One-Mean z-Test

Purpose To perform a hypothesis test for a population mean, μ

Assumptions

1. Simple random sample

2. Normal population or large sample

3. σ known

Step 1 **The null hypothesis is** $H_0{:}\mu = \mu_0$ **, and the alternative hypothesis is**

$$H_a{:}\mu \neq \mu_0 \qquad H_a{:}\mu < \mu_0 \qquad H_a{:}\mu > \mu_0$$
$$\text{or} \qquad \text{or}$$
$$\textbf{(Two tailed)} \qquad \textbf{(Left tailed)} \qquad \textbf{(Right tailed)}.$$

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

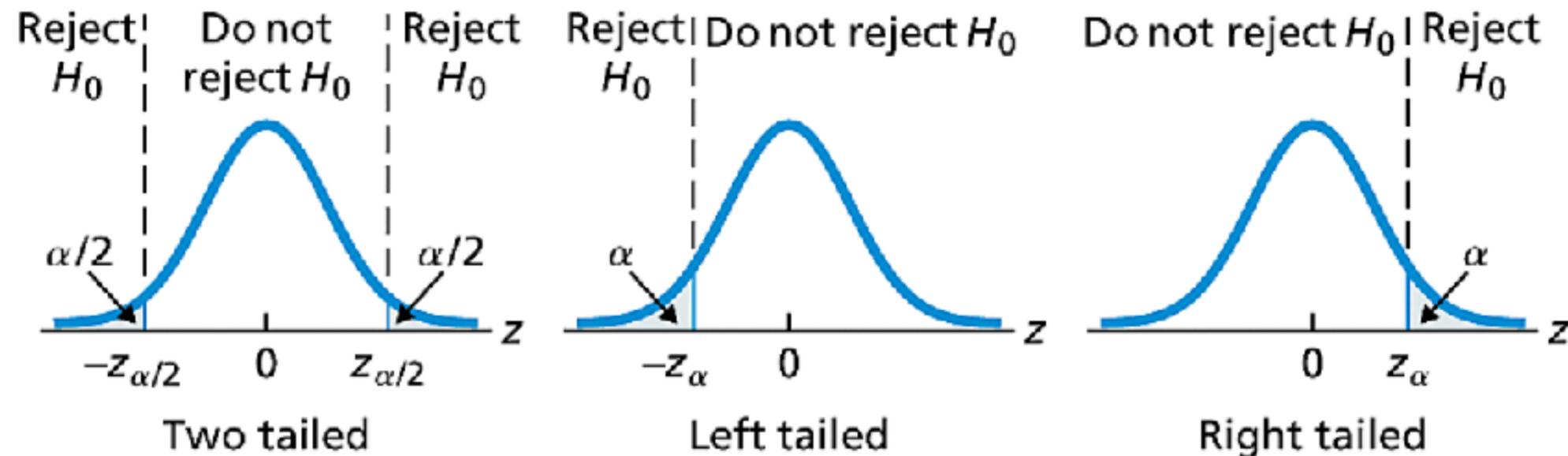$$z = \frac{\bar{x} - \mu_0}{\alpha / \sqrt{n}}$$

and denote that value z_0.

