Prices of New Mobile Homes The U.S. Census Bureau publishes annual price figures for new mobile homes in *Manufactured Housing Statistics*. The figures are obtained from sampling, not from a census. A simple random sample of 36 new mobile homes yielded the prices, in thousands of dollars, shown in Table 8.1. Use the data to estimate the population mean price, μ, of all new mobile homes.

Table 8.1 **Prices ($1000s) of 36 randomly selected new mobile homes**

67.8	68.4	59.2	56.9	63.9	62.2	55.6	72.9	62.6
67.1	73.4	63.7	57.7	66.7	61.7	55.5	49.3	72.9
49.9	56.5	71.2	59.1	64.3	64.0	55.9	51.3	53.7
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

SOLUTION

We estimate the population mean price, μ, of all new mobile homes by the sample mean price, $\bar{x}$, of the 36 new mobile homes sampled. From Table 8.1,

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{2278}{36} = 63.28.$$

Definition 8.1: Point Estimate

A **point estimate** of a parameter is the value of a statistic used to estimate the parameter.

Prices of New Mobile Homes Consider again the problem of estimating the (population) mean price, μ, of all new mobile homes by using the sample data in Table 8.1. Let's assume that the population standard deviation of all such prices is \$7.2 thousand, that is, \$7200. (We might know the population standard deviation from previous research or from a preliminary study of prices. We examine the more usual case where σ is unknown in Section 8.3.)

a. Identify the distribution of the variable $\bar{x}$, that is, the sampling distribution of the sample mean for samples of size 36. $\bar{x} = 63.28$

b. Use part (a) to show that approximately 95% of all samples of 36 new mobile homes have the property that the interval from $\bar{x} - 2.4$ to $\bar{x} + 2.4$ contains μ.

c. Use part (b) and the sample data in Table 8.1 to find a 95% *confidence interval* for μ, that is, an interval of numbers that we can be 95% confident contains μ.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{7.2}{\sqrt{36}} = \frac{7.2}{6} = 1.2$$

$$\mu_{\bar{x}} = \mu$$

$\bar{x} \pm 2.4 \rightarrow 63.28 \pm 2.4$

95% 2 st. dev $2(1.2) = 2.4$

$60.88 \longleftrightarrow 65.68$

95%

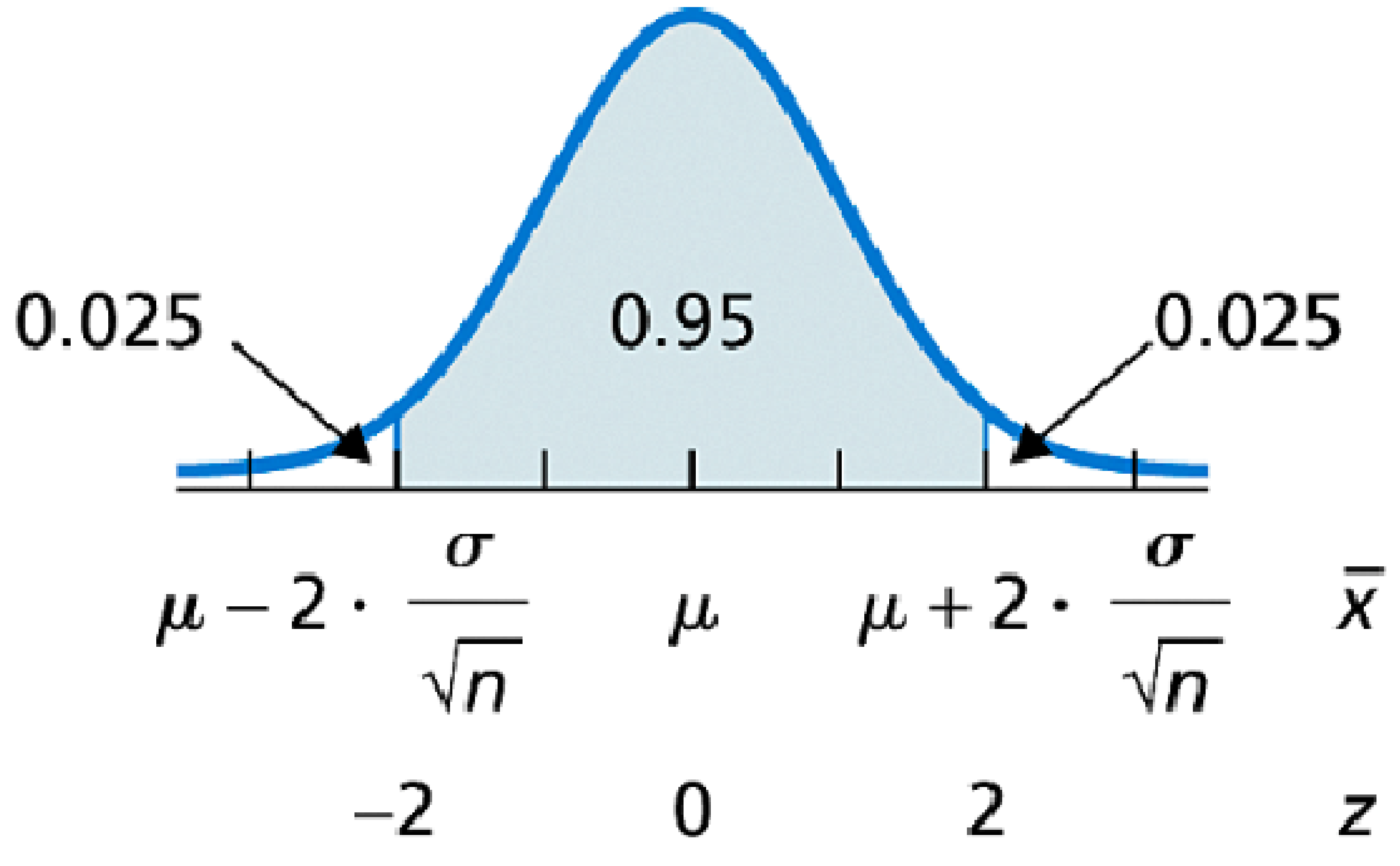
Definition 8.2: Confidence-Interval Estimate

