11 CHAPTER Hypothesis Testing: Single Samples

11.1 Introduction to Hypothesis Testing

11.2 Testing a Hypothesis about a Population Mean with σ Known and Unknown

11.4 Testing a Hypothesis about a Population Proportion

11.6 Practical Significance vs. Statistical Significance

CR Chapter Review

Hypothesis Testing

A **hypothesis** is a statement or claim about a characteristic of one or more populations.

A **hypothesis test** is a procedure, based on sample evidence, that is used to test a statement or claim about a characteristic of one or more populations.

DEFINITION

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Properties of H_0 and H_a

The **null hypothesis**, denoted H_0, is a statement about the value of a population parameter. This statement is assumed to be true unless we find sample evidence that indicates it is not. If the sample evidence is strong enough to indicate it is not true, then we reject the null hypothesis. Otherwise, we fail to reject the null hypothesis.

$H_0 \rightarrow \mu =$

The **alternative hypothesis**, denoted H_a or H_1, is also a statement about the value of a population parameter. It is the statement or claim that we are trying to find evidence to support.

$H_a \rightarrow \mu \quad > \quad < \quad \neq$

DEFINITION

Suppose a potato chip manufacturer is concerned that the bagging equipment is not functioning properly when filling 10-ounce bags. He wants to test a hypothesis that will help determine if there is a problem with the bagging equipment. What is the correct hypothesis?

$H_0 \rightarrow \mu = 10$

$H_a \rightarrow \mu \neq 10$

Martha Brandon wants to assess her candidacy in a forthcoming race for the state senate. She is an incumbent and wants to raise sufficient support to retain her seat. How should she formulate an appropriate hypothesis to determine if there is overwhelming evidence she will retain her seat?

$H_0 \rightarrow \mu = .5$

$H_a \rightarrow \mu > .5$

Mrs. Russell, head product tester for Hathaway Tool Corporation, is testing a newly designed series of bar hooks. The hooks have been designed to give way if they get too hot. The previous design gave way at 240 degrees. Develop a test to determine if the newly designed hooks give way at a higher temperature than the previous design.

$H_0 \rightarrow \mu = 240$

$H_a \rightarrow \mu > 240$

Suppose the average national reading level for high school sophomores is 150 words per minute with a standard deviation of 15. A local school board member wants to know if sophomore students at Lincoln High School read at a level different from the national average for tenth graders. The level of the test is to be set at 0.05. A random sample size of 100 tenth graders from Lincoln High School has been drawn, and the resulting average is 154 words per minute.

$H_0 \to \mu = 150$ $H_a \to \mu \neq 150$

Table 11.2.1 - Hypotheses Concerning a Test about a Single Mean

	Is the population mean different from μ_0?	Is the population mean greater than μ_0?	Is the population mean less than μ_0?
Null Hypothesis, H_0	$\mu = \mu_0$	$\mu = \mu_0$	$\mu = \mu_0$
Alternative Hypothesis, H_a	$\mu \neq \mu_0$	$\mu > \mu_0$	$\mu < \mu_0$
Type of Hypothesis Test	Two-tailed	Right-tailed	Left-tailed

one tailed

Step 3: Validate the assumptions of the hypothesis testing model, identify the appropriate test statistic, and compute its value.

Testing a Hypothesis About a Mean, σ Known

Assumptions:

1. The data is quantitative.
2. The data is obtained via a random sample of size n.
3. The population is normally distributed or the sample size n is large, $n > 30$.
4. The population standard deviation σ is known.

Test statistic:

The test statistic is given by

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

where,

n = the sample size,

$\bar{x}$ = the sample mean,

μ_0 = the population mean (from the null hypothesis), and

σ = the known population standard deviation.

