Chapter 11 Test Review

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Key Topics
Finding Convergance and Divergance of Series
Using the Power Series

EXAMPLE 1 Indicate which test should be used to determine whether the series below converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n-5}{5n+5}$$

Test for Divergence

SOLUTION Since $a_n \to$ 1/5 $\neq 0$ as $n \to \infty$, we should use the ---Select--- .

$$\sum_{n=1}^{\infty} \boxed{\frac{n-5}{5n+5}}$$

$$a_n = \frac{n-5}{5n+5}$$

$$\lim_{n\to\infty} \frac{n-5\,/n}{5n+5\,/n} = \frac{1-5/n}{5+5/n}$$

$$\lim_{n\to\infty} a_n \to \frac{1}{5} \neq 0$$

Diverges

EXAMPLE 2 Indicate which test should be used to determine whether the series below converges or diverges.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^3+5}}{8n^3+6n^2+3}$$

SOLUTION Since a_n is an algebraic function of n, we compare the given series with a p-series. The comparison series for the ---Select--- is $\Sigma\, b_n$, where

$$b_n = \frac{\sqrt{n^3}}{8n^3} = \frac{n^{3/2}}{8n^3} = \frac{1}{}.$$

$a_n < b_n$ b_n = converges a_n = converges

$a_n > bn$ bn = diverges a_n = diverges

a_n ? bn → Limit Comparison test

EXAMPLE 2 Indicate which test should be used to determine whether the series below converges or diverges.

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$\boxed{\text{Limit Comparison}}$

SOLUTION Since a_n is an algebraic function of n, we compare the given series with a p-series. The comparison series for the ---Select--- is $\Sigma\, b_n$, where

$$b_n = \frac{\sqrt{n^3}}{8n^3} = \frac{n^{3/2}}{8n^3} = \frac{1}{}.$$

$$b_n = \frac{\sqrt{n3}}{8n^3} = \frac{n^{3/2}}{8n^3} = \boxed{\frac{1}{8n^{3/2}}} = \frac{1}{8\sqrt{n^3}}$$

$$\frac{\sqrt{n^3+5}}{8n^3+6n^2+3} \overset{?}{\geq} \frac{1}{8\sqrt{n^3}}$$

$p = 3/2 > 1$ converges

EXAMPLE 3 Indicate which test should be used to determine whether the series below converges or diverges.

$$\sum_{n=1}^{\infty} n^4 e^{-5n^5}$$

SOLUTION Since the integral below is easily evaluated, we use the ---Select--- . The Ratio Test also works.

$$\int_1^{\infty} x^4 e^{-5x^5}\, dx$$

Integral Test

$$u = -5x^5$$

$$du = -25x^4\, dx$$

$$\frac{-1}{25}\int e^u\, du$$

EXAMPLE 4 Indicate which test should be used to determine whether the series below converges or diverges.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^4}{n^7 + 5}$$

Alternating Series Test

SOLUTION Since the the series is alternating, we use the ---Select--- .

EXAMPLE 5 Indicate which test should be used to determine whether the series below converges or diverges.

$$\sum_{k=1}^{\infty} \frac{5^k}{k!}$$

SOLUTION Since the the series involves $k!$, we use the ---Select--- . Ratio test

$$\sqrt[k]{\frac{5^k}{k^k}} \leftarrow \text{Root test}$$

EXAMPLE 6 Indicate which test should be used to determine whether the series below converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{5+2^n}$$

SOLUTION Since the the series is closely related to the geometric series $\Sigma\ 1/2^n$, we use the ---Select--- .

$$\sum \frac{1}{2^n} = \left(\frac{1}{2}\right)^n \qquad |r| = 1/2 < 1 \quad \text{converges}$$

$$\frac{1}{5+2^n} < \frac{1}{2^n} \checkmark \qquad \text{Comparison test}$$

Converges

Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{n^7 - 1}{n^8 + 1}$$

- ○ convergent
- ○ divergent

$$\sum_{n=1}^{\infty} \frac{n^7-1}{n^8+1}$$

$$b_n = \frac{n^7}{n^8} = \frac{1}{n}$$

diverges

$p = 1 \ngtr 1$

$$\frac{n^7-1}{n^8+1} \overset{?}{\geq} \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{n^7-1}{n^8+1} \cdot \frac{n}{1} = \frac{n^8-n \;/n^8}{n^8+1 \;/n^8} = \frac{n^8/n^8 - n/n^8}{n^8/n^8 + 1/n^8}$$

$1/n^7$

$$= \frac{1-0}{1+0} = 1 > 0$$

a_n diverges

The Comparison Test Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

(i) If $\sum b_n$ is convergent and $a_n \leqslant b_n$ for all n, then $\sum a_n$ is also convergent.

(ii) If $\sum b_n$ is divergent and $a_n \geqslant b_n$ for all n, then $\sum a_n$ is also divergent.

The Limit Comparison Test Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If

$$\lim_{n \to \infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both diverge.

Test the series for convergence or divergence.

$$\sum_{n=7}^{\infty} \frac{1}{n\sqrt{\ln(6n)}}$$

- ○ convergent
- ○ divergent

The Integral Test Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)\,dx$ is convergent. In other words:

(i) If $\int_1^{\infty} f(x)\,dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.