Confidence interval (CI): An interval of numbers obtained from a point estimate of a parameter.

Confidence level: The confidence we have that the parameter lies in the confidence interval (i.e., that the confidence interval contains the parameter).

Confidence-interval estimate: The confidence level and confidence interval.

One-Mean z-Interval Procedure



(a)

Purpose To find a confidence interval for a populati

Assumptions

Step 1 **For a confidence level of $1-\alpha$, use Tal**

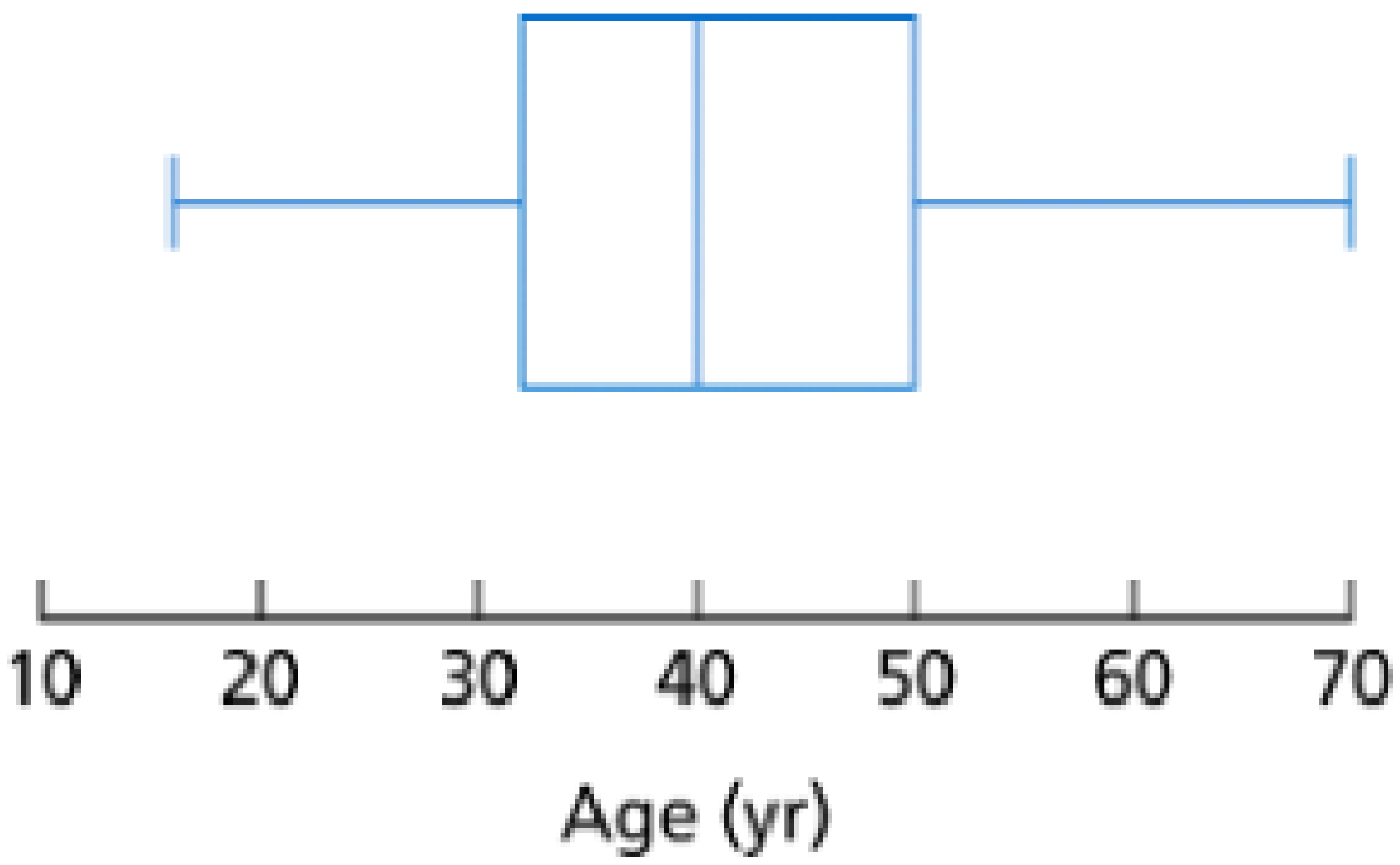
Step 2 **The confidence interval for μ is from**

