6 CHAPTER
Probability, Randomness, and Uncertainty

Conditional Probability

The probability that an event will occur given that some other event has already occurred or is certain to occur, is a **conditional probability**.

DEFINITION

Probability Law 9: Conditional Probability

The **conditional probability** of event A occurring, given that event B has already occurred is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

← Intersection

← probability of given event

The notation $P(A|B)$ is read as *the probability of A given the occurrence of B*. The vertical bar within a probability statement will always mean *given*.

DEFINITION

Suppose a marketing research firm has surveyed a panel of consumers to test a new product and produced the following **cross tabulation** indicating the number of panelists that liked the product, the number that did not like the product, and the number that were undecided.

Market Research Survey				
Age	**Like**	**Not Like**	**Undecided**	**Total**
18–34	213	197	103	513
35–50	193	184	67	444
Over 50	144	219	83	446
Total	**550**	**600**	**253**	**1403**

$$\frac{193}{444}$$

If an individual is between 35 and 50 years old, what is the probability that he or she will like the product?

Out of 300 applicants for a job, 212 are male. Of the male job applicants, 110 have served in the military.

a. What is the probability that a randomly chosen applicant has served in the military, given that he is male?

b. If 152 of the applicants have served in the military, what is the probability that a randomly chosen applicant is male, given that the applicant has served in the military?

a) $\frac{110}{212}$

b) $\frac{110}{152}$

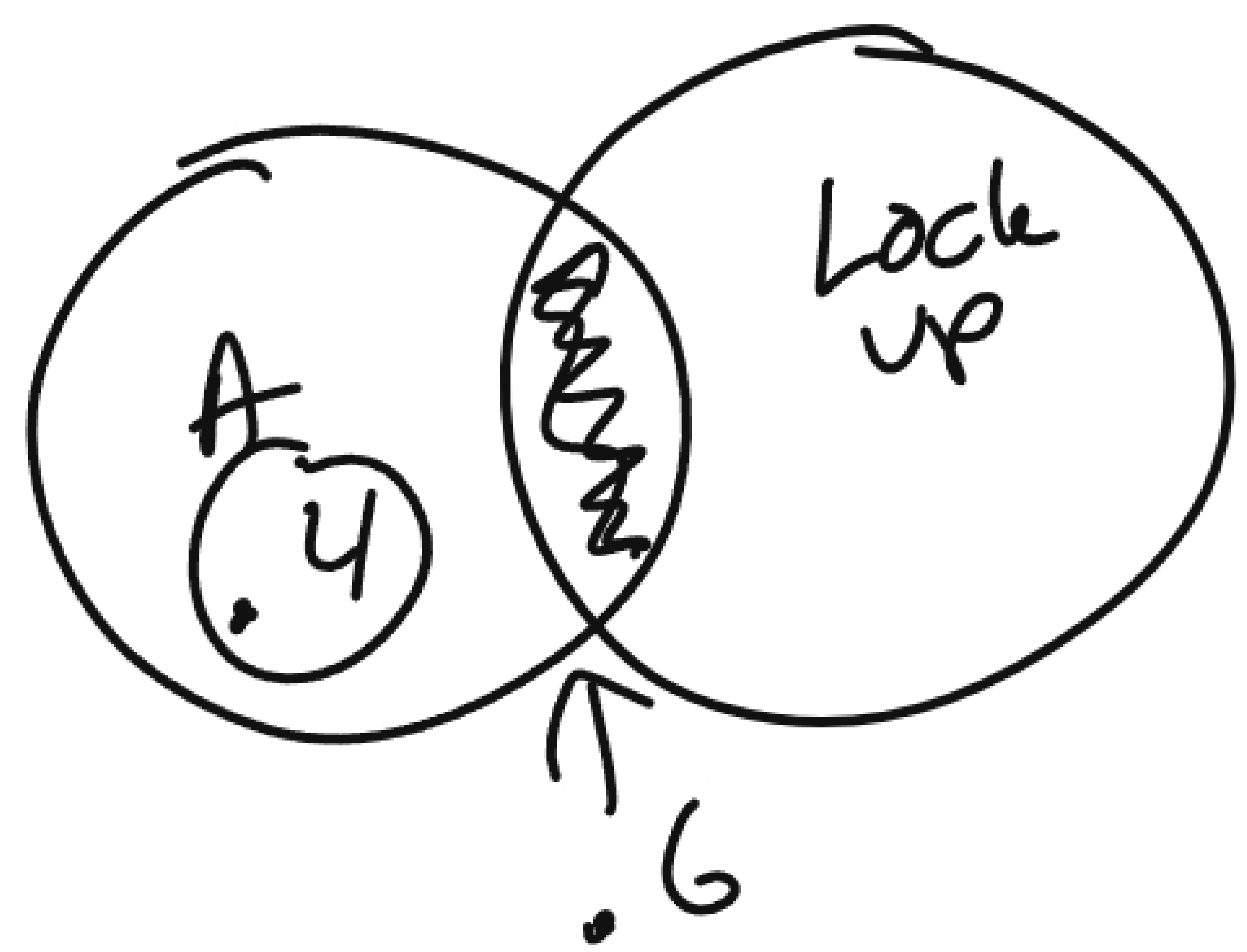
A computer software company receives hundreds of support calls each day. There are several common installation problems, call them A, B, C, and D. Several of these problems result in the same symptom, *lock up* after initiation. Suppose that the probability of a caller reporting the symptom *lock up* is 0.7 and the probability of a caller having problem A and a *lock up* is 0.6.