PROCEDURE

Sig Level → 0.05

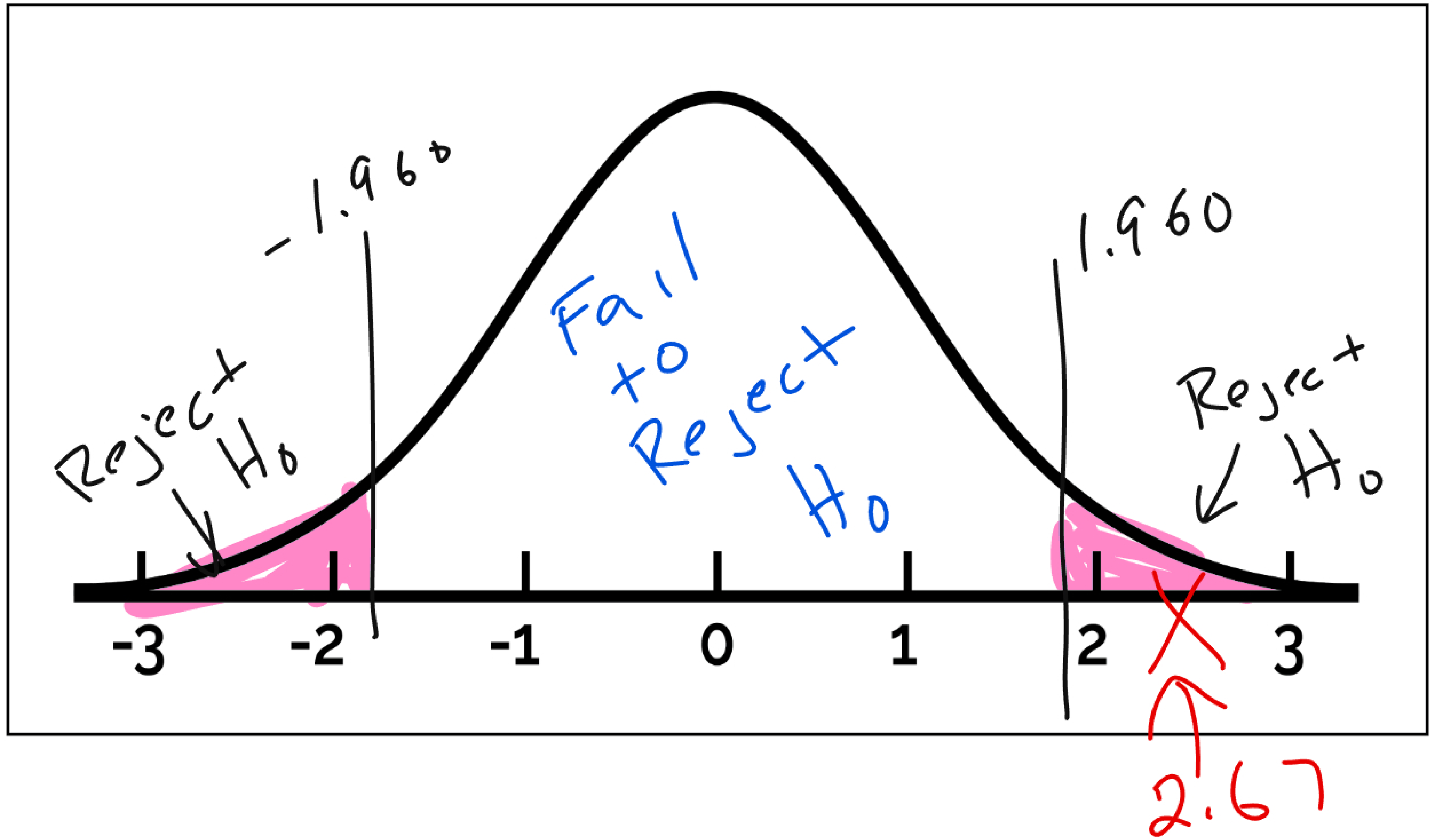
$\mu = 150$

$\sigma = 15$

$x = 154$

$n = 100$

$$z = \frac{154 - 150}{15/\sqrt{100}}$$


-1.960
1.960
Reject H_0
Fail to Reject H_0
Reject H_0
-3
-2
-1
0
1
2
3
2.67

Suppose the average national reading level for high school sophomores is 150 words per minute with a standard deviation of 15. A local school board member wants to know if sophomore students at Lincoln High School read at a level different from the national average for tenth graders. The level of the test is to be set at 0.05. A random sample size of 100 tenth graders from Lincoln High School has been drawn, and the resulting average is 154 words per minute.

Reject H_0

There is statistical evidence that the high school reads a a level different than the national mean

A microprocessor designer has developed a new fabrication process which he believes will increase the usable life of a chip. Currently the usable life is 16,000 hours with a standard deviation of 2500 hours. Test the hypothesis that the process increases the usable life of a chip, at the 0.01 level. A random sample of 1000 microprocessors will be tested. Assume the standard deviation of the life of the new chips will be equal to the standard deviation of the current chips.