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \text{ to } \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}},$$

where $z_{\alpha/2}$ is found in Step 1, n is the sample size, and $\bar{x}$ is computed from the sample data.

Step 3 **Interpret the confidence interval.**

The Civilian Labor Force The Bureau of Labor Statistics collects information on the ages of people in the civilian labor force and publishes the results in *Current Population Survey*. Fifty people in the civilian labor force are randomly selected; their ages are displayed in Table 8.4. Find a 95% confidence interval for the mean age, μ, of all people in the civilian labor force. Assume that the population standard deviation of the ages is 12.1 years.



$$\bar{x} = 41.4$$

$$z_{1-\frac{.95}{2}} = z_{0.025} =$$

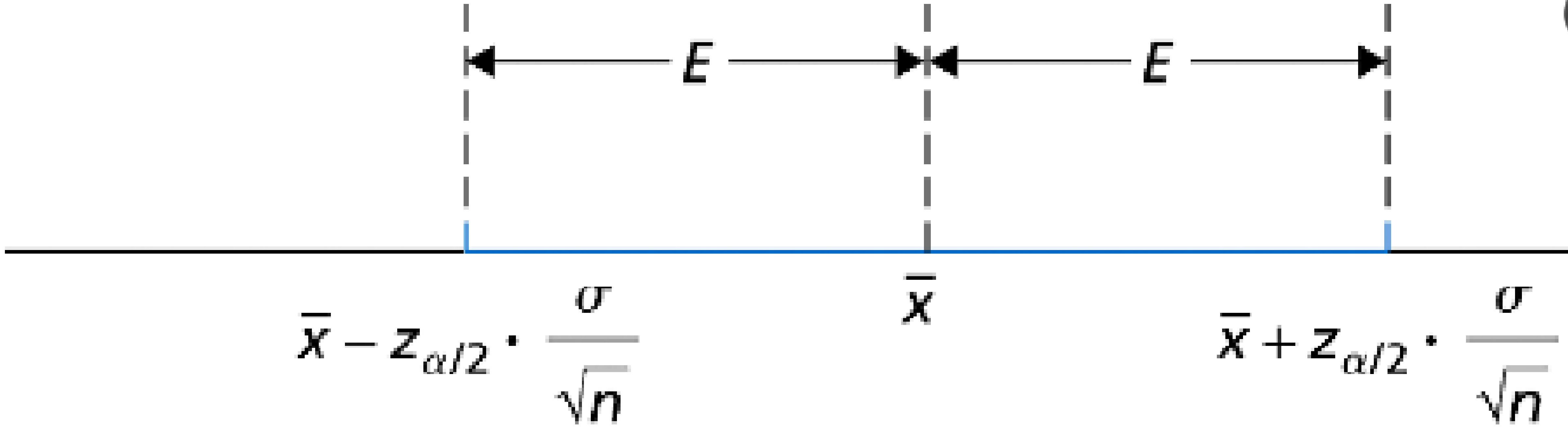
$$z\text{-score} \rightarrow \pm 1.96$$

$$\sigma_{\bar{x}} = \frac{12.1}{\sqrt{50}} = 1.7$$

Formula 8.1: Margin of Error for the Estimate of μ

The margin of error for the estimate of μ is $z_{\alpha/2} \cdot \sigma/\sqrt{n}$, which is denoted by the letter E. Thus,

$$E = z_{\alpha/2} \cdot \sigma/\sqrt{n}.$$



±1.96(1.7)

±3.3

±1.65(1.7)

±2.8

90%

−3.3   41.4 + 3.3

38.1 ⟷ 44.7

95%

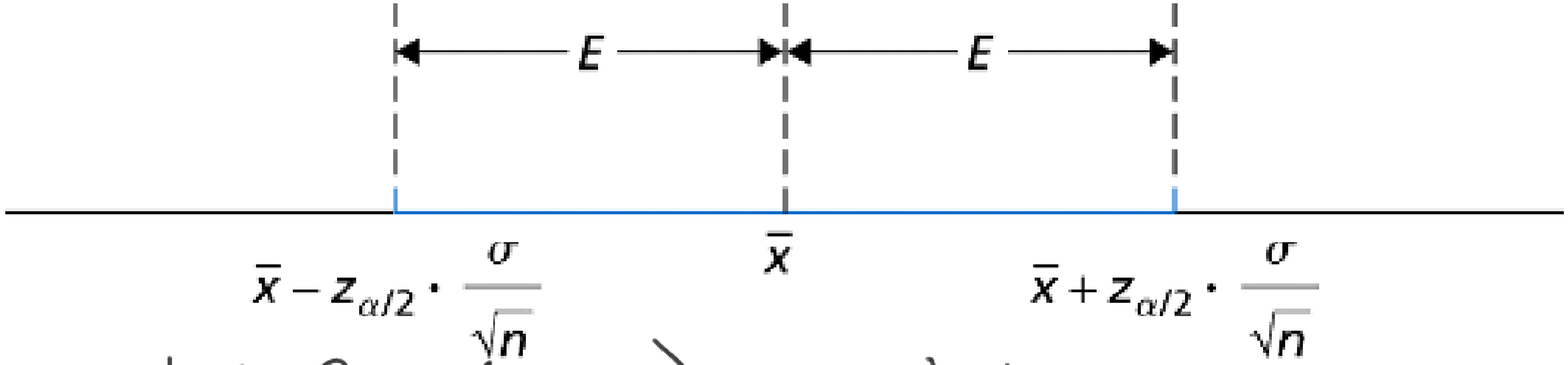
Formula 8.1: Margin of Error for the Estimate of μ

$\bar{x} = 42.2$

The margin of error for the estimate of μ is $z_{\alpha/2} \cdot \sigma/\sqrt{n}$, which is denoted by the letter E. Thus,