CRITICAL-VALUE APPROACH

Step 4 The critical value(s) are

$$\pm z_{\alpha/2} \quad \text{or} \quad -z_{\alpha} \quad \text{or} \quad z_{\alpha} \quad .$$
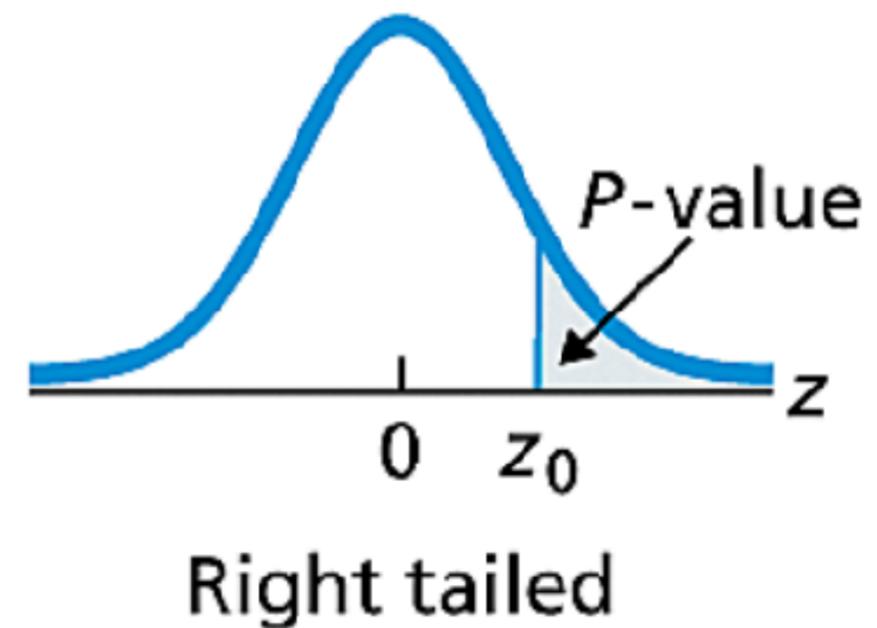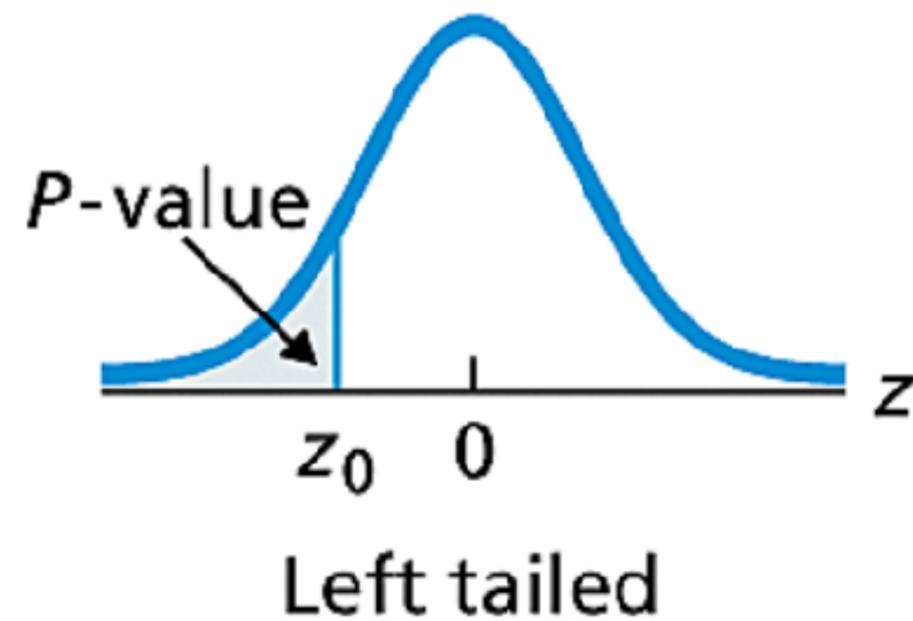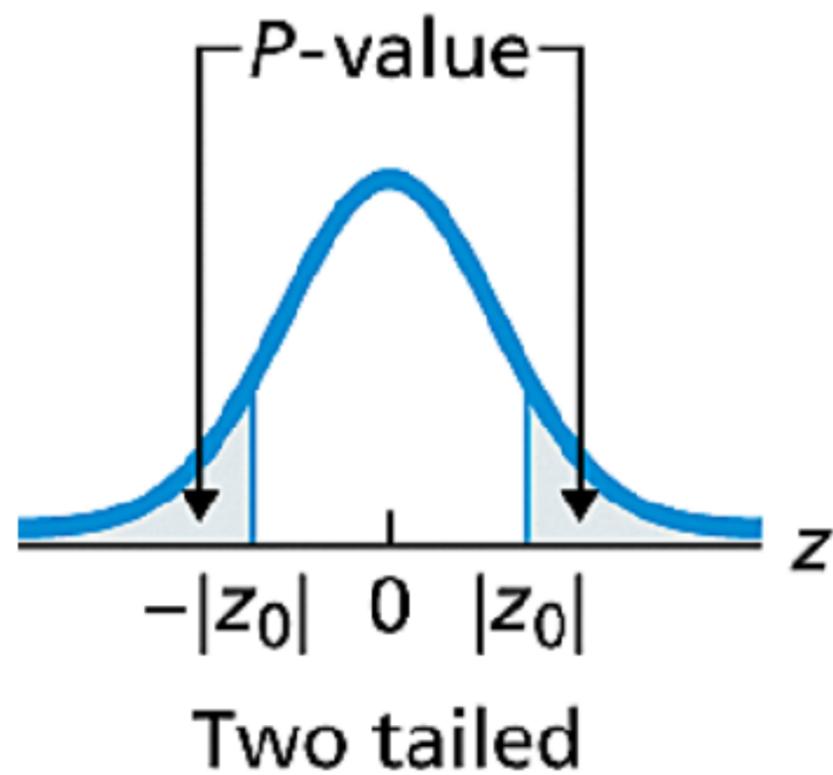$$\text{(Two tailed)} \qquad \text{(Left tailed)} \qquad \text{(Right tailed)}$$

Use Table II to find the critical value(s).

Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

P-VALUE APPROACH

Step 4 Use **Table II** to obtain the *P*-value.

| Two tailed | Left tailed | Right tailed |

Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Key Fact 9.7: When to Use the One-Mean z-Test

- For small samples—say, of size less than 15—the z-test should be used only when the variable under consideration is normally distributed or very close to being so.

- For samples of moderate size—say, between 15 and 30—the z-test can be used unless the data contain outliers or the variable under consideration is far from being normally distributed.

- For large samples—say, of size 30 or more—the z-test can be used essentially without restriction. However, if outliers are present and their removal is not justified, you should perform the hypothesis test once with the outliers and once without them to see what effect the outliers have. If the conclusion is affected, use a different procedure or take another sample, if possible.
- If outliers are present but their removal is justified and results in a data set for which the z-test is appropriate (as previously stated), the procedure can be used.

A half-century ago, the average (U.S.) woman in her 20s was 62.6 inches tall. The heights, in inches, of a random sample of 25 of today's women in their 20s is presented in Table 9.9.
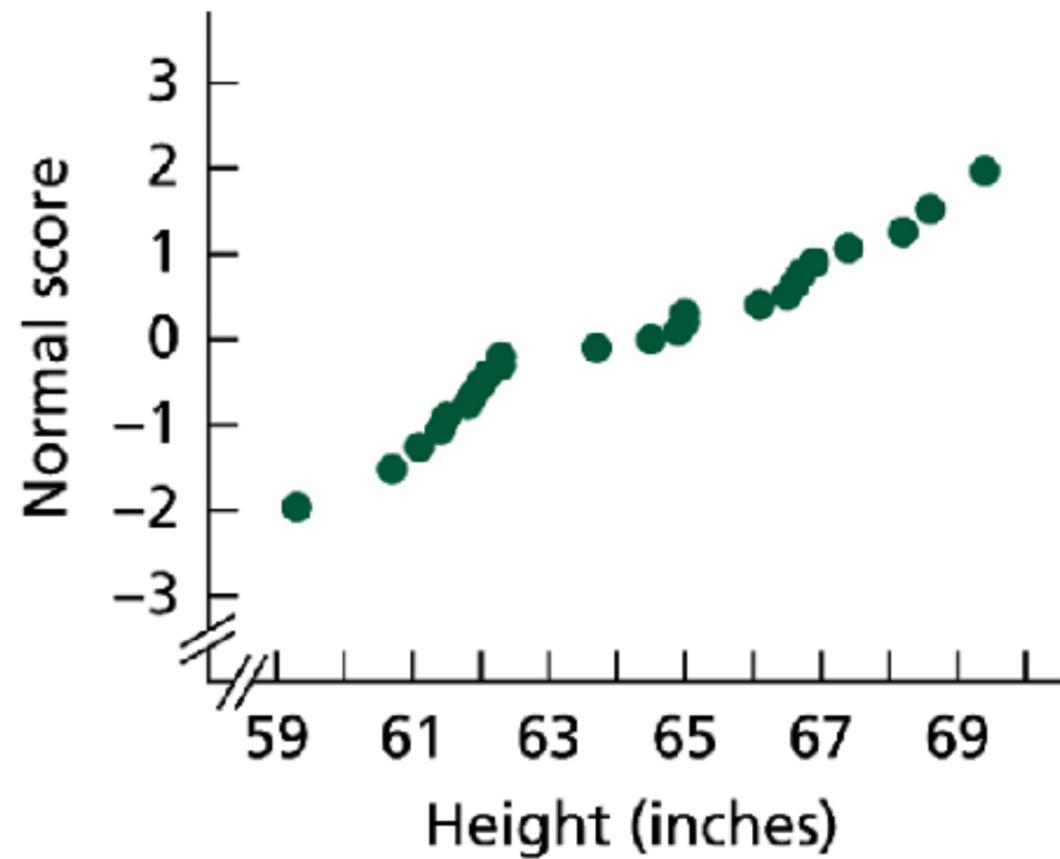
At the 1% significance level, do the data provide sufficient evidence to conclude that the mean height of today's women in their 20s is greater than the mean height of women in their 20s a half-century ago? Assume that the population standard deviation of heights for today's women in their 20s is 2.88 inches.

$$\mu_0 = 62.6, \sigma = 2.88, \text{ and } n = 25. \qquad \bar{x} = 64.24.$$

$H_0 \Rightarrow \mu_0 = 62.6 \qquad H_a: \mu_a > \mu_0,$
$\qquad\qquad\qquad\qquad\qquad$ Right tail