$H_0 \rightarrow \mu = 16000$ sig level 0.01

$H_a \rightarrow \mu > 16000$

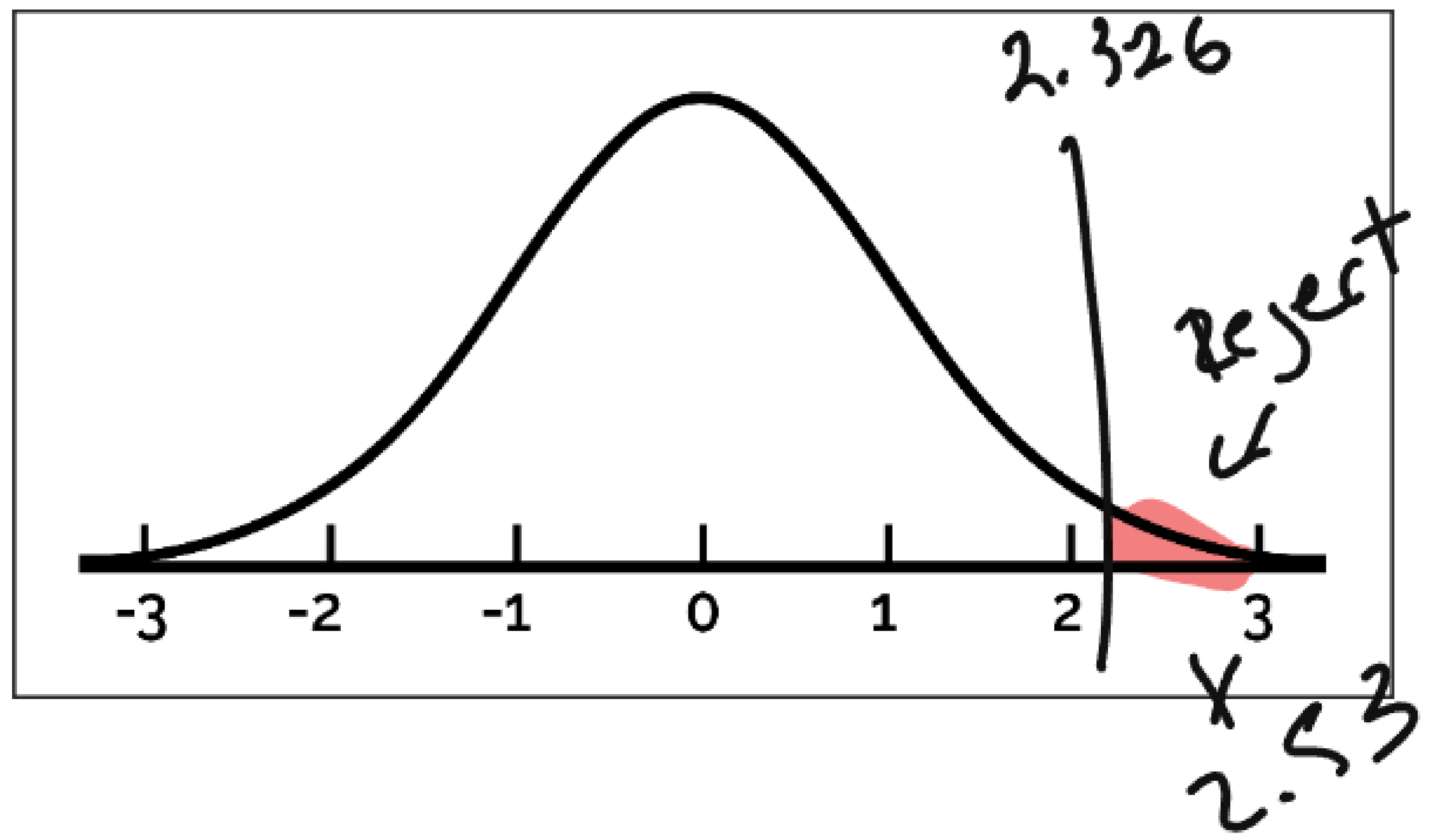
$\mu = 16000$

$\sigma = 2500$

$\bar{x} = 16200$

$n = 1000$

$z = 2.53$



Reject H_0

A personnel researcher has designed a questionnaire she believes will take an average time of 35 minutes to complete. Suppose she randomly samples 20 employees and finds that the mean time to take the test is 29 minutes with a standard deviation of $s = 8$ minutes. Determine if there is sufficient evidence to conclude that the completion time of the newly designed test differs from its intended duration. Conduct the test at the 0.05 level. Assume the sample comes from a normally distributed population.

