k}\widetilde{l}^{\,th}$ product in the marketplace. The $jg^{th}$ element of the matrix $G$ can be expressed as:

(A11) $$g_{jg} = \frac{\partial m_{kl}}{\partial p_{\widetilde{k}\widetilde{l}}} + \frac{\partial m_{\widetilde{k}\widetilde{l}}}{\partial p_{kl}} + \sum_{k'=1}^{K}\sum_{l'=1}^{L_k}[(p_{k'l'} - w_{k'l'})\frac{\partial^2 m_{k'l'}}{\partial p_{kl}\partial p_{\widetilde{k}\widetilde{l}}}]$$

Substituting above expressions into Equation A10, we are able to calculate the profit-maximizing wholesale prices $\left(w_{k1}, w_{k2}, ..., w_{kL_k}\right)$ for each manufacturer.

*Iterative Algorithm of Solving Equilibrium Prices*

Similar to Luo et al. (2007), our procedure for estimating the wholesale and retail prices includes using gradient methods to solve the retailer's profit maximization problem and the manufacturers' profit maximization problem iteratively. The basic idea of the algorithm is below. First, given the initial wholesale prices of the products in the new product line and the current wholesale prices of the incumbent products, the retailer chooses the retail prices for the products in its assortment to maximize its category profit. All the manufacturers (including the focal and the competing manufacturers) then adjust their wholesale prices to maximize their own profits. Next, the retailer re-adjusts the retail prices given the adjusted wholesale prices, and the manufacturers re-adjust the wholesale prices based on the adjusted retail prices. This cycling process continues until the generated prices converge.

*COST MODEL*

In Equation (11), the discount factor $\lambda_{rwl}$ is defined as follows:

(A12) $$\lambda_{rwl} = \begin{cases} 0 & \textbf{\textit{No commonality among products}} \\ \frac{l_{shrw}}{L}\vartheta_{rw} & \textbf{\textit{Otherwise}} \end{cases}$$

When there is no sharing of component $r$ among the products in the product line, the discount factor is zero. When there is component sharing, $\vartheta_{rw}$ represents the degree of cost saving by sharing the $w^{th}$ type of component *r* in the product line. As discussed by Morgan et al. (2001) and Ramdas and Sawhney (2001), $\vartheta_{rw}$ is generally evaluated from historical data on a case-by-case basis. When all the products in the product line adopt the same type of component, the discount factor $\lambda_{rwl}$ is equal to $\vartheta_{rw}$. Otherwise, the discount factor $\lambda_{rwl}$ is a proportion of $\vartheta_{rw}$ depending on the degree of component sharing in the product line. In the current model, this proportion is defined as the number of products sharing the $w^{th}$ type of component *r* (denoted as $l_{shrw}$) divided by the product line breadth. In practice, the definition of this proportion may vary in different applications. A detailed comparison of several commonly used proportion measures can be found in Thevenot and Simpson (2004). As evident, when the degree of component sharing increases, the variable cost of producing each product reduces and the manufacturing process becomes more efficient.

The unit cost of assembling the components into the final product $l$ (denoted as $c_{al}$) is determined by the specific selection of the component type and the combination of the components. A look-up table can be used to obtain the assembly cost. The assembly cost may also decrease when the products share the same components in the product line. In such cases, a discount factor can also be incorporated into the look-up table to reflect the cost saving of the assembly.

The maintenance cost of product $l$ (denoted as $c_{ml}$) is negatively related to the lower bound estimate of product life (Equation A13), which is obtained as an output of the design simulation. In this equation, the values of $cm1,...,cmk$ and the cutoff points of $E1,...,EE$ can be estimated through an examination of the historical data on servicing the products after sales.

(A13) $$c_{ml} = \begin{cases} cm1 & Life(WCS)_l \le E1 \\ cm2 & E1 < Life(WCS)_l \le E2 \\ \dots & \\ cmk & Life(WCS)_l > EE \end{cases}$$

Finally, the salvage cost $c_{sl}$ can be obtained from a look-up table.

