Definition 9.1: Null and Alternative Hypotheses; Hypothesis Test

Null hypothesis: A hypothesis to be tested. We use the symbol H_0 to represent the null hypothesis.

Alternative hypothesis: A hypothesis to be considered as an alternative to the null hypothesis. We use the symbol H_a to represent the alternative hypothesis.

Hypothesis test: The problem in a hypothesis test is to decide whether the null hypothesis should be rejected in favor of the alternative hypothesis.

Null Hypothesis

In this book, the null hypothesis for a hypothesis test concerning a population mean, μ, always specifies a single value for that parameter. Hence we can express the null hypothesis as

$$H_0\colon \mu = \mu_0,$$

where μ_0 is some number.

Alternative Hypothesis

- If the primary concern is deciding whether a population mean, μ, is *different from* a specified value μ_0, we express the alternative hypothesis as

$$H_a\colon \mu \neq \mu_0.$$

  A hypothesis test whose alternative hypothesis has this form is called a **two-tailed test.**

- If the primary concern is deciding whether a population mean, μ, is *less than* a specified value μ_0, we express the alternative hypothesis as

$$H_a\colon \mu < \mu_0.$$

  A hypothesis test whose alternative hypothesis has this form is called a **left-tailed test.**

- If the primary concern is deciding whether a population mean, μ, is *greater than* a specified value μ_0, we express the alternative hypothesis as

$$H_a\colon \mu > \mu_0.$$

  A hypothesis test whose alternative hypothesis has this form is called a **right-tailed test.**

Quality Assurance A snack-food company produces a 454-g bag of pretzels. Although the actual net weights deviate slightly from 454 g and vary from one bag to another, the company insists that the mean net weight of the bags be 454 g.

As part of its program, the quality assurance department periodically performs a hypothesis test to decide whether the packaging machine is working properly, that is, to decide whether the mean net weight of all bags packaged is 454 g.

$H_0 \rightarrow \mu_0 = 454$

$H_a \rightarrow \mu \neq 454$ two-tail

Taller Young Women In the document *Anthropometric Reference Data for Children and Adults*, C. Fryer et al. present data from the *National Health and Nutrition Examination Survey* on a variety of human body measurements, such as weight, height, and size. Anthropometry is a key component of nutritional status assessment in children and adults.

A half-century ago, the average (U.S.) woman in her 20s was 62.6 inches tall. Suppose that we want to perform a hypothesis test to decide whether today's women in their 20s are, on average, taller than such women were a half-century ago.

$H_0 \rightarrow \mu_0 = 62.6$

$H_a \rightarrow \mu > \mu_0$

right tail

Poverty and Dietary Calcium Calcium is the most abundant mineral in the human body and has several important functions. Most body calcium is stored in the bones and teeth, where it functions to support their structure. Recommendations for calcium are provided in *Dietary Reference Intakes*, developed by the Institute of Medicine of the National Academy of Sciences. The recommended adequate intake (RAI) of calcium for adults (ages 19–50 years) is 1000 milligrams (mg) per day.

Suppose that we want to perform a hypothesis test to decide whether the average adult with an income below the poverty level gets less than the RAI of 1000 mg.

