$f(x) = \sin x \qquad f'(x) =$

$$\frac{d}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$\sin(x+h) = \sin(x)\cos(h) + \cos(x)\sin(h)$$

$$\lim_{h \to 0} \frac{\boxed{\sin(x)\cos(h)} + \cos(x)\sin(h) \boxed{- \sin(x)}}{h}$$

$$\lim_{h \to 0} \frac{\boxed{\sin(x)(\cos(h) - 1)} + \cos(x)\sin(h)}{h}$$

$$\lim_{h \to 0} \frac{\boxed{\sin(x)(\cos(h)-1)} + \cos(x)\sin(h)}{h}$$

$$\sin(x)\boxed{\lim_{h \to 0} \frac{\cos(h)-1}{h}} + \cos(x)\boxed{\lim_{h \to 0} \frac{\sin(h)}{h}}$$

$$(0) \qquad\qquad\qquad\qquad (1)$$

$$f(x) = \sin(x) \qquad\qquad f'(x) = \cos x$$

$$\boxed{\frac{d}{dx}(\sin x) = \cos x}$$

$$\boxed{\frac{d}{dx}(\cos x) = -\sin x}$$

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$$f(x) = \tan x = \frac{\sin x}{\cos x} \qquad f'(x) = \sec^2(x)$$

$$n(x) = \sin x \qquad n'(x) = \cos x$$

$$d(x) = \cos x \qquad d'(x) = -\sin x$$

$$f'(x) = \frac{(\cos x)(\cos x) - (-\sin x)(\sin x)}{\cos^2(x)}$$

$$\cos^2 x + \sin^2 x = 1$$

$$f'(x) = \frac{1}{\cos^2 x} = \sec^2 x$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$y = \sin\theta \cos\theta$

$y' =$

$f(x) = \sin x$ $\quad f'(x) = \cos x$

$g(x) = \cos x$ $\quad g'(x) = -\sin x$

$y' = (\cos\theta)(\cos\theta) + (-\sin\theta)(\sin\theta)$

$$\boxed{y' = \cos^2\theta - \sin^2\theta}$$

$$f(x) = \frac{\sec x}{1 + \tan x} \qquad f'(x) =$$

$$n(x) = \sec x \qquad n'(x) = \sec x \tan x$$

$$d(x) = 1 + \tan x \qquad d'(x) = \sec^2 x$$

$$f'(x) \frac{(\sec x \tan x)(1 + \tan x) - (\sec^2 x)(\sec x)}{(1 + \tan x)^2}$$

$$f'(x) \quad \frac{(\sec x \tan x)(1+\tan x) - (\sec^2 x)(\sec x)}{\boxed{(1+\tan x)^2}}$$

$$\sec x \left(\tan x + \cancel{\tan^2 x} \quad -\sec^2 x\right)$$

$$-(1 + \cancel{\tan^2 x})$$

$$f'(x) = \frac{\sec x(\tan x - 1)}{(1+\tan x)^2}$$

$$g(\theta) = e^{\theta}(\tan\theta - \theta)$$

$m(\theta) = e^{\theta}$ $\qquad$ $m'(\theta) = e^{\theta}$

$n(\theta) = (\tan\theta - \theta)$ $\qquad$ $n'(\theta) = \sec^2\theta - 1 = \tan^2\theta$

$$g'(\theta) = e^{\theta}(\tan\theta - \theta) + e^{\theta}(\tan^2\theta)$$

$$= e^{\theta}(\tan^2\theta + \tan\theta - \theta)$$

$$f(t) = te^t \cot t$$

$m(x) = xe^x \qquad m'(x) = e^x + xe^x$

$n(x) = \cot x \qquad n'(x) = -\csc^2 x$

$e^x \cot x + xe^x \cot x - xe^x \csc^2 x$

$f'(x) = (e^x + xe^x)(\cot x) + (-\csc^2 x)(xe^x)$

$f'(x) = e^x(\cot x + x\cot x - x\csc^2 x)$

EXAMPLE 3 An object fastened to the end of a vertical spring is stretched 4 cm beyond its rest position and released at time $t = 0$. (See Figure 4 and note that the downward direction is positive.) Its position at time t is

$$s = f(t) = 4 \cos t$$

Find the velocity and acceleration at time t and use them to analyze the motion of the object.