## Web Appendix C: Implementation Details of Simulation Study

### *PROBLEM SETUP*

We first describe the implementation details of our simulation study when we compared the performances of GA, SA and ATC for the focal manufacturer's design variable configuration problem (first block of Figure 1). When the design space includes both discrete and continuous design variables, the vector of design variables was defined as follows: $y_1$ is a discrete variable with 15 levels, $y_2$ is a discrete variable with 10 levels, $y_3$ is a continuous variable ranging from 3.0 to 10.0, and $y_4$ is a continuous variable ranging from 50.0 to 150.0. The vector of the consumer attributes were obtained from the following responses functions: $x_1 = f_1 (y_1, y_2)$; $x_2 = f_2 (y_1, y_3)$; $x_3 = f_3 (y_3)$; and $x_4 = f_4(y_4)$. We specify the mapping relationships of the first two functions via a set of conditional formulae, which intends to simulate the engineering simulation described in Web Appendix A. The inputs of the formulae were design variables $y_1$, $y_2$, and $y_3$. The outputs of the formulae were the corresponding consumer attributes $x_1$ and $x_2$ and four engineering performance metrics. For each output variable, the formulae provide its nominal value along with its range of variations. Regarding the last two response functions, a simple look-up table was used to establish the mapping relations.

From the marketing side, we assume that the number of segments in the marketplace is the same as the number of products in the product line. The segment sizes were generated from a Dirichlet distribution. Assuming that there were three levels associated with each consumer attribute, we generated the conjoint part-worths and the utility of "no-choice" for each market segment using i.i.d. normal (0, 1) distributions. Individuals within the same segment are assumed to have identical preferences. Among the four consumer attributes, we assume that $x_1$ and $x_2$ are varying attributes with their variations mapped from the set of conditional formulae described above. The initial wholesale prices, the retail markups, and the conjoint part-worths associated with retail prices were randomly selected from truncated normal distributions. When the numbers of products in the product line increased from 1 - 3, to 4 - 5, to 6 – 8, the number of incumbent products in the marketplace were assumed to be 3, 6, and 9 respectively. We randomly selected different incumbent products for each simulation problem.

From the engineering side, we included two feasibility criteria and two robustness criteria. Feasibility criterion #1 required that the upper bound of the product's engineering metric #1 is less than 125. Feasibility criterion #2 required that the lower bound of product's engineering metric #2 is greater than 50. Among the two robustness criteria, robustness criterion #1 required that the range of variation in the product's engineering metric #3 is less than 10, and robustness criterion #2 required that the range of variation in the product's engineering metric #4 is less than 400. In the cost model, design variables $y_1$ and $y_2$ comprised the major components of the product. The corresponding cost and the discount factors associated with component sharing were obtained from a look-up table. The maintenance cost was a function of the lower bound of consumer attribute $x_2$ (we consider this attribute as "product life"). The product's assembly and salvage costs were obtained from look-up tables.

The discounted long-term profit is calculated over a 5-year horizon with 3% discount rate. The market size in each year is randomly generated from a truncated normal distribution. As the length of the product line increases from one to eight, the corresponding fixed cost was randomly selected in an ascending order and each product's production capacity constraint was randomly selected with a descending order. The current category profit of the retailer was randomly generated from a truncated normal distribution. The units of the profit and the market size were in millions.

Under this problem setup, we solved the focal manufacturer's design variable configuration problem using both GA and SA. For each violated constraint, we added a penalty value of 1,000,000,000 to the raw objective value (i.e. estimated product line profit). Note that, as long as the magnitude of this penalty value is large enough to ensure that product lines with violated constraints will always be worse than those without violated constraints, the choice of its specific value is not important.

When we included the ATC method into the comparison, the definition of the design variable vector was revised so that $y_1$ and $y_2$ were also continuous. The mapping relationships from these design variables to consumer attributes were also revised accordingly. In the cost model, we discretize the values of $y_1$ and $y_2$ to obtain look-up tables of component costs, the discount factors, and assembly costs. The other parts of the simulation remain the same as before.