$\mu_0 = 62.6, \sigma = 2.88,$ and $n = 25.$ $\bar{x} = 64.24.$

1% sig. level
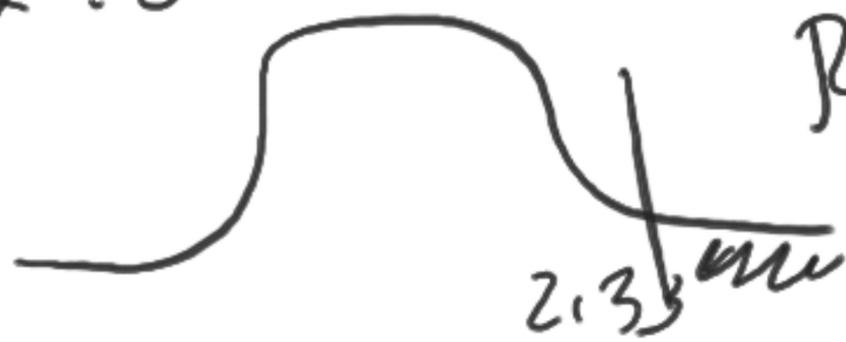
$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{64.24 - 62.6}{2.88/\sqrt{25}}$$

$$\boxed{Z = 2.85}$$

Critical Value
2.33
$2.85 > 2.33$
Reject

2.33

P-value

.9978

Reject

.99 .9978

A simple random sample of 18 adults with incomes below the poverty level gives the daily calcium intakes shown in Table 9.10. At the 5% significance level, do the data provide sufficient evidence to conclude that the mean calcium intake of all adults with incomes below the poverty level is less than the RAI of 1000 mg? Assume that $\sigma = 188$ mg.
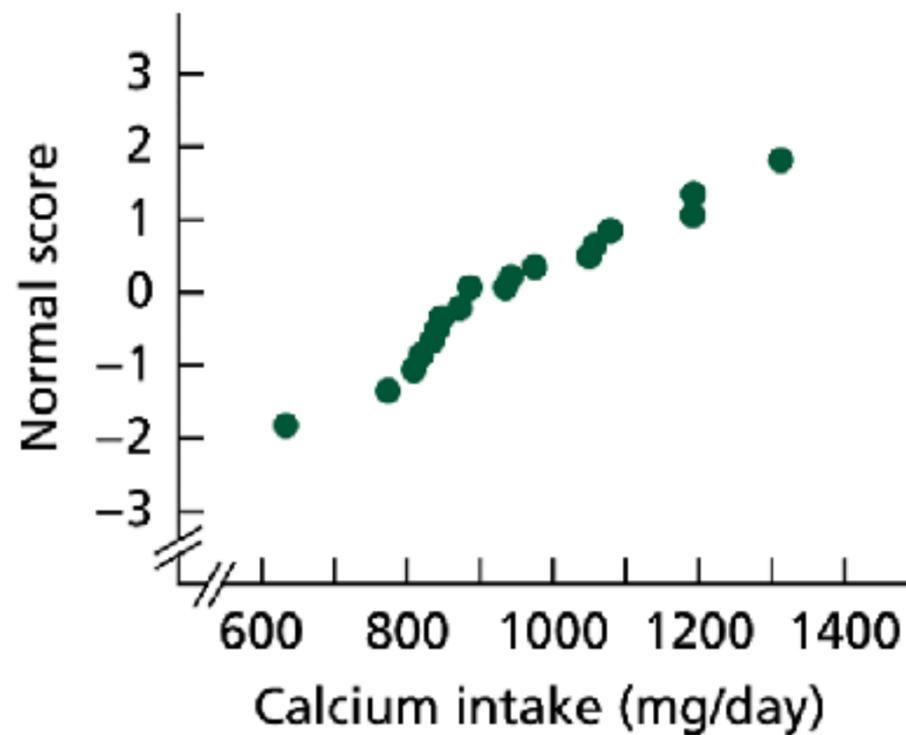
$$\mu_0 = 1000, \sigma = 188, \text{ and } n = 18. t\ \bar{x} = 947.4.$$