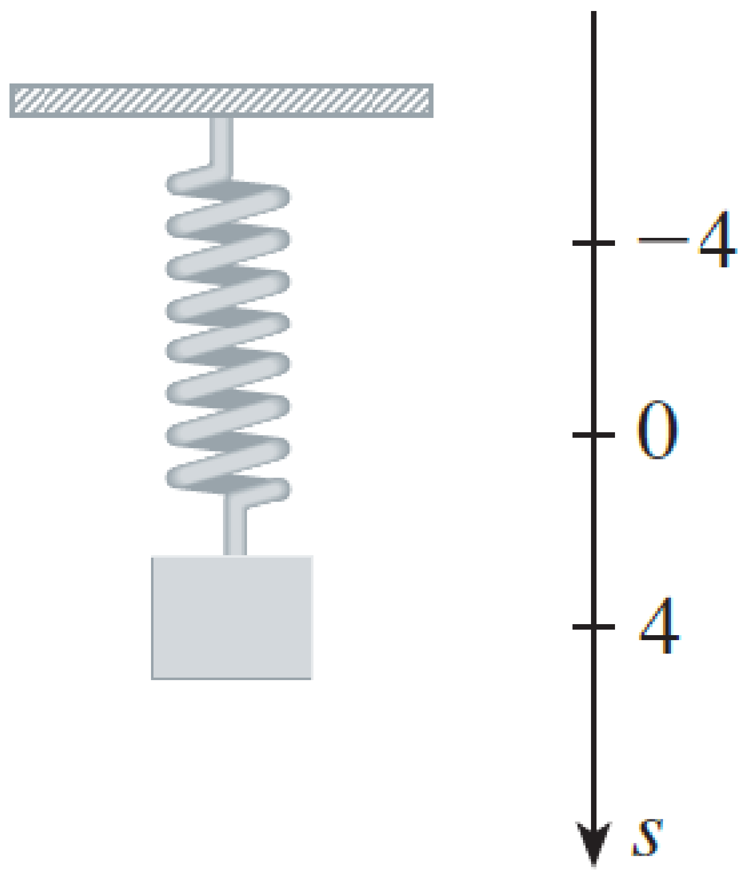


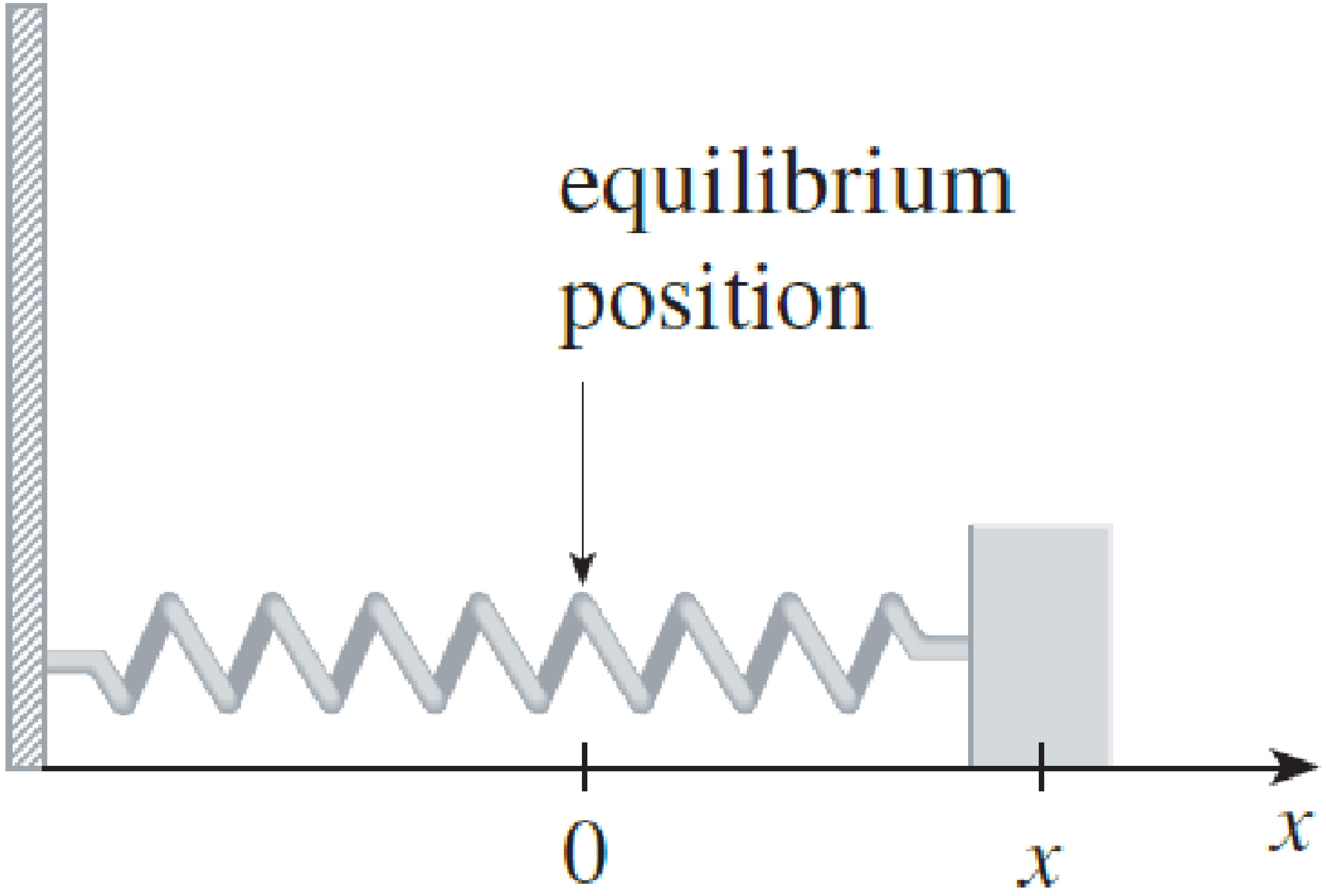
FIGURE 4

$$s = 4 \cos t$$

$$v = s' = -4 \sin t$$

$$a = s'' = -4 \cos t$$

(b) Find the position, velocity, and acceleration of the mass at time $t = 2\pi/3$. In what direction is it moving at that time?



$$x(t) = 8\sin(t)$$

$$v = x'(t) = 8\cos(t) \qquad t = 2\pi/3$$

$$a = x''(t) = -8\sin(t)$$

$$x(2\pi/3) = 8\sin(2\pi/3) = 8\left(\sqrt{3}/2\right) = 4\sqrt{3}$$

$$v(2\pi/3) = 8\cos(2\pi/3) = 8\left(-1/2\right) = -4$$

$$a(2\pi/3) = -8\sin(2\pi/3) = -8\left(\sqrt{3}/2\right) = -4\sqrt{3}$$

An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta}$$

where μ is a constant called the *coefficient of friction*.

(a) Find the rate of change of F with respect to θ.

(b) When is this rate of change equal to 0?

(c) If $W = 50$ lb and $\mu = 0.6$, draw the graph of F as a function of θ and use it to locate the value of θ for which $dF/d\theta = 0$. Is the value consistent with your answer to part (b)?