When we investigated the computation time needed for our overall procedure (the entire Figure 1) under different problem sizes, the vector of the design variables was extended to include more design variables into the design space (with a mix of discrete and continuous variables). The vector of the consumer attributes, the mapping relationships, the cost model, and the configuration of the incumbent products were also extended to reflect these changes in the design space. When applying the gradient method to search for the new prices (the second block of Figure 1), we evaluated the gradient vector after each iteration. If the sum of the absolute values of all the elements in the gradient vector was less than or equal to 0.01, we considered the iteration process to be converged. When the focal manufacturer re-searches a set of design variables to maximize the product line profit based on the adjusted prices, we evaluated the earning of the product line. If the change in the earning levels was less than or equal to $0.01 million, we considered the cycling process to be converged. We also used tighter convergence criteria and found the criteria used here led to essentially the same results while greatly improving the computation efficiency. Table A1 presents a summary of the results across different problem sizes.

Table A1: Average Computation Time by Problem Sizes (Entire Figure 1)

| Number of products | Number of design variables | | |
|---|---|---|---|
| | Small (4, 8) | Medium (12,16) | Large (20,24) |
| Small (1-3) | 2.0 hrs | 2.4 hrs | 3.7 hrs |
| Medium (4-5) | 2.2 hrs | 6.4 hrs | 8.5 hrs |
| Large (6-8) | 2.8 hrs | 7.0 hrs | 10.4 hrs |

*DESCRIPTION OF THE COMPUTATIONAL ALGORITHMS GA, SA, AND ATC*

*Genetic Algorithm (GA)*

The biological process of natural selection provided the original inspiration of GA. It usually starts with a population of random solutions. The "fittest" members of this initial population survive and move on to produce the next generation of solutions. New solutions enter the population through a process of reproduction, crossover and mutation. This process continues until a given stopping condition is reached. This method has been widely used in product line design problems (e.g. Balakrishnan et al. 2004).

Our GA implementation started with an initial population of 200 randomly chosen product lines. The "fitness" of the product line was defined as the expected product line profit minus the penalty value(s) if some constraints were violated. The GA parameters for population replacement, crossover probability, mutation probability and selection type are set as 5, 90%, 5%, and stochastic universal selection. In our implementation, we consider GA to be converged if 30 generations passes without improvement in the best net objective value.

*Simulated Annealing (SA)*

The method of SA is derived from the cooling process a material undergoes after being removed from a heat source. As the material cools, the stress becomes more and more constrained until the material has reached a minimum energy state at a sufficiently low temperature. Belloni et al. (2008) first introduced this method to an optimal product line design problem.

The simulated annealing algorithm starts with a randomly chosen solution. If the point finds a better net objective value, it moves there. Otherwise, it moves to a new point with a probability of $e^{\frac{CV-NV}{T}}$, with $CV$ being the current value of the point, $NV$ being the value of the new point analyzed, and $T$ being the temperature.

In our SA implementation, the initial temperature was set to 100. We adopted a proportional cooling schedule in which the new temperature equaled to the old temperature times 0.9. The number of individual points (i.e. product line candidates) analyzed at each temperature was set to 50. We kept track of the best solution ever reached in each step. SA would stop if no improvement could be found in the net objective value.

*Analytical target cascading (ATC)*

Michalek et al. (2005, 2009) first introduced the ATC-based method into product design problems. Essentially, the ATC method decomposes the design of a product line into 1) a marketing subsystem that determines the consumer attributes of all the products in the product line, and 2) a set of engineering subsystems each searching for a design variable configuration that conforms to the engineering requirements and generates a vector of consumer attributes whose values are as close as possible to those required in the marketing subsystem. These marketing and engineering subsystems are solved iteratively until the convergence criterion is satisfied. Within our context, these subsystems can be expressed as follows:

Engineering subsystem: for each product $l$ ($l$= 1, …, $L_1$) in the focal manufacturer's product line, given $\mathbf{x}_{1l}^{M}$ (the values of consumer attributes required by the marketing subsystem), we select $\mathbf{y}_{1l}$ as follows:

(A14)
$$\begin{aligned}
&\min_{\mathbf{y}_{1l}} \quad \omega(\mathbf{x}_{1l}^{M} - \mathbf{x}_{1l}^{E}) \\
&s.t. \quad 1) \quad \sum_{h=1}^{H}\left(\max[0, g_h(\mathbf{y}_{1l}, \mathbf{pa})]\right) = 0 \\
&\qquad\qquad\qquad with \quad \mathbf{pa}_{LW} \le \mathbf{pa} \le \mathbf{pa}_{UP} \\
&\qquad\quad 2) \quad \left[\max\left(\left| f(_b(\mathbf{y}_{1l}, \mathbf{pa}) - f_b(\mathbf{y}_{1l}, \mathbf{pa}_0)\right|\right)\right] \le \Delta f_b^{\ J} \\
&\qquad\qquad\qquad with \quad b = \mathbf{1}, \ldots, B; \quad \mathbf{pa}_{LW} \le \mathbf{pa} \le \mathbf{pa}_{UP} \\
&where \quad \mathbf{x}_{1l}^{E} = f(\mathbf{y}_{1l})
\end{aligned}$$

We solve the engineering subsystem for each product in the product line in parallel.

Marketing subsystem: given $\mathbf{x}_{1l}^{E}$ (the values of consumer attributes achievable by the engineering subsystem), we solve the following optimization. In Equation (A15), the objective function reflects the designer's tradeoff between maximizing product line profit and minimizing the deviation from the values of consumer attributes achievable by engineering requirements.

(A15)
$$\begin{aligned}
&\max_{\{\mathbf{x}_{11}^{M}, \mathbf{x}_{12}^{M}, \ldots, \mathbf{x}_{1L_1}^{M}\}} \quad \pi_1^m = \left\{ \sum_{t=1}^{T} \left( \sum_{l=1}^{L_1} \left[ (w_{1l} - vc_{1l}) * m_{1l} * D_t \right] / (1+r)^t \right) - F_{1L_1} \right\} - \sum_{l=1}^{L_1} \omega(\mathbf{x}_{1l}^{M} - \mathbf{x}_{1l}^{E}) \\
&s.t. \quad MS_{1l} \times D_t \le W_{1l} \\
&\qquad\;\; \pi^r > \breve{\pi}^r
\end{aligned}$$

Following Michalek et al. (2009), we define $\omega(\mathbf{x}_{1l}^{M} - \mathbf{x}_{1l}^{E}) = \lambda^T(\mathbf{x}_{1l}^{M} - \mathbf{x}_{1l}^{E}) + \left\|\varpi \circ (\mathbf{x}_{1l}^{M} - \mathbf{x}_{1l}^{E})\right\|_2^2$, with $\left\|\varpi \circ (\mathbf{x}_{1l}^{M} - \mathbf{x}_{1l}^{E})\right\|_2^2 = \left\|\varpi \circ (\mathbf{x}_{1l}^{M} \circ \mathbf{x}_{1l}^{M} + \mathbf{x}_{1l}^{E} \circ \mathbf{x}_{1l}^{E} - 2(\mathbf{x}_{1l}^{M} \circ \mathbf{x}_{1l}^{E})\right\|_1$ and "$\circ$" being the Hadamard product. The values of the vectors $\boldsymbol{\lambda}$ and $\varpi$ are updated as follows:

1) Set κ=0, where κ denotes the number of loop iterations. Initialize $\boldsymbol{\lambda}$ and $\varpi$ (all the elements were initialized as 10,000).

2) Solve the engineering subsystem and the marketing subsystem above iteratively.

3) If $\max\left(\left\|\mathbf{x}_{1l}^{M,\kappa-1}-\mathbf{x}_{1l}^{M,\kappa}\right\|,\left\|\mathbf{x}_{1l}^{E,\kappa-1}-\mathbf{x}_{1l}^{E,\kappa}\right\|\right)\leq\sigma$ (set as 0.1 in our implementation), then stop. Otherwise, update λ and $\varpi$ by setting $\lambda^{\kappa+1}=\lambda^{\kappa}+\varpi\circ(\mathbf{x}_{1l}^{M,\kappa}-\mathbf{x}_{1l}^{E,\kappa})$, $\varpi^{\kappa+1}=\eta\varpi^{\kappa}$ (with $\eta$ =1) and go to step 2).

