9.1 The Nature of Hypothesis Testing

One of the most commonly used methods for making such decisions or judgments is to perform a *hypothesis test*. A **hypothesis** is a statement that something is true. For example, the statement "the mean weight of all bags of pretzels packaged differs from the advertised weight of 454 g" is a hypothesis.

Definition 9.1: Null and Alternative Hypotheses; Hypothesis Test

Null hypothesis: A hypothesis to be tested. We use the symbol H_0 to represent the null hypothesis.

H_0

Alternative hypothesis: A hypothesis to be considered as an alternative to the null hypothesis. We use the symbol H_a to represent the alternative hypothesis.

H_a

Hypothesis test: The problem in a hypothesis test is to decide whether the null hypothesis should be rejected in favor of the alternative hypothesis.

Null Hypothesis

In this book, the null hypothesis for a hypothesis test concerning a population mean, μ, always specifies a single value for that parameter. Hence we can express the null hypothesis as

$$H_0: \mu = \mu_0,$$

where μ_0 is some number.

$$H_a: \mu \neq \mu_0.$$

A hypothesis test whose alternative hypothesis has this form is called a **two-tailed test.**

- If the primary concern is deciding whether a population mean, μ, is *less than* a specified value μ_0, we express the alternative hypothesis as

$$H_a: \mu < \mu_0.$$

A hypothesis test whose alternative hypothesis has this form is called a **left-tailed test.**

- If the primary concern is deciding whether a population mean, μ, is *greater than* a specified value μ_0, we express the alternative hypothesis as

$$H_a: \mu > \mu_0.$$

A hypothesis test whose alternative hypothesis has this form is called a **right-tailed test.**

Quality Assurance A snack-food company produces a 454-g bag of pretzels. Although the actual net weights deviate slightly from 454 g and vary from one bag to another, the company insists that the mean net weight of the bags be 454 g.

As part of its program, the quality assurance department periodically performs a hypothesis test to decide whether the packaging machine is working properly, that is, to decide whether the mean net weight of all bags packaged is 454 g.

a. Determine the null hypothesis for the hypothesis test. $H_0 \rightarrow \mu = 454$

b. Determine the alternative hypothesis for the hypothesis test. $H_a \rightarrow \mu \neq 454$

c. Classify the hypothesis test as two tailed, left tailed, or right tailed. 2 tail

Taller Young Women In the document *Anthropometric Reference Data for Children and Adults*, C. Fryer et al. present data from the *National Health and Nutrition Examination Survey* on a variety of human body measurements, such as weight, height, and size. Anthropometry is a key component of nutritional status assessment in children and adults.

A half-century ago, the average (U.S.) woman in her 20s was 62.6 inches tall. Suppose that we want to perform a hypothesis test to decide whether today's women in their 20s are, on average, taller than such women were a half-century ago. $H_0 \to \mu = 62.6$

a. Determine the null hypothesis for the hypothesis test.

b. Determine the alternative hypothesis for the hypothesis test. $H_a \to \mu > 62.6$

c. Classify the hypothesis test as two tailed, left tailed, or right tailed. Rght

Poverty and Dietary Calcium Calcium is the most abundant mineral in the human body and has several important functions. Most body calcium is stored in the bones and teeth, where it functions to support their structure. Recommendations for calcium are provided in *Dietary Reference Intakes*, developed by the Institute of Medicine of the National Academy of Sciences. The recommended adequate intake (RAI) of calcium for adults (ages 19–50 years) is 1000 milligrams (mg) per day.

Suppose that we want to perform a hypothesis test to decide whether the average adult with an income below the poverty level gets less than the RAI of 1000 mg.

a. Determine the null hypothesis for the hypothesis test. $H_0 \ \mu = 1000$

b. Determine the alternative hypothesis for the hypothesis test. $H_a \ \mu < 1000$

c. Classify the hypothesis test as two tailed, left tailed, or right tailed. Left

Golf Driving Distances Jack tells Jean that his average drive of a golf ball is 275 yards. Jean is skeptical and asks for substantiation. To that end, Jack hits 25 drives. The results, in yards, are shown in Table 9.2.

The (sample) mean of Jack's 25 drives is only 264.4 yards. Jack still maintains that, on average, he drives a golf ball 275 yards and that his (relatively) poor performance can reasonably be attributed to chance.

At the 5% significance level, do the data provide sufficient evidence to conclude that Jack's mean driving distance is less than 275 yards? We use the following steps to answer the question.

$H_0 \to \mu = 275$

$H_a \to \mu < 275$

a. State the null and alternative hypotheses.

b. Discuss the logic of this hypothesis test.

