License Plates The license plates of a state consist of three letters followed by three digits.

a. How many different license plates are possible?

b. How many possibilities are there for license plates on which no letter or digit is repeated?

a) 26 26 26 10 10 10

17,576,000

b) 26 25 24 10 9 8

11,232,000

Definition 4.8: Factorials

The product of the first k positive integers (counting numbers) is called $\boldsymbol{k}$ **factorial** and is denoted $\boldsymbol{k!}$. In symbols,

$$k! = k\,(k-1)\cdots 2\cdot 1.$$

We also define $0! = 1$.

$$3! = 3 \times 2 \times 1$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

Formula 4.10: The Permutations Rule

Order matters

The number of possible permutations of r objects from a collection of m objects is given by the formula

$_mP_r$

$$_mP_r = \frac{m!}{(m-r)!}$$

Exacta Wagering In an exacta wager at the race track, a bettor picks the two horses that he or she thinks will finish first and second in a specified order. For a race with 12 entrants, determine the number of possible exacta wagers.

$_nP_r$ $\quad$ $_{12}P_2$

Formula 4.12: The Combinations Rule

order doesn't matter

The number of possible combinations of r objects from a collection of m objects is given by the formula

$_nC_r$ $C(n, r)$

$$_mC_r = \frac{m!}{r!(m-r)!}.$$

$C(69, 4)$

CD-Club Introductory Offer To recruit new members, a compact-disc (CD) club advertises a special introductory offer: A new member agrees to buy 1 CD at regular club prices and receives free any 4 CDs of his or her choice from a collection of 69 CDs. How many possibilities does a new member have for the selection of the 4 free CDs?

864,501

Doing the Calculations Determine the value of each binomial coefficient.

a. $\binom{6}{1}$

b. $\binom{5}{3}$

c. $\binom{7}{3}$

d. $\binom{4}{4}$

$_nC_r$

a) $_6C_1$

b) $_5C_3$

4.253 Los Angeles Dodgers. From the official website of the 2013 Los Angeles Dodgers major league baseball team, we found that there were five active players on roster available to play outfield. Assuming that these five players could play any outfield position, how many possible assignments could manager Don Mattingly have made for the three outfield positions?

$_nP_r \rightarrow {}_5P_3 = 60$

Definition 5.1: Random Variable

A **random variable** is a quantitative variable whose value depends on chance.

Definition 5.2: Discrete Random Variable

A **discrete random variable** is a random variable whose possible values can be listed. In particular, a random variable with only a finite number of possible values is a discrete random variable.

- the sum of the dice when a pair of fair dice are rolled,
- the number of puppies in a litter,
- the return on an investment, and
- the lifetime of a flashlight battery.

Definition 5.3: Probability Distribution and Probability Histogram

Probability distribution: A listing of the possible values and corresponding probabilities of a discrete random variable, or a formula for the probabilities.

Probability histogram: A graph of the probability distribution that displays the possible values of a discrete random variable on the horizontal axis and the probabilities of those values on the vertical axis. The probability of each value is represented by a vertical bar whose height equals the probability.

Mean of a Discrete Random Variable

Recall that, for a variable x, the mean of all possible observations for the entire population is called the *population mean* or *mean of the variable* x. In Section 3.5, we gave a formula for the mean of a variable x:

$$\mu = \frac{\Sigma x_i}{N}.$$

Although this formula applies only to variables of finite populations, we can use it and the language of probability to extend the concept of the mean to any discrete variable. We show how to do so in Example 5.6.

x	$P(X = x)$	$x\,P(x)$
0	0.029	0(0,029) = 0
1	0.049	1(0.049) = 0.049
2	0.078	2(0,078) = 0.156
3	0.155	3(0,155) = 0.465
4	0.212	4)(0,212) = 0.848
5	0.262	5(0.262) = 1.310
6	0.215	6(0.215) = 1.290
		4.118

Key Fact 5.3: Interpretation of the Mean of a Random Variable

In a large number of independent observations of a random variable X, the average value of those observations will approximately equal the mean, μ, of X. The larger the number of observations, the closer the average tends to be to μ.