**Web Appendix D: Implementation Details of the Empirical Application**

The design space is defined as follows: motor type (a discrete variable between 1 and 10), gear box type (a discrete variable between 1 and 6), gear ratio (a continuous variable between 3.5 and 5.0), switch type (a discrete variable between 1 and 4), and housing type (a discrete variable with 2 levels).

Five engineering parameters were identified to represent the uncontrollable variations in the product's usage environment. Table A2 provides their nominal, lower, and upper bound values.

Table A2: Engineering parameters: Nominal, Lower, and Upper Bound Values

| Engineering Parameter | Nominal | Lower bound | Upper bound |
|---|---|---|---|
| Source Voltage (V) | 110 | 95 | 125 |
| Ambient Temperature (C) | 25 | -10 | 50 |
| User Load Bias (lb) | 6 | 3 | 9 |
| Fan CFM Degradation (%) | 0 | 0 | 80 |
| Application Torque Adjustment (%) | 0 | -20 | 20 |

After identifying the set of consumer attributes and attribute levels, we employed orthogonal conjoint design to construct the 18 choice scenarios in the conjoint experiment. Each choice occasion included two products and a "no-choice" option. The respondents were asked to imagine that he/she is on the market for a new tool and the two products shown in each choice occasion were the only products available at the store. We also asked the respondents to assume that the two products were equally equipped for any product features not included in the questionnaire. In addition to the conjoint questions, the respondents were also asked to provide some demographic information.

Because the design of our conjoint study follows the standard procedure used in the literature, our exploratory research identified a set of consumer attributes deemed as the most important at the aggregate level. It is possible that some consumers care about attributes not included in our conjoint study. It is also likely that some attributes in our conjoint study are not important for some consumers. Future research may include an adaptive framework in which the attributes shown to the consumer are personalized to reflect such individual differences.

A finite mixture Bayesian model was used to estimate the conjoint part-worths. Given that we did not have prior belief of whether there is a linear relationship between power amp and consumer preference, we decided to estimate its conjoint part-worth at the three discrete levels. For the same reason, the conjoint part-worths of product life were also estimated at the three discrete levels. The covariate estimates are provided in Table A3. Note that the respondent could select more than one trade if he/she participates in multiple trades. It appears that electricians and carpenters are more likely to belong to segment 2. Additionally, tall people and people with large hands have a higher probability of being members of segment 2, which probably explains the fact that consumers in segment 1 prefer a small girth while consumers in segment 2 prefer a large girth. Finally, the age group of 46-55 is over-represented in segment 2. Given that this age group tends to be the wealthiest among all the age groups, it probably explains why segment 2 consumers are less price-sensitive.

Table A3: Covariate Estimates for Finite Mixture Conjoint Estimation

| | Trade | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Metal fabricator | HVAC | Welder | Maintenance | Concrete | Plumber | Electrician | Carpenter | Others |
| Segment 1 | 0.756 | 0.843 | 0.873 | 0.101 | 0.848 | 0.651 | -0.147 | -4.175 | -0.046 |
| Segment 2 | -0.756 | -0.843 | -0.873 | -0.101 | -0.848 | -0.551 | 0.147 | 4.1754 | 0.046 |

| | Glove size | | | Height | | | |
|---|---|---|---|---|---|---|---|
| | Small | Medium | Large | <= 5’7” | 5’8” to 5’11” | 6’ to 6’3’’ | >= 6’4” |
| Segment 1 | 3.275 | 0.364 | -3.639 | 0.263 | 0.251 | -0.100 | -0.413 |
| Segment 2 | -3.275 | -0.364 | 3.639 | -0.263 | -0.251 | 0.100 | 0.413 |

| | Age | | | |
|---|---|---|---|---|
| | 18 to 35 | 36 to 45 | 46 to 55 | 56 or older |
| Segment 1 | 0.342 | 0.180 | -0.637 | 0.116 |
| Segment 2 | -0.342 | -0.180 | 0.637 | -0.116 |

Table A4 gives the consumer attribute specifications of the competitive products. Lab testing was used to obtain their estimated costs and the ranges of variations related to each product’s power amp and product life. The procedure described in our consumer preference model was then used to derive the utility of these products and the market share estimates. In order to assess the external validity of our consumer preference model, we compared these estimates with the actual market share data provided by our industrial partner. Using the minimum discrimination information statistic (*MDI*) (Kullback, Kupperman, and Ku 1962), we could not reject the null hypothesis that the predicted and the observed market shares are realizations from the same underlying multinomial distribution (*MDI* = 7.23, *d. f.* =4, *p*-value = 0.12).

