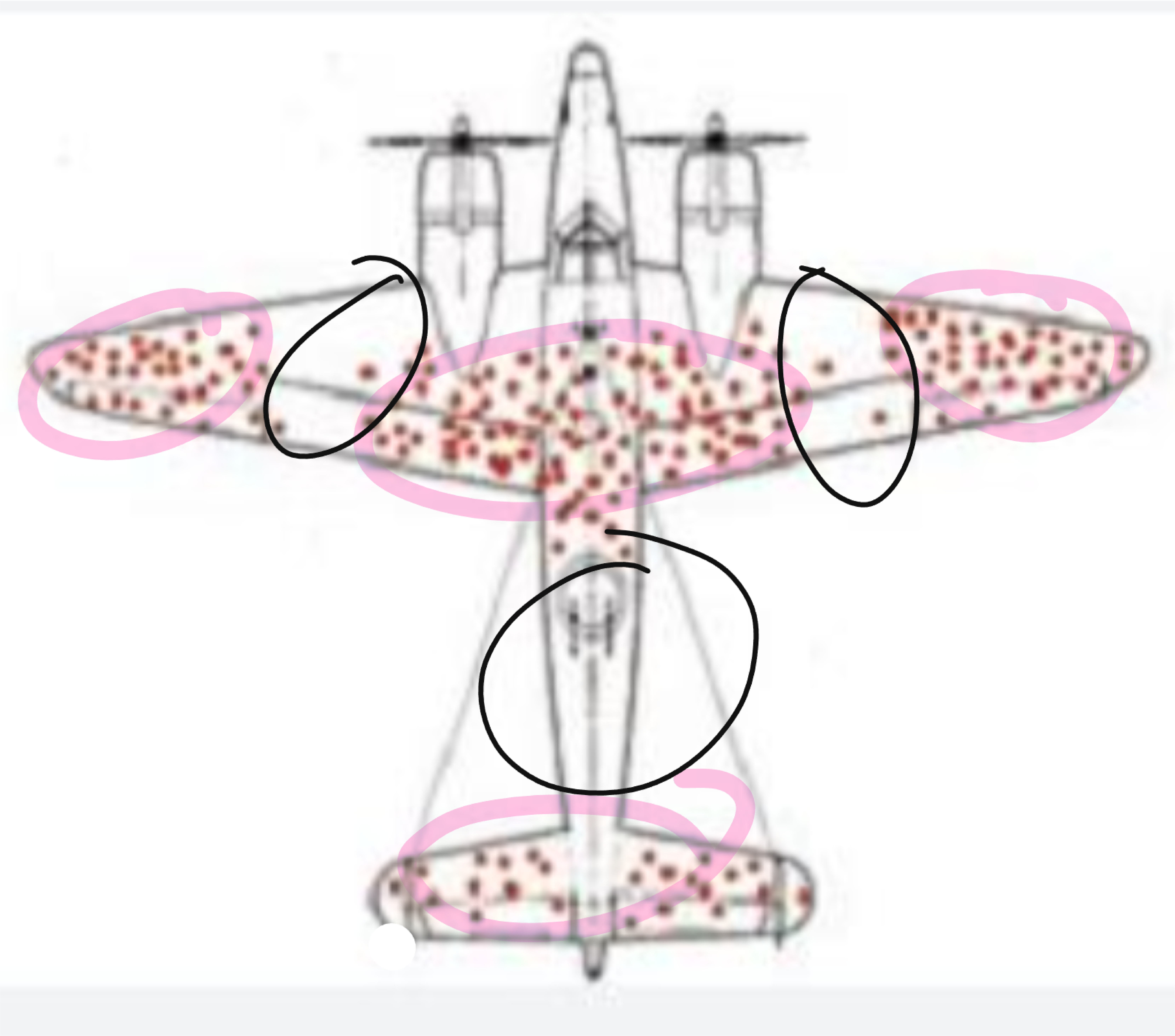
"By a small sample, we may judge the whole piece."
— Miguel de Cervantes from *Don Quixote de la Mancha*

9 CHAPTER Samples and Sampling Distributions



Biased Sample

A **sample is biased** if it overrepresents or underrepresents some segment(s) of the population.

