"If you are distressed by anything external, the pain is not due to the thing itself, but to your estimate of it; and this you have the power to revoke at any moment."

— Marcus Aurelius Antoninus (121 AD - 180 AD)

10 CHAPTER Estimation: Single Samples

Statistical Inference

Using properly drawn sample data to draw conclusions about the population is called **statistical inference**.

DEFINITION

What is an Estimator?

What is meant by the terms **estimator** and **estimate**?

Estimator

An **estimator** is a strategy or rule that is used to estimate a population parameter. If the rule is applied to a specific set of data, the result is an **estimate**.

DEFINITION

Table 10.1.1 - Point Estimators

Point Estimator	Parameter Being Estimated	Point Estimate
$\bar{x}$	μ	$\bar{x} = 12.7$
$\hat{p}$	p	$\hat{p} = 0.37$
s	σ	$s = 6.4$

Sampling Distribution of $\overline{x}$

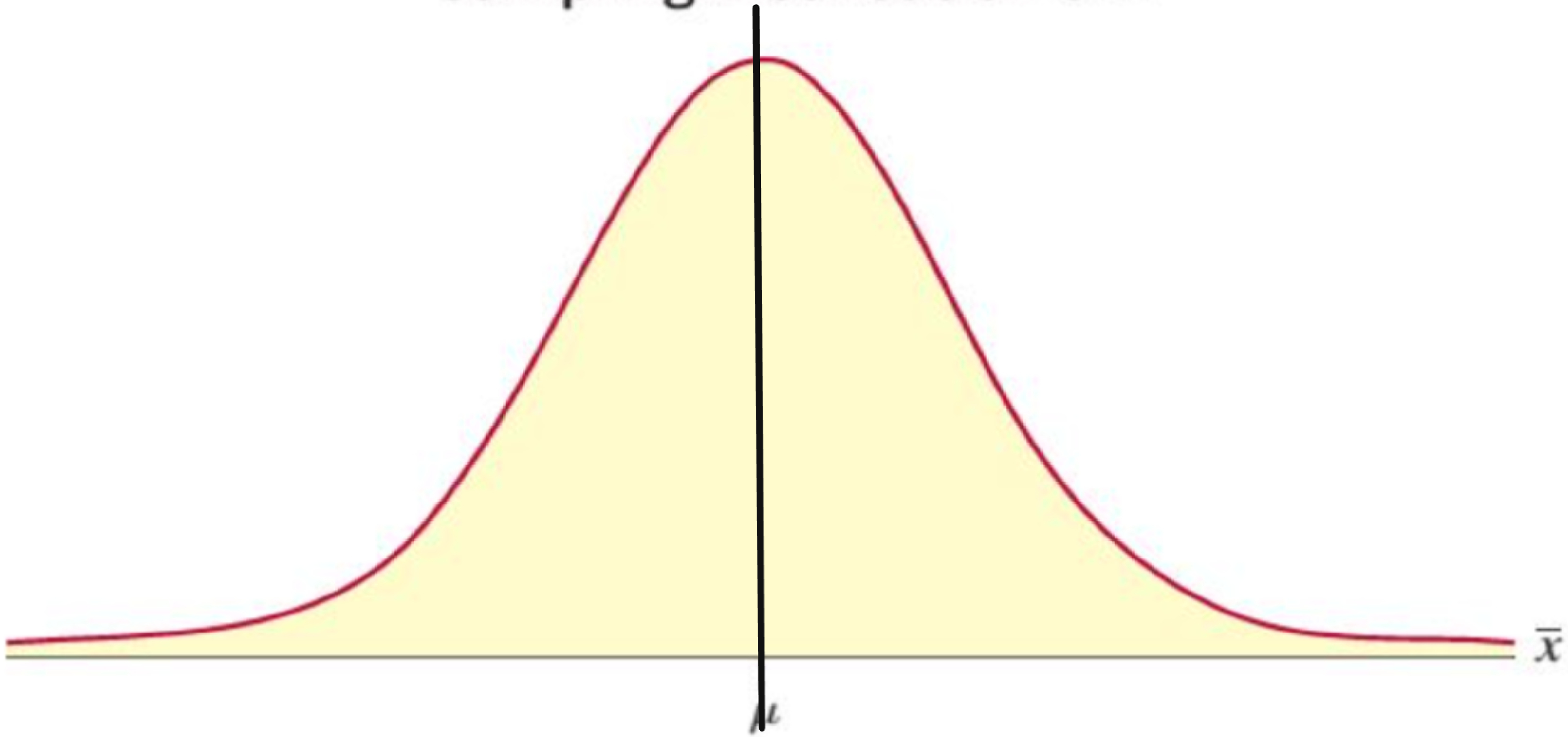


Figure 10.2.1

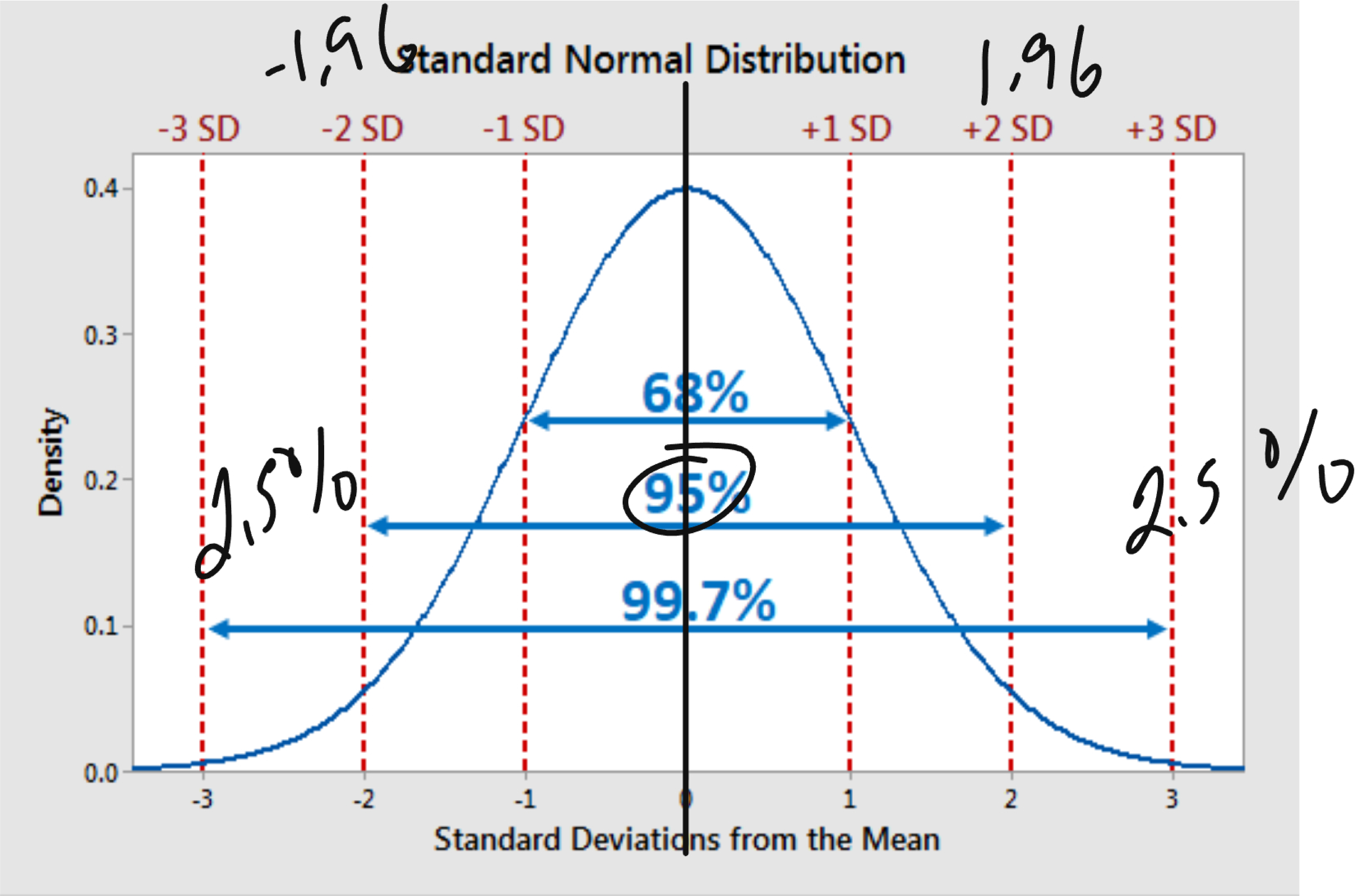

Standard Normal Distribution
-3 SD
-2 SD
-1 SD
+1 SD
+2 SD
+3 SD
-1,96
1,96
68%
95%
99.7%
2,5%
2.5%
Density
0.4
0.3
0.2
0.1
0.0
-3
-2
-1
0
1
2
3
Standard Deviations from the Mean