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta}$$

$n(\theta) = \mu W \qquad n'(\theta) = 0$

$d(\theta) = \mu \sin\theta + \cos\theta \qquad d'(\theta) = \mu \cos\theta - \sin\theta$

$$F'(\theta) = \frac{(0)(2m) - (\mu \cos\theta - \sin\theta)(\mu W)}{(\mu \sin\theta + \cos\theta)^2}$$

$$\frac{\mu W(\sin\theta - \mu\cos\theta)}{(\mu\sin\theta + \cos\theta)^2}$$

$$F' = \frac{\mu W(\sin\theta - \mu\cos\theta)}{(\mu\sin\theta + \cos\theta)^2} = 0$$

$$\sin\theta - \mu\cos\theta = 0$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\mu\cos\theta}{\cos\theta}$$

$$\tan\theta = \mu \rightarrow \tan^{-1}\mu = \theta$$

$$\arctan\mu = \theta$$

20. $g(z) = \dfrac{z}{\sec z + \tan z}$

$$g'(z) = \frac{(\sec z + \tan z)(1) - z(\sec z \tan z + \sec^2 z)}{(\sec z + \tan z)^2}$$

$$\frac{(\sec z + \tan z)(1) - z \sec z(\tan z + \sec z)}{(\sec z + \tan z)^2}$$

$$\frac{(1 - z \sec z)(\sec z + \tan z)}{(\sec z + \tan z)^2} = \frac{1 - z \sec z}{\sec z + \tan z}$$