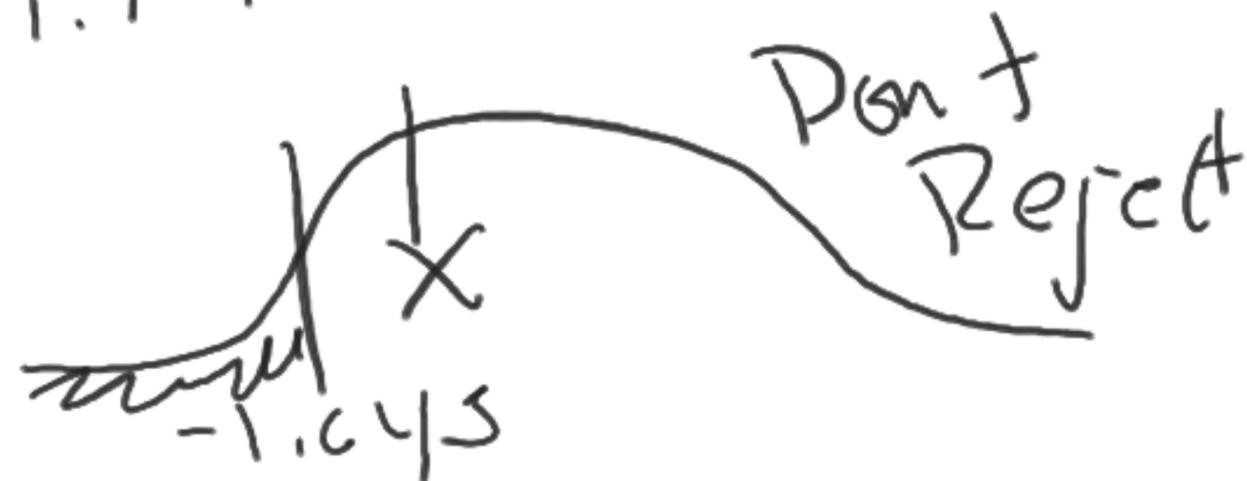$$H_0 \Rightarrow \mu_0 = 1000 \qquad H_a \Rightarrow \bar{x} < \mu_0$$
$$\text{Left tail}$$

$$\alpha = 5\%$$

Normal score vs. Calcium intake (mg/day)

$\mu_0 = 1000$, $\sigma = 188$, and $n = 18$. $\bar{x} = 947.4$.

$H_0 \Rightarrow \mu_0 = 1000$ $H_a \Rightarrow \bar{x} < \mu_0$

Left tail

$\alpha = 5\%$
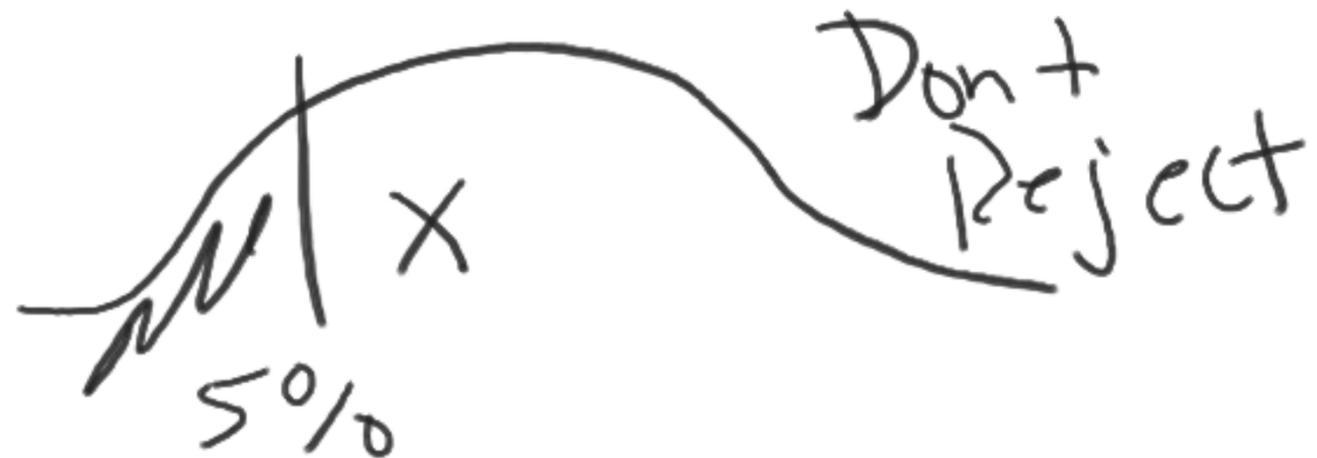
$$z = \frac{947.4 - 1000}{188/\sqrt{18}} = -1.19$$