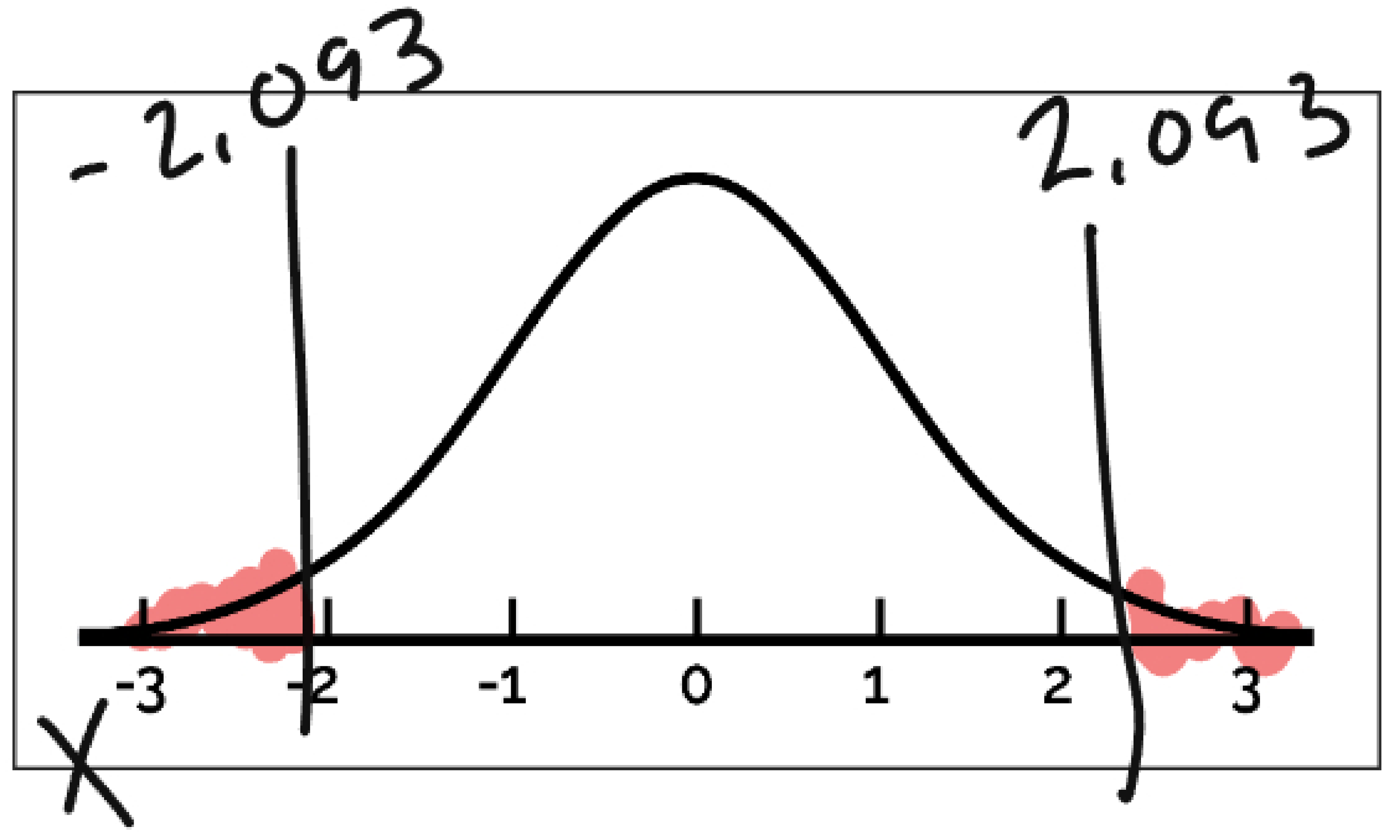
$H_0 \to \mu = 35$

$H_a \to \mu \neq 35$

sig Level 0.05

$n = 20$

d.f. = 19



$\mu = 35$

$\bar{x} = 29$

$s = 8$

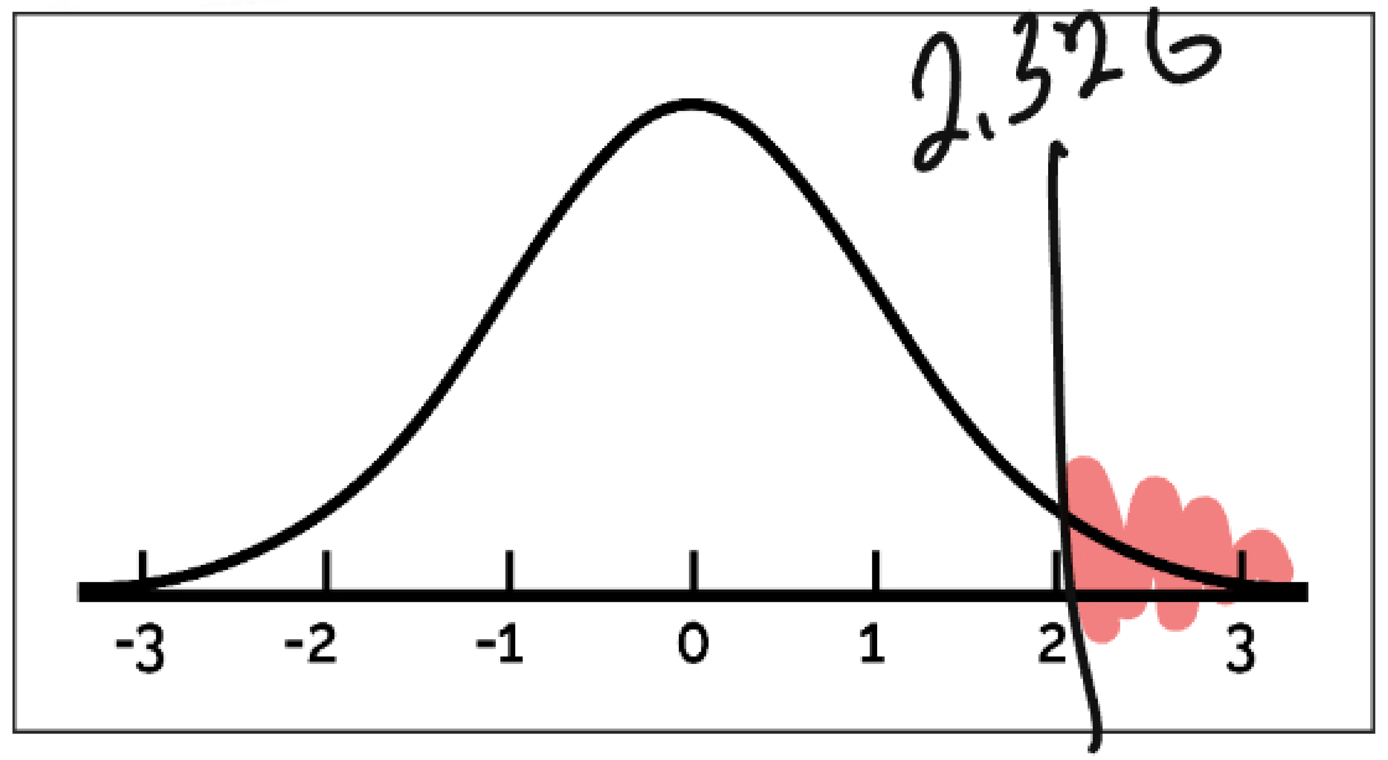
$n = 20$

$t = -3.35$

Reject H_0

Hurricane Andrew swept through southern Florida causing billions of dollars of damage. Because of the severity of the storm and the type of residential construction used in this semitropical area, there was some concern that the average claim size would be greater than the historical average hurricane claim. Historically, the average claim size was $24,000 with standard deviation $2400. Several insurance companies collaborated in a data gathering experiment. They randomly selected 84 homes and sent adjusters to settle the claims. In the sample of 84 homes, the average claim was $27,500.