Table A4 also provides the equilibrium retail and wholesale prices after the entry of the final product line. Interestingly, some incumbents chose to increase wholesale prices while others chose to decrease wholesale prices. This is consistent with the findings of Hauser and Shugan (1983) and Luo et al. (2007). Depending on the distribution of the consumers’ tastes and the market segment the new product line is targeting, a price increase could be optimal for some incumbents.

Table A4: Competitive Products

| brand | power amp (nominal) | product life (nominal) | switch | girth | retail price (before) | wholesale price (before) | market share (before) | retail price (after) | wholesale price (after) | market share (after) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 9 | 110 | 3 | 2 | 99.00 | 79.12 | 3.66% | 127.65 | 102.35 | 2.39% |
| 1 | 5.5 | 90 | 2 | 1 | 59.00 | 49.44 | 31.82% | 72.92 | 57.59 | 25.74% |
| 2 | 12 | 150 | 1 | 1 | 129.00 | 103.87 | 4.32% | 128.18 | 106.76 | 2.82% |
| 2 | 8.5 | 105 | 3 | 2 | 89.00 | 70.50 | 38.66% | 86.00 | 66.69 | 25.35% |
| 3 | 6 | 80 | 1 | 1 | 79.00 | 63.38 | 8.74% | 75.27 | 57.95 | 5.71% |
| No-choice | | | | | | | 12.80% | | | 8.36% |

In the product line optimization, the production capacity constraint was set as follows: when there is a single product in the product line, the production volume constraint is 400,000 units; when there are two products in the product line, the production volume constraint is 300,000 units per product; when there are three products in the product line, the production volume constraint is 250,000 units per product. Given that new products in this category are typically introduced every four or five years, the focal manufacturer's goal is to maximize its discounted long-term profit over a five-year horizon at a discount rate of 3% (this discount rate was chosen based on the average consumer price indices published by the Bureau of Labor Statistics in the last 5 five years). The discounted long-term category profit of the retailer over a five-year horizon before the entry of the new product line was estimated to be $233.6 million. If the retailer does not benefit from an increase in its category profit with the introduction of the new product line, we will penalize the product line in the optimization. The penalty value and the convergence criteria used in the empirical application are the same as the ones used in the simulation study. We also tried to use tighter convergence criteria and the final earning levels did not change much.

Some robustness checks could be conducted to evaluate how sensitive the final earning level is to the model parameter specifications (such as discount factor, capacity constraints, fixed cost estimates etc). Note that because multiple product lines may generate identical (or highly similar) profits and the focal manufacturer's ultimate goal is to maximize its profit, we evaluated the robustness of the final solution by the earning level associated with the product line rather than the closeness of the solutions. We performed the following robustness checks as an example. Similar checks could be conducted on other parameters. We first ran the optimization multiple times when the discount factor related to girth type varied from 0.2 to 0.4 with a step size of 0.04 (the original discount factor is 0.3). The product lines with 3 products remained to be the most profitable. Additionally, the earning of the focal manufacturer remained within the range of $51.2 to $53.8 million under such variations. A similar robustness check was conducted on capacity constraint. Let the capacity constraint used in the optimization being *W*, we varied the level of the constraint from (*W*- 200,000) to (*W*+200,000) with a step size of 8,000 units. The product lines with 3 products were still the most profitable. And the earning levels varied from $50.7 to $53.5 million. As evident, despite the fact that the earning of the final product line was influenced by the specifications of the model parameters, the final earning level obtained in our optimization was not overly sensitive to these parameter specifications.

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