To Determine a Percentage or Probability for a Normally Distributed Variable

Step 1 **Sketch the normal curve associated with the variable.**

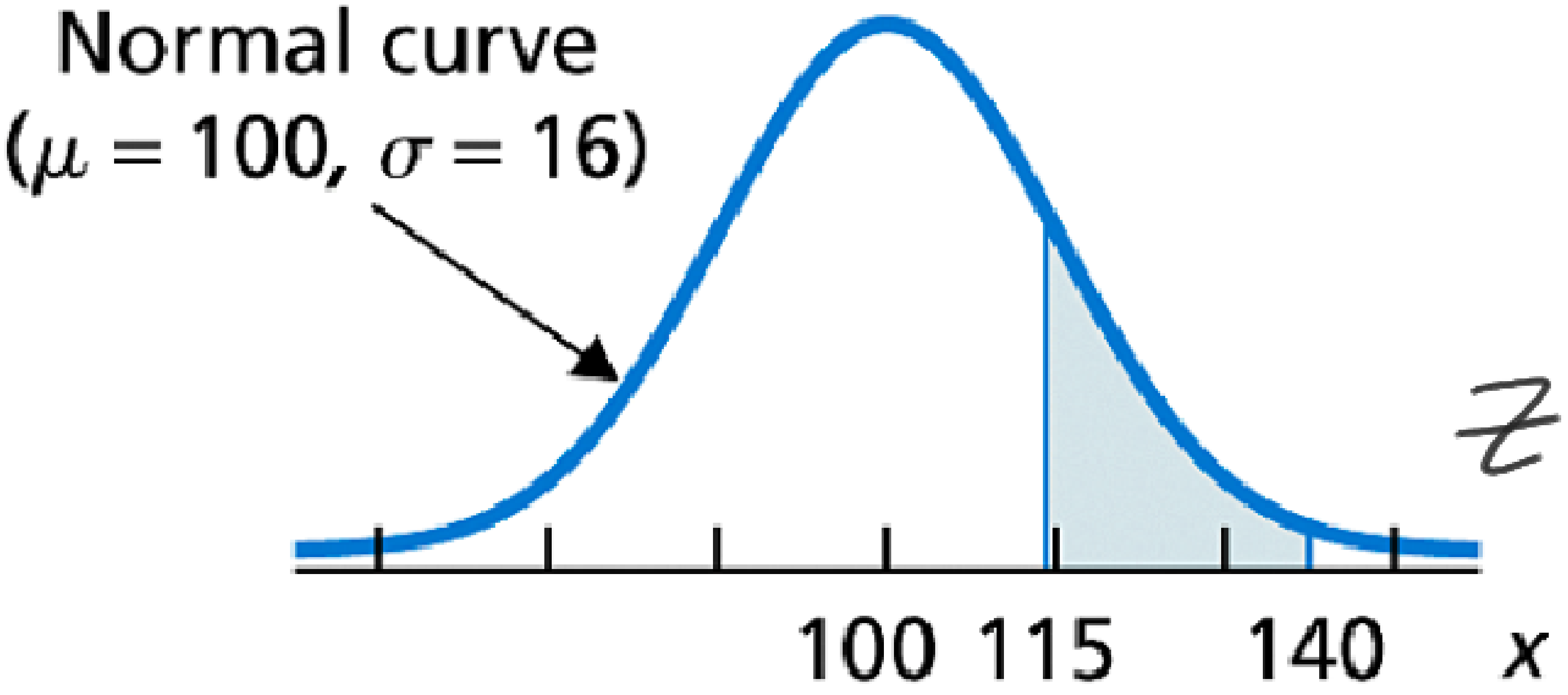
Step 2 **Shade the region of interest and mark its delimiting *x*-value(s).**

Step 3 **Find the *z*-score(s) for the delimiting *x*-value(s) found in Step 2.**

Step 4 **Use Table II to find the area under the standard normal curve delimited by the *z*-score(s) found in Step 3.**

$$z = \frac{x - \mu}{\sigma}$$

Intelligence Quotients Intelligence quotients (IQs) measured on the Stanford Revision of the Binet–Simon Intelligence Scale are normally distributed with a mean of 100 and a standard deviation of 16. Determine the percentage of people who have IQs between 115 and 140.