$H_0 \rightarrow \mu_0 = 1000$

$H_a \rightarrow \mu < \mu_0$

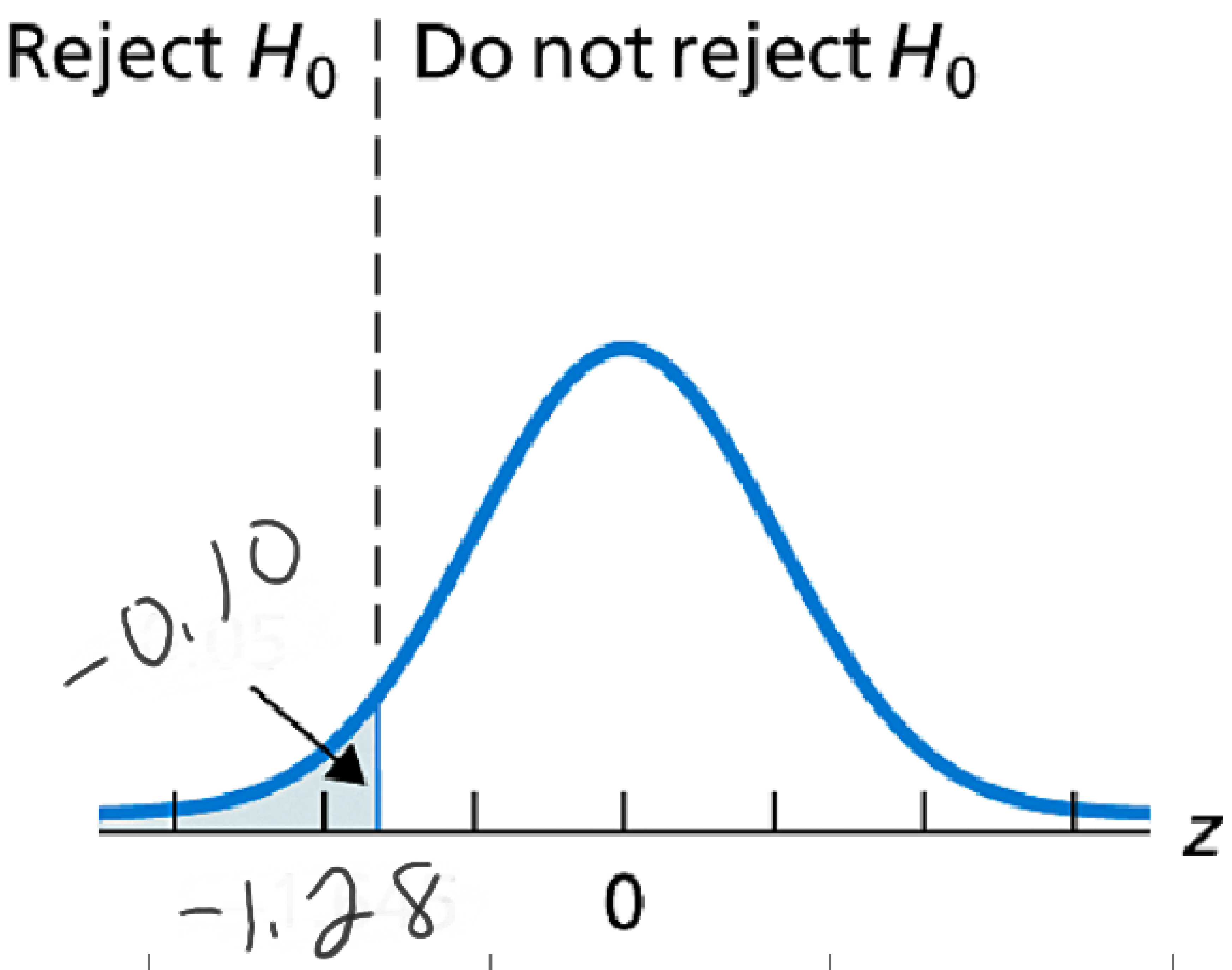
Left tailed

Basic Logic of Hypothesis Testing

Take a random sample from the population. If the sample data are consistent with the null hypothesis, do not reject the null hypothesis; if the sample data are inconsistent with the null hypothesis and supportive of the alternative hypothesis, reject the null hypothesis in favor of the alternative hypothesis.

Golf Driving Distances Jack tells Jean that his average drive of a golf ball is 275 yards. Jean is skeptical and asks for substantiation. To that end, Jack hits 25 drives. The results, in yards, are shown in Table 9.2. $\bar{x} = 264.4$

Table 9.2 Distances (yards) of 25 drives by Jack $\sigma_{\bar{x}} = 4$



297
279
265
243
295

$H_0 \rightarrow \mu_0 = 275$

$H_a \rightarrow \mu < \mu_0$

Left tail test

$\boxed{\begin{array}{l} 90\% \\ -1.28 \text{ z-score} \end{array}}$

$$\frac{264.4 - 275}{4} = -2.65$$

Definition 9.4: Rejection Region, Nonrejection Region, and Critical Values

Rejection region: The set of values for the test statistic that leads to rejection of the null hypothesis.

Nonrejection region: The set of values for the test statistic that leads to nonrejection of the null hypothesis.

Critical value(s): The value or values of the test statistic that separate the rejection and nonrejection regions. A critical value is considered part of the rejection region.