a. Given that the caller reports a lock up, what is the probability that the cause is problem A?

b. What is the probability that the cause of the malfunction is not problem A given that the caller is experiencing a lock up?

a) $\frac{0.6}{0.7} = \frac{6}{7}$

b) $1 - \frac{6}{7} = \frac{1}{7}$



Independent

Two events, A and B, are **independent** if and only if

$$P(A|B) = P(A) \text{ and } P(B|A) = P(B).$$

DEFINITION

Probability Law 10: Multiplication Rule for Independent Events

If two events, A and B, are independent, then

$$P(A \cap B) = P(A) \cdot P(B).$$

If n events, $A_1, A_2, \ldots, A_n$, are independent, then

$$P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1) \cdot P(A_2) \cdot \ldots \cdot P(A_n).$$

DEFINITION

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

A coin is flipped, a die is rolled, and a card is drawn from a standard deck of 52 cards. Find the probability of getting a tail on the coin, a five on the die, and a jack of clubs from the deck of cards.

$$\frac{1}{2} \times \frac{1}{6} \times \frac{1}{52} = \frac{1}{624}$$

This is an actual case that stirred up quite a controversy.

People vs. Collins (1968)

On June 18, 1964, at about 11:30 AM, Mrs. Juanita Brooks was assaulted and robbed while walking through an alley in the San Pedro area of Los Angeles. Mrs. Brooks described her assailant as a young woman with a blond pony tail. At about the same time, John Bass was watering his lawn and witnessed the assault. He described the assailant as a Caucasian woman with dark-blond hair. As she ran from the alley she jumped into a yellow automobile driven by a black man with a mustache and a beard.

Several days later the police arrested two individuals based on the descriptions provided by the assailant and the witness. The two suspects were eventually charged with the crime. During the trial the prosecution called a professor of mathematics to testify. The prosecutor set forth the following probabilities for the characteristics of the assailants.

Assailant Characteristics Data

Characteristic	Probability
Yellow automobile	0.10
Man with mustache	0.25
Girl with ponytail	0.10
Girl with blonde hair	0.33
Black man with beard	0.10
Interracial couple in a car	0.001

How did the prosecution use these probabilities to argue its case?

In a production process, a product is assembled by using four different independent parts (A, B, C, and D). In order for the product to operate properly, each part must be free of defects. The probabilities of the parts being nondefective are given by $P(A) = 0.9$, $P(B) = 0.7$, $P(C) = 0.8$, and $P(D) = 0.9$.

a. What is the probability that all four parts are defective?

b. What is the probability that the product does not work?