sig Level 0.01



H_0 $\mu = 24000$ H_a $\mu > 24000$

$\mu = 24000$

$\bar{x} = 27500$

$\sigma = 2400$

$n = 84$

$z = 13.37$

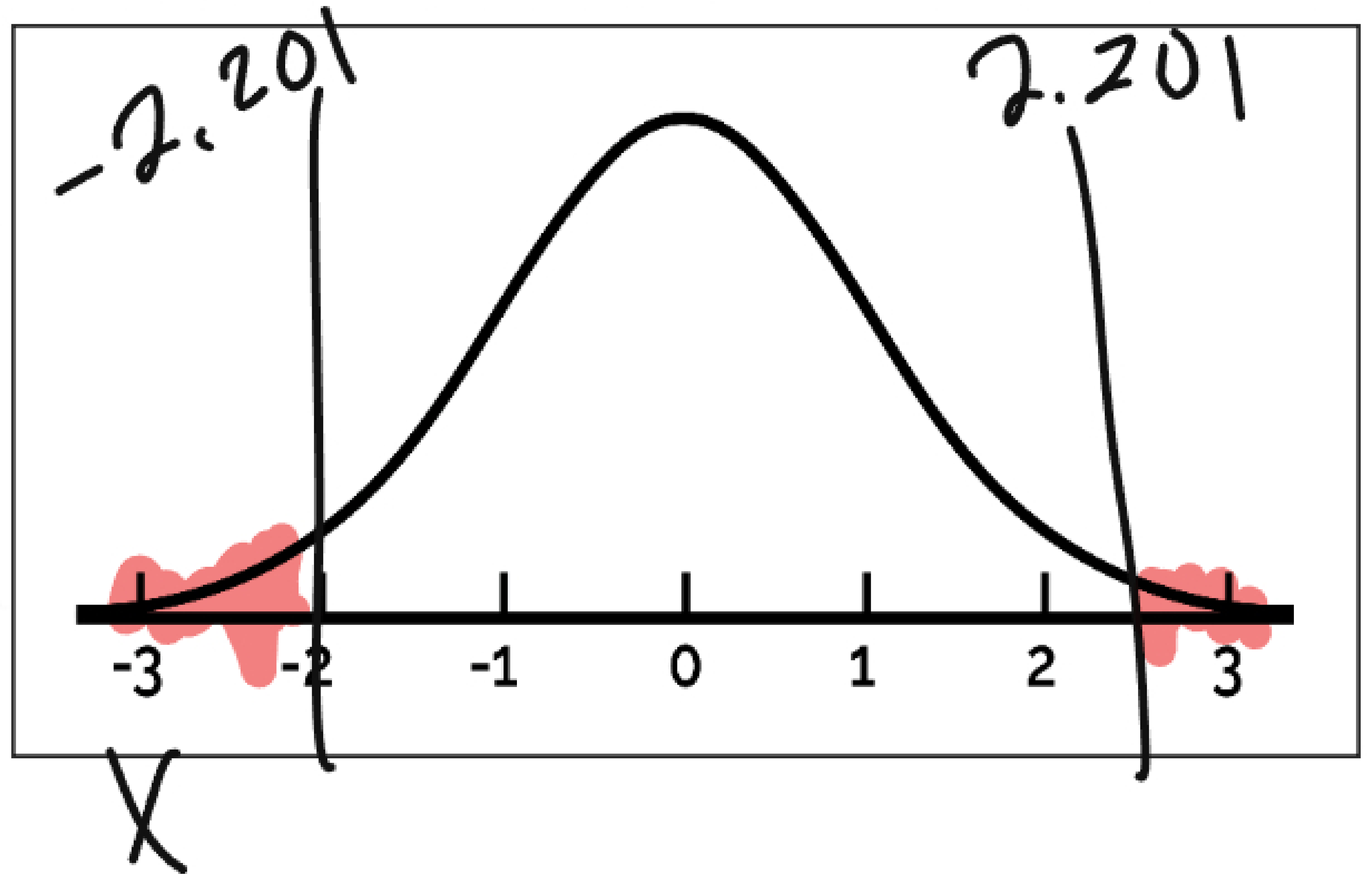
Reject H_0

Researchers studying the effects of diet on growth would like to know if a vegetarian diet affects the height of a child. The researchers randomly select 12 vegetarian children that are six years old. The average height of the children is 42.5 inches with a standard deviation of 3.8 inches. The average height for all six year old children is 45.75 inches.

