Chapter 5
Discrete Random Variables

Chapter Outline

Definition 5.1: Random Variable

A **random variable** is a quantitative variable whose value depends on chance.

Definition 5.2: Discrete Random Variable

A **discrete random variable** is a random variable whose possible values can be listed. In particular, a random variable with only a finite number of possible values is a discrete random variable.

Siblings x	Frequency f	Relative frequency
0	8	0.200
1	17	0.425
2	11	0.275
3	3	0.075
4	1	0.025
	40	1.000

$P(X=1) = 0.425$

$P(X=3) = 0.075$

- $\{X = x\}$ denotes the event that the random variable X equals x.
- $P(X = x)$ denotes the probability that the random variable X equals x.

Definition 5.3: Probability Distribution and Probability Histogram

Probability distribution: A listing of the possible values and corresponding probabilities of a discrete random variable, or a formula for the probabilities.

Probability histogram: A graph of the probability distribution that displays the possible values of a discrete random variable on the horizontal axis and the probabilities of those values on the vertical axis. The probability of each value is represented by a vertical bar whose height equals the probability.

Siblings *x*	Frequency *f*	Relative frequency
0	8	0.200
1	17	0.425
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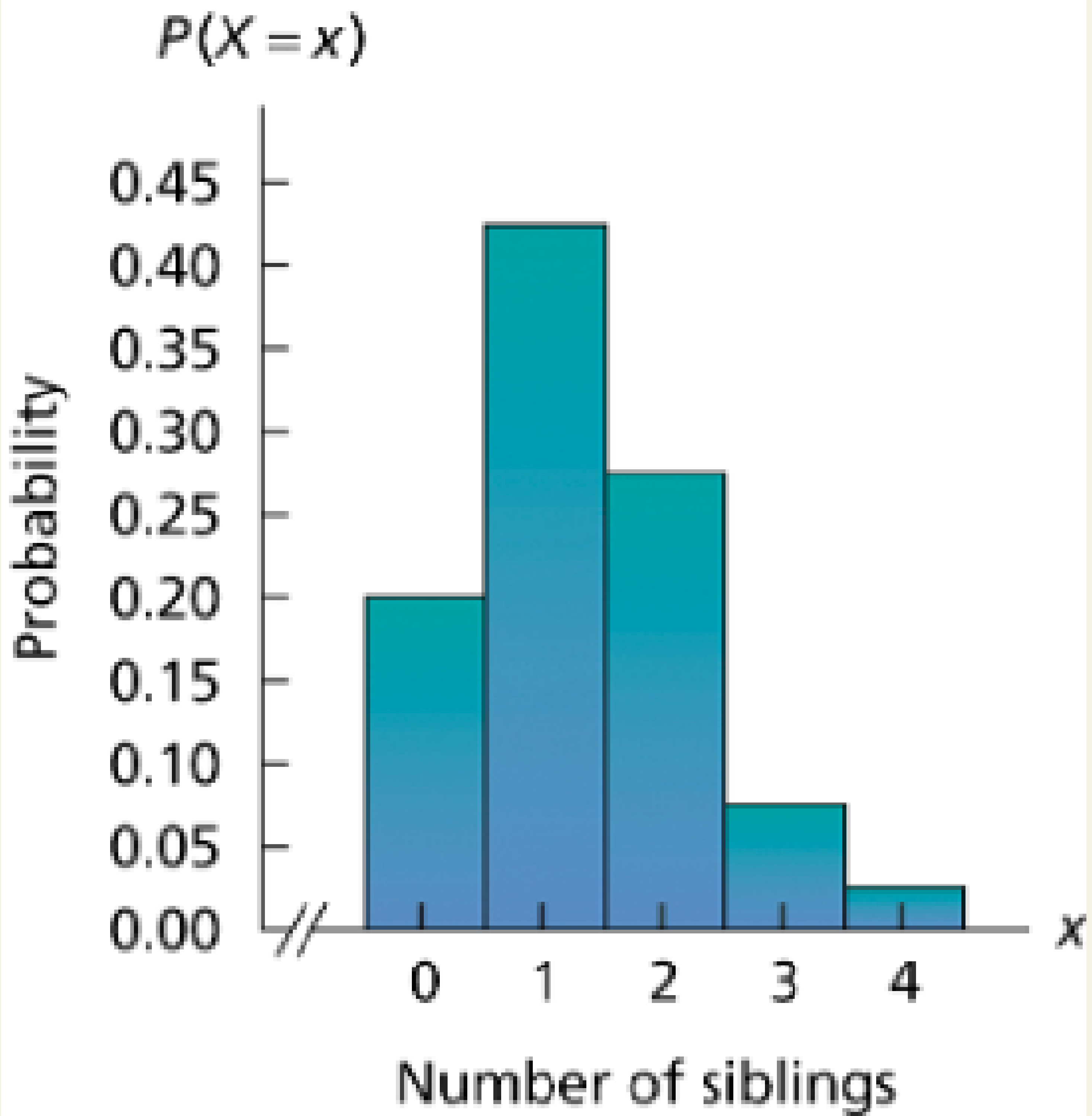
a. Determine the probability distribution of the random variable X.

b. Construct a probability histogram for the random variable X.



a. Use random-variable notation to represent the event that exactly two heads are tossed.

b. Determine $P(X = 2)$.

c. Find the probability distribution of X.

d. Use random-variable notation to represent the event that at most two heads are tossed.

e. Find $P(X \le 2)$.

