Name: Answer Key Date: _____

Calculus 2 Exam Chapter 7 Strategies of Integration

1) $\int x^4 \ln x \, dx$

$u = \ln x \qquad v = x^5/5$

$du = \frac{1}{x} dx \qquad dv = x^4 \, dx$

$$\int x^4 \ln x \, dx = \frac{x^5}{5} \ln x - \int \frac{x^5}{5} \cdot \frac{1}{x} \, dx$$

$$= \boxed{\frac{x^5}{5} \ln x - \frac{x^5}{25} + C}$$

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2) $\int x \csc^2 x \, dx$

$u = x$ $\quad v = -\cot x$

$du = dx$ $\quad dv = \csc^2 x \, dx$

$$\int x \csc^2 x \, dx = -x \cot x - \int -\cot x \, dx$$

$$= \boxed{-x \cot x + \ln|\sin x| + C}$$

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3) $\int (\sin^2 2x)(\cos^2 2x)\,dx$

$u = 2x$

$\frac{1}{2}du = dx$

$\frac{1}{2}\int \cos^2 u \sin^2 u\, du$

$\frac{1}{2}\int (\cos u \sin u)^2\, du$

$\frac{1}{2}\int \frac{1}{2}(\sin 2u)^2\, du = \frac{1}{4}\int \sin^2 2u\, du$

$\frac{1}{4}\int \frac{1}{2}(1-\cos 4u)\,du = \frac{1}{8}\int 1-\cos 4u$

$\frac{1}{8}\int 1\,du - \frac{1}{8}\int \cos 4u\, du$

$\frac{1}{8}u - \frac{1}{32}\sin 4u + C$

$\frac{1}{4}x - \frac{1}{32}\sin 8x + C$

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4) $\int (\tan x)(sec^3 x)dx$

$$\int \tan x \sec^3 x \, dx = \int \tan x \sec x \sec^2 x \, dx$$

$u = \sec x$

$du = \sec x \tan x \, dx$

$$= \int u^2 \, du$$

$$= \frac{1}{3}u^3 + C = \boxed{\frac{1}{3}\sec^3 x + C}$$

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5) $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

$$\int \frac{x^2}{\sqrt{9-x^2}} = \int \frac{9\sin^2\theta}{3\cos\theta}\,3\cos\theta\,d\theta$$

$x = 3\sin\theta \qquad \sqrt{9-x^2} = 3\cos\theta$

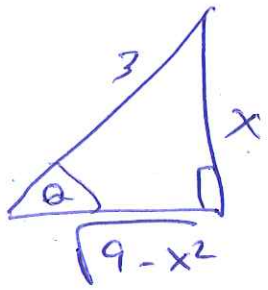
$dx = 3\cos\theta\,d\theta$

$$= 9\int \sin^2\theta\,d\theta = 9\int \frac{1}{2}(1-\cos 2\theta)\,d\theta$$

$$= \frac{9}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

$$= \frac{9}{2}\theta - \frac{9}{4}(2\sin\theta\cos\theta) + C$$

$$= \boxed{\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{x}{2}\sqrt{9-x^2} + C}$$



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6) $\int \frac{\sqrt{x^2-4}}{x}\,dx$

$x = 2\sec\theta$ $\qquad \sqrt{x^2-4} = 2\tan\theta$

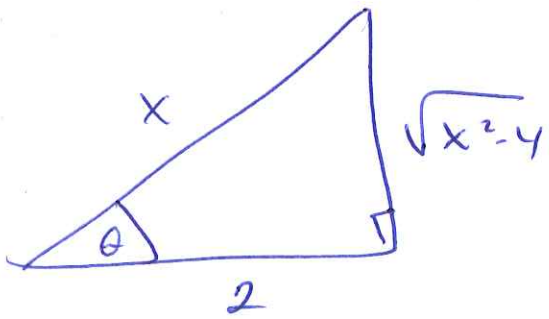
$dx = 2\sec\theta\tan\theta\,d\theta$

$$\int \frac{\sqrt{x^2-4}}{x}\,dx = \int \frac{2\tan\theta}{2\sec\theta}\,2\sec\theta\tan\theta\,d\theta$$

$$= 2\int \tan^2\theta\,d\theta$$

$$= 2\int \sec^2\theta - 1\,d\theta$$

$$= 2\tan\theta - 2\theta + C$$



$$= \sqrt{x^2-4} - 2\sec^{-1}\left(\frac{x}{2}\right) + C$$

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7) $\int \frac{5x+1}{(2x+1)(x-1)}\,dx = \frac{A}{2x+1} + \frac{B}{x-1}$

$5x+1 = A(x-1) + B(2x+1)$

$5x+1 = Ax - A + 2Bx + B$

$5x+1 = (A+2B)x + (-A+B)$

$x=1 \quad 6 = 3B \rightarrow B = 2$

$x=-1/2 \quad -3/2 = -3/2\,A \rightarrow A = 1$

$\int \frac{1}{2x+1}\,dx + \int \frac{2}{x-1}\,dx$

$\frac{1}{2}\ln|2x+1| + 2\ln|x-1| + C$

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8) $\int \frac{10}{(x-1)(x^2-9)}\,dx = \frac{A}{x-1} + \frac{B}{x-3} + \frac{C}{x+3}$

$10 = A(x^2-9) + B(x-1)(x+3) + C(x-1)(x-3)$

$x=3 \quad 10 = 12B \rightarrow 5/6 = B$

$x=1 \quad 10 = -8A \rightarrow -5/4 = A$

$x=-3 \quad 10 = 24C \rightarrow 5/12 = C$

$$-5/4 \int \frac{1}{x-1}\,dx + 5/6 \int \frac{1}{x-3}\,dx + \frac{5}{12} \int \frac{1}{x+3}\,dx$$

$$-5/4 \ln|x-1| + 5/6 \ln|x-3| + 5/12 \ln|x+3| + C$$

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9) $\int \frac{e^x}{(1-e^x)^2}\,dx$

$u = 1 - e^x$

$du = -e^x\,dx$

$$-\int \frac{1}{u^2}\,du = -\int u^{-2}\,du$$

$$= \frac{1}{u} + C$$

$$= \boxed{\frac{1}{1-e^x} + C}$$

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10) $\int (sin^2 x)(\cos x)dx$

$u = \sin x$

$du = \cos x \, dx$

$\int u^2 \, du$

$\frac{u^3}{3} + C = \boxed{\frac{\sin^3 x}{3} + C}$