$df = 11$

Sig Level 0.05

$H_0 \rightarrow \mu = 45.75 \quad H_a \rightarrow \mu \neq 45.75$



$\mu = 45.75$

$\bar{x} = 42.5$

$s = 3.8$

$n = 12$

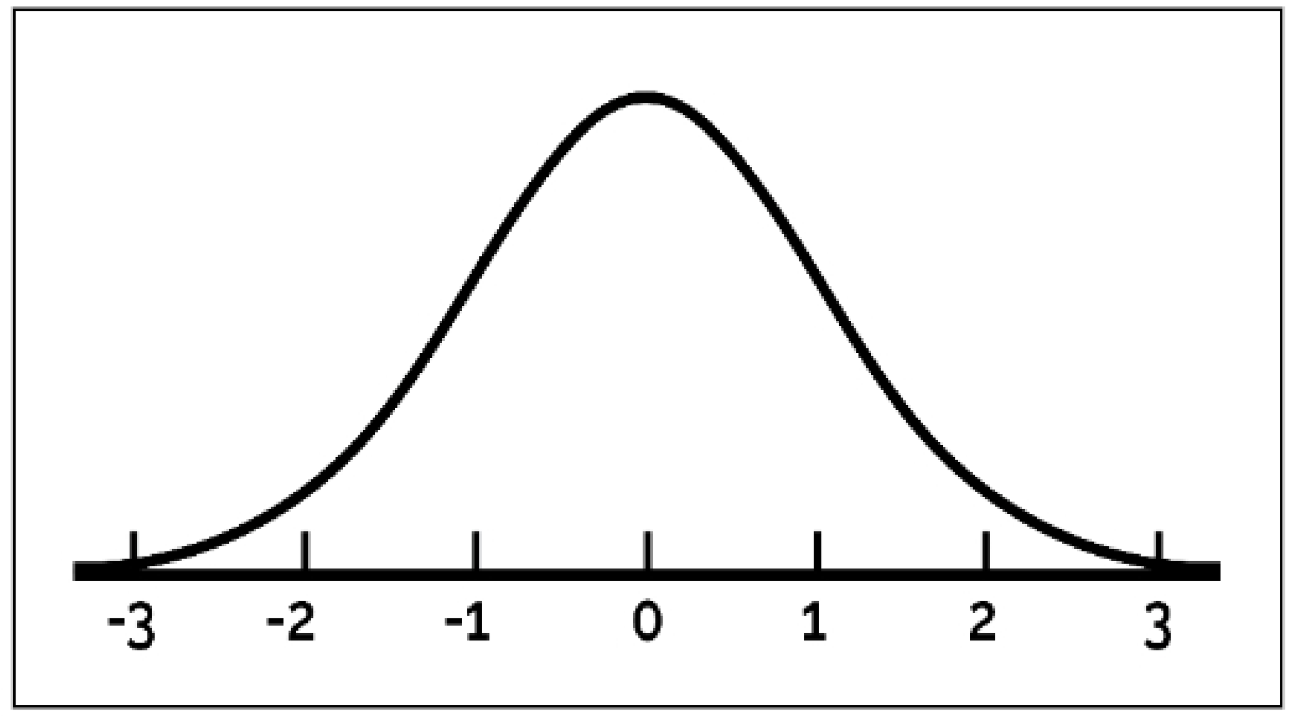
$t = -2.96$

Reject Ho

The director of the IRS has been flooded with complaints that people must wait more than 45 minutes before seeing an IRS representative. To determine the validity of these complaints, the IRS randomly selects 400 people entering IRS offices across the country and records the times that they must wait before seeing an IRS representative. The average waiting time for the sample is 55 minutes with a standard deviation of 15 minutes.

a. What is the population being studied?

b. Are the complaints substantiated by the data at $\alpha = 0.10$?