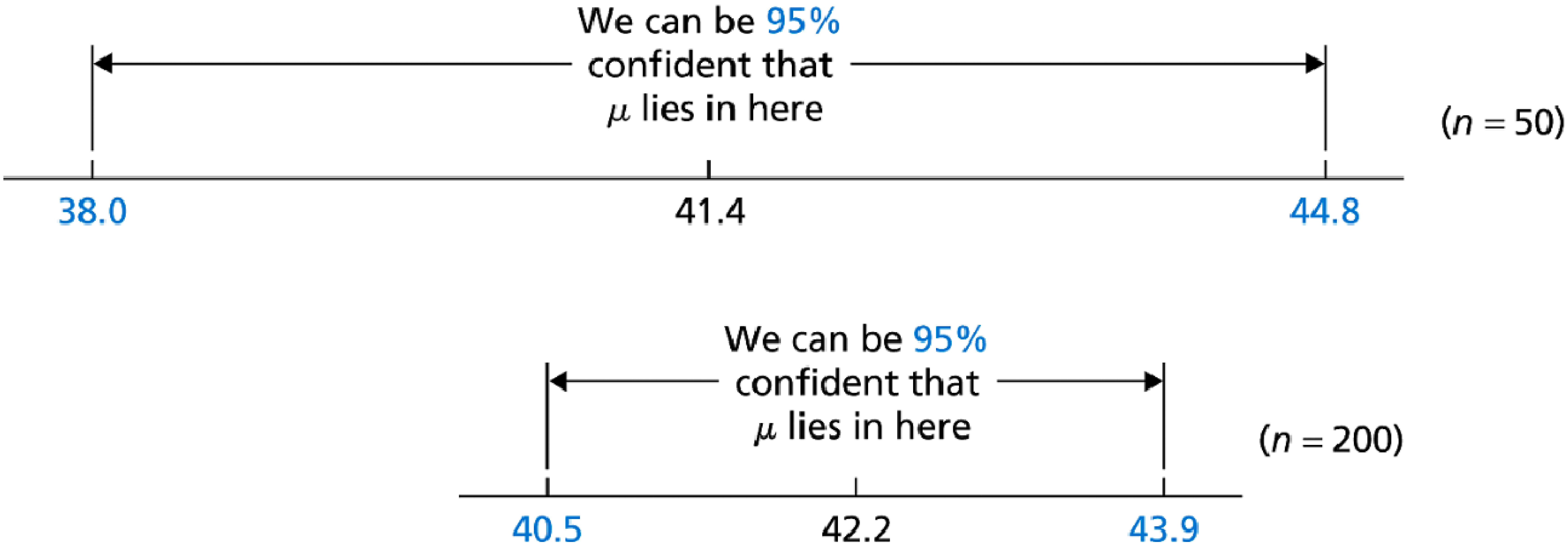
$\sigma_{\bar{x}} = \frac{12.1}{\sqrt{200}} = .86$

$$E = z_{\alpha/2} \cdot \sigma/\sqrt{n}.$$



$95\% \pm 1.96(.86) \rightarrow \pm 1.69$

$90\% \pm 1.65(.86) \rightarrow \pm 1.42$


We can be 95% confident that μ lies in here
(n = 50)
38.0
41.4
44.8
We can be 95% confident that μ lies in here
(n = 200)
40.5
42.2
43.9

Key Fact 8.3: Confidence and Accuracy

For a fixed sample size, decreasing the confidence level decreases the margin of error and, hence, improves the accuracy of a confidence-interval estimate.

Key Fact 8.4: Sample Size and Accuracy

For a fixed confidence level, increasing the sample size decreases the margin of error and, hence, improves the accuracy of a confidence-interval estimate.

Formula 8.2: Sample Size for Estimating μ

The sample size required for a $(1-\alpha)$-level confidence interval for μ with a specified margin of error, E, is given by the formula

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2,$$

rounded up to the nearest whole number.

The Civilian Labor Force Consider again the problem of estimating the mean age, μ, of all people in the civilian labor force.

a. Determine the sample size needed in order to be 95% confident that μ is within 0.5 year of the point estimate, $\bar{x}$. Recall that $\sigma = 12.1$ years.

b. Find a 95% confidence interval for μ if a sample of the size determined in part (a) has a mean age of 43.8 years.