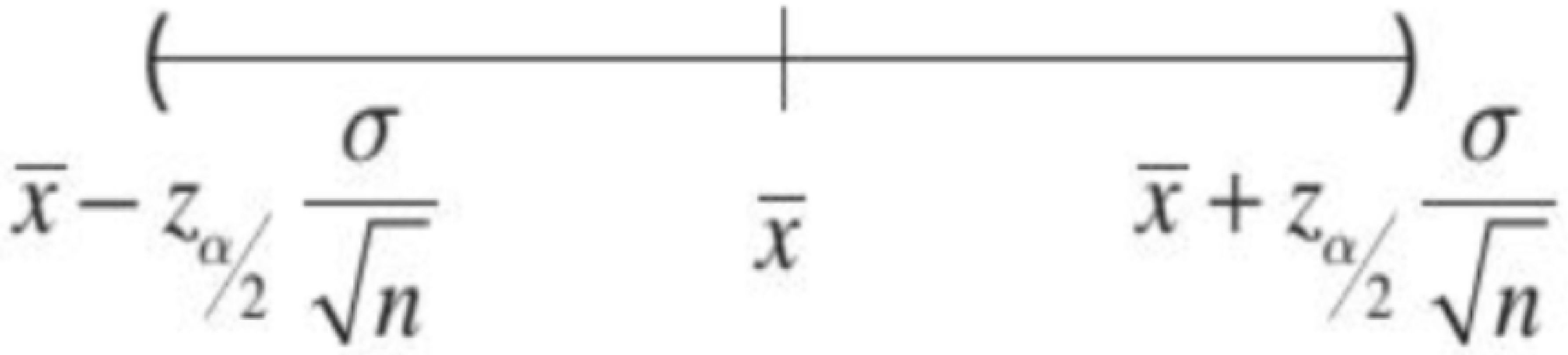
Table 10.2.1 - Critical Values of z	
Confidence $(1 - \alpha)$	$z_{\alpha/2}$
0.80	1.28
0.90	1.645
0.95	1.96
0.99	2.575

100(1 − α)% Confidence Interval for the Population Mean, σ Known

If σ is known and the sample is drawn from a normal population or $n > 30$, a **100(1 − α)% confidence interval for the population mean** is given by

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}.$$

FORMULA



$$\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}.$$

A random sample of 100 car engines has a mean weight of 425 pounds. Construct 80%, 90%, 95%, and 99% confidence intervals for the population mean if the standard deviation of the population is 900.

C.I. ⇒ 80% $\qquad z_{\alpha/2} = 1.28$

$n = 100 \qquad \bar{x} = 425 \qquad \sigma = 900$

$$425 - 1.28\left(\frac{900}{\sqrt{100}}\right) = 309.8$$

$$425 + 1.28\left(\frac{900}{\sqrt{100}}\right) = 540.2$$

A random sample of 100 car engines has a mean weight of 425 pounds. Construct 80%, 90%, 95%, and 99% confidence intervals for the population mean if the standard deviation of the population is 900.

C.I. ⇒ 95% $Z_{\alpha/2} = 1.96$

$n = 100$ $\bar{x} = 425$ $\sigma = 900$

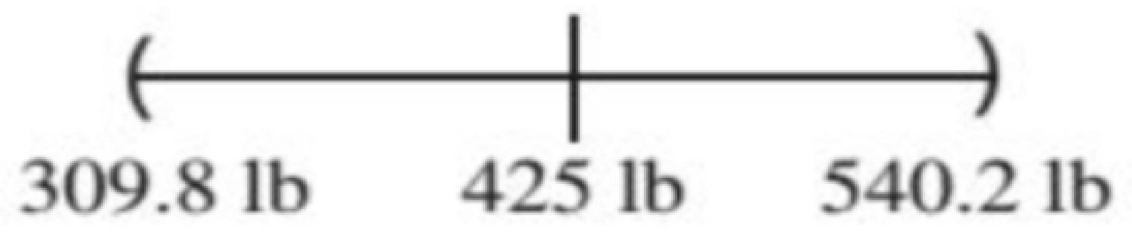
$$425 - 1.96\left(\frac{900}{\sqrt{100}}\right) = 248.6$$

$$425 + 1.96\left(\frac{900}{\sqrt{100}}\right) = 601.4$$

A random sample of 100 car engines has a mean weight of 425 pounds. Construct 80%, 90%, 95%, and 99% confidence intervals for the population mean if the standard deviation of the population is 900.