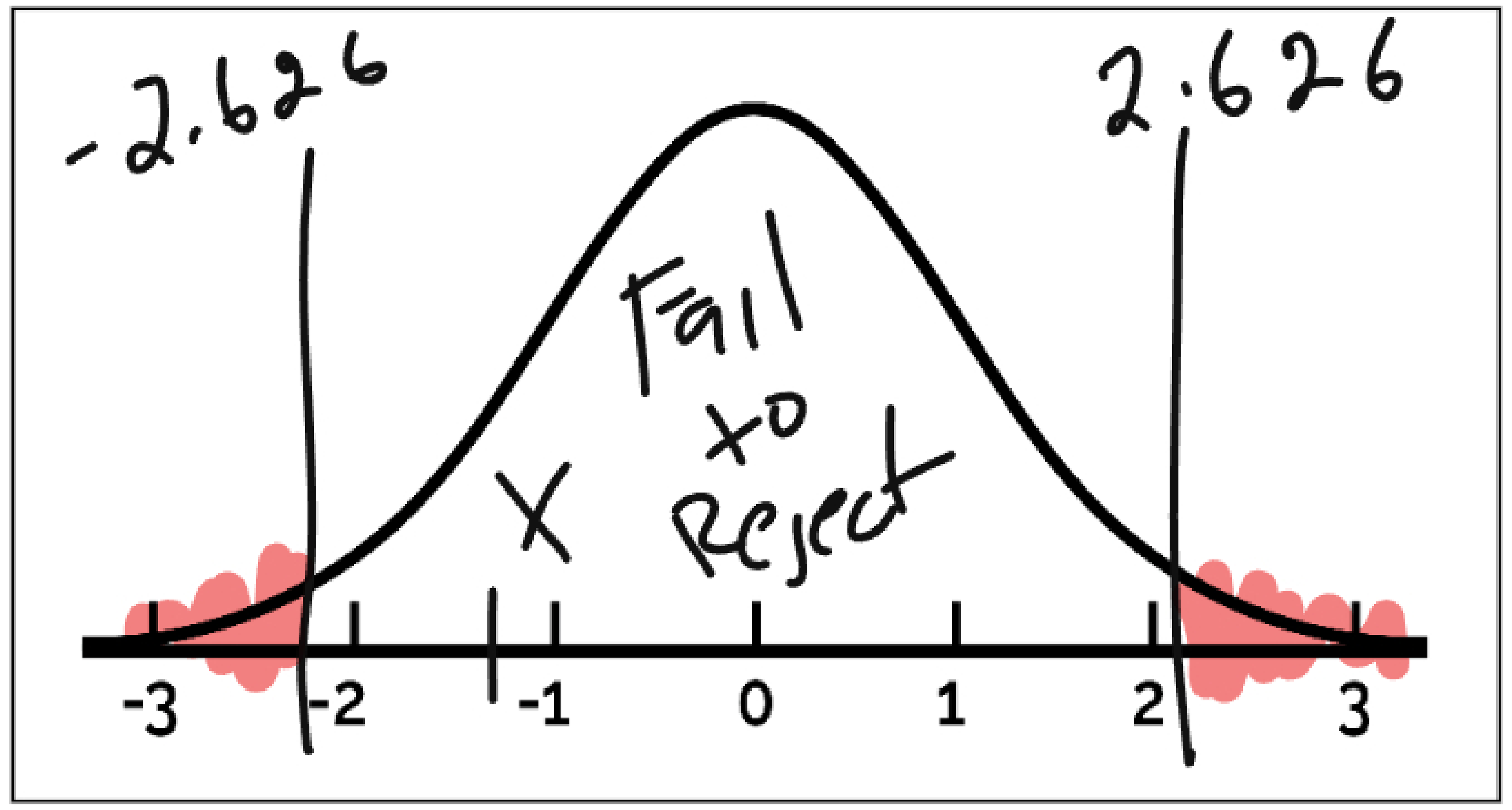


The nutrition label for Oriental Spice Sauce states that one package of the sauce has 1190 milligrams of sodium. To determine if the label is accurate the FDA randomly selects two hundred packages of Oriental Spice Sauce and determines the sodium content. The sample has an average of 1167.34 milligrams of sodium per package with a sample standard deviation of 252.94 milligrams.

0.01 sig Level

$H_0 \rightarrow \mu = 1190$

$H_a \rightarrow \mu \neq 1190$



$\mu = 1190$

$\bar{x} = 1167.34$

$s = 252.94$

$n = 200$

$t = -1.27$