Key Fact 9.3: Obtaining Critical Values

Suppose that a hypothesis test is to be performed at the significance level α. Then the critical value(s) must be chosen so that, if the null hypothesis is true, the probability is α that the test statistic will fall in the rejection region.

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}},$$

Determine the critical value(s) for a one-mean z-test at the 5% significance level ($\alpha = 0.05$) if the test is

95% C.L.

$z_{0.05}$

a. two tailed. ± 1.96

b. left tailed. -1.645

c. right tailed. 1.645

CRITICAL-VALUE APPROACH TO HYPOTHESIS TESTING

Step 1 State the null and alternative hypotheses.

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic.

Step 4 Determine the critical value(s).

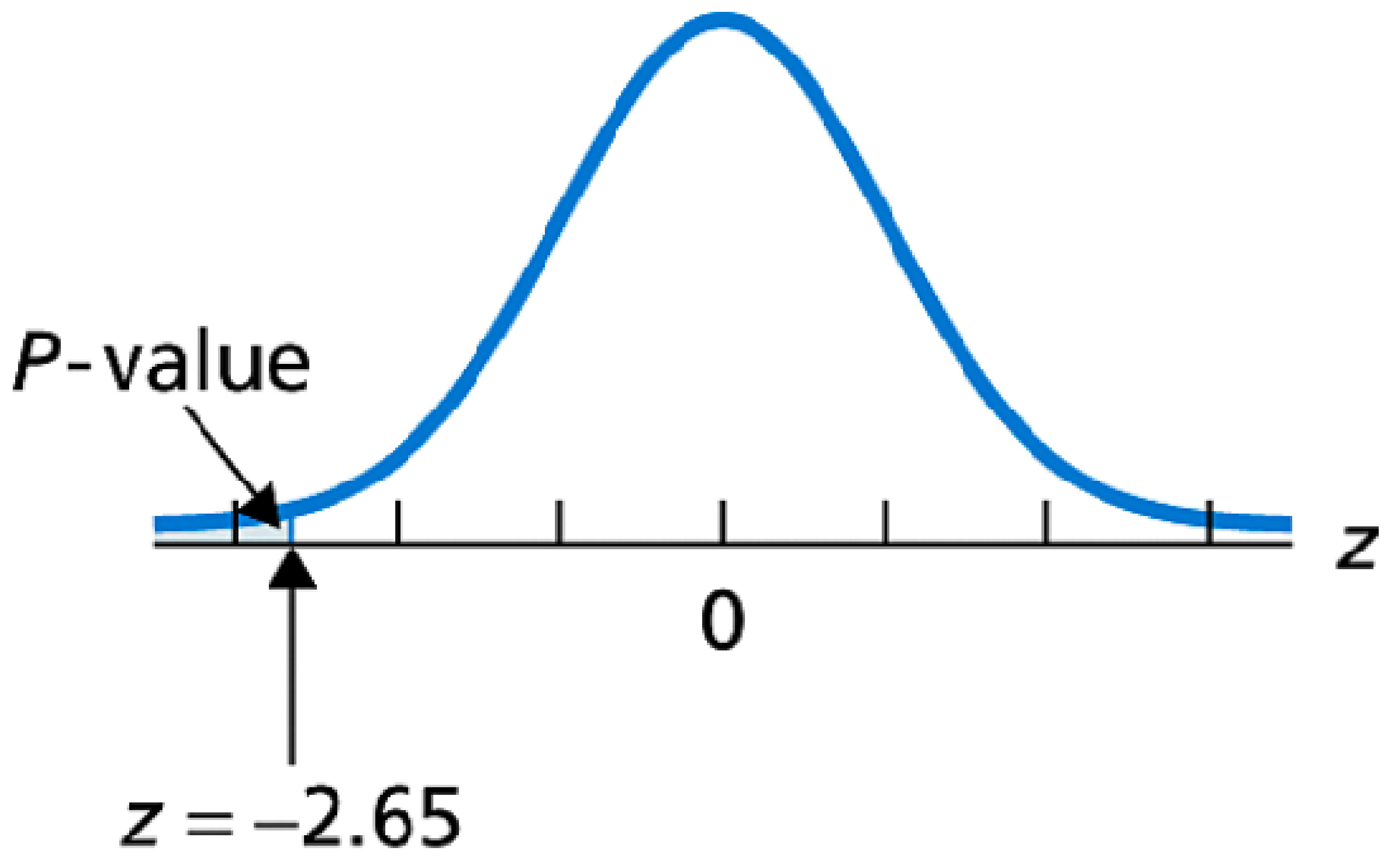
Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the result of the hypothesis test.