DEFINITION

Sampling Frame

A **sampling frame** is a list which identifies all members of the population.

DEFINITION

Simple Random Sample

A **simple random sample** from a finite population is one in which every possible sample of the same size n has the same probability of being selected.

DEFINITION

Students in a marketing class have been asked to conduct a survey to determine whether or not there is a demand for an insurance program at a local college. The students decide to randomly select students from the local college and mail them a questionnaire regarding the insurance program. Of the 150 surveys that were mailed, 50 students responded to the following survey item: *Pick the category which best describes your interest in an insurance program.*

Survey Responses	
Category	**% of Responses**
Very Interested	50
Somewhat Interested	15
Interested	10
Not Very Interested	5
Not At All Interested	20

Sampling Distribution of a Statistic

The **sampling distribution of a statistic** (such as the sample mean or sample proportion) is the probability distribution of all values of the statistic when all possible samples of size *n* are taken from a population.

DEFINITION

Point Estimator

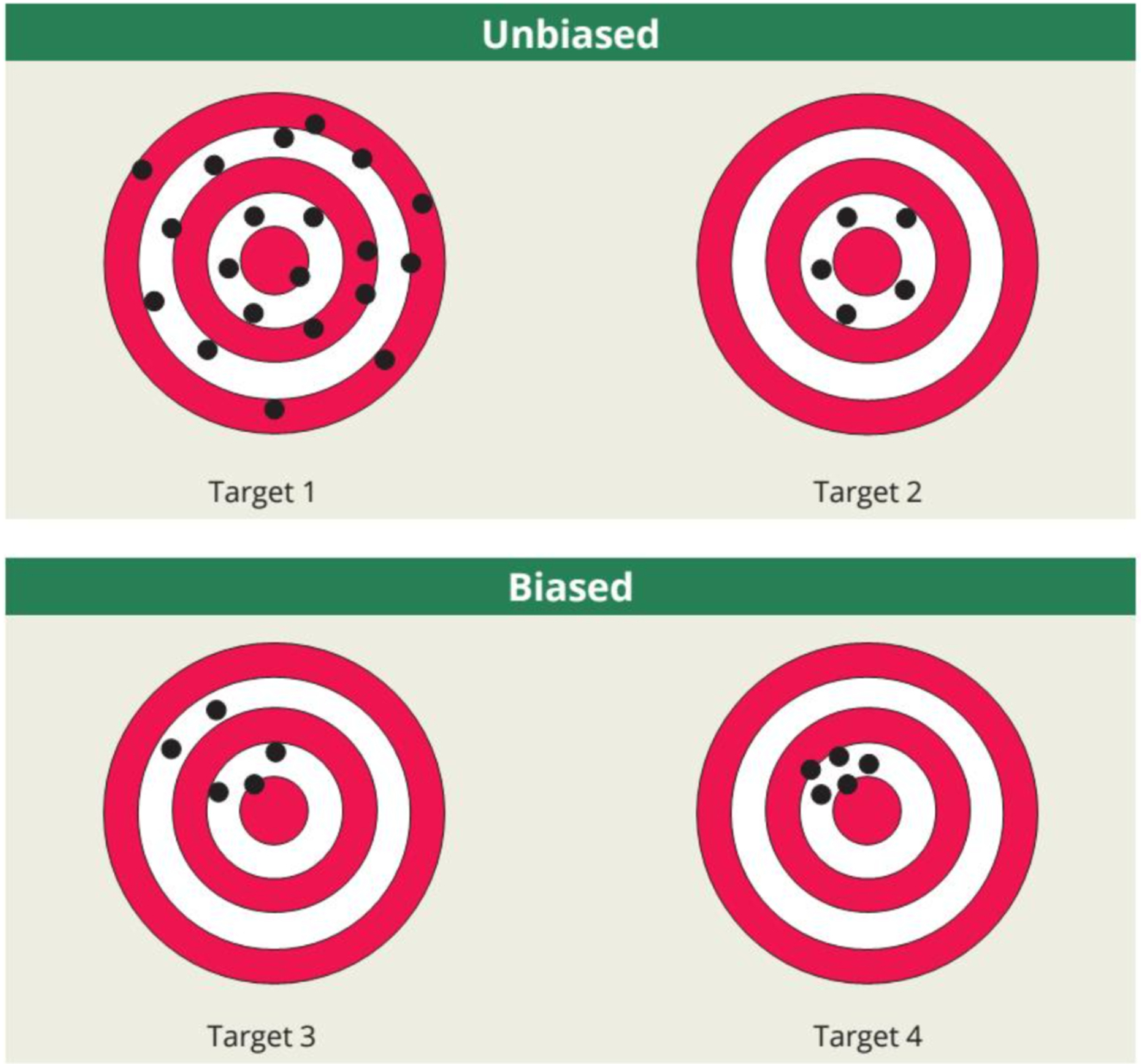
A **point estimator** is a single-valued estimate calculated from the sample data, which is intended to be close to the true population value.