$$n = \left(\frac{1.96(12.1)}{0.5}\right)^2 = 2250$$

$$43.3 \leftarrow 43.8 \rightarrow 44.3$$

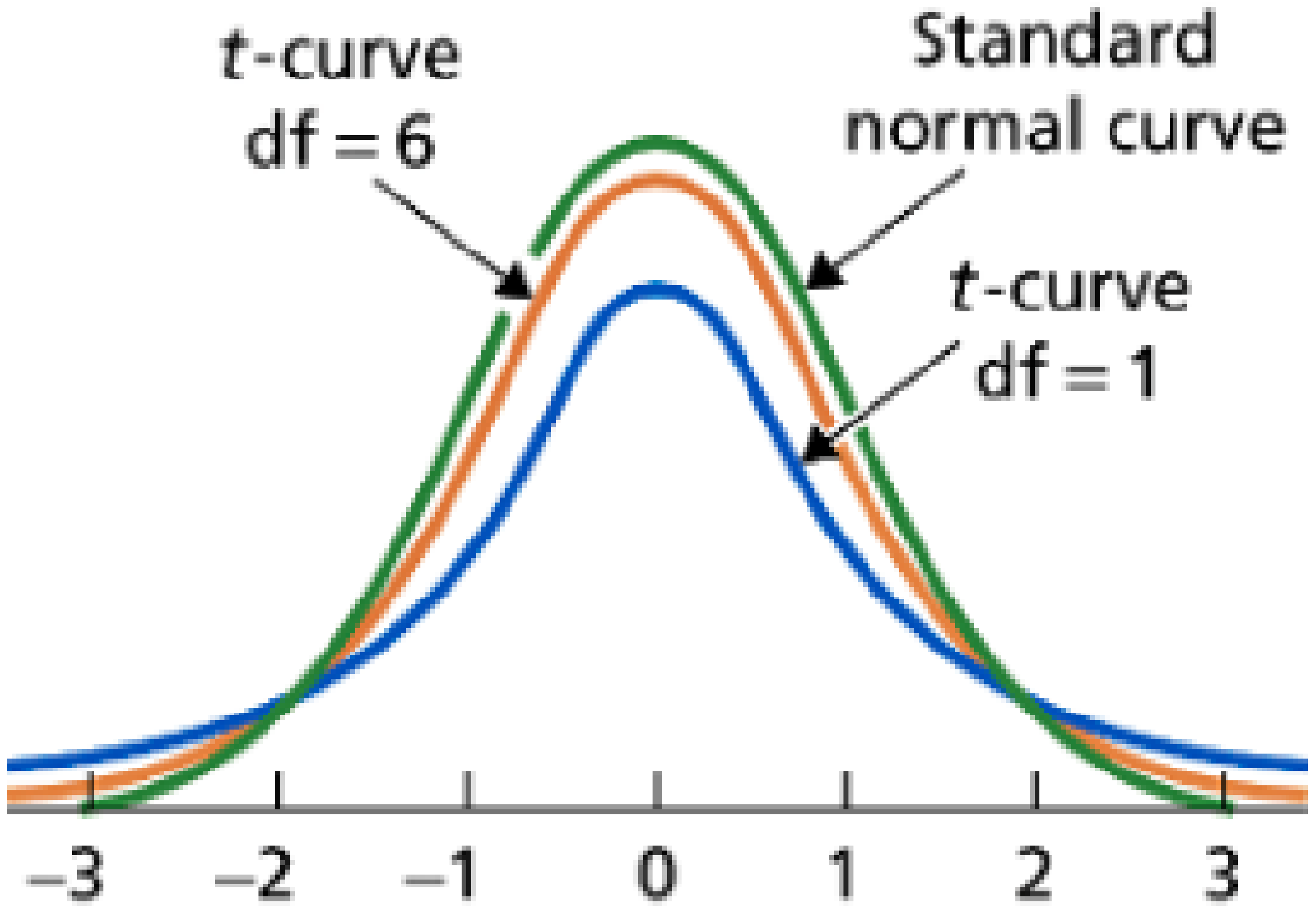
95%

Key Fact 8.5: Studentized Version of the Sample Mean

Suppose that a variable x of a population is normally distributed with mean μ. Then, for samples of size n, the variable

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

has the t-distribution with $n - 1$ degrees of freedom.



Key Fact 8.6: Basic Properties of t-Curves

Property 1: The total area under a t-curve equals 1.