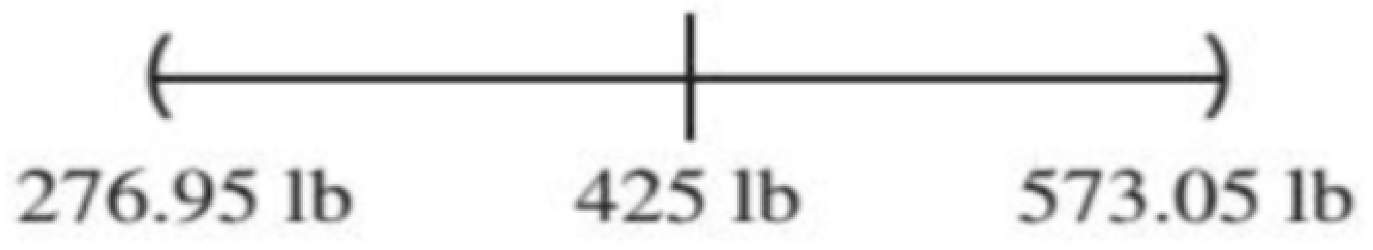
80% Confidence Interval

$$425 \pm 1.28 \cdot \frac{900}{\sqrt{100}} \text{ or } 309.8 \text{ to } 540.2$$



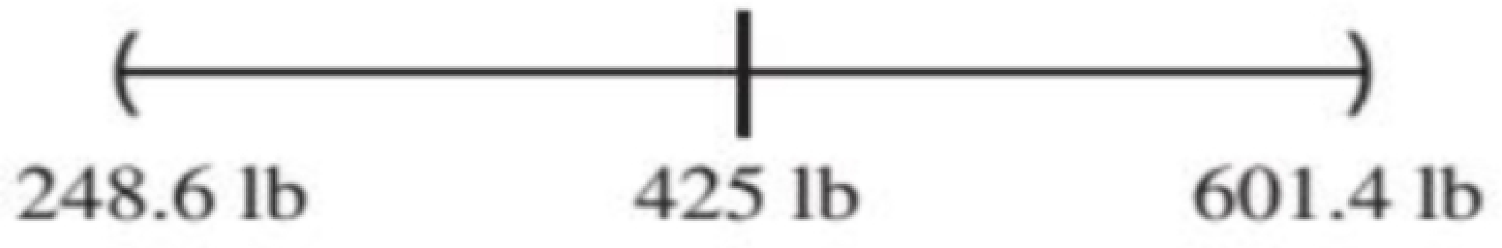
90% Confidence Interval

$$425 \pm 1.645 \cdot \frac{900}{\sqrt{100}} \text{ or } 276.95 \text{ to } 573.05$$



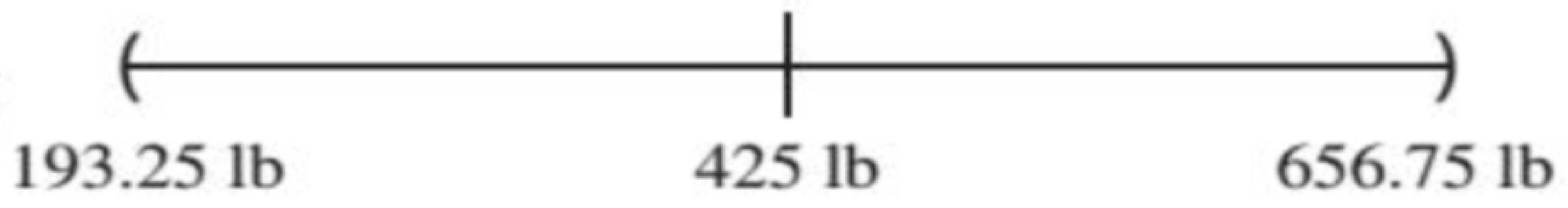
95% Confidence Interval

$$425 \pm 1.96 \cdot \frac{900}{\sqrt{100}} \text{ or } 248.6 \text{ to } 601.4$$