(ii) If $\int_1^{\infty} f(x)\,dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

$$\int_7^{\infty} \frac{1}{x\sqrt{\ln(6x)}}\,dx$$

$$u = \ln(6x)$$

$$du = 6 \cdot \frac{1}{6x} = 1/x\,dx$$

$$\int_7^{\infty} \frac{1}{\sqrt{u}}\,du = 2u^{1/2} + C = 2\sqrt{\ln(6x)}\Big]_7^{\infty}$$

$$2\sqrt{\ln(\infty)} - 2\sqrt{\ln 42} \to \infty$$

Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{\sin(7n)}{1+6^n}$$

○ convergent

○ divergent

1 Definition A series $\Sigma\, a_n$ is called **absolutely convergent** if the series of absolute values $\Sigma\, |a_n|$ is convergent.

$$\left|\frac{\sin 7n}{1+6^n}\right| \le \frac{1}{1+6^n} \le \frac{1}{6^n} = \left(\frac{1}{6}\right)^n$$

$$\left|\frac{\sin 7n}{1+6^n}\right| \le \left(\frac{1}{6}\right)^n \qquad r = 1/6 < 1$$

converges $\qquad$ converges

Find the radius of convergence, R, and interval of convergence, I, of the series.

$$\sum_{n=0}^{\infty} \frac{(x-12)^n}{n^2+1}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left| \frac{\frac{(x-12)^{n+1}}{(n+1)^2+1}}{\frac{(x-12)^n}{n^2+1}} \right|$$

$$\frac{(x-12)\cancel{(x-12)^n}}{\cancel{(x-12)^n}}$$

$$= \lim_{n\to\infty} \left| \frac{(\boxed{x-12})(n^2+1)}{n^2+2n+2} \right|$$

Find the radius of convergence, R, and interval of convergence, I, of the series.

$$\sum_{n=0}^{\infty} \frac{(x-12)^n}{n^2+1}$$

$$\lim_{n\to\infty} \frac{n^2+1}{n^2+2n+2} = \frac{1+0}{1+0+0} = 1$$

$$\lim_{n\to\infty} \frac{(x-12)(n^2+1)}{n^2+2n+2} \to |x-12|$$

$$|x-12| < 1$$

$$-1 < x-12 < 1$$

$$+12 \quad +12 \quad +12$$

$$11 < x < 13$$

Radius $= 1$

Interval $(11, 13)$

Find the radius of convergence, R, and interval of convergence, I, of the series.

$$\sum_{n=0}^{\infty} \frac{(x-12)^n}{n^2+1}$$

$11 < x < 13$

$x = 11$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ converges

A.S.T. $\lim_{n \to \infty} \frac{1}{n^2+1} \to 0$ ✓

$b_{n+1} \leq b_n$ ✓

$x = 13$ $\quad \sum_{n=0}^{\infty} \frac{1}{n^2+1}$ Converges

Comparison $\frac{1}{n^2}$ $p = 2 > 1$ converges

$\frac{1}{n^2+1} \leq \frac{1}{n^2}$ ✓

$\boxed{\text{Interval } [11, 13]}$

Find the radius of convergence, R, of the series.

$$\sum_{n=1}^{\infty} \frac{10^n x^n}{n^4}$$

$R =$ $1/10$

$$a_n = \frac{10^n x^n}{n^4}$$

Find the interval, I, of convergence of the series. (Enter your answer using interval notation.)

$I =$

$$\lim_{n\to\infty} \left| \frac{10^{n+1} x^{n+1}}{(n+1)^4} \cdot \frac{n^4}{10^n x^n} \right| = 10|x| \cdot \left(\frac{n}{n+1}\right)^4 \to 10|x|$$

$$10|x| < 1$$

$$-1/10 < x < 1/10$$

$$|x| < \boxed{1/10}$$

Radius

Find the radius of convergence, R, of the series.

$$\sum_{n=1}^{\infty} \frac{10^n x^n}{n^4}$$

$R =$

$-1/10 \le x \le 1/10$ $\boxed{[-1/10, 1/10]}$

Find the interval, I, of convergence of the series. (Enter your answer using interval notation.)

$I =$

$x = -1/10$ $\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$ $\quad$ A.S.T. $\lim_{n \to \infty} \frac{1}{n^4} \to 0$ ✓

$10^n(-1/10)^n$ $\quad$ converges $\quad \frac{1}{(n+1)^4} < \frac{1}{n^4}$ ✓

$(-1)^n$

$x = 1/10$ $\quad \sum_{n=1}^{\infty} \frac{1}{n^4}$ $\quad P = 4 > 1$ converges

Find a power series representation for the function. Determine the interval of convergence.

$$f(x) = \frac{1}{4+x}$$

$$\frac{1}{1-r} = \sum_{n=0}^{\infty} ar^n \qquad \frac{1}{4+x} = \frac{1}{4\left(1+\frac{x}{4}\right)} = \frac{1}{4\left(1-\left(\frac{-x}{4}\right)\right)}$$

$$\frac{1}{4}\left(\frac{1}{1-\left(\frac{-x}{4}\right)}\right) = \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{-x}{4}\right)^n = \boxed{\frac{1}{4}\sum_{n=0}^{\infty}(-1)^n\left(\frac{x}{4}\right)^n}$$

$$|r|<1 \qquad \left|\frac{x}{4}\right|<1 \qquad -1<\frac{x}{4}<1$$

$$-4<x<4$$

$$\boxed{(-4,\ 4)}$$