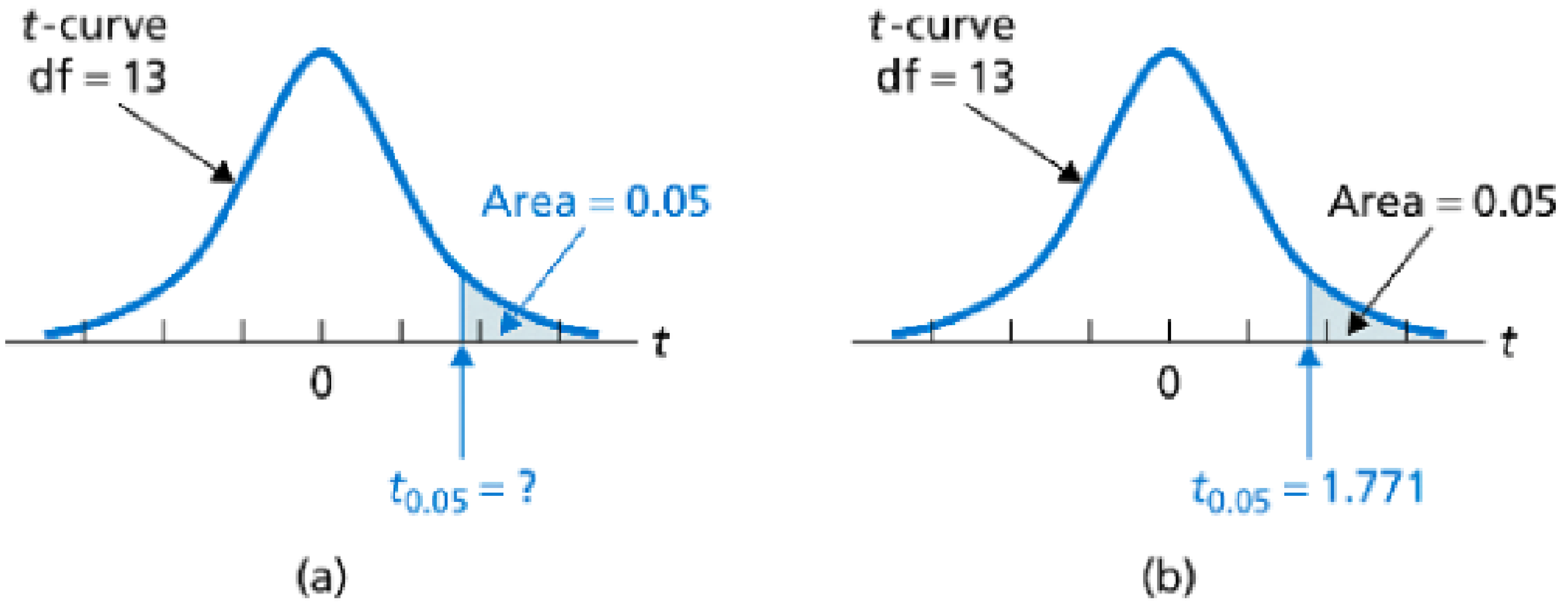
Property 2: A t-curve extends indefinitely in both directions, approaching, but never touching, the horizontal axis as it does so.

Property 3: A t-curve is symmetric about 0.

Property 4: As the number of degrees of freedom becomes larger, t-curves look increasingly like the standard normal curve.

For a t-curve with 13 degrees of freedom, determine $t_{0.05}$; that is, find the t-value having area 0.05 to its right, as shown in Fig. 8.9(a).

Figure 8.9



Finding the t-value having area 0.05 to its right

1. Simple random sample
2. Normal population or large sample
3. σ unknown

Step 1 **For a confidence level of $1 - \alpha$, use Table IV to find $t_{\alpha/2}$ with $df = n - 1$, where n is the sample size.**

Step 2 **The confidence interval for μ is from**

$$\bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \quad \text{to} \quad \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}},$$

where $t_{\alpha/2}$ is found in Step 1 and $\bar{x}$ and s are computed from the sample data.

Step 3 **Interpret the confidence interval.**

Pickpocket Offenses The Federal Bureau of Investigation (FBI) compiles data on robbery and property crimes and publishes the information in *Population-at-Risk Rates and Selected Crime Indicators*. A simple random sample of pickpocket offenses yielded the losses, in dollars, shown in Table 8.6. Use the data to find a 95% confidence interval for the mean loss, μ, of all pickpocket offenses.

$n = 25$ $\bar{x} = 513.32$ $s = 262.23$

$t_{95\%\ DF24} = 2.064$ $\frac{262.23}{\sqrt{25}} = 52.45$

$513.32 \pm 2.064(52.45)$

$405.1 \longleftrightarrow 621.6$

Chicken Consumption The U.S. Department of Agriculture publishes data on chicken consumption in *Food Consumption, Prices, and Expenditures*. Table 8.7 shows a year's chicken consumption, in pounds, for 17 randomly selected people. Find a 90% confidence interval for the year's mean chicken consumption, μ.

Table 8.7 Sample of year's chicken consumption (lb)

57	69	63	49	63	61
72	65	91	59	0	82
60	75	55	80	73	

$n = 16$ $df = 15$

$\bar{x} = 67.1$ $s = 11.1$ 90%

$t = 1.753$

$$67.1 \pm 1.753\left(\frac{11.1}{\sqrt{16}}\right)$$

$62.2 \longleftrightarrow 72$ 90%