99% Confidence Interval

$$425 \pm 2.575 \cdot \frac{900}{\sqrt{100}} \text{ or } 193.25 \text{ to } 656.75$$



An analyst is interested in investigating the average durability of the bladder within the soccer balls that his company manufactures. Suppose a random sample of 100 soccer balls is selected, and the balls are put through a pressure test to determine the PSI at which the bladder will burst. It is known from past experiences that the population standard deviation of the pressure at which a ball bursts is 13.25 PSI. The sample mean PSI necessary to pop a soccer ball is found to be 147.58 PSI. Calculate a 95 percent confidence interval for the population mean PSI necessary to pop a soccer ball.

$n = 100$ $\sigma = 13.25$ $\bar{x} = 147.58$

C.I. → 95% → $z_{a/2} = 1.96$

$$147.58 \pm 1.96\left(\frac{13.25}{\sqrt{100}}\right)$$

$= 144.983$

$= 150.177$

An analyst is interested in investigating the average durability of the bladder within the soccer balls that his company manufactures. Suppose a random sample of 100 soccer balls is selected, and the balls are put through a pressure test to determine the PSI at which the bladder will burst. It is known from past experiences that the population standard deviation of the pressure at which a ball bursts is 13.25 PSI. The sample mean PSI necessary to pop a soccer ball is found to be 147.58 PSI. Calculate a 95 percent confidence interval for the population mean PSI necessary to pop a soccer ball.

$n = 100$ $\sigma = 13.25$ $\bar{x} = 147.58$

C.I. → 99% → $z_{\alpha/2} = 2.575$

$$147.58 \pm 2.575\left(\frac{13.25}{\sqrt{100}}\right)$$

$= 144.168125$

$= 150.991875$

Sample Size Determination for Estimating a Population Mean

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{E} \right)^2$$

FORMULA

Error

Suppose that a quality control manager at Argon Chemical Company wishes to measure the average amount of cleaning fluid the company is placing in their 12 ounce bottles. From previous studies, they believe their population standard deviation to be 0.3 ounces. How large a sample must be taken in order to be 95% confident of estimating the mean cleaning fluid in a 12 ounce bottle to within 0.05 ounces?

139 Bottles

C.I. = 95% → $z_{\alpha/2} = 1.96$

$\sigma = 0.3$ $E = 0.05$

$$n = \left(\frac{(1.96)(0.3)}{0.05} \right)^2 = 138.2976$$

Suppose a sample of 410 randomly selected radio listeners revealed that 48 listened to WXQI. Find a 95% confidence interval for the proportion of radio listeners that listen to WXQI.

$\hat{p} = \frac{48}{410}$

$\sigma = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{48/410(1-48/410)}{410}}$

C.I. = 95% $z_{a/2} = 1.96$

$= 0.08595205273$

$= 0.1481942887$

$\hat{p} \pm z_{a/2}(\sigma)$

$0.086 \leq p \leq 0.148$

8.6% 14.8%

The clinical testing of drugs involves many factors. For example, patients that have been given placebos, which are harmless compounds that have no effect on the patient, often will still report that they feel better. Assume that in a study of 500 random subjects conducted by the Poppins Sucre Drug Company, the percentage of patients reporting improvement when given a placebo was 37%.

$CI = 99\% \quad 2.575$

$\hat{p} = .37$

$\sigma = \sqrt{\frac{.37(1-.37}{500}}$

$.37 \pm 2.575(\sigma)$

$= 0.3144014625$

$= 0.4255985375$

$.314 \leq P \leq .426$

Sample Size Determination for Estimating a Population Proportion

The sample size necessary to estimate the population proportion to within a particular error with a certain level of confidence is given by

$$n \approx \frac{z_{\alpha/2}^2 \, \hat{p}\left(1-\hat{p}\right)}{E^2},$$

Using Example 10.3.2, suppose that radio station WXQI desires to estimate the proportion of the market they hold with a maximum error of 0.01 and a confidence of 0.95. How large a sample would be required to estimate the fraction of listeners to within the desired level of accuracy? Since we don't know the true population proportion, let's assume the previous point estimate of 0.1171 is the true proportion.

$$n = \frac{(1.96)^2(0.1171)(1-0.1171)}{(0.01)^2} \approx 3972$$