$P(A)^c = .1 \quad P(B)^c = .3 \quad P(C)^c = .2 \quad P(D)^c = .1$

a) $(.1)(.3)(.2)(.1) = .0006$

b) $1 - (.9)(.7)(.8)(.9) = .5464$

What is the probability of drawing a king and then a queen from a standard deck if the cards are drawn $\boxed{\textit{without replacement}}$?

$$\underset{K}{\frac{4}{52}} \quad \underset{Q}{\frac{4}{51}} = \frac{16}{2652} = 0.006$$

$$\underset{Q}{\frac{4}{52}} \quad \underset{Q}{\frac{3}{51}} = \frac{12}{2652}$$

Suppose you draw two cards out of a standard deck without replacement. What is the probability that you draw the ace of spades and then another spade?

$$\frac{1}{52} \cdot \frac{12}{51} = \frac{12}{2652}$$

Drug usage in the workplace costs employers incredible amounts of money each year. Drug testing potential employees has become so prevalent that drug users are finding it extremely hard to find jobs. Drug tests, however, are not completely reliable. The most common test used to detect drugs is approximately 98% accurate. To decrease the likelihood of making an error, all potential employees are screened through two tests, which are independent, and each has about 98% accuracy.

a. If a person were drug-free, what is the probability he or she would fail both tests?

b. If a person were a drug user, what is the probability he or she would pass both tests?

The Fundamental Counting Principle

E_1 is an event with n_1 possible outcomes and E_2 is an event with n_2 possible outcomes. The number of ways the events can occur in sequence is $n_1 \cdot n_2$. This principle can be applied for any number of events occurring in sequence.

PROCEDURE

$\boxed{}\ \boxed{}\ \boxed{} - \boxed{}\ \boxed{}\ \boxed{}\ \boxed{}$

10 10 10 10 10 10 10

10,000,000

Most nonpersonalized license plates in the state of Utah consist of three numbers followed by three letters (excluding I, O, and Q). How many license plates are possible?

$\boxed{10}\ \boxed{10}\ \boxed{10}\ \boxed{23}\ \boxed{23}\ \boxed{23}$

N L

12,167,000

$\boxed{23}\ \boxed{23}\ \boxed{10}\ \boxed{10}\ \boxed{10}\ \boxed{10}\ \boxed{10}$

52,900,000

Permutation

A **permutation** is a specific order or arrangement of objects in a set. There are $n!$ permutations of n unique objects.

DEFINITION

Permutation

The number of permutations of n unique objects in which k are selected at a time and repetition is not allowed is given by

$$_nP_k = \frac{n!}{(n-k)!}$$

$_nP_r$

Note that some alternate notations for permutations that you may see are P_k^n and $P(n,k)$.

FORMULA

they finish in first, second, and third place?

$n = 7$ total number

$_nP_r$

$r = 3$ How many chossen

$_7P_3 = 210$

$_{20}P_5 =$

Distinguishable Permutations

If given n objects, with n_1 alike, n_2 alike, … , n_k alike, then the number of **distinguishable permutations** of all n objects is $\dfrac{n!}{(n_1!n_2!n_3!\dots n_k!)}$ **FORMULA**

How many distinguishable permutations can be made from the word *Mississippi*?

$$\frac{11!}{1!\,4!\,4!\,2!} = 34650$$

Combinations

A **combination** is a collection or grouping of objects where the order is ***not*** important.

DEFINITION

Combination

The number of combinations of n unique objects selecting k at a time and repetition is not allowed is given by

$$_nC_k = \frac{n!}{(n-k)!k!}$$

nCr

Note that some alternate notations for combinations that you may see are C_k^n and $C(n,k)$.

FORMULA

In South Carolina's *Palmetto Cash 5* lottery, a player selects five different numbers from 1 to 38 (inclusive). If the numbers selected match the player's numbers in any order, the player wins.

a. What is the total number of winning combinations?
b. What is the probability of winning?

a) $_nC_r = {}_{38}C_5 = 501{,}942$

b) $1/501{,}942$

A DJ needs to select 6 songs from a CD containing 12 songs to compose an event's musical lineup. How many different lineups are possible?

${}_{12}C_{6}$

How many 5 card hands can be dealt from a deck of 52 cards?

$$ {}_{52}C_5 $$

Kara was born on 11/21/1992. She would like to make an eight-digit password using all of the digits in her birth date. How many different eight-digit passwords could she create?

$$\frac{8!}{4!\,2!\,2!}$$

The engineering club at a local high school must choose 2 representatives from each of the sophomore, junior, and senior classes to attend a national convention. If there are 6 sophomores, 5 juniors, and 7 seniors in the club, in how many ways can the group be chosen for the convention?