Golf Driving Distances Jack tells Jean that his average drive of a golf ball is 275 yards. Jean is skeptical and asks for substantiation. To that end, Jack hits 25 drives. The results, in yards, are shown in Table 9.2.

Table 9.2 Distances (yards) of 25 drives by Jack

[illegible]	[illegible]	[illegible]	[illegible]	297
[illegible]	[illegible]	[illegible]	[illegible]	279
[illegible]	[illegible]	[illegible]	[illegible]	265
[illegible]	[illegible]	[illegible]	[illegible]	243
272	279	261	273	295



$Z = -2.65$

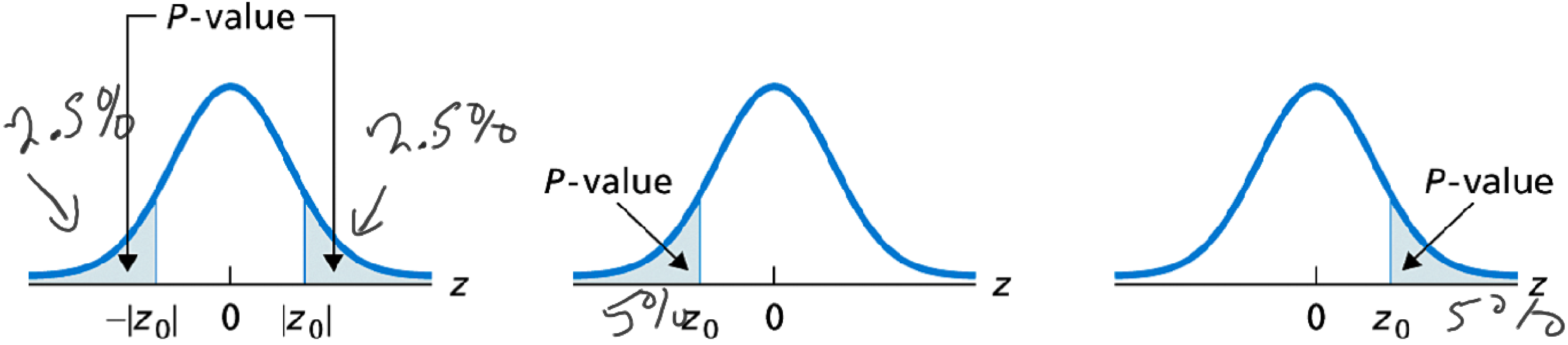
P-value

5% signifigance

% = .0040
= .4%

Reject

5% > .4%



(a) Two tailed (b) Left tailed (c) Right tailed

5%

The value of the test statistic for a left-tailed one-mean z-test is $z = -1.19$.

a. Determine the P-value. .1170

b. At the 5% significance level, do the data provide sufficient evidence to reject the null hypothesis in favor of the alternative hypothesis?

not rejecting

$.1170 > .05$

The value of the test statistic for a right-tailed one-mean z-test is $z = 2.85$.

a. Determine the P-value. $1 - .9978 = .0022$

b. At the 1% significance level, do the data provide sufficient evidence to reject the null hypothesis in favor of the alternative hypothesis?

$.0022 < .01$

Reject

The value of the test statistic for a two-tailed one-mean z-test is $z = -1.71$.

a. Determine the P-value. 0.0436

b. At the 5% significance level, do the data provide sufficient evidence to reject the null hypothesis in favor of the alternative hypothesis?

$0.0436 > 0.025$

Not Reject

P-VALUE APPROACH TO HYPOTHESIS TESTING

Step 1	State the null and alternative hypotheses.
Step 2	Decide on the significance level, α.
Step 3	Compute the value of the test statistic.
Step 4	Determine the *P*-value, *P*.
Step 5	If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.
Step 6	Interpret the result of the hypothesis test.