$\mu_{\bar{x}} = 275$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{25}}$

$\sigma_{\bar{x}} = 4$

$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - 275}{4}$

$\frac{264.4 - 275}{4} = -2.65$

$5\% \rightarrow z = -1.65$

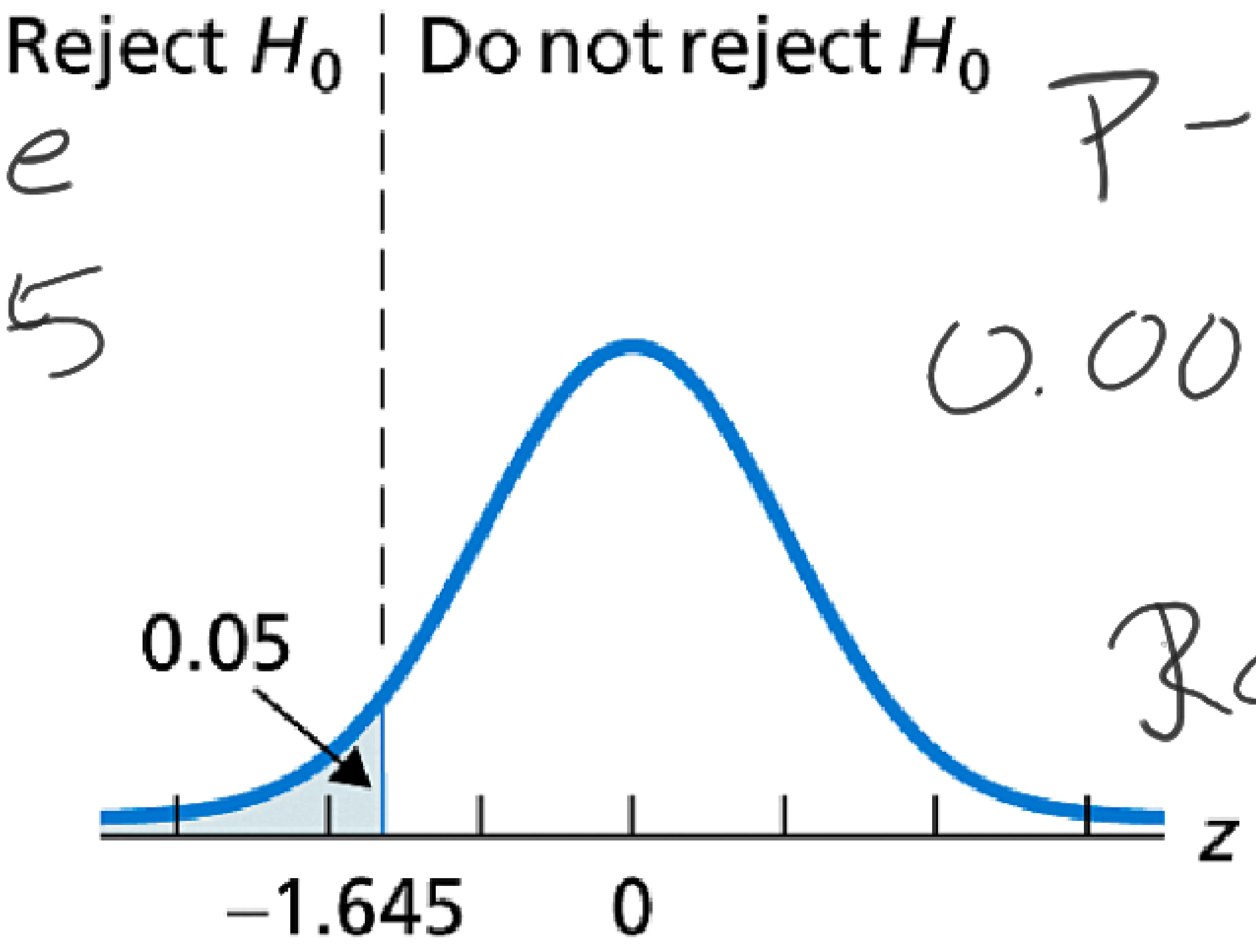
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
–1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
–1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
–1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559