HHH THH

HHT THT

HTH TTH

HTT TTT

$(X = x)$

a) $(X = 2)$

b) $P(X = 2) = \frac{3}{8} = 0.375$

$P(x=0) \rightarrow 1/8 = 0.125$

$P(x=1) \rightarrow 3/8 = 0.375$

$P(x=2) \rightarrow 3/8 = 0.375$

$P(x=3) \rightarrow 1/8 = 0.125$

H	H	H	T	H	H
H	H	T	T	H	T
H	T	H	T	T	H
H	T	T	T	T	T

a. Use random-variable notation to represent the event that exactly two heads are tossed.

b. Determine $P(X = 2)$.

c. Find the probability distribution of X.

d. Use random-variable notation to represent the event that at most two heads are tossed.

e. Find $P(X \leq 2)$.

HHH THH
HHT THT
HTH TTH
HTT TTT

d) $(X \leq 2)$

e) $P(X \leq 2) = 0.875$

Definition 5.4: Mean of a Discrete Random Variable

The **mean of a discrete random variable** X is denoted μ_X or, when no confusion will arise, simply μ. It is defined by

$$\mu = \Sigma x P\left(X = x\right).$$

The terms **expected value** and **expectation** are commonly used in place of the term *mean*.

x	$P(X = x)$
0	0.029
1	0.049
2	0.078
3	0.155
4	0.212
5	0.262
6	0.215

$$\mu = \sum x P(X=x) =$$
$$4.118$$

$0(0.029) = 0$

$1(0.049) = 0.049$

$2(0.078) = 0.156$

$3(0.155) = 0.465$

$4(0.212) = 0.848$

$5(0.262) = 1.310$

$6(0.215) = 1.290$

Definition 5.5: Standard Deviation of a Discrete Random Variable

The **standard deviation of a discrete random variable** X is denoted σ_X or, when no confusion will arise, simply σ. It is defined as

$$\sigma = \sqrt{\Sigma(x - \mu)^2 P(X = x)}.$$

The standard deviation of a discrete random variable can also be obtained from the computing formula

$$\sigma = \sqrt{\Sigma x^2 P(X = x) - \mu^2}.$$

x	x^2	$P(X=x)$
0	0	0.029
1	1	0.049
2	4	0.078
3	9	0.155
4	16	0.212
5	25	0.262
6	36	0.215

$x^2(P(X=x))$

0

0.049

0.312

1.395

3.392

6.550

7.740

$\sum x^2 P(X=x) = 19.438$

$\sigma = \sqrt{19.438 - 4.118^2}$

$\sigma = 1.575$

5.31 Space Shuttles. The random variable X is the crew size of a randomly selected shuttle mission between April 12, 1981 and July 8, 2011. Its probability distribution is as follows.

x	2	4	5	6	7	8
$P(X = x)$	0.030	0.022	0.267	0.207	0.467	0.007
$xP(X=x)$	0.060	0.088	1.335	1.242	3.269	0.056
$x^2P(X=x)$	0.120	0.352	6.675	7.452	22.883	0.448

$\mu = 6.050$

$\sigma = \sqrt{37.930 - 6.050^2}$

$= 1.152$

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \boxed{{}_nC_r}$$

$$\binom{7}{3} \rightarrow {}_7C_3 = \boxed{nCr\,(7,3)}$$

7 nCr 3

Mortality Mortality tables enable actuaries to obtain the probability that a person at any particular age will live a specified number of years. Insurance companies and others use such probabilities to determine life-insurance premiums, retirement pensions, and annuity payments.

According to tables provided by the National Center for Health Statistics in *Vital Statistics of the United States*, a person of age 20 years has about an 80% chance of being alive at age 65 years. Suppose three people of age 20 years are selected at random.

a = alive

d = dead

aaa   ada   daa   dda

aad   add   dad   ddd

$P(aaa) = (.8)(.8)(.8) = .512$

$P(aad) = (.8)(.8)(.2) = .128$

$P(ada) = .128$

$P(add) = .032$

$P(daa) = .128$

$P(dad) = .032$

$P(dda) = .032$

$P(ddd) = .008$

$P(a) = .80$

$P(d) = .20$

$P(x=2) = .384$

$.128 + .128 + .128$

$P(x=0) = .008$

$P(x=1) = .096$

$P(x=2) = .384$

$P(x=3) = .512$

Formula 5.1: Binomial Probability Formula

Let X denote the total number of successes in n Bernoulli trials with success probability p. Then the probability distribution of the random variable X is given by

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n.$$

(nCr)

The random variable X is called a **binomial random variable** and is said to have the **binomial distribution** with parameters n and p.

a. exactly two.

b. at most one.

c. at least one.

d. Determine the probability distribution of the number alive at age 65.

$P = .80$

$n = 3$

a) ${}_3C_2(.80)^2(1-.80)^1 = .384$

b) $P(x=0) + P(x=1) = 0.104$

c) $1 - P(x=0) = 0.992$

Formula 5.2: Mean and Standard Deviation of a Binomial Random Variable

The mean and standard deviation of a binomial random variable with parameters n and p are

$$\mu = np \quad \text{and} \quad \sigma = \sqrt{np(1-p)},$$

respectively.

$n = 3$

$p = .80$

$\mu = 3(.80) = 2.4$

$\sigma = \sqrt{3(.80)(.20)} = 0.693$

5.78 Gestation Periods. The probability is 0.314 that the gestation period of a woman will exceed 9 months. In six human births, what is the probability that the number in which the gestation period exceeds 9 months is