Critical

$z = -1.19$ $5\% \Rightarrow -1.645$

Don't Reject

-1.645

P-Value

$z = -1.19 \Rightarrow .1170 \Rightarrow 11.7\%$

Don't Reject

5%

One common estimate of mean top speed for cheetahs is 60 mph. Table 9.11 📄

gives the top speeds, in miles per hour, for a sample of 35 cheetahs.
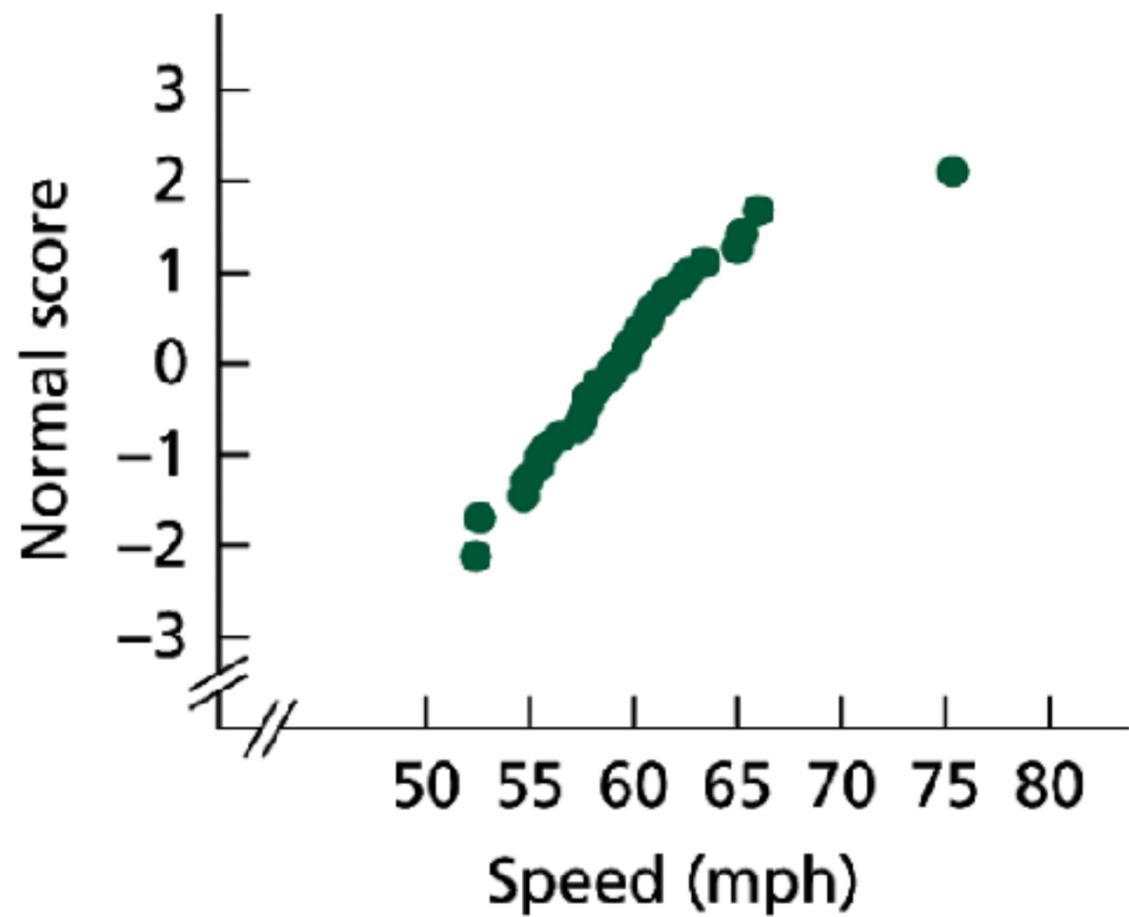
At the 5% significance level, do the data provide sufficient evidence to conclude

that the mean top speed of all cheetahs differs from 60 mph? Assume that the

population standard deviation of top speeds is 3.2 mph.

$$\mu_0 = 60, \sigma = 3.2, \text{ and } n = \overset{34}{\cancel{35}}. \; \bar{x} = 59.526.$$

$$H_0 \Rightarrow \mu_0 = 60 \qquad H_a \Rightarrow \bar{x} \neq \mu_0$$

$$5\% \; SL$$

57.3	57.5	59.0	56.5	61.3
57.6	59.2	65.0	60.1	59.7
62.6	52.6	60.7	62.3	65.2
54.8	55.4	55.5	57.8	58.7
57.8	60.9	~~75.3~~	60.6	58.1
55.9	61.6	59.6	59.8	63.4
54.7	60.2	52.4	58.3	66.0

$\mu_0 = 60$, $\sigma = 3.2$, and $n = \cancel{35}$ 34. $\bar{x} = 59.526$.