DEFINITION

Unbiased

If the average value of an estimator equals the population parameter being estimated, the estimator is said to be **unbiased**.

DEFINITION



Unbiased
Target 1
Target 2
Biased
Target 3
Target 4

Unbiased Estimators

1. The sample mean, $\bar{x}$, is an unbiased estimator of μ.
2. The sample proportion, $\hat{p}$, is an unbiased estimator of p.
3. The sample variance, s^2, is an unbiased estimator of σ^2.

PROPERTIES

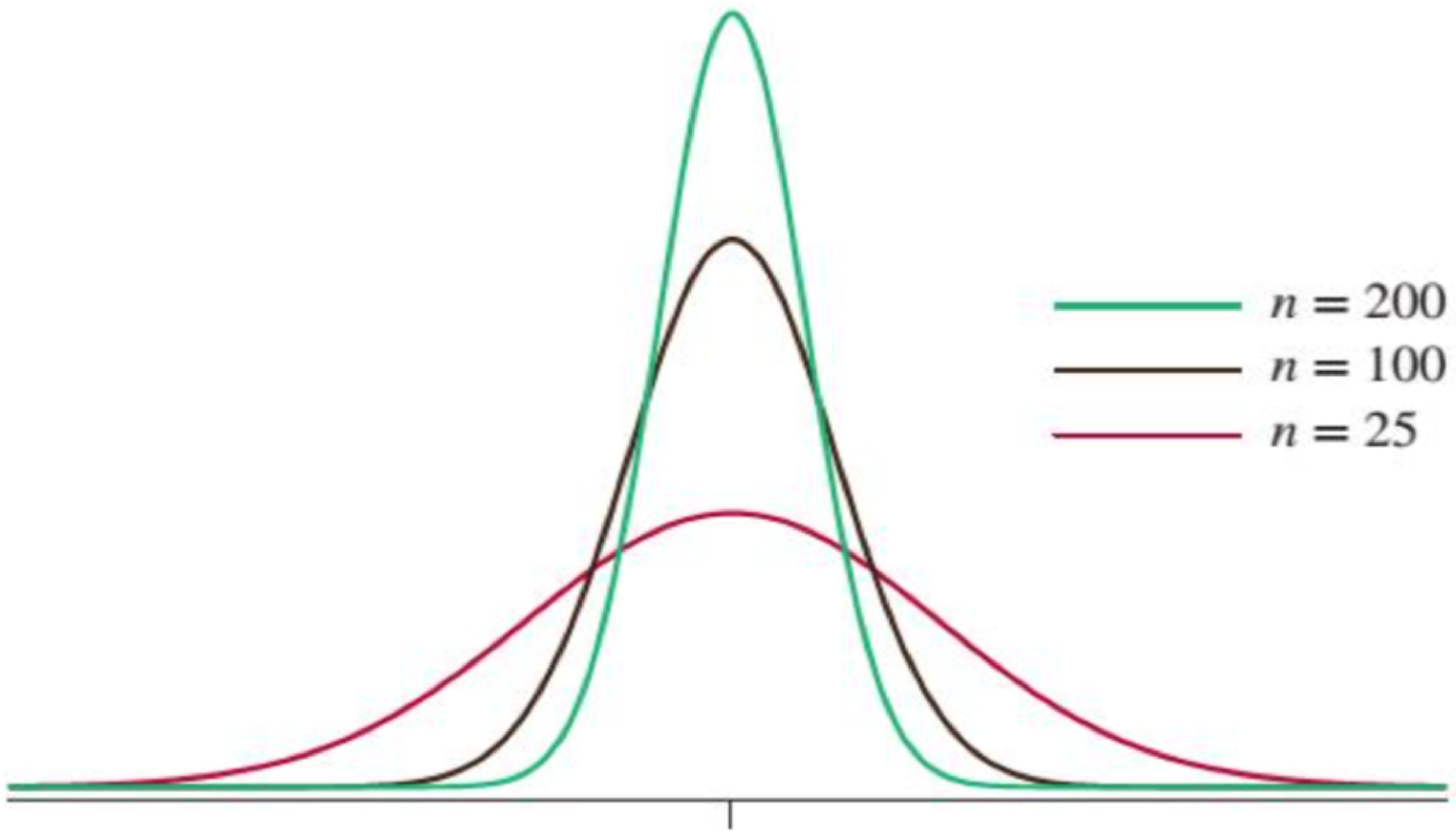
The Central Limit Theorem

If a sufficiently large random sample (i.e., $n > 30$) is drawn from a population with mean μ and standard deviation σ, the distribution of the sample mean will have the following characteristics.

1. An approximately normal distribution regardless of the distribution of the underlying population.
2. $\mu_{\bar{x}} = E(\bar{x}) = \mu$ (The mean of the sample means equals the population mean.)
3. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ (The standard deviation of the sample means equals the standard deviation of the population divided by the square root of the sample size.)

DEFINITION

Sampling Distribution of the Sample Mean, $\sigma = 2500$



Suppose a population has a mean of 30 and a variance of 25. If a sample size of 100 is drawn from the population, what is the probability that the sample mean will be larger than 31?

$\mu = 30$

$\sigma = \sqrt{25} = 5$

$* \; n = 100$

$P(\bar{x} > 31) = .0228$

$$Z = \frac{x - \mu}{\sigma / \sqrt{n}} = \frac{31 - 30}{5 / \sqrt{100}} = 2$$

Suppose a population has a mean of 30 and a variance of 25. If a sample size of 100 is drawn from the population, what is the probability that the sample mean will be larger than 31?