$$z(x=115) = \frac{115-100}{16}$$

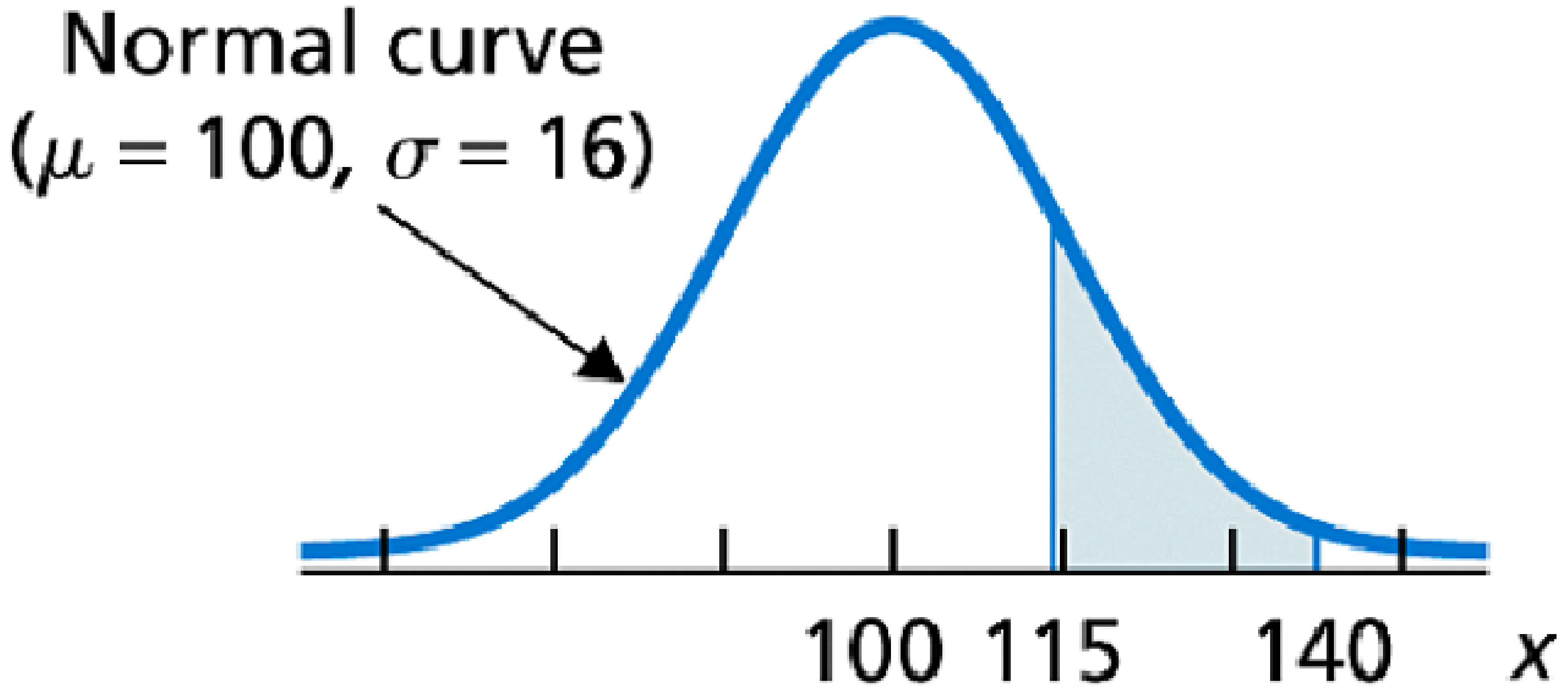
$$= .94$$

$$z(x=140) = \frac{140-100}{16}$$

$$= 2.5$$

$.94 \rightarrow .8264$

$2.5 \rightarrow .9938$



$.9938 - .8264 = .1674$

16.74%

6.99 New York City 10-km Run. As reported in *Runner's World* magazine, the times of the finishers in the New York City 10-km run are normally distributed with mean 61 minutes and standard deviation 9 minutes.

a. Determine the percentage of finishers who have times between 50 and 70 minutes.