$H_0 \Rightarrow \mu_0 = 60$ $H_a \Rightarrow \bar{x} \neq \mu_0$
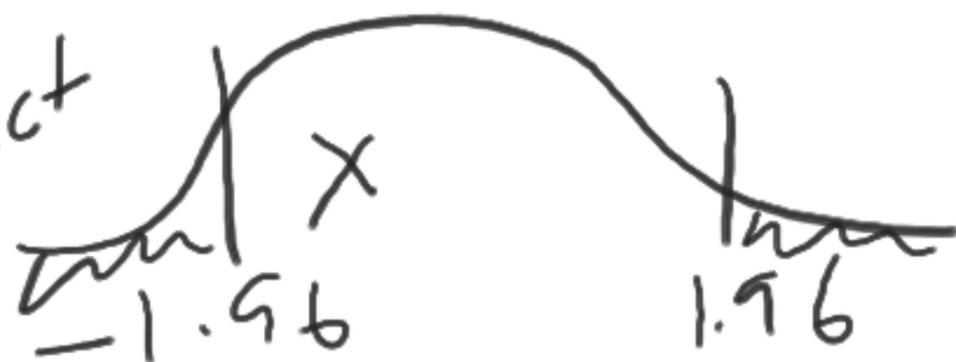
5% S.L.

$$Z = \frac{59.526 - 60}{3.2/\sqrt{34}} = -0.86$$
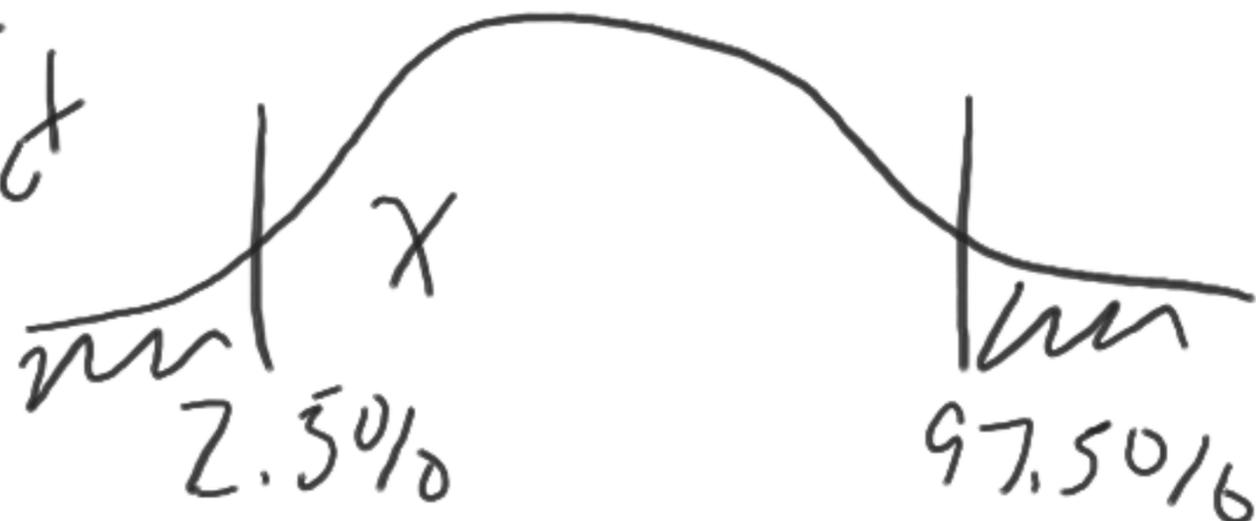

Critical $\quad \alpha = .05/2$

$-0.86 \qquad \pm 1.96$

Don't
Reject

$-1.96 \qquad 1.96$

P-Value $\qquad -0.86 \rightarrow 19.49$

Don't
Reject

2.50%   97.50%

For example, the manufacturer of a new car, the Orion, claims that a typical car gets 26 miles per gallon. We think that the gas mileage is less. To test our suspicion, we perform the hypothesis test

$$H_0: \mu = 26 \text{ mpg (manufacturer's claim)}$$
$$H_a: \mu < 26 \text{ mpg (our suspicion)},$$

We take a random sample of 1000 Orions and find that their mean gas mileage is 25.9 mpg. Assuming $\sigma = 1.4$ mpg, the value of the test statistic for a z-test is $z = -2.26$. This result is statistically significant at the 5% level. Thus, at the 5% significance level, we reject the manufacturer's claim.