$\mu = 30$

$\sigma = \sqrt{25} = 5$

* $n = 1000$

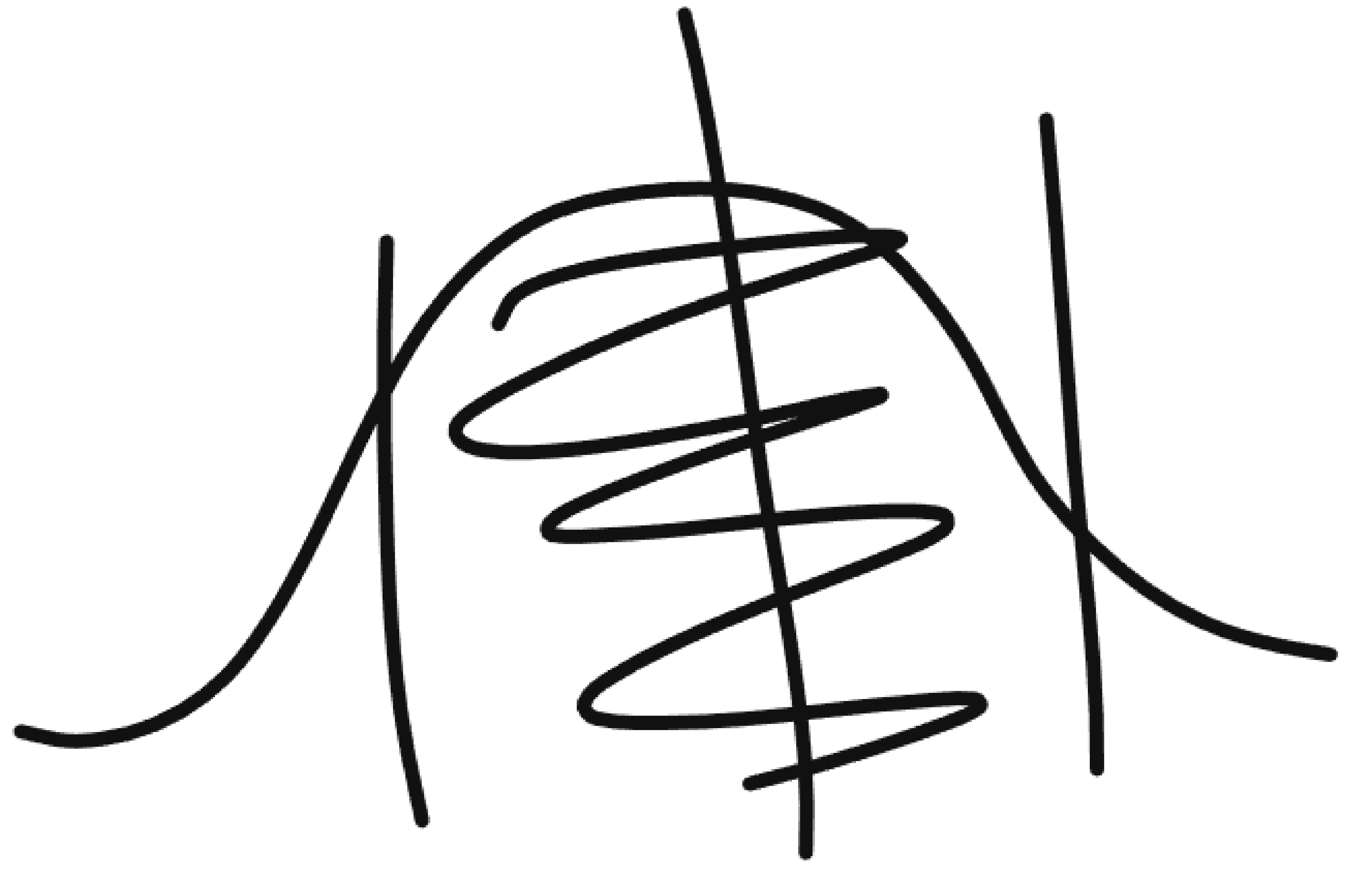
$P(\bar{x} > 31) = .0000001$

$$Z = \frac{x - \mu}{\sigma / \sqrt{n}} = \frac{31 - 30}{5 / \sqrt{1000}} = 6.32$$

A company fills bags with fertilizer for retail sale. The weights of the bags of fertilizer have a normal distribution with a mean weight of 15 lb and standard deviation of 1.70 lb.

a. What is the probability that a randomly selected bag of fertilizer will weigh between 14 and 16 pounds?

$\mu = 15$ $\sigma = 1.70$



$P(14 \leq X \leq 16)$

$X = 14$ $z = -0.59$ $.2776$

$X = 16$ $z = 0.59$ $-.7224$

$> .4448$

A company fills bags with fertilizer for retail sale. The weights of the bags of fertilizer have a normal distribution with a mean weight of 15 lb and standard deviation of 1.70 lb.

b. If 35 bags of fertilizer are randomly selected, find the probability that the average weight of the 35 bags will be between 14 and 16 pounds.

$\mu = 15$ $\sigma = 1.70$

$n = 35$ $P(14 \le X \le 16) = .9994$

$X = 14$ $\frac{14 - 15}{1.70/\sqrt{35}} = -3.48$ $.0003$

$X = 16$ $= 3.48$ $.9997$

The turkeys found in a particular county have an average weight of 15.6 pounds with a standard deviation of 4.00 pounds. Forty-five turkeys are randomly selected for a county fair.

a. Find the probability than 14.5 pounds.

$$\frac{14.5 - 15.6}{\frac{4}{\sqrt{45}}} = -1.844756081$$

$P(x<14.5) = .0329$

b. What is the probabilit more than 17 pounds?

$$\frac{17 - 15.6}{\frac{4}{\sqrt{45}}} = 2.347871376$$

$P(x>17) = .0094$

c. Find the probability that the average weight of the turkeys will be between 13 and 18 pounds.

$$\frac{13 - 15.6}{\frac{4}{\sqrt{45}}} = -4.360332556$$

$$\frac{18 - 15.6}{\frac{4}{\sqrt{45}}} = 4.024922359$$

.9999999