Definition 5.5: Standard Deviation of a Discrete Random Variable

The **standard deviation of a discrete random variable X** is denoted σ_X or, when no confusion will arise, simply σ. It is defined as

$$\sigma = \sqrt{\Sigma (x-\mu)^2 P(X=x)}.$$

The standard deviation of a discrete random variable can also be obtained from the computing formula

$$\sigma = \sqrt{\Sigma x^2 P(X=x) - \mu^2}.$$

x	$P(X = x)$
0	0.029
1	0.049
2	0.078
3	0.155
4	0.212
5	0.262
6	0.215

$\mu = 4.118$

$\sigma \sqrt{\Sigma x^2 - \mu^2}$

$0^2(0.029)$
$1^2(0.049)$
$2^2(0.078)$
$3^2(0.155)$
$4^2(0.212)$
$5^2(0.262)$
$6^2(0.215)$

19.438

$\sigma = \sqrt{19.438 - (4.118)^2}$

$\sigma = 1.575$

5.39 Roulette. An American roulette wheel contains 38 numbers: 18 are red, 18 are black, and 2 are green. When the roulette wheel is spun, the ball is equally likely to land on any of the 38 numbers. Suppose that you bet $1 on red. If the ball lands on a red number, you win $1; otherwise you lose your $1. Let *X* be the amount you win on your $1 bet. Then *X* is a random variable whose probability distribution is as follows.

x	1	–1
$P(X = x)$	0.474	0.526

1(0.474) (-1)(.526)

0.474 -.526

-0.052

5.41 Homeowner's Policy. An insurance company wants to design a homeowner's policy for mid-priced homes. From data compiled by the company, it is known that the annual claim amount, X, in thousands of dollars, per homeowner is a random variable with the following probability distribution.

x	0	10	50	100	200
$P(X = x)$	0.95	0.045	0.004	0.0009	0.0001

Definition 5.8: Bernoulli Trials

Repeated trials of an experiment are called **Bernoulli trials** if the following three conditions are satisfied:

1. The experiment (each trial) has two possible outcomes, denoted generically s, for **success**, and f, for **failure**.
2. The trials are independent, meaning that the outcome on one trial in no way affects the outcome on other trials.
3. The probability of a success, called the **success probability** and denoted p, remains the same from trial to trial.

Formula 5.1: Binomial Probability Formula

Let X denote the total number of successes in n Bernoulli trials with success probability p. Then the probability distribution of the random variable X is given by

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n.$$

The random variable X is called a **binomial random variable** and is said to have the **binomial distribution** with parameters n and p.