73.01%

b. Determine the $\boxed{\text{percentage of finishers}}$ who have times less than 75 minutes. 94.06%

$z(50) = \frac{50-61}{9} = -1.22$ Area $\rightarrow$ .1112

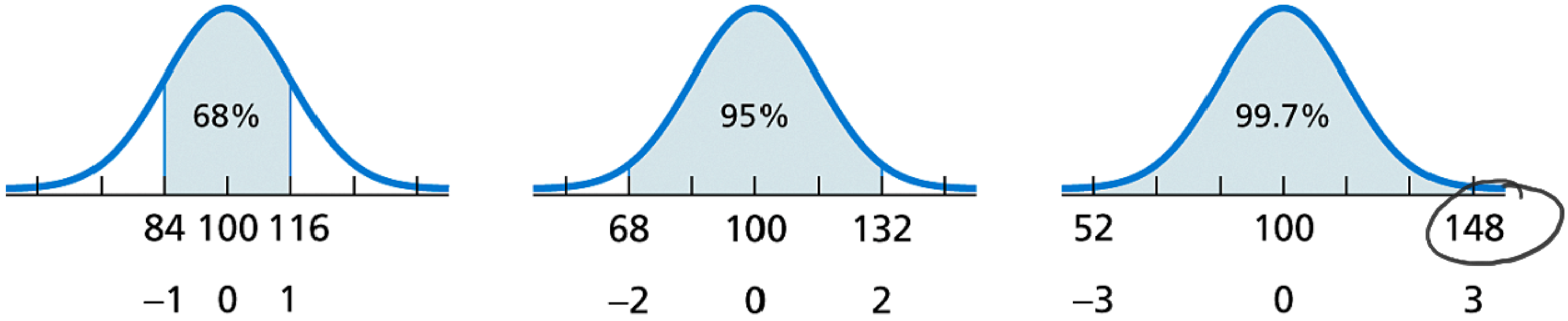
$z(70) = \frac{70-61}{9} = 1$ Area $\rightarrow$ .8413

$z(75) = \frac{75-61}{9} = 1.56$ Area $\rightarrow$ .9406

Intelligence Quotients Apply the empirical rule to IQs.

68 - 95 - 99.7

1 2 3



.15%

To Determine the Observations Corresponding to a Specified Percentage or Probability for a Normally Distributed Variable

Step 1 Sketch the normal curve associated with the variable.

Step 2 Shade the region of interest.

Step 3 Use Table II to determine the z-score(s) delimiting the region found in Step 2.

Step 4 Find the x-value(s) having the z-score(s) found in Step 3.

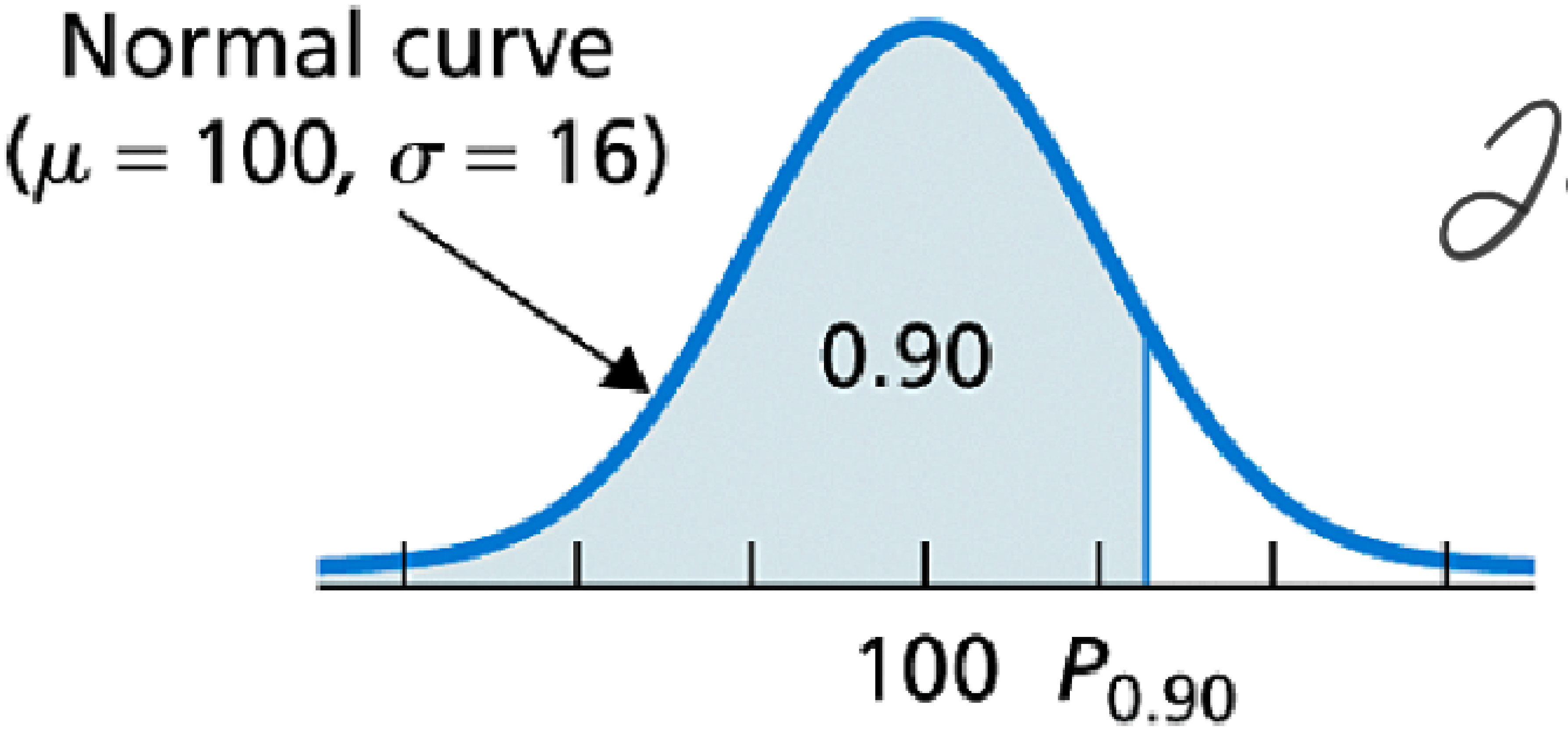
Intelligence Quotients Obtain and interpret the 90th percentile for IQs.

