

For problems number 1 – 6 find the critical numbers and identify if they are minimum or maximum then identify which interval the function is concave up or concave down.

1) $y = x^3 + 3x^2$

2) $y = \frac{x-x^2}{2-3x+x^2}$

3) $y = (x - 4)\sqrt[3]{x}$

4) $y = \sin^3 x$

5) $y = \frac{1}{1+e^{-x}}$

6) $y = x(\ln x)^2$

Use l'Hospital's rule (if needed) to find the limit given in questions 7-8

7) $\lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x - 4}$

8) $\lim_{x \rightarrow 0} \sin 5x \csc 3x$

9) A box with a square base and an open top must have a volume of $32,000\text{cm}^3$. Find the dimensions of the box that minimize the amount of material used.

10) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.