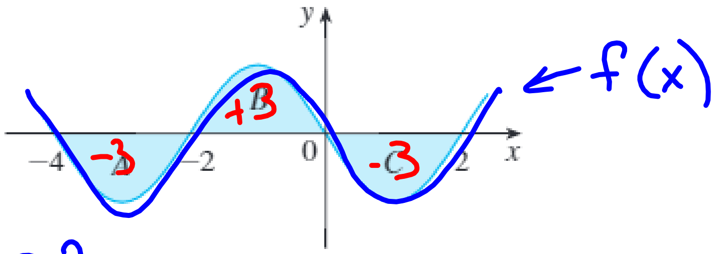
53. Each of the regions A, B, and C bounded by the graph of f and the x-axis has area 3. Find the value of

$$\int_{-4}^{2} [f(x) + 2x + 5]\, dx$$



$$\int_{-4}^{2} [f(x) + 2x + 5]\, dx$$

$$\boxed{\int_{-4}^{2} f(x)} + \int_{-4}^{2} 2x + \int_{-4}^{2} 5$$

$$-3 + \frac{\cancel{2}x^2}{\cancel{2}}\Big]_{-4}^{2} + 5x\Big]_{-4}^{2}$$

$$-3 + (2^2 - (-4)^2) + (5(2) - 5(-4))$$

$$-3 + (4 - 16) + (10 + 20)$$

$$-3 + (-12) + 30$$

$$-15 + 30$$

$$15$$

15. $\int (2 + \tan^2\theta)\, d\theta$

$$\int 2 + (\sec^2\theta - 1)\, d\theta$$

$$\int 1 + \sec^2\theta\, d\theta$$

$$\int 1 + \int \sec^2\theta\, d\theta$$

$$\boxed{\theta + \tan\theta + C}$$

15. $\int \cos^3\theta \sin\theta \, d\theta$

$u = \cos\theta$

$\int -u^3 \, du$

$du = -\sin\theta \, d\theta$

$-du = \sin\theta \, d\theta$

$-\frac{1}{4}u^4$

$$\boxed{-\frac{1}{4}(\cos^4\theta) + C}$$

17. $\displaystyle\int \frac{e^u}{(1-e^u)^2}\,du$

$x = 1 - e^u$

$dx = -e^u\,du$

$-dx = e^u\,du$

$$-\int \frac{1}{x^2}\,dx$$

$$-\int x^{-2}\,dx \Rightarrow -\frac{x^{-1}}{-1} = \frac{1}{x} = \boxed{\frac{1}{1-e^u} + C}$$

45. $\int \frac{1+x}{1+x^2}\,dx$

$$\int \frac{1}{1+x^2}dx + \boxed{\int \frac{x}{1+x^2}\,dx}$$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\downarrow$$

$$\tan^{-1}x + \frac{1}{2}\int \frac{1}{u}\,du$$

$$+\frac{1}{2}\ln|u|$$

$$\tan^{-1}x + \frac{1}{2}\ln(1+x^2) + C$$

47. $\int x(2x+5)^8\,dx$

$$\frac{1}{2}\int \frac{1}{2}(u-5)\,u^8\,du$$

$$\frac{1}{4}\int u^9 - 5u^8\,du$$

$$\frac{1}{4}\left(\frac{u^{10}}{10} - \frac{5u^9}{9}\right)+C$$

$u = 2x+5$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$u = 2x + 5$

$-5 \qquad -5$

$\frac{u-5}{2} = \frac{2x}{2}$

$\frac{1}{2}(u-5) = x$

$$\boxed{\frac{1}{40}(2x+5)^{10} - \frac{5}{36}(2x+5)^9 + C}$$

67. $\int_1^2 x\sqrt{x-1}\,dx$

$u = x - 1$

$u + 1 = x$

$du = dx$

If $x = 1$ $u = 1 - 1 = 0$

$x = 2$ $u = 2 - 1 = 1$

$$\int_0^1 (u+1)\,u^{1/2}\,du$$

$$\int_0^1 u^{3/2} + u^{1/2}\,du$$

$$\left.\frac{2u^{5/2}}{5}\right]_0^1 + \left.\frac{2u^{3/2}}{3}\right]_0^1$$

$$\frac{2}{5} + \frac{2}{3} = \boxed{\frac{16}{15}}$$

69. $\int_{e}^{e^4} \frac{dx}{x\sqrt{\ln x}}$

$u = \ln x$

$du = \frac{1}{x} dx$

$u = \ln(e) = 1$

$u = \ln(e^4) = 4$

$\int_1^4 \frac{1}{\sqrt{u}} du$

$\int_1^4 u^{-1/2} du$

$2u^{1/2} \Big]_1^4$

$2(4)^{1/2} - 2(1)^{1/2}$

$4 - 2$

$\boxed{2}$

71. $\int_0^1 \frac{e^z + 1}{e^z + z}\, dz$

$u = e^z + z$

$du = e^z + 1 \; dz$

$z = 1 \quad u = e^1 + 1$

$z = 0 \quad u = e^0 + 0 = 1$

$$\int_1^{e^1+1} \frac{1}{u}\, du$$

$$\ln u + C \Big]_1^{e^1+1}$$

$$\ln(e+1) - \ln(1)$$

$$\boxed{\ln(e+1)}$$

73. $\displaystyle\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$

$u = 1+\sqrt{x} \Rightarrow u-1 = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

$2(u-1)\,du = dx$

$x=1 \quad u = 1+\sqrt{1} = 2$

$x=0 \quad u = 1+\sqrt{0} = 1$

$$\int_1^2 \frac{2(u-1)}{u^4}\,du$$

$$2\int_1^2 \frac{1}{u^3} - \frac{1}{u^4}\,du$$

$$2\int_1^2 u^{-3} - u^{-4}\,du$$

$$2\left(\frac{u^{-2}}{-2}\Big]_1^2 - \frac{u^{-3}}{-3}\Big]_1^2\right)$$

$$2\left(\left(-\frac{1}{8} - \frac{-1}{2}\right) - \left(-\frac{1}{24} - \frac{-1}{3}\right)\right)$$

$$2\left(\frac{3}{8} - \frac{7}{24}\right) = 2\left(\frac{2}{24}\right) = \frac{4}{24} = \boxed{\frac{1}{6}}$$