$P_{.90} \rightarrow \quad z = 1.28$

$$1.28 = \frac{x - 100}{16}$$

$$20.48 = x - 100$$

$$\boxed{120.48 = x}$$



6.102 Elephant Pregnancies. G. Wittemeyer et al. studied demographic data on African elephants living in Kenya in the article "Comparative Demography of an At-Risk African Elephant Population" (*PLOS ONE*, Vol. 8. No. 1). Based on this study, we will assume that the time between pregnancies of the African elephant in Kenya, for elephants that have more than one calf, is normally distributed with mean 4.01 years and standard deviation 0.94 years. Determine the percentage of such times that are

a) Between 3 and 5 years →

b) Greater than 5 years →

c) 90th percentile →

$$z(3) = \frac{3-4.01}{0.94} = -1.07 \rightarrow .1423$$

$$z(5) = \frac{5-4.01}{0.94} = 1.05 \rightarrow .8531$$

$$P_{.90} = 1.28 \rightarrow \quad 1.28 = \frac{x-4.01}{0.94}$$

$$\boxed{= 5.21}$$

$3 \Leftrightarrow 5 \rightarrow 0.7108 \qquad 71.08\%$

$> 5 \rightarrow 0.1469 \qquad 14.69\%$

(20 points) According to *JAVMA News*, a publication of the American Veterinary Medical Association, roughly 60% of U.S. households own one or more pets. Four U.S. households are selected at random.

$$({}_nC_r)(p)^r(1-p)^{n-r}$$

$$({}_4C_1)(.60)^1(.40)^3 = .1536 \rightarrow 15.36\%$$

1000 households → 300 have pets

$$({}_{1000}C_{300})(.60)^{300}(.40)^{700}$$

Mortality The probability is 0.80 that a person of age 20 years will be alive at age 65 years. Suppose that 500 people of age 20 are selected at random. Determine the probability that

a. exactly 390 of them will be alive at age 65.

b. between 375 and 425 of them, inclusive, will be alive at age 65.

$$\mathrm{nCr}(500{,}390)(.80)^{390}(.20)^{110} = 0.02337876182$$

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}$$

To Approximate Binomial Probabilities by Normal-Curve Areas

$n = 500 \qquad p = .80$

Step 1 Find n, the number of trials, and p, the success probability.