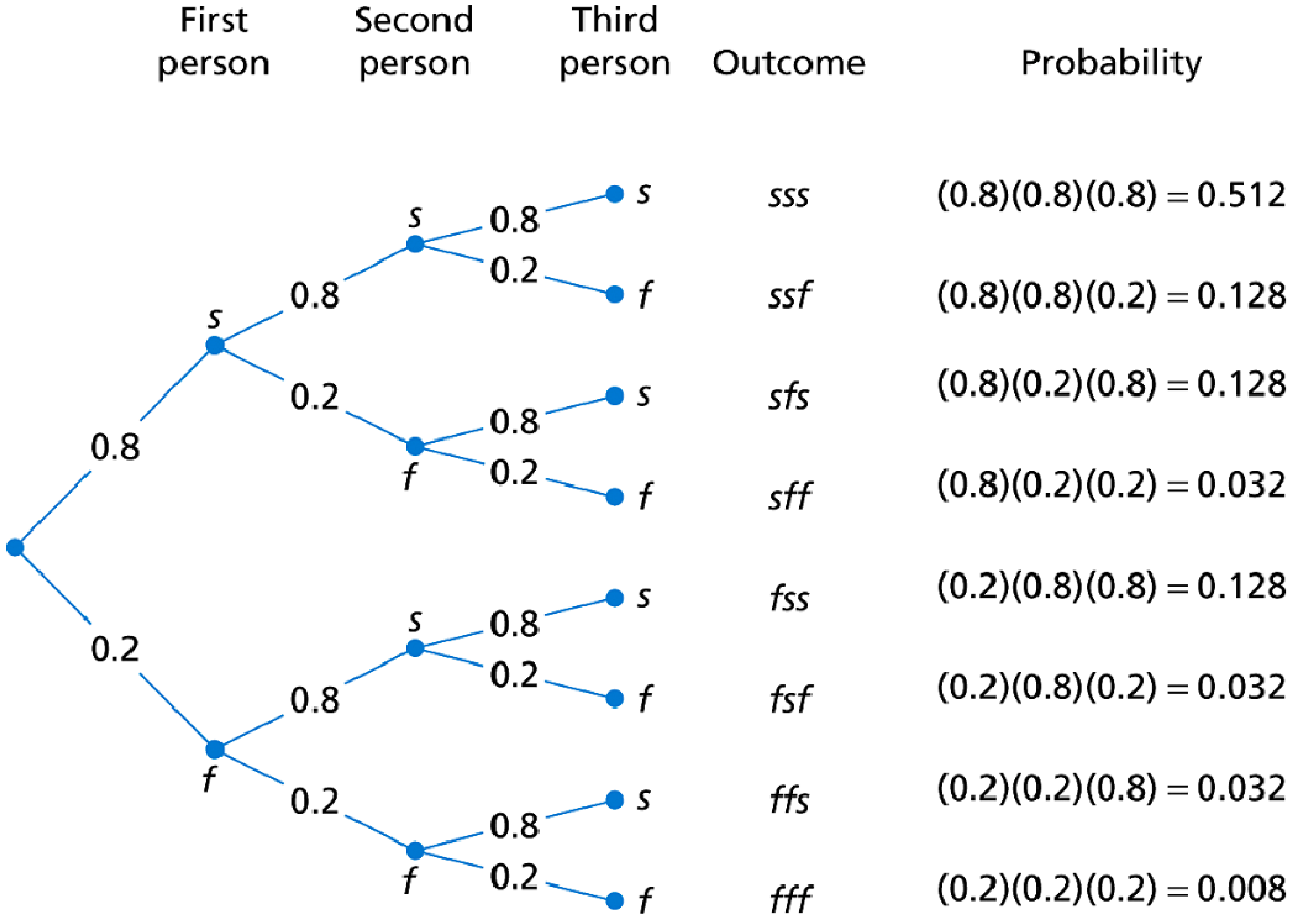
Mortality According to tables provided by the National Center for Health Statistics in *Vital Statistics of the United States*, there is roughly an 80% chance that a person of age 20 years will be alive at age 65 years. Suppose that three people of age 20 years are selected at random. Find the probability that the number alive at age 65 years will be

a. exactly two.

b. at most one.

c. at least one.

d. Determine the probability distribution of the number alive at age 65.



First person
Second person
Third person
Outcome
Probability
0.8
0.2
s
f
0.8
0.2
0.8
0.2
s
f
s
f
0.8
0.2
0.8
0.2
0.8
0.2
0.8
0.2
s
f
s
f
s
f
s
f
sss
ssf
sfs
sff
fss
fsf
ffs
fff
(0.8)(0.8)(0.8) = 0.512
(0.8)(0.8)(0.2) = 0.128
(0.8)(0.2)(0.8) = 0.128
(0.8)(0.2)(0.2) = 0.032
(0.2)(0.8)(0.8) = 0.128
(0.2)(0.8)(0.2) = 0.032
(0.2)(0.2)(0.8) = 0.032
(0.2)(0.2)(0.2) = 0.008