Figure 9.2

Critical Value

$-2.65 < -1.65$

Reject



P-Value

$0.0040 < 0.05$

Reject

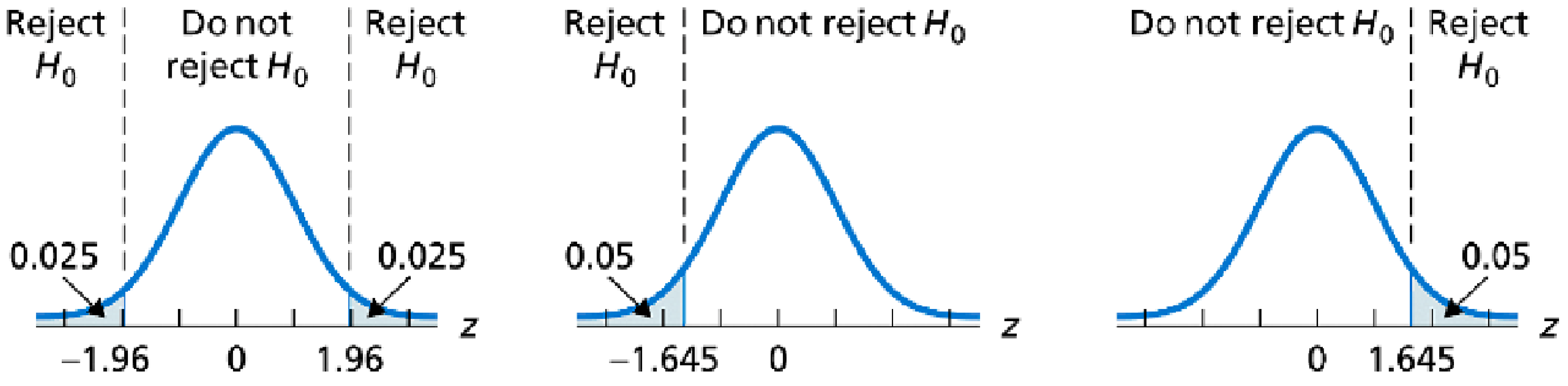
Criterion for deciding whether to reject the null hypothesis

Determine the critical value(s) for a one-mean z-test at the 5% significance level ($\alpha = 0.05$) if the test is

a. two tailed.

b. left tailed.

c. right tailed.



(a) Two tailed (b) Left tailed (c) Right tailed

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A half-century ago, the average (U.S.) woman in her 20s was 62.6 inches tall. The heights, in inches, of a random sample of 25 of today's women in their 20s is presented in Table 9.9.

At the 1% significance level, do the data provide sufficient evidence to conclude that the mean height of today's women in their 20s is greater than the mean height of women in their 20s a half-century ago? Assume that the population standard deviation of heights for today's women in their 20s is 2.88 inches.

The mean of the sample of today's women is 64.24

$H_0 \; \mu = 62.6$ $\quad H_a \rightarrow \mu > 62.6$ $\boxed{1\%}$

$\bar{x} = 64.24 \quad \mu_{\bar{x}} = 62.6 \quad \sigma_{\bar{x}} = \frac{2.88}{\sqrt{25}} = .58$

$$z = \frac{64.24 - 62.6}{.58} = 2.83$$

Critical value

$2.33 > 2.83$

Reject

P-value Reject

$.0023 > .01$

$.9977$

Poverty and Dietary Calcium Calcium is the most abundant mineral in the human body and has several important functions. Most body calcium is stored in the bones and teeth, where it functions to support their structure. Recommendations for calcium are provided in *Dietary Reference Intakes*, developed by the Institute of Medicine of the National Academy of Sciences. The recommended adequate intake (RAI) of calcium for adults (ages 19–50 years) is 1000 milligrams (mg) per day.

A simple random sample of 18 adults with incomes below the poverty level gives the daily calcium intakes shown in Table 9.10. At the 5% significance level, do the data provide sufficient evidence to conclude that the mean calcium intake of all adults with incomes below the poverty level is less than the RAI of 1000 mg? Assume that $\sigma = 188$ mg.

The mean of the sample is 947.4

$H_0 \rightarrow \mu = 1000$ $\qquad$ $H_a \rightarrow \mu < 1000$

$\boxed{5\%}$

$\bar{x} = 947.4$ $\qquad$ $\mu_{\bar{x}} = 1000$

$$\sigma_{\bar{x}} = \frac{188}{\sqrt{18}} = 44.3$$

$$z = \frac{947.4 - 1000}{44.3} = -1.19$$

P-Value

$.1170 > .05$

Accept Null

Clocking the Cheetah The cheetah is the fastest land mammal and is highly specialized to run down prey. According to the online document "Cheetah Conservation in Southern Africa" (*Trade & Environment Database (TED) Case Studies*, Vol. 8, No. 2) by J. Urbaniak, the cheetah is capable of speeds up to 72 mph.

One common estimate of mean top speed for cheetahs is 60 mph. Table 9.11 gives the top speeds, in miles per hour, for a sample of 35 cheetahs.

At the 5% significance level, do the data provide sufficient evidence to conclude that the mean top speed of all cheetahs differs from 60 mph? Assume that the population standard deviation of top speeds is 3.2 mph.

The mean of the sample is 59.526 on 34 cheetahs.

$H_0 \rightarrow \mu = 60$ $H_a \rightarrow \mu \neq 60$

5%

$\bar{x} = 59.526$ $\mu_{\bar{x}} = 60$

$\sigma_{\bar{x}} = \frac{3.2}{\sqrt{34}} = 0.55$

$$z = \frac{59.526 - 60}{0.55} = -0.86$$

$.1949 > .025$ accept null