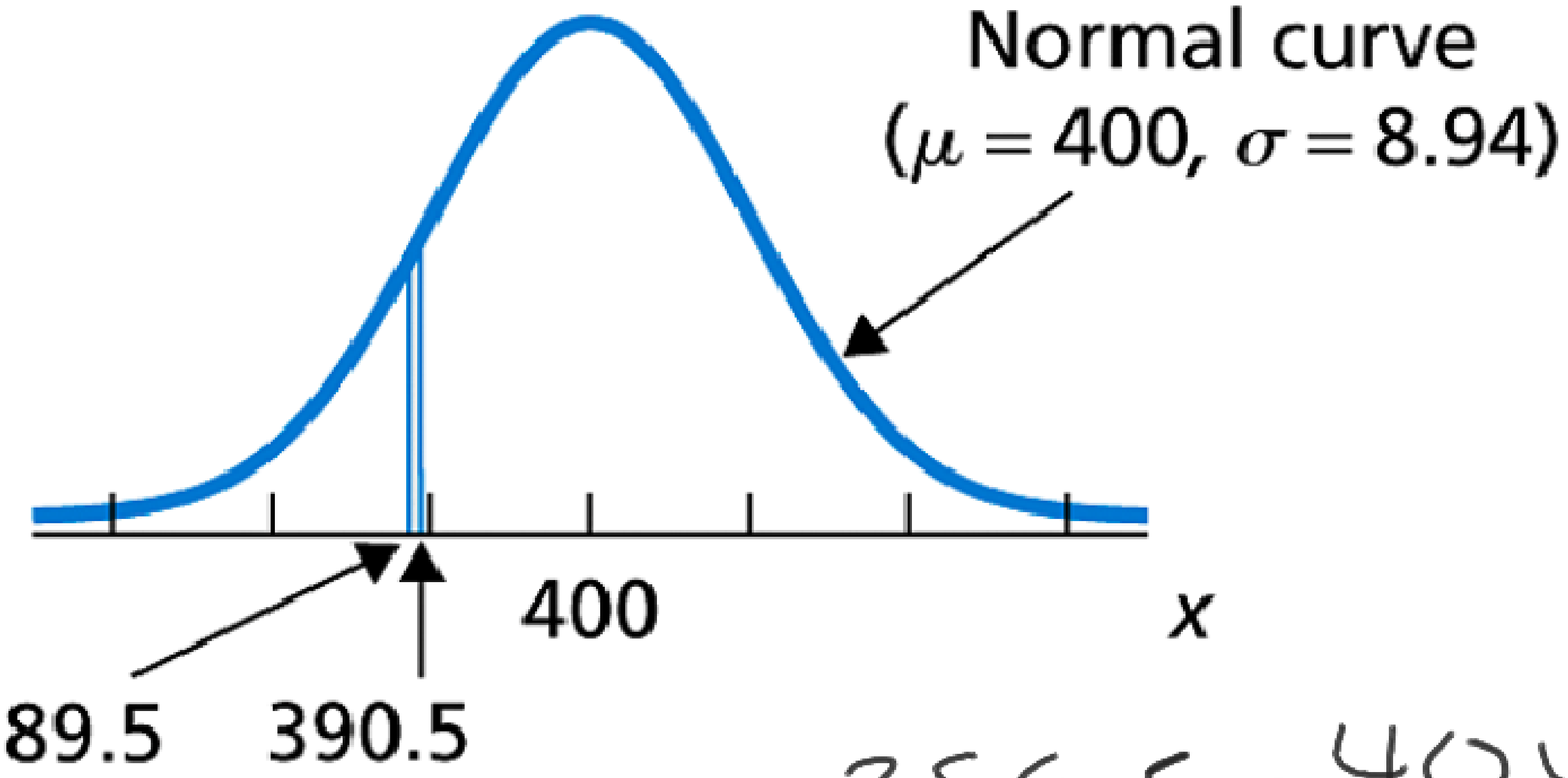
Step 2 Continue only if both np and $n(1-p)$ are 5 or greater.

Step 3 Find μ and σ, using the formulas $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.

Step 4 Make the correction for continuity, and find the required area under the normal curve with parameters μ and σ.

$500(.80) = 400 \qquad 500(.20) = 100$

$\mu = 400 \qquad \sigma = \sqrt{500(.80)(.20)} = 8.94$



extaly 390

$389{,}5 \longleftrightarrow 390{,}5$

$$z(389.5) = \frac{389{,}5-400}{8{,}94} = -1.17 \to .1210$$

$$z(390{,}5) = \frac{390.5-400}{8{,}94} = -1.06 \to .1446$$

0.0236

2.36%

from 375 $\leftrightarrow$ 425

$z(374.5) = -2.85 \rightarrow .0023$

$z(425.5) = 2.85 \rightarrow .9978$

$.9955$

$\boxed{99.55\%}$