9.87 Serving Time. According to the Bureau of Crime Statistics and Research of Australia, as reported on *Lawlink*, the mean length of imprisonment for motor-vehicle-theft offenders in Australia is 16.7 months. One hundred randomly selected motor-vehicle-theft offenders in Sydney, Australia, had a mean length of imprisonment of 17.8 months. At the 5% significance level, do the data provide sufficient evidence to conclude that the mean length of imprisonment for motor-vehicle-theft offenders in Sydney differs from the national mean in Australia? Assume that the population standard deviation of the lengths of imprisonment for motor-vehicle-theft offenders in Sydney is 6.0 months.

$H_0 \Rightarrow \mu = 16.7$

$H_a \Rightarrow \mu \neq 16.7$ two tail

$\mu = 16.7 \qquad \sigma = 6 \qquad n = 100 \qquad \bar{x} = 17.8$
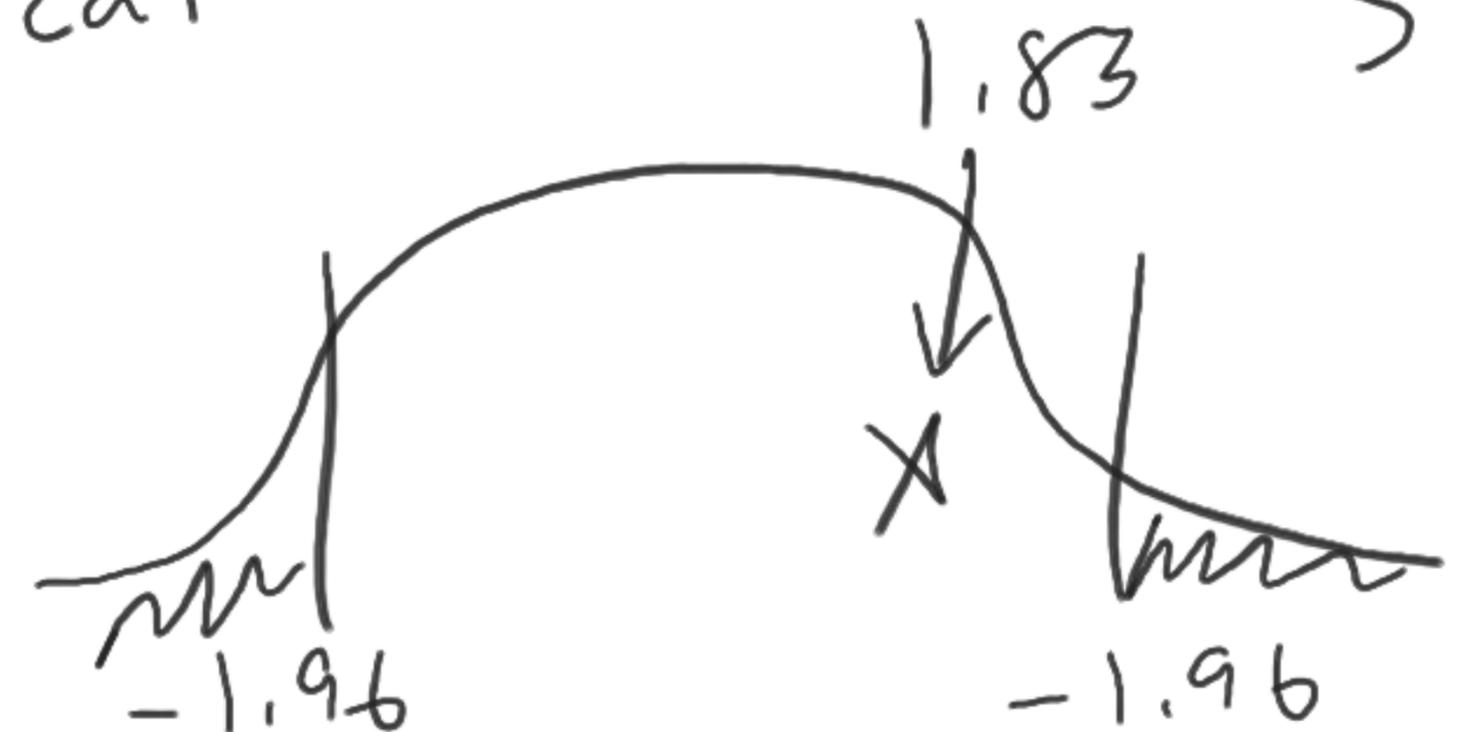
$H_0 \Rightarrow \mu = 16.7$ $H_a \Rightarrow \mu \neq 16.7$ 5%
two tail

$\mu = 16.7$ $\sigma = 6$ $n = 100$ $\bar{x} = 17.8$

$$Z = \frac{17.8 - 16.7}{6/\sqrt{100}} = 1.83$$

Critical

1.83 $5\% = \pm 1.96$

Don't
Reject

-1.96 -1.96